

The Mathematical Journey of Yamaguchi Kanzan

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It was probably too late to be up working on a math puzzle, Kanzan thought, but he didn't care. His teacher, Hasegawa sensei,¹ had given him a new book of problems, and he couldn't put it down. It was filled with geometrical puzzles from all across Japan. Circles, squares, and ellipses, shaded in red, black, and yellow with the most vibrant pigments available, seemed to leap off the pages as young Kanzan made his calculations.



The problem wasn't easy, but Kanzan wouldn't be discouraged. Finally, he worked it out; "I've got it! The radius of this little circle, it must be 4!" he exclaimed. That was enough for now, he thought. With the puzzle solved, he put down his book of problems, the *Shimpeki Sampo*, and headed off to bed.²

Long ago, long before Kanzan's time, the mathematicians of Japan would spend their days pondering the latest discoveries to make their way across the sea from China. They studied geometry, and the remarkable algebra of the celestial unknown³, from the Chinese classics. But those days were no more. The Tokugawa clan had taken over the island country⁴, and under their strict rule, *no one* was allowed in or out.⁵ Now, Japan was completely cut off from the outside world.



But the mathematicians wouldn't let that stop them. In the great capital of Edo⁶, where Kanzan studied, mathematics thrived. If they couldn't read math from the outside world, the mathematicians thought, they would have to make their own⁷. Painted on wooden tablets, and hung in shrines and temples across the nation, Japan's mathematicians placed the toughest, the strangest, and the most beautiful geometry puzzles they could think up.



They were as much offerings to the gods as they were challenges to the other mathematicians: “who, out there, can solve this?”⁸ These problems, called *sangaku*, were the puzzles that filled Kanzan's new problem book, and they were the puzzles that would one day inspire him to set out on a journey unlike any that had come before.

As Kanzan grew older, he found he was no longer content to study only the mathematics of Edo. He wanted to master the techniques of mathematicians from even the most far-flung reaches of the country. He decided to set out on a journey across Japan. He would travel up the country and then back down again, to the far southern port of Nagasaki, visiting every temple and shrine he could find along the way, and recording in his journal the sangaku problems he found there.⁹



He would bring them back to Edo, and publish¹⁰ them all together, for every mathematician to see. He had travelled to see sangaku before, but this would be a journey that put all his others to shame.¹¹

It was a warm summer day when he was ready to set off. His friends had all come to wish him a great journey, and leave him their farewell haiku. They knew it would be a long while before he returned to Edo.¹²



He soon found himself in the castle town of Takasaki.¹³ This town, he recalled, was the home of the mathematician Ono Eiju. Eiju was a student of the great Fujita Sadasuke, one of the most famous mathematicians in all of Japan's history. As it turned out, Sadasuke was also the father of the very man who wrote that book of problems that had captivated Kanzan so many years ago.¹⁴



When Kanzan arrived in Takasaki, without hesitation, he crossed the shallow river and scaled the steep Usui pass to reach Kumano shrine, where the sangaku were hung. Eiju wasn't in, but a sangaku tablet hung suspended from the rafters, just as Kanzan had hoped.

Upon closer inspection, Eiju had left a message on the tablet; he was not the original author. "Mr. Tsunoda, who hung this sangaku, visited me and told me that he eagerly wanted to study mathematics and so I introduced him to my teacher in Edo"¹⁵ the tablet read. Even though it was not Eiju's creation, the problem was just as wonderful as any Eiju had devised. Kanzan took it down in his journal, and was on his way.

Some days later, Kanzan arrived at the bank of the sprawling Chikuma river. On the other side sat the town of Hachiman, his next destination. He took the ferry across, and as they got closer and closer to Hachiman, he found he could begin to make out the sounds of a festival. People had come from far and wide to attend, and the town was bustling with excitement. Kanzan, however, was here for more than the festivities. He wanted to find his next sangaku. While the other festival-goers celebrated, Kanzan slipped away to search for it.



He soon discovered just what he sought, in a little village nearby, where he stayed for the night. Never one to stop for too long, the next day, he was off again.

After a bit of travel, Kanzan found himself in the city of Nagano, home to the biggest temple he had seen yet: Zenkoji. Kanzan had wanted to visit Zenkoji since the day he set off,¹⁶ and the time had finally come. After arriving in Nagano, he spent the night in the temple's guest house.¹⁷ It was packed with visitors who had come to worship at Zenkoji, but there was still just enough room for him.

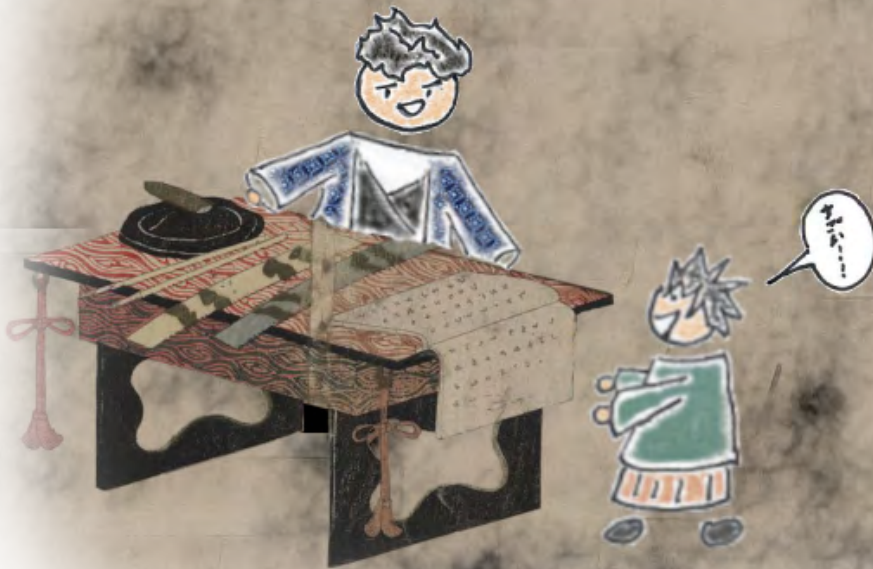


The next day, he got up early to search for sangaku. He was in luck; in the temple he found four new tablets, with new problems for his journal. As he began to copy them down, one in particular caught his eye. It had already been solved, years earlier, and the solution he saw was incredible.¹⁸ An enormous equation spanned the back of the tablet. Kanzan looked over it in amazement, and when he got to the end, there was a note left by author¹⁹. He was only the second person to solve the problem, and in the original solution, the equation was 100 times more complex! Kanzan could barely believe it, but he was relieved that at least he didn't have to copy such a monstrous solution down. After recording his sangaku, Kanzan was once again on his way.



By the time Kanzan reached the village of Yada, he was exhausted from his travels. It had been weeks since he left Edo, and he wanted nothing more than a rest. Since his journey had begun, the hot summer days had given way to chilly fall evenings, and travel was getting difficult. Kanzan decided he would rest in Yada until the spring, when his travels could begin anew.

Nearby Yada lived an old friend of Kanzan's, Eguchi Shinpachi. While he rested, Kanzan went to visit him. Much to Kanzan's surprise, Shinpachi's son Tamekichi was fascinated by the problems that Kanzan had collected. Eagerly, Tamekichi ask if he could become Kanzan's student.²⁰ He had never received such a request before, but he accepted. Tamekichi would become the first student of the Yamaguchi school, Kanzan's own school of mathematics.



Spring had arrived, and it was time for Kanzan to set out on his travels once again. Bidding Shinpachi and Tamekichi farewell, Kanzan set off once again. His journey took him miles and miles, up and down the coast of Japan. He made his way through tiny villages, and cities so big he could barely believe his eyes²¹. He passed through castle-towns and down perilous stretches of sea.²²



He crossed a marvelous bridge, which curved up over the water with no supports beneath it. Whenever he saw a temple along the way, he hurried in to record the sangaku in his old journal. He met with old friends and fellow mathematicians at every stop, telling stories of his journey and challenging each other with the hardest problems they had come across.

After weeks of travel, Kanzan reached the city of Osaka. He had come to see Osaka's famous castle —and, of course, search for sangaku. The castle's great walls and golden adornments were unlike anything Kanzan had ever seen.²³ As he stared up at the huge building, a small paper leaflet blew along the ground by his feet. He picked it up. It was an advertisement for an astronomy lecture²⁴, by the mathematician Takeda Atsunoshin, who lived in Osaka. Kanzan just had to attend. After the lecture, the two began to talk. Atsunoshin was very impressed with Kanzan's journal, and gave him yet another problems to write down. But Kanzan couldn't stay long. Nagasaki was close by, and his journey would soon be complete.

Kanzan now arrived at the inland sea.²⁵ This was all that stood between him and the last leg of his journey. He crossed the waters by ferry, landing at the base of a huge hill. He had heard that at the top there was a shrine. In a shrine, knew Kanzan, there would be sangaku. He scaled the steep slopes, up and up, and finally made his way to the top. Indeed, a little shrine sat gingerly atop the hill. Inside, another problem for his journal. Next, to Nagasaki!

Stepping into the city of Nagasaki, Kanzan's excitement grew. The city was bustling with energy. Trading ships sailed up and down the coast, bringing with them as many new ideas as they brought goods.²⁶ Kanzan wanted to explore it all. He could barely fit everything into his short stay, but of course, he made time to collect his sangaku. Though he could hardly pull himself away,²⁷ he knew he couldn't stay long.



He had done it. He traveled to the very farthest edge of Japan. He had filled his journal with the hardest, most amazing, and most puzzling problems he could find. His journal was nearly full, and so, just as soon as he had arrived, he had to depart. It was time to make the long trek back to Edo.

Through mountains, grasslands, and seaside, Kanzan made his winding trip home to the capital. He collected a last few sangaku as he went, but his determination to reach home drove him forward at a steady pace. When he arrived, he planned, he would publish his journal of sangaku for all the mathematicians of Edo to read. He had already gone as far as to place ads in all the popular mathematics books,⁹ so no one would miss it. But in the end, Kanzan never did publish his journal. The reason why? History doesn't say. Maybe he didn't think he had collected problems enough yet. Maybe he had so many he couldn't fit them all in one book. Like many things, we may never know.

...but, there are some things history does tell us. The last entry in Kanzan's sangaku journal, you see, simply says:

“I have enjoyed these travels. After some rest, I will plan my next sangaku journey.”



Footnotes

¹ Yamaguchi Kanzan studied in Edo⁶ at the school of Hasegawa Hiroshi.

² It is likely, though not specifically known, that Yamaguchi would have studied the *Shimpeki Sampo* in his mathematical training, given its popularity and the time of its publishing (1790). It is, unfortunately, impossible to know if he every stayed up into the early hours of the morning studying it, as we have suggested, but it adds a bit of necessary characterization.

³ The term *algebra of the celestial unknown* refers to the 天元術 (Chinese *tiān yuán shù*, Japanese *tengenjutsu*), a technique for finding solutions to polynomials, first attested in China in the 13th century.

⁴ The term “island country” (Japanese *shimaguni*) is a frequent metaphor for Japan in Japanese.

⁵ An external trade relationship was maintained with the Netherlands via the tiny port of Dejima during this period, but there was allowance only for movement of very select people and goods through in and out.

⁶ Modern day Tokyo

⁷ Mathematics, of course, had been produced in Japan prior to the closing of the country (*sakoku*) in 1636. However, it is often considered that *wasan*, the specific Japanese mathematical tradition, took off in its own right after this point.

⁸ According to Fukagawa and Pedoe in *Japanese Temple Geometry Problems*, many *sangaku* were left without proof, perhaps as an implicit challenge to other mathematicians to attempt a proof for themselves.

⁹ In an 1837 advertisement for his would-be compendium of these problems, it is written “Mathematician Yamaguchi has traveled all over Japan for six years, from the spring of 1816[7] to the winter of 1821. With many distant mathematicians, he has discussed new technical methods of solving mathematical problems. If you buy this book, then you will be able to know and obtain without traveling the new technical methods of solving problems of far-away mathematicians.” This translation, as well as those of excerpts from Yamaguchi’s journal seen below, is due to Fukagawa and Rothman, *Sacred Mathematics: Japanese Temple Geometry*.

¹⁰ Though Yamaguchi did succeed in compiling a large number of problems in his journals, he was never able to have them published.

¹¹ Yamaguchi had made two prior journeys to collect sangaku, and would go to make three more. His third journey, however, was the biggest by far, on which he collected more problems than on all his other five combined.

¹² This journey would last for 2 years. In his journal, Yamaguchi writes “22nd of July, 1820: Many friends came to see me off in Edo for my long travels and left me send-off haiku.”

¹³ “Castle towns” were former fortified settlements which had become prefectural administrative centers by the Tokugawa period.

¹⁴ This was Fujita Kagen.

¹⁵ This is (a slightly abridged version of) a quote directly relayed in Yamaguchi’s journal.

¹⁶ From Yamaguchi’s journal: “[w]orshipping at Zenkoji is one aim of my travels.”

¹⁷ “Guest house” refers to the designated area of a temple for visiting devotees to stay overnight. According to his journal, Kanzan was only allowed to stay after getting (presumably, purchasing) a pass from the local inn. Several of the other worshippers, who couldn’t obtain passes, were simply kicked out.

¹⁸ This is a well-known problem in the history of Japanese mathematics [Fukagawa and Rothman], referred to as the Gion shrine problem. The original solution, due to Tsuda Nobuhisa, involved a polynomial of degree 1024 in several variables. This was later simplified to a degree 10 solution in a single variable by Ajima Naonobu. This is the solution Yamaguchi encountered, but unfortunately, he never wrote it down.

¹⁹ Actually left by Saito Mitsukuni, who hung the tablet, but may or may not have been the original author.

²⁰ From Yamaguchi’s journal: “[w]hile I have been staying with my friend Eguchi Shinpachi, his son Tamekichi has asked me to teach him math and to make him a student of the Yamaguchi school.”

²¹ From Yamaguchi’s journal: “I have arrived at the big castle town of Kanazawa. This town is so big that it had [a] hundred thousand houses.”

²² “There is a big river the Kurobe near the inn and I looked at the beautiful bridge 36 ken [70 m] long, which crosses the river without any supports in the middle.” From Yamaguchi’s journal.

²³ “I visited Osaka castle, which is beautiful beyond description.” From Yamaguchi’s journal.

²⁴ “When I entered Osaka, I visited a mathematician Takeda Atsunoshin, who was a disciple of Ban Shinsuke. Takeda is an astronomer, too. In the town, I saw leaflets about an astronomy lecture by Takeda.” From Yamaguchi’s journal. The exact details of his finding the leaflet go unremarked upon, and so are fictionalized in the present story.

²⁵ This refers to the Seto Inland Sea (瀬戸内海).

²⁶ Japan’s lone international port of Dejima was located in Nagasaki.

²⁷ “As much as I am leaving my heart with Nagasaki, I have set off to Kurume where I visited the Takarao shrine and recorded a sangaku problem.” From Yamaguchi’s journal.

Sources

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