Power Sharing "Discontinuities": Legitimacy, Rivalry, and Credibility

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Abstract

Power-sharing arrangements between a leader and a popular outsider can be mutually beneficial and threatening. The literature has focused primarily on the former's trade-off where a leader gains legitimacy when sharing power with a respected outsider but also subsequently creates a rival who could challenge their rule. Yet, this outsider also faces a simultaneous trade-off between power and credibility in acquiescing to the leadership. I incorporate both coinciding trade-offs in developing a formal model to examine such power-sharing arrangements which have been prevalent historically and currently. I illustrate a "discontinuity" in optimal power-sharing where a leader either shares nothing or shares a specific amount to compensate the rival for their lost credibility. Counterintuitively, I further show that the leader should share more power with less trustworthy rivals to reduce their strong incentive to challenge. I then revisit the Investiture Controversy in Medieval Europe using these insights from the model.

Power-sharing agreements play an important role in governance. Coalition governments, where ministerial portfolios are shared across coalitional partners, arise in both parliamentary and presidential systems (Cheibub et al., 2004). At the end of a civil war, establishing a credible power-sharing agreement can be vital to ensure that the two warring sides do not fall back into violence (Walter, 2002). In autocracies, the tenure of leadership is often dependent on how the dictator chooses to divide power between themselves, a chosen elite, and the military (Svolik, 2009, 2012). Differently to power sharing amongst actors that hold government positions or actors that deliver compliance through force, sometimes political leaders "share" their resources with popular outsiders in exchange for their approval in order to obtain "legitimacy."

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This problem presents trade-offs for both the leader and the outsider. First, although the leader can increase their support by obtaining legitimacy from sharing power, the leader is also simultaneously creating a potential "rival" that can turn around and challenge them for all the power. Furthermore, the probability that the rival wins in such a challenge is plausibly increasing in the amount of power that is shared with them. Thus, the leader gains legitimacy but also sets themselves up for a potential takeover. Second, the rival often derives their popularity from their independence. Complying with a leader might allow this popular outsider to acquire more power (pecuniary and otherwise), but it would also diminish their standing amongst their followers if it involves straying too far from their ideals. The rival would then need to net out the effects of the loss in followers and the increase in power when deciding whether to pose a challenge to the existing leader. Therefore, when deciding to share power, the leader needs to calculate the optimal amount of power to share such that they gain legitimacy but do not incentivize the rival to challenge them.

Historical and modern cases reveal that the threat of a takeover and the concern for credibility were real issues. The history of the Papacy in Medieval Europe (Blumenthal, 1988; Morrison, 1971), the Shia clerics in the Safavid Empire (Abisaab, 2004), and the Catholic Church in Latin America (Schwaller, 2011) all include stories illustrating the trade-offs described above vis-à-vis a leader facing a religious official. Similarly, the recent past and current interactions between Russia and China's political leadership and religious institutions depict both mutually beneficial but potentially mutually threatening relationships (Koesel, 2014). In all these cases, political leaders used religion as a source of legitimacy but were also aware of the threat it could pose in undermining their power. Concurrently, the religious officials in these historical and modern episodes acknowledged their increased rents and perquisites that came with supporting the political leaders, but they were also worried about their subsequent loss in credibility in the eyes of their original followers. This latter concern sometimes drove religious establishments to challenge the leadership that initially empowered

them. Thus, political leaders need to account for both the dynamics of legitimacy and rivalry when deciding how to optimally share power.

In this way, this paper differs from the existing literature on power-sharing which has tended to focus on only one of the trade-offs: the fact that sharing power can empower a potential rival. Yet, the simultaneous second trade-off - the fact that certain "legitimacy-granting" institutions can potentially lose credibility from such quid pro quo relationships - has been less explored. A substantial amount of notable power-sharing relationships contains both of these strategic tensions, and it is thus necessary to account for both dynamics in order to fully appreciate the foundations, outcomes, and peculiarities of such decision-making processes.

This paper develops a model of power-sharing incorporating both of these trade-offs. Counter-intuitively, I depict how a leader should be more concerned with sharing too little power as opposed to sharing too much. The rival needs to be compensated sufficiently in order to discourage them from undertaking the gamble of challenging the leader. With this formal model, I illustrate how a "discontinuity" exists where the rival challenges and attacks only if the shared power is less than a critical point. The leader, in equilibrium, would share exactly this critical amount of power or choose not to have any power-sharing arrangement at all.

After this initial depiction, I then examine the nature of this discontinuity by studying the power-sharing problem in the presence of uncertainty in order to portray the more realistic environment where actors have imperfect information[1]. This allows us to derive further conditions for how and when power-sharing could lead to conflict when there is imperfect

[1]I also study the problem when the leader's decision space is in two dimensions as leaders usually also have more than one instrument that they can use to "buy" the rival's support. In this case, the leader can choose both policy and the amount of power-sharing. I show that the leader has a range of options to choose from in order to satisfy a rival. The leader can choose either to move the policy closer to the rival's ideal point and give less power or to move the policy closer to their own ideal point and give more power to the rival. The main point remains that the rival needs to be compensated for their loss in credibility from supporting the leader. The analysis and proof are in the appendix.

information. Uncertainty arises because the rival has an incentive to misrepresent their preferences in order to extract more power (i.e. more concessions) from the leader. Under uncertainty, I derive a comparative static which illustrates that when the leader trusts the rival less, the leader should actually share more power with them. This is because in order to placate an untrustworthy rival who is highly incentivized to betray the leader, the leader needs to compensate them by sharing more power with them.

I then apply the model to the Investiture Controversy in Medieval Europe where leaders shared power with a religious official - the Pope. Both King William I of England (a.k.a. William the Conqueror) and Emperor Henry IV of the Holy Roman Empire wanted to keep the Church close to the state but were also concerned with maintaining influence over their Church by keeping the right to investiture - the control over appointments of bishops and abbots. This issue eventually led to a power struggle between Pope Gregory VII and Henry IV, but did not lead to such a struggle between Pope Gregory VII and William I. Why did Gregory VII ask for more power? Why did Gregory VII choose to challenge and enter into conflict with Henry IV? Why did Gregory VII treat William I with more leniency? I use the model as a means to understand these historical events.

The paper is organized as follows. First, I discuss the two strategic tensions in more detail and briefly discuss the contribution of this paper in the context of the related literature. Second, I present the model and its results under the different scenarios. Third, I revisit the Investiture Controversy using insights from the model. Fourth, I discuss some critiques of the model. Finally, I conclude.

The two strategic tensions

This paper analyzes a specific but important understudied problem in power-sharing. The core of the problem involves two strategic trade-offs - one for the leader and one for the

potential popular partner who commands a following of supporters. First, the leader can share power with a popular individual to gain legitimacy, but is also potentially empowering a rival that can end up challenging them. Second, the popular person or institution, who accepts the leader's terms, might acquire more power in return but also loses credibility with their initial supporters. Thus, the "popularity" of the individual could be severely weakened if they show acquiescence to the leader. Studying this specific problem would advance our understanding of power-sharing because the simultaneity of the two trade-offs have not received extensive attention. The first trade-off has been studied before in isolation, but the second trade-off has been hardly covered in the existing literature as a simultaneous phenomenon in the context of this problem.

This strategic tension behind the first trade-off has been examined in the context of monarchies and dictatorships. Herz (1952, p.30) depicts this dilemma in the "crown-prince problem"[2] where a clear system of succession needs to be implemented in order to appease an elite constituency. Yet, a dictator who attempts to groom a successor might inadvertently create a rival who could possibly attempt to usurp power and Brownlee (2007); Kurrild-Klitgaard (2000); Tullock (1987); and Kokkonen and Sundell (2014) study how a system of primogeniture can help overcome this problem. Similarly, Svolik (2009, 2012) also portrayed this trade-off in dictatorships in two scenarios - power-sharing with an elite and using the military to control the population. In Svolik's (2009) framework, the dictator can attempt to engage in opportunistic behavior to consolidate more power. In an environment where the dictator's actions are not perfectly visible, the elite then decides whether or not to stage a coup. In the other dictatorship problem, the autocrat can build up the military in order to repress any civil unrest. However, in return, the military will

[2]In order to stay in power, dictators rely on loyalty from a certain elite. Yet, the elite need to be assured that they will continue to be rewarded for their loyalty even after the dictator dies. Therefore, the autocrat will need to construct a system of succession that will assure the elite about their future rents

demand concessions under the threat of potentially intervening and replacing the autocrat. Svolik (2012) uses a formal model to study the conditions under which the military will intervene in politics. Thus, the problem of creating a potential rival has been examined but, in all these models, the leader's chosen partner does not incur any disutility from supporting the leader. In other words, the second trade-off is ignored. For example, in Svolik's (2009; 2012) models, there is no direct cost to the elite or military for complying with the leader. This might seem reasonable in the context of power-sharing with the military and elite, but there are many notable power-sharing situations where both trade-offs would materialize.

Leaders cannot always go and "buy" citizen support (Bueno de Mesquita et al., 2003; Dekel et al., 2008), or use the means of force and coercion to compel compliance (Svolik, 2012). The cost of directly buying support or using force might be large and unfeasible, and a leader's long-term survival is likely to require more than physical coercion and instead depend on the citizenry perceiving the leader to be the "legitimate" ruler. Legitimacy is a convoluted topic and a contested term which can have a multitude of meanings (François and Sud, 2006; Lipset, 1959). Yet, broadly, legitimacy is defined in terms of the evaluations of the relevant population. A political leader can be seen as legitimate depending on whether their values are perceived to fit in with "the primary values" of society (Lipset, 1959, pp.86-87). Thus, the extent of a leader's legitimacy is directly proportional to the number of supporters of the leader.

To acquire such legitimacy, leaders might need to rely on another popular individual to deliver them followers and supporters. This situation can arise in the context of a leader sharing power with religious officials or when a leader shares power with the influential head of an ethnic group. These partners, in their capacity as representatives of important social constructs, provide sources of legitimacy for the leader. Yet, compromising one's

independence and acquiescing to the leader can alienate the initial supporters of the legitimizing partner and weaken the partner's standing in the community. Thus, it is under these scope conditions, where a leader needs to rely on a popular individual to deliver them supporters, that both strategic tensions would arise for the leader and their legitimacy-granting rival.

The takeover threat for the leader and the credibility concerns for the popular outsider are common features in real world power-sharing pacts. Thus, in contrast to the other literature, I develop a more thorough model that captures trade-offs for both the leader and the legitimacy-granting agent.

One prominent example where this framework is particularly pertinent is when a leader and religious official attempt to enter into a mutually beneficially arrangement. Power-sharing arrangements have often involved leaders and religious officials (Cosgel and Miceli, 2013). The heads of religions often command a large amount of followers due to their important standing within their communities. Chaney (2013) presents historical evidence from Egypt that suggested that religious officials and political leaders cooperated during economic shocks through a quid pro quo. The influential religious official controlled the unrest, while the autocrat allocated more resources towards religious structures and was less likely to replace the religious official. Similarly, leaders in Russia and China have patronized their "indigenous" and even foreign religious institutions, with the goal of receiving the latter's approval which in turn provides the embellishment of legitimacy for the leader's rule (Koesel, 2014). These cases exemplify the immediate cost of obtaining legitimacy - a leader would give up some resources to the religious officials in order to gain the support of the religious followers.

The independence of the religious official and institution is an important feature of their appeal - they, after all, should first be a representative of God and not the state.

Submitting to a secular political leader might thus compromise the religious official's standing and cost them followers. The risk of losing credibility is a real concern. Soon after China's Communist Revolution, the Protestant organization, the Three-Self Patriotic Movement, sought to develop closer ties with the state which alienated a substantial amount of the Protestant community who viewed these state-religious relations as a betrayal to their faith (Koesel, 2014, p.138). In the Safavid Empire, people became disillusioned with the Shia clerics when they entered and complied in arrangements with the Shah (Abisaab, 2004, pp.96-97). A considerable proportion of the French Catholic laity and even Bishops became disillusioned with the Church after the "Vichy-Church embrace" in France during the Second World War (Paxton, 2001, p.153). In a similar vein, Greif and Rubin (2015) also illustrate this dynamic in England during the post-English Reformation era where Monarchs started using Parliament as a source of legitimacy (instead of the Anglican Church) and thus increased the powers of Parliament. The Monarch was the head of the Anglican Church, and this institution was subsequently seen as having less legitimizing power than the relatively more independent (and hence more legitimizing) Parliament.

In other words, these religious officials lost followers and support amongst the citizens when they developed a closer relationship with the state. Therefore, groups that can grant legitimacy are likely to suffer some cost from complying with the leaders. Subsequently, it is not surprising that there have been particular episodes where the religious official later turned on the leader that initially empowered them. In Poland, the Catholic Church played a large symbolic and functional role in the anti-Communist opposition. Yet the basis for the Church's influence in this confrontation was through its initial cooperation with the State, where the latter granted the former certain power and privileges while the former, in return, promised (at least tacit) support and no obstruction to the latter's rule (Hruby, 8283, pp.320-321). The Serbian Orthodox Church was initially strengthened by the

warming of relations with President Slobodan Milosevic, but this modus vivendi was short lived as the Church eventually turned against Milosevic in favor of the opposition (Crawshaw and Peric-Zimonjic, 2000; Higgins and Block, 1999). In this paper, I discuss another historical episode - the Investiture Controversy - where an empowered religious official (the Pope) challenged a political leader (the Holy Roman Emperor).

Aside from religious contexts, another situation where both trade-offs play a role is the one where a leader might require the consent of the head of an ethnic group as a means to deliver support. This is exemplified well in India, a country where the central government has often had to rely on the help of legitimizing agents in Jammu and Kashmir.

Given the contentious nature with which the then Maharajah of Kashmir, Hari Singh, acceded to India[3], the legitimacy of the central government's rule was clearly questionable and they thus needed a partner to help deliver the support of Kashmiris. The National Conference Party in Kashmir, founded by the popular Sheikh Abdullah, played this role. The Indian Government overlooked any corrupt practices of this political party as long as they did not raise the idea of secession (Ganguly, 1997, p.39). Thus, a mutually beneficial power-sharing arrangement was created.

This arrangement did not mean that the central government could necessarily always trust their "legitimacy-granting" agent. For example, the Indian Government arrested Sheikh Abdullah at different points in time if there was any suspicion that the latter was agitating for secession. Thus, the central government acted preemptively to contain the potential threat of their empowered rival.

[3]Kashmir was one the few princely states where the religion of the ruler (Hinduism) was different from the religion of the majority of the population (Islam). The ruler, Hari Singh, delayed his decision on whether to join Pakistan or India after Partition. In response, violence broke out and Pakistani-backed forces entered Kashmir. The Maharajah asked India for help, and the latter agreed on the condition that the former had to sign an Instrument of Accession to India. Hari Singh agreed to the terms and Indian forces entered Kashmir which prompted the First Kashmir War.

Aside from this tension, the second-trade off was also present in this relationship. The close political ties between the National Conference Party and the government delegitimized the former in the eyes of young Kashmiris. Eventually, by the mid 1980's, the party leader Farooq Abdullah (the successor to the relatively more popular Sheikh Abdullah) was seen simply as "a stalking-horse" for the Congress Party in New Delhi (Ganguly, 1997, p.92).

Even though the two trade-offs might not apply in all situations, the above discussion reveals that both strategic tensions are present in many prominent, renowned cases of power-sharing. In the next section, I develop a model that incorporates both trade-offs, and I thus provide a more complete framework with which to understand the balancing acts that occur behind such power-sharing agreements.

Model

In the power-sharing story, the leader cares about two factors - the power they hold and legitimacy (i.e. the number of their supporters) - and the rival cares about two factors - the power that is shared with them and the number of their followers. Whether the leader is a democratically elected one or an autocrat, both types rely on support to stay in power and to pursue their policy agenda[4]. Therefore, it is reasonable to assume a leader would care about the size of their support base. A leader can "gain" legitimacy by sharing power with a rival. However, the leader suffers two costs from this action - one immediate and one potential. The immediate cost is the power they hand over to the rival. The potential cost is the possibility that the rival might take the offered power, and then turn around and challenge the leader for all the power. Furthermore, the probability that the rival wins in

[4]The size of this "support" group will vary under different circumstances (see Bueno de Mesquita et al. 2003).

such a challenge would be increasing in the amount of power that is initially shared with them. The more power that is shared, the easier it is for the rival to capture the rest of it.

Consequently, the rival also makes a choice when receiving the power. They can either comply with the leader or stage a challenge. In complying with the leader, the rival then has to promote the leader's preferred policy. However, the rival also cares about maintaining supporters who are composed of citizens. The citizens' ideal policy is different from the leader's ideal policy, and the citizens will join or leave the rival depending on the rival's promoted policy. The rival will have fewer followers if they promote the leader's ideal policy instead of the citizens' ideal policy as they lose some credibility from supporting the leader. Thus, the rival also faces a trade-off: they comply and gain power but they lose followers in this process. In light of these dynamics, the leader needs to calculate the optimal amount of power to share with the rival. The model below represents this power-sharing story.

Actors, actions, payoffs, and sequence

I make the following main assumptions about the actors and their objectives. There are two main strategic actors - the leader and the popular rival group leader (rival hereafter). The leader wants to maximize the number of supporters and the proportion of power they hold. The leader can share power ($x \in [0, 1]$) with the rival in order to gain the support of the rival's followers (i.e. gain legitimacy). The leader's utility is linear in both the number of supporters and the share of power they hold. The leader wants to implement policy p, but the citizens' preferred policy is q where $p \neq q$, and policy is a uni-dimensional space.

The rival's utility is linearly increasing in both the amount of followers and the amount of power that they obtain from the leader (x). If the leader shares power with the rival, the rival can always choose to attack and challenge the leader for total power. The probability that the rival wins is increasing in the proportion of power that is initially shared with them.

The citizens are not typical game-theoretic actors but instead behave as voters do in models of electoral competition[5]. There is a continuum of citizens who choose to join the rival or not depending on the rankings of their utilities. The proportion of citizens who support (i.e. join) the rival group are composed of the people who choose to follow the rival if the valuation from joining is greater than some reservation utility μ. Everyone has the same reservation utility. The valuations of joining the rival (δ) in the population are distributed according to some distribution function $F(\delta)$.

If the rival remains independent from the leader, then only those who have a valuation larger than the reservation utility will join the rival group. In other words, the proportion of the population that choose to join the rival is $1 - F(\mu)$. I will assume that the remaing proportion of the population ($F(\mu)$) support the leader but this is not important for the results and analysis[6]. The point is just to convey that the leader is likely to have some underlying support amongst citizens that do not follow the rival.

If the rival accepts the power sharing deal with the leader, then they have to promote policy p as well which is different from the citizens' ideal policy q. The followers who then join the rival have to accept and comply with policy p as well. Citizens suffer a cost from supporting policies that they dislike. As a result, only those who have a belief valuation such that $\delta - (p - q)^2 \geq \mu$ will join the rival. The proportion of the citizens that join the rival then becomes $1 - F(\mu + (p - q)^2)$ when the rival becomes a "servant of the state." This is the trade-off that the rival faces when deciding whether to comply and promote the leader's policy - they lose followers but gain power. I will refer to this diminished standing cost as the "credibility cost."

The utility functions for the rival are denoted by the letter $U_{(\cdot)}$, with the subscript

[5]I thank the anonymous reviewer for this useful characterization.

[6]As this level of support is constant, it cancels out in the analysis and thus does not affect the main results or comparative statics.

indicating the rival's chosen action. The utility for the rival from remaining independent and from acquiescing to the state (i.e. complying) are respectively:

$$U_I = 1 - F(\mu) \tag{1}$$

$$U_{Co} = 1 - F(\mu + (p-q)^2) + x \tag{2}$$

I am assuming that the utility is linear in both the number of followers and the share of power that the rival obtains. Furthermore, I am assuming that those followers that drop out from the rival group when the rival chooses to promote policy p (i.e. $F(\mu + (p-q)^2) - F(\mu)$) do not instead simply support the leader (this will be discussed below).

The rival also has the option of rising up and challenging the leader for total power. The probability of successfully challenging the leader is denoted by the jointly continuous function $\Pi(s, x)$. This probability is increasing in the amount of the rival's supporters (s) and increasing in the amount of power that is shared with the rival (x). I also assume $\Pi(s, 0) = 0$. In other words, the rival will only challenge the leader if the latter initially decides to empower the former. Once the rival decides to attack, they no longer have to support the government policy p. Thus, the utility for the rival from accepting x and then challenging the leader for power is:

$$\begin{aligned} U_{Ch} &= 1 - F(\mu) + x + \Pi(1 - F(\mu), x)[1 - x] + (1 - \Pi(1 - F(\mu), x))[-x] \\ &= 1 - F(\mu) + \Pi(1 - F(\mu), x) \end{aligned} \tag{3}$$

Clearly, in order for the rival to not attack, equation (2) needs to be at least as great as equation (3). I assume also that for all values of s, $0 < \partial\Pi(s, x)/\partial x < 1$. If $\partial\Pi(s, x)/\partial x > 1$, then equation (3) would always be greater than equation (2), and thus the rival would always want to challenge the leader. In such a case, the leader would never want to share power in the first place.

Some may take issue with two assumptions that are being made in the model. The first being that the rival can comply then choose to revolt and still gain back all their initial supporters. The second being that the rival does not suffer any direct cost for challenging the leader. In the discussion section of this paper, I explain in more detail why these modeling assumptions (among others) should not be a concern[7].

The leader has the choice of whether to share power with the rival or not. As mentioned before, the leader cares about the amount of power they hold and their level of support. The utility functions for the leader are denoted by the letter $V_{(\cdot)}$ with the subscript indicating the leader's chosen action. Assuming the rival does not challenge for power, the leader faces the following utilities:

$$V_{NS} = F(\mu) + 1 \tag{4}$$

$$V_{Co} = F(\mu) + (1 - x) + (1 - F(\mu + (p - q)^2)) \tag{5}$$

Equation (4) is the utility for the leader if they decide not to share any power with the rival. Equation (5) is the utility for the leader if they decide to share an amount x with the rival, and the latter complies. In this second case, the leader now has less power (the second component of the equation), but they gain the supporters associated with the rival (the third component of the equation). I will refer to this gain in supporters as the "legitimacy-gain." As stated above, it is assumed (inconsequentially for the analysis) that the leader always has a certain guaranteed level of support from a segment of the citizens ($F(\mu)$). However, those followers who drop out from the rival group when the rival complies with the leader (i.e. $F(\mu + (p - q)^2) - F(\mu)$) do not instead simply support the leader. I assume they drop out

[7]First, if we incorporated into the model the idea that a rival cannot capture back all the followers they lost, then the optimal level of power-sharing changes but the dynamics (explained in the propositions) still hold. Second, including a direct cost would only introduce thresholds where the dynamics hold when the cost of challenging is less than a certain cut-point. For costs of challenging above this cut-point, the leader will not share any power (or only a negligible amount) as the rival would still comply when $x = 0$.

of the game entirely and do not support either the leader or the rival. This is a conservative assumption because if it did not hold (i.e. if those who dropped out from the rival group instead simply supported the leader), then this would imply that the leader could push all the rival group followers into their support group by adopting a policy p that is extremely different from the citizen's ideal policy q.

Furthermore, if the rival accepts x and then challenges the leader for power, then the utility of the leader from this perspective is[8]:

$$\begin{aligned} V_{Ch} &= F(\mu) + (1-x) + \Pi(1-F(\mu), x)[x-1] + (1-\Pi(1-F(\mu), x))[x] \\ &= F(\mu) + 1 - \Pi(1-F(\mu), x) \end{aligned} \tag{6}$$

By comparing equations (4) and (6), it is clear that in equilibrium, the leader would not share power if they knew they were going to be challenged.

The initial static game proceeds in the following sequence. First, the leader observes the reservation utility (μ), the citizens' ideal policy (q), and the distribution of valuations within the society for following the rival ($F(\delta)$). Then, the leader chooses the amount of power to share (x) with the rival. The solution concept will be a sub-game perfect equilibrium.

Analysis

As the leader would want to minimize the amount of power they share, it is optimal for the leader to give the rival just enough power such that the rival is indifferent between challenging and complying. Thus, by equating equations (2) and (3) together, we obtain the

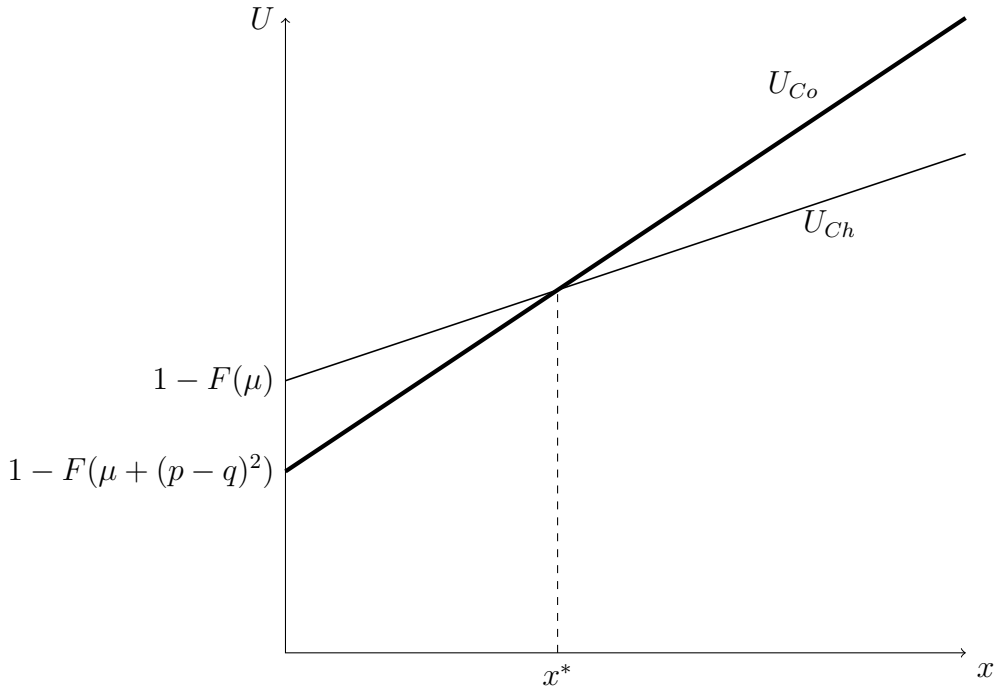
[8]Similar to the rival, I do not include a direct cost for the leader in the model when they are challenged by the rival. In the discussion section of this paper, I explain in more detail why this omission should not be a concern. Including such a cost would not affect three out of the four propositions at all and only affects Proposition II slightly by introducing thresholds for Proposition II*iii*. Proposition II*iii* holds when this cost of challenging is less than a certain cut-point and does not hold when the cost of challenging is greater than this cut-point.

following optimal value of x^* for the leader to choose:

$$x^* = F(\mu + (p-q)^2) - F(\mu) + \Pi(1 - F(\mu), x^*) \tag{7}$$

Figure 1 illustrates potential plots for equations (2) and (3) and labels the value of x that equates these two lines as the optimal value x^* described above in (7). For this graph, $\Pi(1 - F(\mu), x)$ is linear in x but this is not a necessary requirement. The assumption that $\partial\Pi(s, x)/\partial x < 1$ is important for obtaining this value. If this assumption did not hold, then the leader would never want to share power. The difference in the intercepts is the loss in followers (i.e. the credibility cost) if the rival promotes the leader's preferred policy p instead of the citizens' preferred policy q.

Figure 1: *The Discontinuity*



As shown in Figure 1 above, if $\tilde{x}$ is offered to the rival by the leader, then the rival will only challenge the leader for power if $\tilde{x} < x^*$. Otherwise, the rival will comply with the

leader.

I label the "discontinuity" as the distance between the origin and x^* on the horizontal axis. For all values of x below x^*, the utility from challenging is greater than the utility from complying. Thus, the leader can only lose full power when the rival is given too little. Giving more than x^* on the other hand, does not incentivize the rival to challenge as the utility from complying is always greater.

In comparing equations (4) and (5), it is clear that the leader only benefits from sharing power when $x \leq 1 - F(\mu + (p-q)^2)$. In words, the leader benefits from sharing power if the support gained is at least as big as the share of power conceded. Therefore, this puts a constraint on the optimal amount of x^* to share and this constraint is described below in Proposition I: the leader would either give exactly x^* or nothing.

Proposition I - The Discontinuity

In equilibrium, the leader will offer the following amount of power $(\hat{x})$ *to the rival*

If $x^* \leq 1 - F(\mu + (p-q)^2)$ *then* $\hat{x} = x^*$

If $x^* > 1 - F(\mu + (p-q)^2)$ *then* $\hat{x} = 0$

where x^* *is defined in Equation (7).*

This, at first, might seem the same as the standard "take-it-or-leave-it" bargaining game where, in equilibrium, the leader is sharing power with the rival as cheaply as possible in order to avert a conflict. However, in terms of power-sharing, Proposition I is actually a counter-intuitive result and the "discontinuity" illustrates a key difference between the conventional framework and my model. Conventionally, we would think that in a power-sharing arrangement, where a rival is increasingly made stronger with the more power that is shared with them, the leader would be concerned with sharing too much

power. This is the logic that is presented in Svolik (2012) when he looks at a dictator sharing resources with a military. This concern is portrayed below in Figure 2. The utility of the leader reaches an optimum at x^* but then jumps down to V_{Ch} (the utility from being challenged) when power-shared is above some critical threshold (x^*) because then the rival always engages in a challenge after this threshold. Below this threshold, the rival complies and thus the leader receives V_{Co} (the utility from the rival complying) in this segment of the graph.

However, the results from my model suggest the opposite dynamic which is portrayed below in Figure 3. Below the critical threshold (x^*), the leader is always challenged by the rival and thus only obtains the utility V_{Ch}. However, once the threshold is crossed, the utility for the leader jumps up to a higher level V_{Co} where the rival complies. Thus, even though the probability of the rival winning in a challenge is increasing in the level of power shared (x), my model indicates that the leader should still be more worried about sharing too little power as opposed to sharing too much.

Figure 2: *The conventional framework*

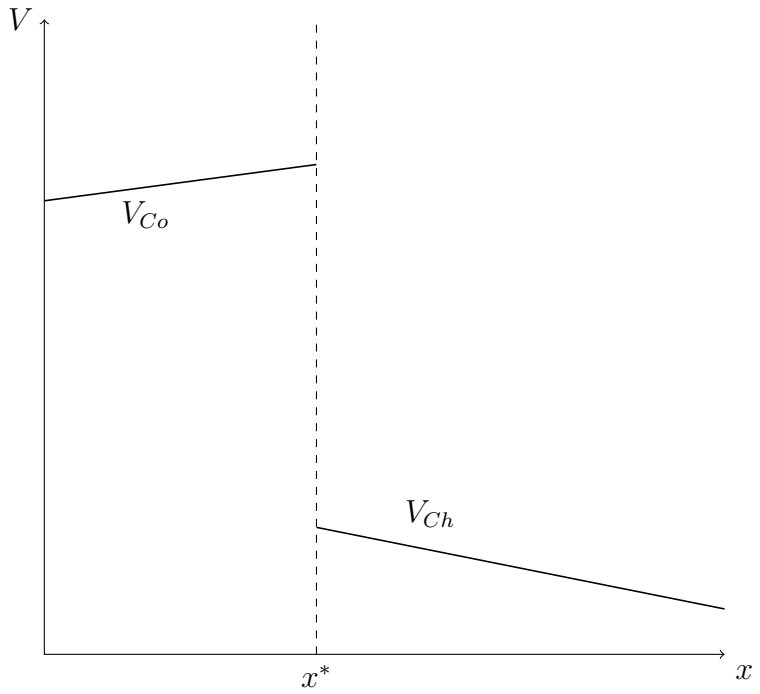
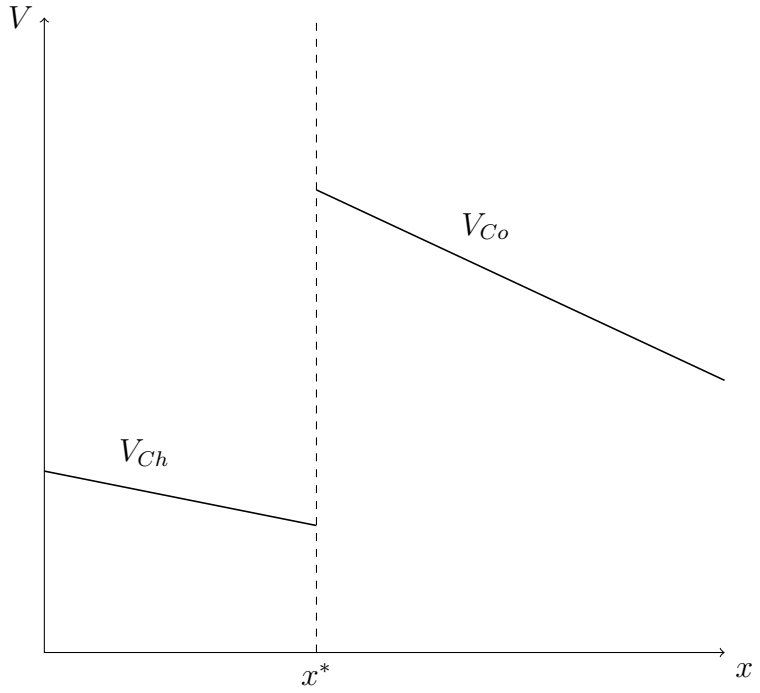


Figure 3: *The discontinuity model*



The intuition behind the discontinuity is based on the trade-off that the rival faces in accepting a power-sharing deal. Even though they acquire more power, it comes at a cost - they lose credibility and thus lose followers. With this cost in mind, the rival needs to be sufficiently compensated in order for the power-sharing deal to dissuade them from raising a challenge. If we only account for the trade-off involving the leader (obtaining legitimacy but creating a potential rival), then we are in the conventional framework world (as shown in Figure 2). Yet, this standard framework ignores the trade-off that the rival faces and thus misses the fact that a leader would need to account for the rival's cost in credibility when designing any power-sharing agreement. As described before, there are many prominent historical and modern cases of power-sharing where the rival faced such credibility costs.

When we account for the rival's trade-off, simultaneously with the leader's trade-off, a discontinuity arises where a leader would either gives exactly x^* (as specified in Proposition I) or share no power at all. If no power is shared, then the rival is unable to pose a challenge to the leader. If the leader shares a level of power between 0 and x^*, then the rival is not being sufficiently compensated for their loss in followers (the credibility cost). Thus, the rival is incentivized to take the gamble and mount a challenge against the leader within this interval of power-sharing. However, once the level of power-sharing is x^* or higher, the rival is sufficiently compensated for their loss and they find it better to accept the higher level of power-shared and acquiesce to the leader, than to engage in the lottery of challenging the leader. In trying to establish a power-sharing arrangement within this world where both the leader and the rival face their own respective trade-offs, the leader is attempting to achieve the multiple goals of gaining legitimacy, holding on to as much power as possible, and preventing any challenge to their rule. In pursuit of these somewhat conflicting goals, the intuition above reveals that there will be a jump, or discontinuity, in optimal power-sharing levels where the leader either implements no power-sharing or exactly an x^* level of power-sharing (as shown in Figure 3). In sum, the substantive implication of this result is

that the rival needs to be sufficiently compensated in order to dissuade them from staging a challenge against the leader, but the leader would not want to share more than necessary.

It might seem counter-intuitive that a leader would be concerned with sharing too little power as opposed to sharing too much power but, in a later section, I will discuss the Investiture Controversey in Medievel Europe to lend support to this argument. In 1075 in Medieval Europe, Pope Gregory VII banned lay investiture and also claimed that he had the right to depose secular leaders in his *Dictatus Papae*. Emperor Henry IV refused to accept these claims, and the two entered into a conflict. This case will depict how legitimizing agents need to be compensated adequately such that they are disincentivized to challenge the leadership.

Adding uncertainty

Now that we have characterized and established the intuition behind the discontinuity, we would want to further examine the nature of this construct. In this section, we are able to derive conditions for how and when power-sharing could lead to conflict by studying how the relevant actors would operate under imperfect information - a more realistic environment as information asymmetries are bound to be fairly pervasive in such power-sharing arrangements. Specifically, we will now consider the situation where the leader cannot perfectly observe the amount the rival values power relative to maintaining followers (i.e. maintaining credibility). The solution concept will now be a bayesian nash equilibrium where the equilibrium strategy for the leader will maximize their expected payoff given their beliefs about the type of the rival.

If uncertainty exists, it is most plausibly in this format. As stated before, the rival's utility is composed of two arguments - the amount of power shared with the rival and the number of their followers. Both the leader and the rival would be engaging with the

population so it is likely that they know the population characteristics regarding their valuations and their ideal policy. Therefore, in the interplay between a leader and a rival, uncertainty is most likely to arise in the form where the leader is unsure about the rival's relative preferences between power and maintaining followers. The leader cannot just simply ask the rival to declare their true disposition as the rival has an incentive to misrepresent their preferences in order to extract more power from the leader.

The utility of the rival from complying with the leader is now taken to be:

$$U_{Co} = 1 - F(\mu + (p-q)^2) + x\varepsilon \tag{8}$$

where ε is a random variable that has distribution G with support $[\underline{\kappa}, \overline{\kappa}]$ where $0 < \underline{\kappa} < 1$. The leader only knows G and the support but does not know the exact value of ε. Values of $\varepsilon > 1$ indicate that the rival prefers power to maintaining followers, and this one-sided preference becomes stronger as ε increases. The opposite occurs when $0 < \varepsilon < 1$, as now the number of followers is more important to the rival than holding power.

The play of the game is as follows. First, nature determines the preferences of the rival (ε). Second, the leader decides how much power to share with the rival. If the leader decides not to share any power, then the game is over. If the leader shares a positive amount of power, then the rival decides how to act (whether to comply or challenge). Given the utility from challenging (equation 3), the rival chooses to comply iff

$$\begin{aligned} 1 - F(\mu + (p-q)^2) + x\varepsilon &\geq 1 - F(\mu) + \Pi(1 - F(\mu), x)\varepsilon \\ \Rightarrow \varepsilon &\geq \frac{F(\mu + (p-q)^2) - F(\mu)}{x - \Pi(1 - F(\mu), x)} \end{aligned}$$

Let us call this threshold $\hat{\kappa}(x)$. For simplicity, throughout this section, we will assume that for all $x \in [0, 1]$,

$$\hat{\kappa}(x) = \frac{F(\mu + (p-q)^2) - F(\mu)}{x - \Pi(1 - F(\mu), x)} \in (\underline{\kappa}, \overline{\kappa}).$$

Therefore, the leader solves the following maximization problem:

$$\underset{x}{\text{maximize}} \int_{\underline{\kappa}}^{\hat{\kappa}(x)} g(\varepsilon) * V_{Ch} \, \mathrm{d}\varepsilon \quad + \quad \int_{\hat{\kappa}(x)}^{\overline{\kappa}} g(\varepsilon) * V_{Co} \, \mathrm{d}\varepsilon$$

As V_{Ch} and V_{Co} do not depend on ε, the above integral can be simplified into:

$$\underset{x}{\text{maximize}} \; G\left(\frac{F(\mu + (p-q)^2) - F(\mu)}{x - \Pi(1 - F(\mu), x)}\right) * V_{Ch} +$$

$$\left[1 - G\left(\frac{F(\mu + (p-q)^2) - F(\mu)}{x - \Pi(1 - F(\mu), x)}\right)\right] * V_{Co}$$

Under uncertainty, the leader has to face a slightly modified trade-off from the one they faced under perfect information. The more power they share, the more likely the rival will comply with them and offer them support. However, by allocating more power to the rival, the probability of the rival winning in a challenge also increases.

The first order condition (shown in the appendix) implies that the discontinuity cut-point x^* will be an interior maximum under certain conditions. The model in this paper will explore three of those conditions. First, I will assume that ε is distributed uniformly over $[\underline{\kappa}, \overline{\kappa}]$. Second, I also assume that the second derivative $\partial^2\Pi/\partial x^2 > 0$[9]. Third, and as stated in Proposition I, the leader will only share power if the support gained is at least as big as the share of power lost (i.e. $x^* \leq 1 - F(\mu + (p-q)^2)$). These are not necessary conditions (except for the third one) but these are sufficient conditions for the existence of an interior maximum. We will assume that these three conditions hold in the next section dealing with the comparative statics.

[9]This assumption might raise concerns as it is conventional to assume diminishing marginal returns. Thus, a suitable alternative for this condition is that $\partial^2\Pi/\partial x^2 < 0$ but the magnitude of this second derivative is sufficiently small.

Comparative statics with uncertainty

We want to explore how the optimal amount of power that is shared (x^*) changes with the standard deviation and expected value of ε. In the conventional framework (see Figure 2), even under uncertainty, the leader should always err on the side of giving less power than more power to the rival. On the other hand, under uncertainty in my model, I show that in certain conditions the leader might prefer to increase the amount of power shared in order to placate the rival and to prevent them from staging a challenge.

Standard Deviation

The standard deviation is a measure of the uncertainty with regards to the value of ε (how much the leader values power relative to maintaining followers). We would want to analyze how the optimal level of power shared varies with this uncertainty. As I show below, the relationship between the optimal value of power-sharing and uncertainty is not necessarily monotonic. Let σ and E be respectively the standard deviation and the expected value of ε.

Using the implicit function theorem (details in the appendix), we can see that $\frac{dx^*}{d\sigma}$ will be positive (negative) iff

$$[1 - F(\mu + (p-q)^2)] \left[\frac{F(\mu + (p-q)^2) - F(\mu)}{(x^* - \Pi(1 - F(\mu), x^*))^2} \right] < (>) E \tag{9}$$

The left-hand side of the above inequality is decreasing in x^* and is not defined at $x^* = 0$. This leads to the next proposition that I label as the "Uncertainty Comparative Static."

Proposition II - Uncertainty Comparative Static

i. If the legitimacy-gain is small and the credibility-cost is large, then the optimal amount of power shared (x^*) decreases as uncertainty increases.

ii. Even if the legitimacy-gain is large, if the probability of a rival winning in a challenge is sufficiently high then the optimal amount of power shared (x^*) decreases as uncertainty increases.

iii. If the probability of the rival winning is sufficiently low, and both the legitimacy-gain and credibility-cost are large, then the relationship between the discontinuity and uncertainty is non-monotonic. If the initial discontinuity cut-off point is high (low), then increases in uncertainty lead to increases (decreases) in the level of power sharing.

The calculations behind Proposition II are in the appendix. This proposition basically states that as uncertainty about the rival's preferences increases, the leader could end up actually sharing more power with the rival under certain conditions. It is important to note that the important factors in the proposition are the legitimacy-gain $(1 - F(\mu + (p - q)^2))$ and credibility-cost $(F(\mu + (p - q)^2) - F(\mu))$ variables. Thus, to interpret the proposition, we need to account for how these two elements compare to each other in magnitude.

If the legitimacy-gain is small and the credibility-cost is large, then the discontinuity cut-off point will always be decreasing as uncertainty increases. In this case, the rival has a higher cost from complying and thus, has an increased incentive to challenge the leader. Also, the amount of followers the leader obtains from sharing power is relatively small so the leader has less to gain even if the rival complies. Given these circumstances, the leader would prefer to reduce the amount of power shared as uncertainty increases in order to avoid making the rival any stronger in case of a challenge. This is depicted in part i. of Proposition II.

Furthermore, if the probability of the rival winning in a challenge is sufficiently high, then the discontinuity cut-off point decreases as uncertainty increases. Even though it may be the case that the legitimacy-gain is large, the high risk of losing in a challenge dominates and

thus the leader shares less as uncertainty increases. This is depicted in part ii. of Proposition II.

Under the assumption that the probability of the rival winning is sufficiently low, Part iii. of Proposition II illustrates how the relationship between the discontinuity and uncertainty is non-monotonic when both the legitimacy-gain and credibility-cost are sufficiently large. In this case, the rival has a high cost from complying but the leader also has a large amount of followers to gain if the rival does comply. However, the way the discontinuity cut-off point changes with uncertainty depends on the initial conditions. If the initial discontinuity cut-off point is high, then increases in uncertainty lead to increases in the level of power-sharing. On the other hand, if the initial discontinuity cut-off point is low, then increases in uncertainty lead to decreases in the level of power-sharing. This result can be understood in light of why initial power-sharing is high or low in the first place. Recall that p and q are assumed to be exogenous and fixed. From the first order condition (see the appendix), initial power sharing is higher when the credibility-cost for the rival is large (i.e. $F(\mu + (p-q)^2) - F(\mu)$ is large), and if the leader has a large legitimacy-gain opportunity (i.e. $1 - F(\mu + (p-q)^2)$ is large). Even though the rival has more of an incentive to challenge, the leader prefers to increase his chances of gaining the support of the large amount of followers by sharing more power when uncertainty increases. This higher level of shared power reflects the fact that the rival needs to be recompensed for their relatively large credibility cost. On the other hand, initial power sharing is lower when the rival's credibility cost and the leader's legitimacy gain are relatively smaller. In this case, as uncertainty increases, the leader would prefer to share less power as the gain in supporters is not enough to overcome the increased risk of making the rival stronger.

Expected Value

Next, we would want to understand how the optimal level of power-sharing changes when the leader's trust in the rival changes. In this section, I examine how the discontinuity cut-off point varies with the expected value of ε which measures, in expectation, how much a rival values the amount of power-shared relative to the amount of their followers. Using the implicit function theorem again (with details in the appendix), we can derive $\frac{dx^*}{dE}$ which leads to the following proposition that I label as the "Compensation Comparative Static."

Proposition III - Compensation Comparative Static

If the expected relative valuation of power (E) increases, then the amount of power shared (x^*) decreases.

This implies that if the rival is likely to put relatively more (less) value in the amount of power than in the amount of their supporters, then the leader should optimally share less (more) power with the rival. When ε is low, the rival has an increased incentive to challenge the leader. The rate of return from an incremental increase in the number of followers is higher than the rate of return from the same incremental increase in the level of power. Therefore, this proposition reveals that when the leader trusts the rival less, the rival gets more power, and when the leader trusts the rival more, the rival gets less power. This is because when the rival values power less relative to their number of followers, there is a larger credibility cost for the rival to comply with the leader as the loss in followers is more pronounced. To compensate them for this larger cost of complying, the leader needs to give the rival more power. If the leader gave them too little, then the rival would prefer to challenge. Again, the leader would be worried about giving too little (as opposed to giving too much) and this increases the level of power shared even though the probability of the

rival winning in a challenge will increase as a result. On the other hand, when the rival values power more, then the leader can afford to share less power with them as there is a higher chance of the rival complying. The loss in followers has a lower effect, and thus the compensation required is also lower. In sum, this comparative static illustrates that in order to placate an untrustworthy rival who is highly incentivized to betray the leader, the leader needs to compensate them by sharing more power with them. However, the leader also has the outside option of not sharing any power so we would also need to check for the conditions for when this action is optimal.

Outside Option

The leader has the outside option of choosing not to share any power at all. In this case, they receive the utility V_{NS}. Thus, if the expected utility from sharing power is less than V_{NS}, the leader would choose this outside option.

It can be shown that the leader would choose the outside option iff

$$\frac{1 - F(\mu + (p-q)^2) - x^*}{\Pi(1 - F(\mu), x^*) + 1 - F(\mu + (p-q)^2) - x^*} \leq G\left(\frac{F(\mu + (p-q)^2) - F(\mu)}{x^* - \Pi(1 - F(\mu), x^*)}\right) \tag{10}$$

The right-hand side of the inequality is the probability, at the discontinuity cut-off point, that the rival's relative valuation of power is too small such that they choose to challenge. The left-hand side of the inequality captures the size of the successful challenge probability relative to the "profit" of the leader from sharing power. This leads to the next proposition:

Proposition IV - Outside Option

The outside option of staying independent is more likely to be used by the leader when:

1. *The legitimacy-gain for the leader is small.*
2. *The leader's optimal policy (p) is further away from the citizens' ideal policy (q).*

The details of the proof are in the appendix, and the intuition is fairly straightforward. If the leader has less followers to gain from power-sharing $(1 - F(\mu + (p - q)^2))$ then the legitimacy-gain from the relationship becomes increasingly weaker. Legitimacy, in turn, would becomes less important to achieve, while the uncertainty means that there is still a chance that power-sharing could result in a challenge. Additionally, if the legitimacy gain becomes smaller then that is equivalent to $F(\mu + (p - q)^2)$ becoming larger, which would increase the credibility cost $(F(\mu + (p - q)^2) - F(\mu))$. This increased credibility cost also increases the probability that the rival would want to stage a challenge at the optimal power-sharing level (x^*). Subsequently, the outside option of no power sharing becomes more attractive for the leader.

When the leader's optimal policy (p) is further away from the citizens' ideal policy (q), then this increases the interval of valuations for which the rival would want to challenge the leader. Furthermore, this would also shrink the legitimacy-gain for the leader which, from the first part of the proposition, we know increases the value of the outside option. Thus, the leader is less inclined to share power under these circumstances.

Investiture Controversy

According to the compensation comparative static examined in Proposition III, when the rival values acquiring power less than maintaining credibility (i.e. keeping followers), then the leader will have to share more resources with the rival in order to prevent them from challenging in any power-sharing arrangement. In this way, the leader compensates the rival for the latter's highly valued loss in credibility by sharing more power. The opposite occurs when the rival values power more than maintaining credibility, and under such circumstances, the rival would comply even if they received less power as their loss in

credibility is not as significant. In sum, those rivals that are less trustworthy and more likely to care about their standing in the community need to be offered more power. In this section, we will lend support to this counter-intuitive logic by examining the case of the Investiture Controversy.

The Investiture Controversy is a good case to examine this particular comparative static for two main reasons. First, the power-sharing arrangement involves leaders and a religious official which means the case falls under the scope conditions of the model. The religious official in this narrative is Pope Gregory VII, and the two political leaders are the Holy Roman Emperor Henry IV and King William I of England. Second, the case allows us to compare two power-sharing arrangements involving the same religious official at around the same time period. Thus, I am more convincingly able to control for additional confounding variables that I would otherwise not be able to do if I chose different religious officials or focused on different time periods. Yet, even though the time period and religious figure is held somewhat constant, the outcomes for the two power-sharing leaders were different. Both Henry IV and William I chose to not yield to the Papacy, yet why did Gregory VII choose to challenge and enter into conflict with Henry IV but treat William I with more leniency? I use the insights from the model as a means to understand this important historical event.

In using the model to interpret this case, we are conducting a type of "analytic narrative" by focusing on the comparative statics and off-the-path behavior of the model (Bates et al., 1998; Levi and Weingast, 2016). Specifically, we try to understand the actions of key actors and the subsequent events during the Investiture Controversey through the lens of the compensation comparative static.

Pope Gregory VII and Emperor Henry IV

In light of the uncertainty surrounding such power-sharing arrangements, we would need to first determine the relative valuation of power to credibility for Gregory VII vis-à-vis Henry IV (see Equation 8). In other words, we need to ascertain whether ε was high or low before proceeding to predictions on the optimal level of power-sharing to prevent conflict. Advocacy for monastic reform had begun well before Pope Gregory VII. When Leo IX became Pope, "renewal of religious life" had become an important concern for many monks, clergy, and laity (Blumenthal, 1988, p.64). Two of the specific concerns were simony (selling of church offices) and clerical marriage. At the 1049 synod in the Lateran Palace, Pope Leo IX proclaimed that all consecrations by Simony were invalid and declared that all wives and concubines of priests were "to become property of the Roman Church" (Blumenthal, 1988, p.74). Thus, the concern for restoring credibility appeared to be a prime concern for Leo IX, and his successors seemed to continue on this reform trajectory. For example, in 1059, Pope Nicholas II's Papal Election Decree stated that the College of Cardinals, not the German emperors and secular authorities, would elect the Pope (Ellis, 1971, p.92). When Gregory VII became Pope, he shared similar views as his predecessors regarding reform (Blumenthal, 1988, p.114). In reference to the model, these reforms suggest that the expected value of ε for Gregory VII was low as maintaining credibility had becoming increasingly important for the Pope. Thus, according to the compensation comparative static, the religious official would need to receive more power in order to be compensated for their increased aversion to losing credibility.

Gregory VII had already succeeded in reforming the sexual and financial corruption of the clergy, but now "extended his field of battle" to investiture (Toynbee, 1971, pp.115-116). In fact, in his 1075 *Dictatus Papae*, Gregory VII banned lay investiture but also claimed that he should have the right to depose secular leaders. With reference to the model, the Papacy was asking for a higher level of x (i.e. a higher share of power). Given that we were

in a world where Gregory VII was placing a higher value on maintaining credibility (lower ε), the compensation comparative static suggests that more power needed to be shared with Gregory VII in order to placate him and dissuade him from challenging Henry IV. However, Henry IV refused to accept these claims. The "discontinuity" equilibrium implies that anything less than the critical threshold would result in the rival challenging the leader, and this should be the off-the-path outcome as the leader would have preferred to abstain from sharing power than being challenged (compare equations (4) and (6)). Thus, when Henry refused to accept the terms of *Dictatus Papae*, the off-the-path outcome was realized and subsequently Gregory VII excommunicated and deposed Henry IV while Henry IV declared that Gregory VII was deposed, and the two entered into conflict.

The right to investiture was not a trivial issue. Formally, investiture somewhat resembled the system of feudalism involving nobles and vassals, where the bishop or abbot promised fealty and was granted royal protection in return (Blumenthal, 1988, p.36). This exemplifies a power-sharing arrangement which was mutually beneficial to all actors. The agreement allowed the episcopate to gain territorial power while the monarch received "God's" approval (Blumenthal, 1988, pp.34-36). As a result, by 1075, a substantial proportion of Western Christendom civil administration was controlled by clerics (and they held it by feudal right) so a ban on lay investiture would severely reduce the secular power's authority over a considerable part of their dominion, and would transform the Church into "a civil as well as an ecclesiastical *imperium in imperio* [empire inside an empire]" (Toynbee, 1971, p.118). Thus, the Emperor had a strong incentive to maintain as much control over investiture as possible, as ceding this right to the Pope would represent a significant forfeiture in power.

The excommunication initially led to some significant agitations from important segments in society against Henry IV which would eventually lead to the Great Saxon Revolt (Blumenthal, 1988, pp.122-125). The loss of legitimacy from the excommunication

mobilized certain Saxon Princes to challenge the rule of Henry IV. In comparison to this opposition, Henry IV had the weakest faction and thus he realized that he needed to have the excommunication lifted in order to improve his chances of overcoming this resistance (Blumenthal, 1988, pp.122-123). In line with Proposition IV, the legitimacy-gain was important so Henry IV was willing to somewhat submit to the Church as it helped to reduce the strength of the opposition. Henry IV went to the Pope and displayed penitence and humility in a famous episode called the *Road to Canossa* and was reconciled with the Church (Blumenthal, 1988, p.123). Thus, the leader (Henry IV) was operating in a world where, as Proposition IV states, the legitimacy gained from not incurring the Pope's disapproval was more important than the outside option of ignoring the Church completely.

The Papacy initially attempted to remain a neutral arbiter between the Henry and the opposition Saxon Princes (Blumenthal, 1988, p.125). Yet, Henry IV refused to abide by the Papacy's requests, and eventually Gregory VII excommunicated him again in 1080 in favor of the the anti-King Rudolf of Rheinfeld who promised obedience to the Papacy (Emerton, 1932, pp.149-152; Cowdrey, 1998, pp.197-198). The Papacy was still operating in a world where they valued their credibility highly. Thus, given this low ε, the compensation comparative static implies that more power needed to be ceded to the Pope in order to compensate him for his acquiescence and in turn prevent conflict from arising. However, Henry IV was still unwilling to follow these terms even after his first excommunication and the *Road to Canossa*. Thus, unsurprisingly, the Emperor and Papacy again enter into conflict with Henry being excommunicated again. The Investiture Controversy is eventually resolved later in 1122 with the Concordat of Worms.

Pope Gregory VII and King William I

The quid pro quo relationship between leader and religious official is also portrayed in this case as well. William I felt cheated out of the Crown of England after the death of Edward the

Confessor, when Harold II was instead declared the heir to the Crown. Intent on violently challenging this succession, William I looked towards the Papacy for support. Similar to Henry IV facing the Saxon agitations, William I was in a world where the legitimacy gained from the Church's support outweighed the option of staying disconnected from the Pope (as described in Proposition IV). Thus, a power-sharing deal was made between leader and religious official.

William I wanted approval from Rome for his invasion of England and, in return, he promised then Pope Alexander II (Gregory VII's predecessor) that "he would bring the English Church and clergy into closer submission to the Roman See" (Ellis, 1971, p.93). However, soon after the invasion, William I made it clear to the Papacy that he was the ultimate person in charge and, similar to Henry IV, refused to yield to Gregory VII.

The amount of power-shared, x, was reduced below the initial offer that William I had hinted at prior to his invasion. In addition to declaring his unwillingness to do homage to Gregory VII, William I declared that the Pope's recognition in England was contingent on his consent, that papal bulls or letters could not enter England without royal permission, that councils could not enact canons or forbid anything without his permission, and that bishops or ecclesiastics could not excommunicate any of his barons without William I's permission (Ellis, 1971, pp.93-94). Gregory VII complained about these declarations but, unlike with Henry IV, he did not challenge William I. This is puzzling given that this was occurring around the same time Gregory VII had entered into a conflict with Henry IV over the latter's refusal to submit to the former's demands.

This raises the question of why Gregory VII treated William I with much more leniency? The insights from the compensation comparative static can help us understand this different approach. Catholic Historian Ellis (1971, p.94), argues that at that time Gregory VII was already engaged in a struggle against Henry IV and thus, in need of the support and goodwill of William I, chose not to insist on any rigid rules. The Pope's bargaining position became

further weakened when the anti-Pope was elected (as this gave William I the credible threat to switch his recognition of the Church's chief religious figure) and, in light of all these cumulative dangers, Gregory VII accepted less and granted more freedom to William I (Ellis, 1971, pp.94-95). Therefore, the Pope adopted a different posture towards William I as the Pope's relative valuation (ε) between credibility and power vis-à-vis William I was different from that which he had had with Henry IV. In terms of the model, the expected value of ε for the Papacy increased as William's support was an important asset in their fight against Henry IV. The compensation comparative static illustrates than when this occurs, then the rival religious official would be willing to settle for less power and still comply with the leader. In other words, the required compensation to prevent conflict decreases. Thus, in accordance with the compensation comparative static, the Papacy accepted less from William I and still chose to comply.

In sum, I have argued that the different outcomes in the relationship between the Papacy and Henry IV and the Papacy and William I can be understood from the perspective of the compensation comparative static. In the first relationship, the Papacy increasingly valued credibility and thus Henry IV's unwillingness to capitualte and increase the amount of power-shared from its initial level resulted in conflict. This could have been avoided if sufficiently more power was given to Gregory VII. Once the Papacy had entered into this conflict, he required support. Thus, with regards to William I, he highly valued the support-related resources that came with power-sharing. In this situation, William I could refuse to yield to the Pope and the Pope would accept these terms and not turn to conflict (as he had done with Henry IV).

Discussion

In this section, I will address some of the expected critiques of the model. As I argue below, these critiques either do not affect the results or only introduce threshold points where, below a certain cut-off point, the "discontinuity" dynamic still holds. Above the cut-off point, either no power would be shared at all or only some parts of the proposition no longer hold (e.g. Proposition II *iii*).

First, I do not incorporate a cost to fighting for either the rival or the leader (e.g., I do not subtract exogenous cost terms from Equations (3) and (6)) but if we accounted for these costs then this either only introduces thresholds for some of the propositions or does not have an effect on the propositions. When the cost of war for the rival is smaller than a certain amount (specifically, smaller than $F(\mu + (p-q)^2) - F(\mu)$), then the "discontinuity" still exists. If the cost of war is greater than this threshold then the leader would not share power with the rival (or share a negligible amount), as the rival would still comply anyway since conflict is too costly. The results under uncertainty would be affected in the same way, where power-sharing occurs (and the subsequent comparative statics hold) when the cost to fighting is below the threshold, while no power is shared when the cost is above the threshold.

When we incorporate a cost of conflict for the leader in the model, then this does not affect the results in Proposition I. It only further emphasizes the point that, in equilibrium, the leader would not share power if they knew they were going to be challenged. With uncertainty, incorporating a cost of conflict for the leader does not change Proposition III, and only affects Proposition II by introducing thresholds. If the cost of conflict is smaller than a certain amount, then Propositions II*i*, *ii* and *iii* still hold. If the cost of conflict is greater than a certain amount, then only Proposition II*iii* would no longer hold.

Proposition II*i* and Proposition II*ii* specified the conditions under which increases in uncertainty lead to decreases in the level of power-sharing. Proposition II*iii*, on the other hand, specified the conditions under which increases in uncertainty can lead to increases in the level of power sharing. When the cost of conflict for the leader becomes sufficiently large, the Proposition II*iii* conditions are no longer important, and thus increases in uncertainty would always lead to decreases in the level of power sharing. Furthermore, the results described in Proposition IV regarding the usage of the outside option are not affected by this cost of conflict for the leader.

Second, I have assumed the rival is purely opportunistic and does not suffer directly (ideologically) from the policy in place. The rival only suffers a cost (in terms of lost followers) from the policy in place when they comply with the leader. However, again, even if we included a disutility component (e.g., $-(p-q)^2$) that accounted for the difference between the rival's ideal policy and the implemented policy in Equations (1), (2), and (3), then this would only put bounds on the existence of the discontinuity illustrated in Figure 1. The intercept for the U_{CO} line would decrease but the slope would remain the same. The intercept for the U_{CH} line would decrease by the same amount but the slope would change and increase by a magnitude of $(p-q)^2$. which is the square of the distance between the leader's ideal point and society's ideal point. The illustrated discontinuity point still exists as long as $\partial\Pi/\partial x < 1/(1+(p-q)^2)$. If this inequality is not satisfied then the utility from challenging is always greater than the utility of complying and thus the leader would not share any power with the rival in equilibrium. The results under uncertainty would be affected in a similar way, where power will be shared (and the subsequent comparative statics hold) when this inequality holds while no power will be shared when the inequality is not satisfied. Proposition IV concerns the leader so it is not affected by the inclusion of this disutility for the rival.

Similarly, the rival could suffer a cost from complying and then revolting. For example, they might not be able to capture back all the followers that they lost. We can incorporate this cost in Equation (3), by making it such that they only gain back $1 - F(\mu + \gamma)$ followers and not $1 - F(\mu)$ followers when they choose to challenge. We will make $0 < \gamma < (p - q)^2$, so that the rival has less followers than they would have had if they remained independent but more followers than they would have if they were still complying. In Figure 1, this would result in the intercept for the U_{Ch} line decreasing, but it would still be above the intercept of the rival's utility from complying $(1 - F(\mu + (p - q)^2))$. Furthermore, the slope of U_{Ch} would also decreases as the number of supporters falls from $1 - F(\mu)$ to $1 - F(\mu + \gamma)$. Therefore, the discontinuity would still hold but the optimal x^* would be lower. The cost from complying and then revolting would have no effect on any of the other Propositions.

Third, I have assumed that all citizens have the same preferences over policy. Even if we relaxed this assumption and allowed some citizens to prefer the leader's preferred policy, p, instead of q, this would not change the results. In terms of Figure 1, this would increase the intercept on U_{Co}, and decrease the intercept on U_{Ch}. As long as the intercept of U_{Ch} is above the intercept of U_{Co}, the discontinuity still exists. Furthermore, propositions II and III derived in the case of uncertainty, and proposition IV concerning the outside option would still hold. If the intercept of U_{Ch} fell below the intercept of U_{Co}, then the leader would not share any power with the rival and the rival would still comply and promote policy p. This is because, in this situation, the majority of citizens support policy p over q, and so the rival would have an incentive to promote policy p in order to maintain a larger amount of followers.

Fourth, if the leader cares about the policy implemented then they should care about

the policy enacted if they are replaced when they are challenged. This would involve adding a component to Equation (6) that accounts for this disutility when the leader loses. Under the assumption that the rival would implement the preferred policy of the citizens, q, the cost could take the form of $-(p-q)^2$. However, similar to the discussion above about including a direct cost to conflict, this would only further emphasize the point that, in equilibrium, the leader would not share power if they knew they were going to be challenged. With uncertainty, adding this policy cost for the leader does not change Proposition III, and only affects Proposition II by introducing thresholds. If the policy cost is smaller than a certain amount, then Propositions IIi, ii, and iii still hold. If the policy cost is greater than a certain amount then only Proposition IIiii would no longer hold. As explained above, this would imply that increases in uncertainty always leads to decreases in the level of power-sharing. Additionally, Proposition IV is not affected by incorporating this policy cost.

Finally, I have assumed that the people who fall away from the rival when the rival supports the leader ($F(\mu + (p-q)^2) - F(\mu)$) drop out of the game entirely and do not support either the leader or the rival. This raises the question of whether another rival would spring up to capture these citizens who have dropped out. This is outside the scope of this paper but is a worthwhile extension of this model. It suggests that the number of rivals will be determined endogenously with rivals competing amongst each other to gain followers and power.

Conclusion

I have presented a power-sharing model that illustrates why and how a leader would want to create a potential rival for power. By sharing power with a popular outsider, the leader

can gain legitimacy for their rule. However, at the same time, the leader wants to make sure that this new rival does not end up challenging them for total power. On the other hand, the rival can gain power by accepting the arrangement but at a cost to their credibility. By accounting for the trade-offs for both the leader and the rival, I have shown that, contrary to conventional wisdom, a leader should worry about sharing too little power as opposed to sharing too much power as the rival needs to be compensated for their loss in credibility. I depict a "discontinuity" in the optimal level of power-sharing and study this power-sharing problem in the presence of uncertainty. In the appendix, I also analyze the situation where the leader makes a choice over two dimensions of policy and power-sharing and show a fairly straight-forward dynamic. In this power-sharing deal, the leader can either choose to move the policy closer to the rival's ideal policy and share less power, or the leader can share more power and bring the policy closer to their ideal policy. Yet even with two dimensions, the main point remains - the rival will need to be compensated for their submission to the leader.

One of the key results from the model is the compensation comparative static. This counterintuitive result illustrates how the leader needs to share more power with less trustworthy rivals in order to placate the latter who are highly incentivized to betray the leader for the sake of maintaining their credibility. I then use the insights from this comparative static to revisit a puzzling episode of the Investiture Controversy - why did Pope Gregory VII engage in conflict with Emperor Henry IV but not with King William I?

The model has assumed that the rival group is a homogenous bloc. However, this assumption may need to be relaxed as different actors within a rival group might have divergent preferences. For example, after Gregory VII's *Dictatus Papae*, German Bishops were angry about Gregory VII's "intervention in their dioceses that they renounced their obedience to the Pope...and, furthermore induced King Henry to do the same" (Blumenthal, 1988, pp.118-119). In the presence of multiple heterogenous actors, the game

between a leader and fragmented rival group could potentially lead to different results depending on the circumstances.

More importantly, I have only used a two-period model in this paper, but historical cases (including the Investiture Controversy) seem to imply that a more dynamic relationship is more suitable. The following process seems to occur: (1) power is initially shared, (2) a somewhat stable short-term compliant equilibrium follows, (3) at some time in the future, the rival's preferences change, (4) the initial equilibrium is now unstable, (5) the leader and rival bargain, and (6) a new outcome (compliance or challenge) is now realized. Future research would need to incorporate this dynamic nature of the game in order to better represent these historical cases. However, for now, the game portrayed in this paper is sufficient to illustrate the important trade-offs involved.

I provided some suggestive evidence from the Investiture Controversy to support the model but, moving forward, it would be a reasonable next step to fit the propositions of this paper to the data more rigorously and use either a qualitative or quantitative methodology to verify or disprove the corresponding hypotheses. Although the model suffers from some limitations (as addressed above), the incorporation of the two simultaneous trade-offs (for the leader and the rival) is still a step in the right direction. Future formal research that looks into questions involving the relationship between a leader and a "popular" rival should consider both strategic tensions in order to better understand the calculus of political leaders when it comes to power-sharing.

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Appendix A - Proofs

First order condition under uncertainty

The leader solves the following maximization problem:

$$\underset{x}{\text{maximize}}\ G\left(\frac{F(\mu+(p-q)^2)-F(\mu)}{x-\Pi(1-F(\mu),x)}\right)*V_{Ch}+$$

$$\left[1-G\left(\frac{F(\mu+(p-q)^2)-F(\mu)}{x-\Pi(1-F(\mu),x)}\right)\right]*V_{Co}$$

Therefore, the first order condition implies:

$$g\left(\frac{F(\mu+(p-q)^2)-F(\mu)}{x^*-\Pi(1-F(\mu),x^*)}\right)\left[1-\frac{\partial\Pi(1-F(\mu),x^*)}{\partial x}\right][1-F(\mu+(p-q)^2)-x^*+\Pi(1-$$
$$F(\mu),x^*)]\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{(x^*-\Pi(1-F(\mu),x^*))^2}\right]+G\left(\frac{F(\mu+(p-q)^2)-F(\mu)}{x^*-\Pi(1-F(\mu),x^*)}\right)\left[1-\frac{\partial\Pi(1-F(\mu),x^*)}{\partial x}\right]-1=0$$

Standard deviation analysis

The above first order condition can be rewritten as:

$$\frac{1}{2\sigma\sqrt{3}}\left[1-\frac{\partial\Pi(1-F(\mu),x^*)}{\partial x}\right][1-F(\mu+(p-q)^2)-x^*+\Pi(1-$$
$$F(\mu),x^*)]\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{(x^*-\Pi(1-F(\mu),x^*))^2}\right]\ +$$
$$\frac{1}{2}\left[1-\frac{\partial\Pi(1-F(\mu),x^*)}{\partial x}\right]\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{\sigma\sqrt{3}(x^*-\Pi(1-F(\mu),x^*))}-\frac{E}{\sigma\sqrt{3}}+1\right]-1=0 \tag{11}$$

Let us characterize this first order condition by the equation $Z(x^*(\sigma),\sigma)=0$. We can use the implicit function theorem to derive $\frac{dx^*}{d\sigma}$. We know by the second order conditions of a maximum, that $\frac{\partial Z}{\partial x^*}$ is negative. The partial derivative $\frac{\partial Z}{\partial\sigma}$ is equal to:

$$-\frac{1}{2\sigma^2\sqrt{3}}[1-F(\mu+(p-q)^2)-x^*+\Pi(1-F(\mu),x^*)]\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{(x^*-\Pi(1-F(\mu),x^*))^2}\right]-$$
$$\frac{1}{2\sigma^2\sqrt{3}}\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{(x^*-\Pi(1-F(\mu),x^*))}-E\right]$$

Thus, $\frac{dx^*}{d\sigma}$ will be positive (negative) iff

$$[1-F(\mu+(p-q)^2)]\left[\frac{F(\mu+(p-q)^2)-F(\mu)}{(x^*-\Pi(1-F(\mu),x^*))^2}\right]<(>)E \tag{12}$$

Proposition II

Under the assumption that $[1-F(\mu+(p-q)^2)][F(\mu+(p-q)^2)-F(\mu)]>\rho E$ for some range of $0<\rho<(1-\Pi(1-F(\mu),1))^2$, then let us label $\ddot{x}$ as the amount of power sharing that makes 12 hold with equality.

i. If $[1-F(\mu+(p-q)^2)][F(\mu+(p-q)^2)-F(\mu)]>\rho E$ *for some range of* $0<\rho<(1-\Pi(1-F(\mu),1))^2$, *and* $\ddot{x}>1-F(\mu+(p-q)^2)$ *then* $\frac{dx^*}{d\sigma}<0$ *for all* $x^*\in(0,1-F(\mu+(p-q)^2))$

ii. If $[1-F(\mu+(p-q)^2)][F(\mu+(p-q)^2)-F(\mu)]>\rho E$ *for all* $0<\rho\leq(1-\Pi(1-F(\mu),1))^2$, *then* $\frac{dx^*}{d\sigma}<0$ *for all* $x^*\in(0,1-F(\mu+(p-q)^2))$

iii. If $[1 - F(\mu + (p-q)^2)][F(\mu + (p-q)^2) - F(\mu)] > \rho E$ *for some range of* $0 < \rho < (1 - \Pi(1 - F(\mu), 1))^2$, *and* $\ddot{x} < 1 - F(\mu + (p-q)^2)$ *then:*

a.) $\dfrac{dx^*}{d\sigma} < 0$ *for* $x^* \in (0, \ddot{x})$

b.) $\dfrac{dx^*}{d\sigma} = 0$ *for* $x^* = \ddot{x}$

c.) $\dfrac{dx^*}{d\sigma} > 0$ *for* $x^* \in (\ddot{x}, 1 - F(\mu + (p-q)^2))$

Proposition III

Similar to the standard deviation analysis above, the first order condition can be characterized by the equation $N(x^*(E), E) = 0$. We know by the second order conditions of a maximum, that $\dfrac{\partial N}{\partial x^*}$ is negative. The partial derivative $\dfrac{\partial N}{\partial E}$ is equal to:

$$-\frac{1}{2\sigma\sqrt{3}}\left[1 - \frac{\partial \Pi(1 - F(\mu), x^*)}{\partial x}\right]$$

This partial derivative will always be negative and that establishes Proposition III.

Proposition IV

The important inequality (equation 10) that determines whether the outside option will be used is the following:

$$\frac{-x^* + 1 - F(\mu + (p-q)^2)}{\Pi(1 - F(\mu), x^*) - x^* + 1 - F(\mu + (p-q)^2)} \leq G\left(\frac{F(\mu + (p-q)^2) - F(\mu)}{x^* - \Pi(1 - F(\mu), x^*)}\right)$$

Taking the derivative of both sides with respect to $F(\mu + (p-q)^2)$ indicates that the left hand side is decreasing and the right hand side is increasing. This implies that decreasing $1 - F(\mu + (p-q)^2)$ would have the same effect. This establishes the first part of Proposition IV. Taking the derivative of both sides with respect to $(p-q)^2$ indicates that the left hand side is decreasing, and the right hand side is increasing. This establishes the second part of Proposition IV. It is also worth noting that as both the left-hand side and right-hand side are decreasing in x^*, it is not immediately clear how raising or decreasing x^* would affect the leader's calculus without specifying functional forms.

Appendix B - Choosing two dimensions: policy and power-sharing

Leaders often have more than one instrument available that can be used to "reward" compliance. In this appendix section, we augment the model to allow the leader to choose

both the amount of power to be shared and the policy choice to be implemented. Let us assume that policy choice is on a one-dimensional space, the leader's ideal policy is p, and the rival and population have the same ideal policy of q. Without loss of generality, let us assume throughout that $p < q$. The leader now needs to also choose a policy to implement $(\tilde{p})$. Therefore, assuming utility from a policy is decreasing in its distance from one's ideal point, it must be the case that $\tilde{p} \in [p, q]$.

The leader incurs disutility by choosing a policy different from their ideal point, and this disutility is increasing in the distance between the implemented policy and their preferred one. Furthermore, in choosing on the two dimensions, the leader needs to take into account two constraints. First, the leader would need to make sure that the rival has an incentive to comply (call this the compliance-constraint). Second, the leader would need to make sure that actually entering into a relationship with the rival is better than remaining independent and implementing their preferred policy p (call this the incentive-constraint). This basically means that the leader will only enter into a relationship with the rival when the number of supporters gained from doing so is greater than the disutility from sharing power and adjusting the policy away from their ideal policy p. This decision-making challenge is represented by the following maximization problem:

$$\underset{x,\tilde{p}}{\text{maximize}} \;\; F(\mu) + (1 - x) + (1 - F(\mu + (\tilde{p} - q)^2)) - (\tilde{p} - p)^2$$

$$\text{subject to } 1 - F(\mu + (\tilde{p} - q)^2) + x \geq 1 - F(\mu) + \Pi(1 - F(\mu), x)$$
$$1 - F(\mu + (\tilde{p} - q)^2) \geq x + (\tilde{p} - p)^2$$

The compliance-constraint can be rewritten as $F(\mu) \geq F(\mu+(\tilde{p}-q)^2)-x+\Pi(1-F(\mu), x)$, and the incentive-constraint can be rewritten as $1 \geq F(\mu + (\tilde{p} - q)^2) + x + (\tilde{p} - p)^2$.
Using the Lagrangian, the first order conditions imply:
(wrt x) $-1 + \lambda_1(1 - \frac{\partial \Pi}{\partial x}) + \lambda_2(-1) = 0$
(wrt $\tilde{p}$) $-f(\mu + (\tilde{p} - q)^2) \cdot 2(\tilde{p} - q) - 2(\tilde{p} - p) + \lambda_1(-f(\mu + (\tilde{p} - q)^2) \cdot 2(\tilde{p} - q)) + \lambda_2(-f(\mu + (\tilde{p} - q)^2) \cdot 2(\tilde{p} - q) - 2(\tilde{p} - p)) = 0$
Clearly, from the first order conditions, $\lambda_1 \neq 0$ and therefore the compliance-constraint binds. This is rather intuitive because if the constraint did not bind, then the leader could reduce the amount of power shared and still satisfy the constraint. Furthermore, if the incentive-constraint was satisfied, then it would still be satisfied if there was a reduction in the amount of power shared. On the other hand, λ_2 could possibly equal 0 and the incentive-constraint would not bind. Therefore, the compliance-constraint always binds, and the leader either earns some "rent" from sharing power or earns no rent and is indifferent between sharing power and remaining independent. Let x^* and $\tilde{p}^*$ be the optimal values that solve the maximization problem. Thus, there are two equilibriums to consider:

1. If $1 > F(\mu + (\tilde{p}^* - q)^2) + x^* + (\tilde{p}^* - p)^2$:
   $\lambda_1 > 0, \; \lambda_2 = 0$
   The compliance-constraint binds, but the incentive-constraint does not bind. Therefore x^* and $\tilde{p}^*$ solve the following equation:
   $-F(\mu) + F(\mu + (\tilde{p}^* - q)^2) - x^* + \Pi(1 - F(\mu), x^*) = 0$

2. If $1 = F(\mu + (\tilde{p}^* - q)^2) + x^* + (\tilde{p}^* - p)^2$:
   $\lambda_1 > 0,\ \lambda_2 > 0$
   Both the compliance-constraint and the incentive-constraint bind. Therefore, by combining both constraints, the leader will choose x^* and $\tilde{p}^*$ to solve the following equation:
   $F(\mu) + 2x^* - \Pi(1 - F(\mu), x^*) + (\tilde{p}^* - p)^2 - 1 = 0$

Comparative statics - the trade-off between power and policy

I will now analyze how the discontinuity cut-off point varies as the policy choice varies. Let us look at the first equilibrium where only the compliance-constraint binds. Given a chosen $\tilde{p}$, let $x^*(\tilde{p})$ be the optimal amount of power-sharing that satisfies the compliance-constraint. Using the implicit function theorem, we obtain:

$$\frac{dx^*}{d\tilde{p}^*} = -\frac{f(\mu + (\tilde{p}^* - q)^2)2(\tilde{p}^* - q)}{\frac{d\Pi}{dx} - 1}. \tag{13}$$

The above equation is negative (as we assumed $p < q$ and $\frac{d\Pi}{dx} < 1$).

In the second equilibrium, both the compliance-constraint and the incentive-constraint bind. By combining these constraints into one equation (as done above), and given a chosen $\tilde{p}$, let $x^*(\tilde{p})$ be the optimal amount of power-sharing that satisfies the combination of these constraints. The implicit function theorem reveals that:

$$\frac{dx^*}{d\tilde{p}^*} = -\frac{2(\tilde{p}^* - p)}{2 - \frac{d\Pi}{dx}}. \tag{14}$$

The above equation is negative (as we assumed $p < q$ and $\frac{d\Pi}{dx} < 1$). Therefore, $dx^*/d\tilde{p}^*$ is negative for both equilibriums and this leads to the next proposition.

Proposition V
In satisfying the compliance-constraint, if the leader reduces (increases) the power shared, the leader will then move the policy closer to (further from) the ideal policy of the rival.

Therefore, less power is shared when the leader chooses a policy closer to the rival's preferred policy. The amount that the shared power is reduced depends on the distance between the $\tilde{p}$ and the leader's preferred policy, and the marginal productivity of power for the rival if they chooses to challenge. Intuitively, the larger the distance between $\tilde{p}$ and p, the larger the reduction in the discontinuity cut-off point (i.e. the larger the reduction in the power-shared).

The trade-off is fairly straight forward. The leader chooses either to move the policy closer to the rival's ideal point and shares less power, or the leader chooses to move the policy closer to their own ideal point and gives more power to the rival. In either case, the

main point is that the rival needs to be compensated in some manner for their credibility cost.