Compositional variation of density and seismic velocities in natural peridotites at STP conditions: Implications for seismic imaging of compositional heterogeneities in the upper mantle

Cin-Ty Aeolus Lee
Department of Earth Science, Rice University, Houston, Texas, USA

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[1] Densities and elastic properties of solid natural spinel- and garnet-peridotite samples \( (n = 133) \) at standard temperature \( (T) \) and pressure \( (P) \) (STP) conditions were calculated for compositions ranging from Mg\# (100 \( \times \) Mg/(Mg + Fe)) of 86–94. The physical properties were used to investigate how natural compositional variations control density and seismic velocity. A corresponding set of compositional derivatives \( (d/d\text{Mg\#}) \) of density and seismic velocity is provided. Because the \( P \) and \( T \) derivatives of elastic moduli are very similar for different compositional end-members of peridotitic minerals, the variation of elastic moduli with Mg\# at STP conditions holds at elevated \( P \) and \( T \). Increased Mg\# leads to a significant increase in \( V_S \) because of the sensitivity of mineral shear moduli to this parameter \( (dV_S/d\text{Mg\#} = 0.0143 \pm 0.0009 \text{ km s}^{-1}) \). In contrast, the compressional wave velocity \( (V_P) \) is insensitive to Mg\# and, instead, correlates weakly with increasing olivine abundance at STP conditions. The ratio \( V_P/V_S \) therefore exhibits a significant negative correlation with Mg\# \( (d(V_P/V_S)/d\text{Mg\#} = -0.00407 \pm 0.00038) \).

Because the temperature dependency of \( V_P/V_S \) is small \( (\sim 0.04\%/100^\circ \text{C}) \) compared to the compositional dependency \( (1.7\%/\text{Mg\# \text{ unit}}) \), the variation in \( V_P/V_S \) is a fairly robust measure of compositional variation even when temperature varies. Finally, a new density versus Mg\# parameterization is derived. Combined with a compilation of bulk-rock Mg\#'s of peridotite xenoliths from cratonic lithospheric mantle, it is shown that the intrinsic density of cratic mantle balances to within error its negative thermal buoyancy imposed by its cooler thermal state relative to upwelling asthenospheric mantle.

INDEX TERMS: 1025 Geochemistry: Composition of the mantle; 3909 Mineral Physics: Elasticity and anelasticity; 7207 Seismology: Core and mantle; 8120 Tectonophysics: Dynamics of lithosphere and mantle—general;
KEYWORDS: xenolith, peridotite, seismic velocity, density, lithosphere, tectosphere


1. Introduction

[2] There has been a recent increase in the number of high-resolution seismic tomographic studies of the uppermost mantle with the intention of characterizing small variations in the thermal and compositional structure of the deep lithosphere. The ability to infer bulk composition variations in the uppermost mantle from seismic observations could have important implications for understanding mantle geodynamics and the petrogenesis of various upper mantle reservoirs. For example, not only is it important to delineate the physical differences between continental lithospheric mantle, oceanic lithospheric mantle, and asthenospheric mantle, much insight on the physics and chemistry of continent formation can be gained if subtle compositional variations within the continental lithospheric mantle can be seismically resolved. There is thus a growing need for assessing the degree to which seismic velocities vary as a function of temperature and composition in natural upper mantle samples.

[3] Here the effects of compositional variation on density and isotropic seismic velocities in upper mantle peridotites are investigated by directly studying a group of natural peridotites \( (n = 133) \), spanning the primary compositional range seen in nature \((100 \times \text{Mg}/(\text{Mg + Fe}) = \text{Mg\#} \sim 86–88)\) to highly residual compositions \((\text{Mg\#} \sim 93–94))\). As a first step toward inverting seismic velocity anomalies for compositional variations, compositional derivatives \( (d/d\text{Mg\#}) \) between calculated standard state (STP) seismic velocities and various bulk compositional indices are presented here. As will be discussed, the pressure and temperature derivatives of the compositional end-members of peridotitic minerals are very similar, and therefore the compositional derivatives of elastic moduli with respect to Mg\# at STP conditions are applicable to elevated pressure and temperature. Seismic velocities are calculated using knowledge of mineral compositions, mineral modes, and experimentally...
determined elastic moduli of mineral end-members. The rationalization for confining the velocity calculations to
STP conditions rather than in situ pressures and temperatures
is that the pressure and temperature derivatives of the elastic
moduli for all mineral end-members are not known. How-
ever, the STP compositional derivatives of seismic velocities
are not likely to depend significantly on pressure and
temperature, and therefore the results presented here are
applicable to interpreting relative variations in seismic ve-
clocity at upper mantle conditions.

of natural peridotites differs fundamentally from studies in
which seismic velocities are calculated using theoretical
mineral assemblages in the mantle as determined from
thermodynamic considerations [Jordan, 1979; Kelly et al.,
2003]. In the latter approach, extrapolations of thermody-
namic parameters to high pressure and temperature may
cause systematic biases in calculated mineral modes. In
addition, the thermodynamic approach often requires some
simplification of the compositional space because the ther-
modynamic parameters in more complicated systems have
not yet been fully worked out. Finally, the thermodynamic
approach assumes that the mineral mode assemblage in the
modeled system is in equilibrium (modal equilibrium),
which may or may not be the case, depending on the
prescribed lengthscale of the system. The approach of using
natural peridotites is advantageous in that there is no a priori
assumption of modal equilibrium and there is no need to
rely on any thermodynamic assumptions or simplifications
in calculating mineral modes. In addition, the use of natural
peridotites allows for the assessment of the true natural
variation in peridotite compositions in so far as the sampling
of peridotites (as xenoliths or tectonically obducted massifs)
is representative of the upper mantle. The drawback of
using natural peridotite is the possibility of internal mineral
heterogeneity as opposed to modal disequilibrium) due to
local disequilibrium. Such heterogeneity can give rise to
a difficult-to-quantify error in the calculation of mineral
modes and mineral elastic moduli.

[5] It is important to recognize that other lithologies occur
in the upper mantle, e.g., eclogites, Fe-rich ultramafics (e.g.,
olivine cumulates and/or pyroxenites with Mg# < 86), and
recycled sediment. Although these lithologies are of inter-
est, they are significantly less abundant than peridotites with
Mg#s > 86. As a first step toward understanding composi-
tional variation in the mantle, this manuscript focuses solely
on the abundant peridotite lithologies. The effects of a melt
phase or dissolved hydrogen in silicate phases on elastic
moduli are not discussed here. The effects of anelasticity
(seismic attenuation) and anisotropy are not considered
here.

2. Review of the Composition of Upper Mantle
Peridotites and Minerals

[6] A number of detailed reviews on the compositional
variations of upper mantle peridotites and their constituent
minerals can be found in the literature [Nixon et al., 1981;
Boyd, 1987, 1989; McDonough and Frey, 1989; Menzies,
1989; McDonough, 1990; Menzies et al., 1991; McDonough
and Sun, 1995; Griffin et al., 1999; Poudjom et al., 2001].
However, a brief review summarizing the main composi-
tional features of peridotites and their minerals “in the
context” of seismic velocity calculations is necessary. Seis-
umic velocities of upper mantle peridotites have been esti-
imated previously without accounting for all compositional
effects. This review is intended to show the entire composi-
tional variation in peridotite mineralogies so that more
accurate densities and elastic moduli can be calculated.

2.1. Mineralogic and Major-Element Variations

[7] Peridotites are olivine-rich rocks containing variable
amounts of orthopyroxene, clinopyroxene, and an alumi-
nous phase (garnet or spinel). A number of accessory
phases, such as phlogopite, amphibole, rutile, perovskite,
ilmenite, sulfides, carbonates, melilitic, and/or zircon can
also be found in peridotites. However, since these accessory
phases rarely amount to more than a few percent of the bulk
rock, their contribution to seismic velocities and densities
are ignored here.

[8] The presence of spinel or garnet in a peridotite depends
primarily on pressure and bulk composition. For fertile
peridotites (e.g., those that have experienced very little melt
extraction), the spinel-garnet transition occurs at ~1.5 GPa
(or ~45 km [O’Neill, 1981]). Using the fertile peridotite
composition as a reference, the following metamorphic
facies terminology is defined: peridotites derived from
pressures <1.5 GPa are referred to as “spinel-facies perido-
tites” because spinel is the stable aluminous phase, while
those derived from >1.5 GPa are referred to as “garnet-facies
peridotites” because garnet is the stable aluminous phase. In
refractory peridotites (e.g., those that have experienced
significant melt extraction), the increased Cr content of
residual spinels depresses the spinel-garnet transition so
that the first appearance of garnet is suppressed to pressures as
high as ~3 GPa (or ~90 km; [O’Neill, 1981; Girnis and
Brey, 1999]), depending on the Cr/(Cr + Al) of the spinel.
These refractory peridotites, which contain Cr-rich spinel
instead of garnet, but are otherwise derived from pressures
greater than 1.5 GPa, are referred to here as “garnet-facies
spinel-peridotites” [cf. Lee and Rudnick, 1999].

[9] Figure 1a shows the observed range in peridotite
mineralogy in olivine-orthopyroxene-clinopyroxene wt %
space (excluding cumulates). Yellow squares represent
spinel-peridotites collected from Phanerozoic continental
lithospheric mantle and oceanic mantle. The compositional
array represented by such peridotites is generally taken to
reflect progressive extraction of partial melt [Boyd, 1989].
Figure 1b shows how Mg# and olivine mode of Phanero-
zoi peridotites varies as melt is progressively extracted
(arrow). Primitive peridotites, i.e., those that have experi-
cenced minor amounts of melt extraction, are characterized
by Mg#s of ~86–89 and significant amounts of clinopy-
roxene and orthopyroxene in addition to olivine; such
peridotites are classified as lherzolites. With increasing
degree of melting, clinopyroxene and orthopyroxene modes
decline and Mg# increases in the residual peridotite; perido-
tites with zero to <5 wt % clinopyroxene are classified
as harzburgites. When enough melt has been exhausted
(e.g., >20 wt %) to exhaust not only clinopyroxene but also
orthopyroxene (<10 wt %), an olivine-rich residue (dunite)
is generated. Tables 1a and 1b shows additional composi-
tional and mineralogic information for a small selection of
natural spinel-peridotites and garnet-peridotites.
In summary, progressive melt extraction results in an increase in Mg# and a decrease in orthopyroxene/olivine ratio [cf. Boyd, 1989]. However, in some instances, such as for some Archean peridotite xenoliths from the South African craton, there appears to be a number of peridotites that have excess orthopyroxene for a given Mg# [Boyd, 1989], i.e., they fall to the right of the melt-depletion arrow in Figure 1c. Although it is generally agreed that this excess orthopyroxene is a result of additional petrogenetic processes superimposed on earlier melt extraction events [Boyd, 1989], the responsible petrogenetic process is still hotly debated [Rudnick et al., 1994; Kelemen et al., 1998; Herzberg, 1999]. This paper concerns only the determination of seismic velocities and densities of natural peridotites, so although the petrogenesis of these rocks is irrelevant for the purposes of this manuscript, all peridotite samples regardless of petrogenetic origin are fully incorporated into the database in order to describe the entire variation in upper mantle peridotite.

2.2. Variations in Mineral Composition

Figures 2–4 show the range in compositions typically displayed in the major minerals in peridotites ranging from Mg# ~ 86–94. Figures 2a and 2b show the compositional range displayed by clinopyroxenes and orthopyroxenes in terms of Mg, Fe, and Ca. Figures 3a and 3b show variations in the amount of Cr2O3 and Al2O3 in pyroxenes; garnet-facies pyroxenes typically show lower Al contents because most of the Al in the bulk rock is partitioned into garnet. It will be shown later that Al has a small but significant effect on the elastic moduli of pyroxenes. Figures 2c and 3c show the compositional variation of garnets; peridotitic garnets are invariably pyrope-rich, with small but variable amounts of grossular and almandine components. Figure 4 shows the compositional variation of spinels in spinel-facies and garnet-facies spinel-peridotites (see above for definition of garnet-facies spinel peridotite). Spinel-facies peridotites consist predominantly of aluminous spinels, whereas the spinels in garnet-facies spinel-peridotites are typically Cr-rich [cf. Lee and Rudnick, 1999].

3. Peridotite Data Set

The peridotite database (n = 133) used here for calculating elastic moduli consists of data from the literature, in which bulk-rock and mineral composition data are both reported. The data set comprises obducted massif peridotites and mantle xenoliths erupted through continental lithosphere, representing Archean to Phanerozoic continental lithospheric mantle. Peridotites from Archean lithospheres include South African, Tanzanian, and Siberian cratonic peridotites [Boyd, 1987; Boyd and Mertzman, 1987; Boyd et al., 1993; Lee and Rudnick, 1999]. Peridotites from Proterozoic regions include peridotites from southwestern USA (Colorado Plateau) and ultramafic massifs in western Europe [Ehrenberg, 1982; Frey et al., 1985; Wilshire et al., 1988, 1991]. Those from Phanerozoic regions include southwestern USA and North American Cordilleran xenolith suites [Francis, 1987; Shi et al., 1998].

4. Methods

4.1. Mineral Proportions

Mineral mass proportions were determined by inversion of bulk composition data (XRF) and mineral composition data (electron microprobe). The following oxides
Table 1a. Peridotite Examples: Natural Spinel-Peridotites and Natural Garnet-Peridotites

<table>
<thead>
<tr>
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<td>51.94</td>
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<td><strong>Al₂O₃</strong></td>
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<td>6.10</td>
<td>0.03</td>
<td>57.27</td>
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<tr>
<td><strong>Cr₂O₃</strong></td>
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<tr>
<td><strong>FeO</strong></td>
<td>6.25</td>
<td>2.84</td>
<td>9.93</td>
<td>11.59</td>
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<td>0.36</td>
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<td>89.29</td>
<td>84.51</td>
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<tr>
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<tr>
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**Table 1b. Peridotite Examples: Primitive Models**

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or quenched glasses. These accessory phases typically amount to less than 0.5–1% of the total mass of the rock, and even if present, could have been introduced during late-stage metasomatism or during entrainment in the host magma. Samples were initially omitted if there were large-scale metasomatic effects or heterogeneous mineral compositions described in the literature. Otherwise, samples with residuals (observed bulk composition versus calculated bulk composition) greater than 0.7% were omitted on the assumption that mineral heterogeneity or accessory metasomatic phases were present.

This inverse approach is identical to that taken by Boyd and McAllister [1976]. However, Boyd and McAllister included Cr$_2$O$_3$ as an additional constraint. Cr$_2$O$_3$ data were not available for all samples in the compilation, so it was not used in the mode calculations. In the absence of a comprehensive mineral composition data set, mineral modes can be estimated by calculating the theoretical equilibrium mineral assemblage of a given bulk composition for fixed pressure and temperature using thermodynamic considerations. The first attempt to do such calculations made use of simple mass balance and empirical mass action relationships between the major-element compositions of each mineral phase [Jordan, 1979]. Thermodynamic calculations should in theory yield the “equilibrium” mineral assemblage and mineral compositions if the thermodynamic model is accurate. In many cases, assumptions and simplifications must be used to extrapolate calculations to pressures, temperatures, and bulk compositions of interest. Some of the assumptions made by Jordan include constant equilibrium coefficients (e.g., independent of temperature and pressure) and simplified mineral compositions. Orthopyroxene was assumed to contain no Al$_2$O$_3$, but the Al$_2$O$_3$ content of orthopyroxenes in ultramafic xenoliths is not negligible as up to a few wt % Al$_2$O$_3$ can be present in orthopyroxene depending on temperature and pressure.

The inverse approach taken here gives the most accurate picture of the true mineral assemblage in a peridotite sample as there are no assumptions other than the assumption of mineral homogeneity (see section 4.4). For the purposes of this study, it is assumed that the measured mineral assemblage of a peridotite xenolith or a massif peridotite reflects the state they last resided in the mantle, i.e., there has been no change in mineral assemblage/

Figure 2. (a) Pyroxene ternary diagram in garnet-facies peridotites expressed in mole fraction of respective cation end-member in pyroxene. (b) Same as Figure 2a but for pyroxenes in spinel-facies peridotites. (c) Garnet ternary diagram expressed in mole fraction of respective cation end-member in garnet.
4.2. Density Calculations

[17] The following steps were followed for calculating bulk rock densities. Mineral densities were calculated using mixing models to describe compositional solid solutions as a function of end-member molar volumes (cm\(^3\) mole\(^{-1}\)). The mixing models for olivine, orthopyroxene, garnet, and spinel were taken exactly from the work of Brey et al. [1999] and that of clinopyroxene was constructed analogous to the model of Brey et al. For olivine and pyroxene, linear additivity of molar volumes was assumed. For spinel and garnet, nonideal mixing effects were accounted for as described below. Nonidealities included the effects of inter-site reciprocal reactions and intrasite interactions. Mineral densities were then calculated by dividing the formula weight of the mineral by its molar volume. The density of the bulk rock was then calculated by linear additivity of the densities according to the mass fraction of each mineral. The mass fraction instead of the volume fraction was used because mass fractions are more efficiently determined than volume fractions; the former can be determined by inversion of bulk compositional and mineral data, whereas the latter requires tedious point-counting techniques. The errors introduced by using mass fraction instead of volume fraction are negligible. Molar volumes of mineral end-member components are given in Table 2 (nonideal mixing terms are given in Table 3).

4.2.1. Olivine

[18] Olivine stoichiometry follows (Mg,Fe)\(_2\)SiO\(_4\), hence it is described using the following components: forsterite (Mg\(_2\)SiO\(_4\)) and fayalite (Fe\(_2\)SiO\(_4\)). Mixing between the two end-members was assumed to be ideal, which yields \(V_{\text{Ol}} = V_{\text{Fo}} X_{\text{Mg}} + V_{\text{Fa}} (1 - X_{\text{Mg}})\) where \(X_{\text{Mg}} = \text{Mg}/(\text{Mg} + \text{Fe})\), and Mg and Fe represent the atomic proportions on a 4-oxygen basis.

4.2.2. Spinel

[19] The chemistry of spinels in ultramafic rocks can be expressed as (Mg,Fe)(Cr,Al)\(_2\)O\(_4\). The following end-member components are used to describe spinel: chromite (FeCr\(_2\)O\(_4\)), spinel (MgAl\(_2\)O\(_4\)), and picrochromite (MgCr\(_2\)O\(_4\)). It is assumed that Cr occurs only in the \(+3\) valence state, so it occurs only in the tetrahedral site. The mixing model of Brey et al.

Figure 3. (a) Cr\(_2\)O\(_3\) versus Al\(_2\)O\(_3\) in clinopyroxenes (expressed in wt %). (b) Cr\(_2\)O\(_3\) versus Al\(_2\)O\(_3\) in orthopyroxenes. (c) Cr\(_2\)O\(_3\) versus CaO in garnet.

Figure 4. Molar Mg/(Mg + Fe) versus molar Cr/(Cr + Al) in spinels from spinel-facies and garnet-facies peridotites.
Table 2. Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Molar Volume ((V^0)_{\text{m}}) cm(^3) mole(^{-1})</th>
<th>Bulk Modulus(^a) K(\text{GPa})</th>
<th>Shear Modulus(^a) K(\text{GPa})</th>
<th>Thermal Expansivity(^a) K(\text{K}^{-1})</th>
<th>(\alpha) (\text{K}^{-1}) x 10(^{-5})</th>
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<td></td>
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<td>3.93</td>
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<tr>
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<tr>
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<td></td>
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</tr>
<tr>
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<td>Mg(\text{Si}_2\text{O}_4)</td>
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<td>Fa 46.35</td>
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<td>11.5 -0.0518</td>
</tr>
</tbody>
</table>

\(^a\)Molar volumes are taken from the work of Brey et al. [1999].

\(^b\)Bulk moduli represent averages of compiled data shown in Table 4.

\(^c\)Shear moduli represent averages of compiled data shown in Table 4.

\(^d\)Thermal expansion data are from the work of Fei [1995].

\(^e\)Elastic moduli of Mg-Tschermak pyroxene end-member was determined by unmixing the elastic moduli \((K_e = 115.5 \text{ GPa}, G = 78.1 \text{ GPa})\) of a natural aluminous orthopyroxene \((\text{En}_{0.735}\text{Fs}_{0.086}\text{Di}_{0.041}\text{Mg-Ts}_{0.122})\) from the work of Chai et al. [1997] (see Table 4) using the end-member elastic moduli of Diopside, Enstatite, and Ferrosilite in this table (see text for details).

\(^f\)Values of a bronzite \((\text{Mg}_2\text{Fe}_3\text{Si}_3\text{O}_8)\) are substituted here due to lack of data on mineral end-members (see Table 4). Mg-Ts (Magnesium-Tschermak pyroxene), Mg-Cr-Ts (Magnesium-Chromium-Tschermak pyroxene), Fs (Ferrosilite), En (Enstatite), Di (Diopside), Py (Pyrope), Gr (Grossular), Kn (Knorringite), Alm (Almandine), Uv (Uvarovite), Chr (Chromite), Sp (Spinel), PChr (Picrochromite), Hercy (Hercynite), Fo (Forsterite), Fa (Fayalite).

The last two terms are Margules parameterizations, which account for the nonideality of intrasite interactions.

4.2.3. Garnet

Gemets typical of ultramafic rocks are assumed to have the following chemistry and stoichiometry: \((\text{Ca},\text{Mg},\text{Fe})_2\text{Cr}_2\text{Si}_3\text{O}_{12}\). It is assumed that Cr occurs only in the Al site. The garnet compositions can thus be described by the following components: pyrope \((\text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12})\), grossular \((\text{Ca}_3\text{Al}_2\text{Si}_3\text{O}_{12})\), knorringite \((\text{Mg}_3\text{Cr}_2\text{Si}_3\text{O}_{12})\), and almandine \((\text{Fe}_3\text{Al}_2\text{Si}_3\text{O}_{12})\). The mixing model and molar volume parameters of Brey et al. [1999] were adopted exactly:

\[ V^\text{Gr} = (X_{\text{Mg}} - X_{\text{Cr}}) V_{\text{Py}} + X_{\text{Ca}} V_{\text{Gr}} + X_{\text{Cr}} V_{\text{Kn}} \]

\[ + \left( X_{\text{Mg}} + X_{\text{Cr}} - 1 \right) V_{\text{rch}} + \left( X_{\text{Mg}} - X_{\text{Cr}} \right) \Delta V_I \]

\[ + X_{\text{Mg}} \left( 1 - X_{\text{Mg}} \right) \left[ W^\text{Gr}_{\text{MgFe}} \left( 1 - X_{\text{Mg}} \right) + W^\text{Gr}_{\text{FeMg}} X_{\text{Mg}} \right] \]

\[ + 2 X_{\text{Cr}} \left( 1 - X_{\text{Cr}} \right) \left[ W^\text{Gr}_{\text{CAI}} \left( 1 - X_{\text{Cr}} \right) + W^\text{Gr}_{\text{AC} 6} X_{\text{Cr}} \right], \]

where \(X_{\text{Cr}} = \text{Cr}/(\text{Cr} + \text{Al}), X_{\text{Mg}} = \text{Mg}/(\text{Mg} + \text{Fe})\), and \(\text{Cr}, \text{Al}, \text{Mg}, \text{Fe}\) represent atomic proportions on a 4-oxygen basis. The fourth term represents the reciprocal isosteric mixing term, where \(\Delta V_I\) is the volume change of the reciprocal reaction

\[ (\text{R1}) \quad \text{MgAl}_2\text{O}_4 + \text{FeCr}_2\text{O}_4 = \text{FeAl}_2\text{O}_4 + \text{MgCr}_2\text{O}_4. \]

The last two terms are Margules parameterizations, which account for the nonideality of intrasite interactions.

Table 3. Reciprocal Mixing Terms\(^a\)

<table>
<thead>
<tr>
<th>Molar volume, cm(^3) mole(^{-1})</th>
<th>Spinel</th>
<th>Garnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W^V_{\text{FeMg}})</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>(W^V_{\text{AC} 6})</td>
<td></td>
<td>-0.2</td>
</tr>
<tr>
<td>(W^V_{\text{CAI}})</td>
<td></td>
<td>0.18</td>
</tr>
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</table>

\(^a\)Data are from the work of Brey et al. [1999].
\[ \Delta V_{\text{III}} \] represent the volume changes of the following reactions:

\[
\begin{align*}
\text{Fe}_3\text{Al}_2\text{Si}_3\text{O}_{12} + \text{Mg}_3\text{Cr}_2\text{Si}_3\text{O}_{12} & = \text{Fe}_3\text{Cr}_2\text{Si}_3\text{O}_{12} + \text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12}, \\
\text{Ca}_3\text{Al}_2\text{Si}_3\text{O}_{12} + \text{Mg}_3\text{Cr}_2\text{Si}_3\text{O}_{12} & = \text{Ca}_3\text{Cr}_2\text{Si}_3\text{O}_{12} + \text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12}.
\end{align*}
\]

(R2)

(R3)

The last two terms account for intrasite nonidealities (Table 3).

### 4.2.4. Pyroxenes

[21] Pyroxenes in ultramafic rocks were assumed to have the following composition and stoichiometry: \((\text{Ca}, \text{Mg}, \text{Fe})^2\text{M}_2^2(\text{Mg, Fe, Al, Cr})^4\text{M}_1^4(\text{Si, Al})_2\text{O}_6\). Ca was assumed to occur only in the larger M2 site, Al was assumed to occur both in the octahedral M1 and tetrahedral Si site, and Cr was assumed to occur only in the M1 site. Pyroxene composition can thus be described using the following end-member components: enstatite \((\text{MgMgSi}_3\text{O}_6)\), ferroactinolite \((\text{FeFeSi}_3\text{O}_6)\), Mg-Tschermack \((\text{MgAl}_2\text{Si}_3\text{O}_6)\), Cr-Mg-Tschermack \((\text{MgCrSi}_3\text{O}_6)\), and diopside \((\text{CaMgSi}_2\text{O}_6)\). The mixing model of Brey et al. [1999] for orthopyroxene was modified to describe clinopyroxene as well, yielding

\[
M^P_x = (X_{Mg}^M - X_{Al}^{M1} - X_{Cr}^{M1})V_{En} + \left(1 - X_{Mg}^M + X_{Ca}^M\right)V_{Ps},
\]

\[
+ X_{Al}^{M1}V_{Mg-I} + X_{Cr}^{M1}V_{Cr-I} + X_{Ca}V_{Dn},
\]

\[
+ X_{Al}^{M1}\left(1 - X_{Mg}^M - X_{Ca}^M\right)\Delta V_V, \text{ and } X_{Cr}^{M1}\left(1 - X_{Mg}^M - X_{Ca}^M\right)\Delta V_V + X_{Ca}^{M1}\Delta V_{VI} + V_{\text{ex}},
\]

(4)

where \(X_{Al}^{M1} = (\text{Al} - \text{Cr})/2, X_{Cr}^C = \text{Cr}, \text{ and } X_{Ca}^{M1} = \frac{\text{Ca}}{\text{Mg + Fe + Ca + } X_{Al}^{M1} + \text{Cr} + 1}, \)

\[
X_{Mg}^M = \frac{\text{Mg}}{\text{Mg + Fe}} \left(1 + \frac{X_{Mg}^M}{\text{Mg + Fe} - 1 + X_{Al}^{M1} + \text{Cr}}\right)^{-1},
\]

and \(\Delta V_{Ps}, \Delta V_{V}, \text{ and } \Delta V_{VI}\) represent the volume change for the following respective reactions:

\[
\text{MgAl}_2\text{Si}_3\text{O}_6 + 0.5 \text{FeFeSi}_2\text{O}_6 = \text{FeAl}_2\text{Si}_3\text{O}_6 + 0.5 \text{MgMgSi}_2\text{O}_6,
\]

(R4)

\[
\text{MgCr}_2\text{Si}_3\text{O}_6 + 0.5 \text{FeFeSi}_2\text{O}_6 = \text{FeCr}_2\text{Si}_3\text{O}_6 + 0.5 \text{MgMgSi}_2\text{O}_6,
\]

(R5)

\[
\text{MgAl}_2\text{Si}_3\text{O}_6 + \text{CaMgSi}_2\text{O}_6 = \text{CaAl}_2\text{Si}_3\text{O}_6 + \text{MgMgSi}_2\text{O}_6.
\]

(R6)

Brey et al. showed that the nonideal mixing terms for orthopyroxene are very close to zero. Mixing of end-member components was thus assumed to be ideal for clinopyroxene as well. The nonideal mixing terms, however, are shown here for completeness in case there is a need in the future to update the calculations.

### 4.3. Elastic Parameters and Seismic Velocities

[22] The isotropic adiabatic bulk moduli \((K_S)\) and the shear moduli \((G)\) of peridotites at STP conditions were calculated as follows. Elastic moduli of mineral end-members were taken from compilations by Bass [1995] and Isaak et al. [1993] and from additional literature sources; each of the data points used (and their respective literature sources) is shown in Table 4. The average values for each mineral end-member are tabulated in Table 2, which are the values used here in calculating elastic moduli of impure minerals. The \(K_S\) and \(G\) for each mineral were calculated by assuming linear additivity of the elastic parameters of the end-member mineral components, e.g., \(M = \sum M_i X_i\), where \(M\) is the elastic modulus of the mineral solid solution, \(M_i\) is the elastic modulus of the mineral end-member component, and \(X_i\) is the mole fraction of the mineral end-member component. Olivine solid solution was described using only the forsterite and fayalite components. Garnet was constructed from the following components: Pyrope \((\text{Mg}_3\text{Al}_2\text{Si}_3\text{O}_{12})\), Grossular \((\text{Ca}_3\text{Al}_2\text{Si}_3\text{O}_{12})\), Uvarovite \((\text{Ca}_3\text{Cr}_2\text{Si}_3\text{O}_{12})\), and Almandine \((\text{Fe}_3\text{Al}_2\text{Si}_3\text{O}_{12})\). Spinel was constructed from Chromite \((\text{FeCr}_2\text{O}_4)\), Spinell \((\text{MgAl}_2\text{O}_4)\), and Hercynite \((\text{FeAl}_2\text{O}_4)\) components.

[23] Pyroxenes were constructed with Enstatite \((\text{Mg}_3\text{Si}_2\text{O}_6)\), Ferroactinolite \((\text{Fe}_2\text{Si}_3\text{O}_6)\), Diopside \((\text{CaMgSi}_2\text{O}_6)\), and Mg-Tschermack \((\text{MgAl}_2\text{Si}_3\text{O}_6)\) components. There were no elastic data available for pure Mg-Tschermack or Cr-Tschermack end-members. The elastic moduli of the Mg-Tschermack end-member was inferred from experimentally determined elastic moduli of a natural aluminous orthopyroxene [Chai et al., 1997]. For a natural pyroxene having the composition \(\text{En}_{0.73}\text{Fs}_{0.08}\text{Di}_{0.04}\text{Mg}_{-0.12}\) \((\text{Al}_2\text{O}_3 \approx 4 \text{ wt }%, \text{CaO} \approx 1 \text{ wt }%)\), Chai et al. found that \(K_S = 118.5\) and \(78.1\) GPa, respectively. The elastic moduli of the Mg-Tschermack end-member was extracted by “unmixing” the natural orthopyroxene of Chai et al. using the first five terms in the mixing model of equation (4) and the elastic moduli of Diopside, Enstatite, and Ferroactinolite components shown in Table 2. A \(K_S\) of 181 GPa and a \(G\) of 114 GPa for the pure Mg-Tschermack end-member were obtained (compare with \(K_S = 107\) GPa and \(G = 75.9\) GPa for pure Enstatite; Table 2). These values are considerably higher than those of Tschermak-free orthopyroxenes (see Table 2), indicating that Al substitution in pyroxenes may have a significant effect on elastic moduli. Garnet-facies peridotites, the \(\text{Al}_2\text{O}_3\) content in clinopyroxenes and orthopyroxenes is small (<2 wt %; <4% Mg-Ts), and therefore there is only a small effect on the elastic moduli caused by the incorporation of Mg-Tschermack component. However, for spinel-facies peridotites, the \(\text{Al}_2\text{O}_3\) content ranges between 2 and 6 wt % for orthopyroxene and 2 and 8 wt % for clinopyroxene. For spinel-facies peridotites, failure to account for the Mg-Tschermack component will result in an underestimate of seismic velocities by ~3%. Additional effects may be caused by the incorporation of Cr-Tschermack component, but lack of elastic data on Cr-bearing pyroxenes prevents its consideration in this paper. However, ignoring this component may not introduce significant error into this...
<table>
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<th>Mineral</th>
<th>STP Density (p, g cm(^{-3}))</th>
<th>Isotropic Adiabatic Bulk Modulus (K_0), GPa</th>
<th>(dK_0/dP)</th>
<th>(dK_0/dT), GPa K(^{-1})</th>
<th>Isotropic Shear Modulus (G), GPa</th>
<th>(dG/dP)</th>
<th>(dG/dT), GPa K(^{-1})</th>
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<td></td>
<td>[Levien et al., 1979]</td>
</tr>
<tr>
<td>STD</td>
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<td></td>
<td>0.28</td>
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<td>Forsterite</td>
<td>3.226</td>
<td>128.7</td>
<td>1.40</td>
<td>0.7</td>
<td>1.70</td>
<td>-0.013</td>
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<td>[Duffy et al., 1995]</td>
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<tr>
<td>STD</td>
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<td></td>
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<td></td>
<td></td>
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<td>[Yoneda and Morioka, 1992]</td>
</tr>
<tr>
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<td>3.570</td>
<td>171</td>
<td>92</td>
<td>1.60</td>
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<td></td>
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<td>[Chen et al., 1999]</td>
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<tr>
<td>STD</td>
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<td>98.1</td>
<td></td>
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<td>[Zhang et al., 1999]</td>
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<tr>
<td>Almandine</td>
<td>4.325</td>
<td>177.9</td>
<td>98.1</td>
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<td>[Takahashi and Liu, 1970]</td>
</tr>
<tr>
<td>Chromite</td>
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<td>203</td>
<td>105</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[Hearmon, 1984]</td>
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<tr>
<td>Hycrinite</td>
<td>4.280</td>
<td>210.3</td>
<td>84.40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[Wang and Simmons, 1972]</td>
</tr>
<tr>
<td>Spinell (MgAl(_2)O(_4))</td>
<td>3.585</td>
<td>197.39</td>
<td>-0.014</td>
<td>107.8</td>
<td>-0.008</td>
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<td>[Suzuki et al., 2000]</td>
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<td>[Cynn et al., 1993]</td>
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<tr>
<td>Chromite</td>
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<td>203</td>
<td>105</td>
<td></td>
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<td>[Hearmon, 1984]</td>
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<td>Hycrinite</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>[Cynn et al., 1993]</td>
</tr>
</tbody>
</table>

*The majority of the above parameterizations are directly from compilations by Bass [1975] and Isaak [2001].
study because (1) it is a small component in most pyroxenes and (2) its concentration in pyroxenes remains relatively constant over a wide range in the composition of the host bulk rock.

[24] Assuming that the bulk rock is an isotropic mixture of the mineral constituents, its elastic moduli were calculated using two theoretical mixing models, the Voigt-Reuss-Hill (VRH) average and the Hashin-Shtrikman (HS) average. A good review of the application of these mixing models to earth materials is given by Watt et al. [1976]. The Hashin-Shtrikman average is reported here as it produced tighter bounds on the elastic moduli. For completeness, the relevant equations by Watt et al. are shown here. The upper \((K^*)\) and minimum \((K^-)\) bounds for the bulk modulus are

\[
K^* = K_1 + [A_1/(1 + \alpha_1 A_1)]
\]

\[
K^- = K_n + [A_n/(1 + \alpha_n A_n)],
\]

where \(K_1\) represents the smallest of the \(n\) moduli, \(K_n\) is the largest modulus, and

\[
\alpha_1 = -3/(3K_1 + 4G_1),
\]

\[
\alpha_n = -3/(3K_n + 4G_n),
\]

\[
A_1 = \sum_{i=2}^{n} \frac{X_i}{(K_i - K_1)^{1/2}} \alpha_1,
\]

\[
A_n = \sum_{i=1}^{n-1} \frac{X_i}{(K_i - K_n)^{1/2}} \alpha_n.
\]

For the shear modulus, the maximum \(G^*\) and minimum \(G^-\) bounds are

\[
G^* = G_1 + [B_1/(1 + \beta_1 B_1)]/2,
\]

\[
G^- = G_n + [B_n/(1 + \beta_n B_n)]/2,
\]

where \(G_1\) is the smallest of the \(n\) shear moduli, \(G_n\) is the largest, and

\[
\beta_1 = -3[K_1 + 2G_1]/[5G_1(3K_1 + 4G_1)],
\]

\[
\beta_n = -3[K_n + 2G_n]/[5G_n(3K_n + 4G_n)],
\]

\[
B_1 = \sum_{i=2}^{n} \frac{X_i}{2(G_i - G_1)^{1/2}} \beta_1,
\]

\[
B_n = \sum_{i=1}^{n-1} \frac{X_i}{2(G_i - G_n)^{1/2}} \beta_n.
\]

[25] Compressional \((V_p)\) and shear \((V_s)\) velocities at STP were calculated using the bulk densities and HS averages of the bulk-rock elastic parameters. The largest uncertainty (shown in figures) in seismic velocity determinations was estimated by propagating the uncertainties in mineral modes, elastic moduli, and density.

4.4. Estimation of Errors

[26] Errors on the elastic moduli calculated for natural minerals were estimated by propagating the errors (two standard deviations) for the elastic moduli of the mineral end-member components. Mineral density errors were negligible compared to the possible errors in estimating mineral modes. The errors on estimated mineral modes are difficult to quantify. Ideally, the error on mineral modes can be calculated by propagating the uncertainties in mineral compositions and bulk-rock compositions. These uncertainties are controlled by heterogeneity in the mineral and in the bulk rock rather than by analytical uncertainties, which are relatively small. Mineral-scale heterogeneity, however, is seldom fully quantified in most of the published studies from which the data in this study are derived. The approximate errors on mineral mode calculations were instead estimated by calculating the misfit between the measured bulk composition and that determined from the calculated mineral modes. In so doing, it is assumed that any misfit reflects sample or mineral heterogeneity. Samples with misfits greater than 0.7% were omitted. The “maximum” error on estimated seismic velocity was determined by propagating the maximum estimated error (0.7%) on mineral modes and elastic moduli. A linear regression was applied to fit the density and seismic velocity data to corresponding compositional parameters (e.g., Mg# and olivine mode); no weighting of data points was applied due to the difficulty in estimating errors.

5. Results

5.1. Mineral Modes

[27] Calculated mineral modes for two samples described previously in the literature are shown in Figures 5a and 5b. In general, there is good agreement with Boyd and McAllister’s [1976] calculations, as should be expected because the approaches are similar. The agreement with Jordan’s [1979] calculations is not so good. In the case of PHN1569, a harzburgite, Jordan’s approach results in much higher garnet contents. This is probably due to the fact that the orthopyroxenes in PHN1569 account for only 3% of the bulk Al2O3 content, which explains why, in this case, Jordan’s calculated modes are in agreement.

5.2. Densities

[28] Figures 5b and 5c compare our calculated mineral and bulk densities with the calculated densities of Jordan [1979] and the measured mineral densities of Boyd and McAllister [1979]. The mineral densities calculated here agree better than Jordan’s densities with the measured mineral densities of Boyd and McAllister. However, for reasons not understood, the garnet and clinopyroxene densities for PHN1569 calculated here and by Jordan are about 1% higher than the measured garnet and clinopyroxene...
densities of Boyd and McAllister. Nevertheless, the bulk densities calculated here agree well with the bulk densities of Boyd and McAllister. Jordan’s bulk densities for PHN1569 and PHN1611 are overestimated because of an overestimation of garnet mode in the former and an overestimation of garnet density in the latter.

Figure 6 shows how STP bulk density (g cm\(^{-3}\)) correlates with garnet (Figure 6a) and spinel (Figure 6b) mode. In Figure 6a, garnet-peridotites have been given different symbols in order to denote the bulk Mg\# of the peridotite (large symbols for Mg\# > 91, small symbols for Mg\# < 91). When the garnet-peridotite data are taken as a whole, bulk densities roughly correlate with garnet mode. This is an expected result as garnet, being the densest phase in garnet-peridotites, controls the bulk density. However, if only garnet-peridotites with Mg\# > 91 are considered, the correlation with garnet mode is poor. This is due to the fact that at high Mg\#s, garnet mode is reduced, and the variation in orthopyroxene/olivine ratio is likely to exert more of a control on bulk density.

Figure 6b shows how density varies as a function of spinel mode in spinel-facies peridotites and in garnet-facies spinel-peridotites. As defined earlier, garnet-facies spinel-peridotites represent peridotites derived from pressures greater than \(\sim 1.5\) GPa but otherwise contain Cr-rich spinel. As expected, when the data are taken collectively, there appears to be a rough positive correlation between bulk density and spinel mode. The garnet-facies spinel-peridotites tend to be slightly denser than spinel-facies peridotites owing to the fact that Cr-rich spinel is denser than aluminous spinel.

Figures 7a and 7b show bulk densities of garnet-peridotites and garnet-facies spinel-peridotites as a function of bulk Mg\# and olivine Mg\#. There is a strong negative correlation between bulk density and the two Mg\#s. Regression and fit parameters are shown in Table 5. This negative correlation is mainly due to the fact that mineral densities are correlated with Fe content, and to a lesser extent, to the fact that garnet mode decreases with increasing Mg\#. Jordan’s [1979] empirical correlation is also shown for reference. The correlation presented here differs slightly from Jordan’s but is to within error the same. This correlation differs from a previous attempt by this author to determine a density-Mg\# empirical fit on natural samples.
5.3. Seismic Velocities

Figure 6. (a) STP bulk density calculations of garnet-peridotites versus garnet mode; vertical bar represents the estimated largest 2σ error for density calculations. (b) Same as in Figure 6a, but spinel-facies spinel-peridotites and garnet-facies spinel-peridotites have been superimposed. Error bar as in Figure 6a.

[Lee et al., 2001]. In that particular paper, the density data set was limited to garnet-facies Cr-rich spinel-peridotites, a few garnet-peridotites, and some Fe-rich dunites from Tanzania. The combination of Fe-rich dunites and garnet-facies spinel-peridotites resulted in a less negative T dependence of bulk density for spinel-facies peridotites. There appears to be a rough negative correlation between T and Mg# for spinel-facies peridotites.

Figures 7c and 7d are reproduced from Figures 7a and 7b after adding the spinel-facies peridotites. It can be seen that these peridotites, owing to their aluminous (as opposed to Cr-rich) spinels, have slightly lower densities than the garnet-facies spinel-peridotites at a given Mg#. There appears to be a rough negative correlation between bulk composition and Mg# for spinel-facies peridotites.

5.3. Seismic Velocities

Figures 8a–8d show how STP elastic moduli (KS and G) calculated from mineral end-member moduli for natural minerals compare with direct experimental measurements on natural minerals. In most cases, the calculated moduli agree with the direct observations to within error. For the purposes of this paper, it is assumed that the calculated elastic moduli provide a reasonably accurate prediction of the elastic moduli of natural samples in the absence of direct experimental observation of all natural compositions of interest. The most important feature of Figure 8 is that over the compositional range of natural residual peridotites (Mg# ~ 88–93), there is a much stronger dependence of shear modulus (G) on Mg# than there is for bulk modulus (Ks); over the compositional range of interest, G varies by over 2% whereas Ks varies by no more than 1%

The STP Vp and Vs calculated using these elastic moduli and density determinations are shown in Figures 9–13. In Figures 9–13, only garnet-facies peridotite data have been linearly regressed. Linear regression parameters and their associated standard errors (2σ) are shown in Table 5. No attempt to regress the spinel-facies data was made due to the small number of data points. Figure 9 shows how Vs varies with bulk Mg# for garnet-facies peridotites and spinel-facies peridotites. It can be seen that there is a strong positive correlation of Vs on bulk Mg# for garnet-facies peridotites but only a weak dependence for spinel-facies peridotites. Shear wave velocities vary by ~2.5% between Mg# 86 and 94. There appears to be no correlation between Vs and olivine mode (Figure 10a). In contrast to Vs, there is almost no correlation between Vp and bulk Mg# (Figure 11). This is due to the stronger dependence of shear modulus on Mg# than bulk modulus on Mg#. Instead, there appears to be a weak positive correlation between Vp and olivine mode (Figure 10b) due to the different Ks values of olivine and orthopyroxene at STP conditions. However, at elevated pressures and temperatures, olivine and orthopyroxene will have similar Ks due to the fact that the pressure derivative of Ks (dKs/dP) for orthopyroxene is about two times higher than in olivine [Webb and Jackson, 1993]. For example, using the pressure and temperature derivatives of Ks shown in Table 2, it can be seen that Ks for forsterite olivine is 132.8 GPa and for enstatitic orthopyroxene is 128.8 GPa at 5 GPa and 1200°C (compare with 128.8 for forsterite and 106.7 for enstatite at STP conditions). Thus it is likely that the weak correlation between Vp and olivine mode at STP conditions (Figure 10b) disappears completely at upper mantle conditions.

It is important to note that the pressure and temperature derivatives of the elastic moduli of the mineral end-member components are very similar (Tables 2 and 4) and the entire compositional range in upper mantle peridotites investigated here is small (Mg# = 86–94). Therefore the chemical derivatives (d/dMg#) of the elastic moduli derived here for STP conditions hold at elevated pressure and temperature.

6. Discussion

6.1. Prospects and Limitations of Using Seismic Velocities to Detect Upper Mantle Compositional Variations

[Webb and Jackson, 1993] Figures 12 and 13 and Table 5 show the following STP total differentials for garnet-facies peridotites: d ln Vp/d ln Mg#, d ln Vs/d ln Mg#, and d ln Vp/d ln Mg# = (d ln Vp + d ln Vs)/2, where Mg# = 100 × Mg/(Mg + Fe). Only garnet-facies peridotites are included in this calculation as the data set for spinel-facies peridotites is too small and too scattered to construct meaningful compositional
parameters. These total differentials express the entire compositional variation of density and seismic velocity in natural peridotites in terms of Mg#. Clearly, other variables besides Mg# also control density and seismic velocity (e.g., mineral modes), but some of these variables are correlated with Mg# and/or exhibit only minimal control relative to Mg#. Thus the effect of other variables has been incorporated into the total differentials with respect to Mg#. Expressed in this manner, these total differentials should simplify the compositional inputs into various geophysical and geodynamic models. It is assumed that total differentials calculated at STP conditions are relatively constant over upper mantle conditions (although the absolute values of densities and seismic velocities obviously change as a function of $P$ and $T$).

The question that arises is whether densities and seismic velocities are meaningfully sensitive to compositional variations in mantle peridotites. Figures 13a and 13c indicate that density and $V_S$ are strongly negatively and positively correlated with Mg#, respectively. Figure 12 shows that $V_P/V_S$ is negatively correlated with Mg#. In contrast, $V_P$ exhibits no correlation with Mg# and should be considered useless in determining compositional variations in mantle peridotites.

### Table 5. Garnet-Facies Peridotites

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>SE</th>
<th>$b$</th>
<th>SE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = a \text{ (Mg#)} + b$</td>
<td>-0.0144</td>
<td>±0.0010</td>
<td>4.66</td>
<td>±0.091</td>
<td>0.67</td>
</tr>
<tr>
<td>$V_S = a \text{ (Mg#)} + b$</td>
<td>0.0143</td>
<td>±0.0009</td>
<td>3.53</td>
<td>±0.084</td>
<td>0.71</td>
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<tr>
<td>$V_P = a \text{ (olivine mode)} + b$</td>
<td>0.00232</td>
<td>±0.00029</td>
<td>8.13</td>
<td>±0.021</td>
<td>0.39</td>
</tr>
<tr>
<td>$\ln \rho = a \text{ (Mg#)} + b$</td>
<td>-0.00431</td>
<td>±0.00030</td>
<td>1.60</td>
<td>±0.030</td>
<td>0.67</td>
</tr>
<tr>
<td>$\ln V_S = a \text{ (Mg#)} + b$</td>
<td>0.00296</td>
<td>±0.00019</td>
<td>1.31</td>
<td>±0.018</td>
<td>0.71</td>
</tr>
<tr>
<td>$V_P/V_S = a \text{ (Mg#)} + b$</td>
<td>-0.00407</td>
<td>±0.00038</td>
<td>2.09</td>
<td>±0.035</td>
<td>0.54</td>
</tr>
</tbody>
</table>

\(^{a}\) Mg# = 100 × Mg/(Mg + Fe)\text{bulk}, olivine mode = wt %, $V$ = km s\(^{-1}\), $\rho$ = g cm\(^{-3}\).
In Figure 14a, the relative importance of Mg# and temperature on $V_S$ and $V_P/V_S$ for garnet-facies peridotites is shown, where $V_S$ and $V_P$ at different temperatures have been estimated using existing experimental data on mineral thermal expansivities [Fei, 1995] and elastic moduli after correction for temperature using the first temperature derivatives of the bulk moduli (Table 2). It is found that $dV_S/dT$ is approximately $-0.00033$ km s$^{-1}$ °C$^{-1}$. Combined with $dV_S/d$Mg# = 0.0150 (Figure 9a and Table 5), it can be shown that the relative effects of Mg# and temperature on a change in $V_S$ is given by $\Delta$Mg# ≈ $-0.022$ $\Delta$T. This means that an ~1% increase in $V_S$ could be due to either an “increase” in Mg# by five units (e.g., from fertile lherzolite (Mg# = 88) to refractory harzburgite (Mg# = 93)) or a “decrease” in temperature by ~220°C. It is thus concluded that although $V_S$ is sensitive to compositional variations, considerable care must be taken to separate the effects of temperature. In cases where xenolith thermobarometry or surface heat flow measurements provide an independent estimate of temperature at depth, it may be possible to use $V_S$ anomalies alone to infer bulk composition variations. An important corollary of this study is that variations in bulk Mg# of garnet-facies peridotites impose larger relative perturbations in $V_S$ (~2.5%) than in $V_P$ (~2%). This means that, for a given pressure and temperature, any variation in the bulk Mg# of a garnet-facies peridotite that is significant enough to yield a $V_P$ perturbation should yield a larger

Figure 8. Comparison between observed and calculated mineral elastic moduli at STP conditions. (a) Isotropic adiabatic bulk modulus $K_S$ (GPa) of olivine versus % forsterite content (% forsterite = molar 100 $\times$ Mg/(Mg + Fe) in olivine); symbols represent actual experimental data on natural samples; thick solid line represents predicted value based on linear average of end-member compositions; dotted lines define the 2σ error resulting from uncertainties in the end-member moduli. (b) Shear modulus $G$ (GPa) of olivine versus % forsterite content. (c) Adiabatic isotropic bulk modulus $K_S$ of orthopyroxene versus % enstatite content (% enstatite = molar 100 $\times$ Mg/(Mg + Fe) in orthopyroxene). (d) Shear modulus $G$ of orthopyroxene versus % enstatite content.

Figure 9. STP bulk shear wave velocities ($V_S$) versus Mg# for garnet and spinel-facies peridotites. Solid regression line is for garnet-facies peridotites only (garnet-peridotites and garnet-facies spinel-peridotites). Black vertical bar represents the estimated largest 2σ error on seismic velocity calculations.
perturbation in $V_S$ (assuming no melt or metasomatic phases).

[39] Along these lines, it appears that the $V_P/V_S$ ratio may prove useful in detecting compositional variations. Figure 14b shows how this ratio varies as a function of temperature (at constant pressure) and Mg# “in the absence of a melt phase or incipient melting” (e.g., below the solidus). The temperature dependence of $V_P/V_S$ is very

Figure 10. (a) STP bulk shear wave velocities and (b) $P$ wave velocities versus olivine mode. Regression line in Figure 10b is for garnet-facies peridotites only. Black vertical bar represents the estimated largest propagated 2σ error on seismic velocity calculations.

Figure 11. STP bulk $P$ wave velocities ($V_P$) versus bulk Mg#. Black vertical bar represents error as in Figure 10.

Figure 12. The ratio $V_P/V_S$ at STP conditions plotted against bulk Mg#. Regression line is for garnet-facies peridotites only.

Figure 13. Natural logarithm of STP density and seismic velocities versus bulk Mg# for garnet-facies peridotites only. (a) Natural logarithm of density, ln $\rho$, versus Mg#. (b) Natural logarithm of $P$ wave velocity (ln $V_P$) versus Mg#. (c) Natural logarithm of shear wave velocity (ln $V_S$) versus Mg#. Regression lines are through all data.
small (<0.04%/100°C) compared to the dependence on Mg# (~1.7%/Mg# unit). The relative effects of temperature and Mg# change on \( V_p/V_S \) is roughly \( \Delta \text{Mg#} \sim -0.1 \Delta T \). It can be seen from Figure 12b that this ratio is much more sensitive to composition than it is to temperature.

Recent studies have observed small velocity perturbations within cratonic lithospheric mantle [James et al., 2001; Shirey et al., 2002]. James et al. report 1% perturbations in \( V_p \) within the South African cratonic lithospheric mantle, which was interpreted by Shirey et al. to be due to compositional differences within the South African lithospheric mantle. Shirey et al. observe that the low \( V_p \) regions coincide with the presence of eclogitic- and lherzolitic-type diamond inclusions, whereas the remainder of the lithospheric mantle is characterized by high \( V_p \) and harzburgitic inclusions. They therefore suggest that the low \( V_p \) regions are associated either with a locally refertilized lithospheric mantle (yielding a decrease in bulk Mg# by the infiltration of melts) or the presence of eclogite. On the basis of the study presented herein, it is not likely that refertilization of a refractory peridotite (e.g., a harzburgite with Mg# of 93) to yield lherzolitic compositions within the range of primitive-type mantle (Mg# ~ 88) is capable of significantly lowering \( V_p \) (e.g., Figure 11a). As just discussed, changes in bulk Mg# result in a larger decrease in \( V_S \) than in \( V_p \), which suggests that this particular velocity anomaly is not due to a decrease in bulk Mg# alone. It is important to note that in the same study [James et al., 2001; Shirey et al., 2002], the region of significant \( V_p \) perturbation was not accompanied by any significant \( V_S \) perturbation. As suggested by Shirey et al., other possibilities, such as the presence of garnet pyroxenite and/or eclogites, should be investigated. However, given the high bulk and shear modulus of garnet, too much garnet will result in very large positive velocity perturbations.

6.2. Density of Lithospheric Mantle

[41] The density-Mg# correlation in Figure 7a can be used to determine whether cratonic lithospheric mantle is neutrally buoyant, as suggested by Jordan [1978, 1988]. Jordan recognized that continental lithosphere is thicker than oceanic lithosphere, owing to the colder thermal state of the former. The colder thermal state of continental lithospheric mantle should result in a mass excess due to the effects of thermal contraction. However, continental lithospheres appear to be more stable than oceanic lithospheres, implying paradoxically that continental lithospheric mantle must be stabilized despite their colder temperatures. Jordan [1978, 1988] hypothesized that continental lithospheric mantle was composed of refractory peridotite, which is intrinsically less dense than asthenospheric mantle due to previous extraction of meltable components. Jordan formalized this hypothesis into the “isopycnic hypothesis,” which states that the compositional buoyancy of the lithospheric mantle offsets the negative thermal buoyancy associated with the lithosphere’s colder temperatures:

\[
\frac{\Delta \rho_{\text{Mg#}}}{\rho_0} = -\alpha \Delta T_Z,
\]

where \( \Delta \rho_{\text{Mg#}}/\rho_0 \) is the compositional buoyancy term relative to a reference state (the asthenospheric mantle), \( \alpha \) is the thermal expansivity, and \( \Delta T_Z \) is the temperature difference between the continental lithospheric mantle and the asthenosphere at a particular depth \( Z \). In Figure 15a, xenolith thermobarometric data from several literature sources are shown along with a geotherm appropriate for a surface heat flow of 40 mW m\(^{-2}\) [Rudnick et al., 1993] and a 1350°C potential adiabat (assuming a 15°C km\(^{-1}\) gradient). The 40 mW m\(^{-2}\) geotherm is believed to be typical of Archean cratons and is broadly consistent with the xenolith thermobarometry [Rudnick et al., 1993]. In Figure 15a, \( \Delta T_Z \) would represent the difference in temperature between these two curves at a given depth. Figure 15b shows what \( \Delta \rho_{\text{Mg#}}/\rho_0 \) is required to be in order for the lithospheric mantle to be neutrally buoyant at every depth, given a 40 mW m\(^{-2}\) continental geotherm.

[42] The empirical relationship between bulk density and bulk Mg# in Figure 7a (\( \rho = -0.0144 \text{ Mg#} + 4.66; \) cf., \( \rho = \))
The equation used to reexpress the isopycnic condition (modified from equation (2) in the work of Jordan [1988]) can be used to reexpress the isopycnic condition:

\[
\Delta \text{Mg}_\# = -\frac{\rho_0 \alpha}{(d\rho/d\text{Mg}_\#)} \Delta T.
\]

(12a)

Using an average bulk thermal expansivity of \(\alpha\) at elevated temperatures (3.9 \(\times\) 10\(^{-5}\) \(^\circ\)C\(^{-1}\) calculated by combining the thermal expansivities of each mineral at temperatures of \(\sim\)1000\(^\circ\)C using the temperature parameterizations of Fei [1995] at constant pressure), a \(d\rho/d\text{Mg}_\#\) of \(-0.0143\) (Figure 7a), and a reference asthenospheric mantle composition of \(\text{Mg}_\#\) 89 (\(\rho_0 = 3.377\) g cm\(^{-3}\)), the above proportionality relationship between density and temperature for isopycnic conditions is

\[
\Delta \text{Mg}_\# = \Delta T/(108\,^\circ\text{C}).
\]

(12b)

This relationship is shown in Figure 15c (curve \(L_{89}\)). This relationship is significantly different from Jordan’s relationship

\[
\Delta \text{Mg}_\# = \Delta T/(187\,^\circ\text{C}).
\]

(12c)

of \(\Delta T/(187\,^\circ\text{C})\) (curve \(J_{89}\) in Figure 15c) due to the slightly different density-\(\text{Mg}_\#\) relationship derived in this study and a higher thermal expansivity used by Jordan (\(\alpha = 3 \times 10^{-5}\) \(^\circ\)C\(^{-1}\) with the density-\(\text{Mg}_\#\) relationship of this study results in \(\Delta T/(141\,^\circ\text{C})\). The effect of pressure on thermal expansivity is to lower the expansivity by \(\sim 0.85\) for a pressure increase in \(\sim 4\) GPa (this factor was estimated using the Anderson-Grüneisen parameter of olivine and the Birch-Murnaghan equation of state [Fei, 1995]). This changes equation 12b to \(\Delta \text{Mg}_\# = \Delta T/(124\,^\circ\text{C})\), which is bracketed by \(L_{89}\) and \(J_{89}\).

Figure 15d shows a compilation of peridotite \(\text{Mg}_\#\)s for various Archean cratons [Lee et al., 2001]. As most cratonic mantle peridotites are derived from pressures
between 3 and 6 GPa, it can be seen from Figures 14c and 14d that cratonic peridotites are on average neutrally buoyant if the density-Mg\# parameterization of this study is adopted ($L_{0\theta}$). On the other hand, if Jordan’s parameterization is used ($J_{0\theta}$), the high Mg\#s of some cratonic peridotites (Mg\# $\sim 93$) will be positively buoyant. If an asthenospheric mantle composition of Mg\# $\sim 88$ is assumed instead of Mg\# $\sim 89$, the isopycnic curves will accordingly shift to the right as shown in Figure 15c (curves $L_{SS}$ and $J_{SS}$). Using the Mg\#-density parameterization of this study, cratonic mantle would still be close to neutrally buoyant ($L_{0\theta}$), but adopting Jordan’s density parameterization would result in an even more extremely buoyant cratonic mantle ($J_{0\theta}$).

[45] Given the uncertainty in the Mg\# of the asthenospheric mantle and in the Mg\#-density parameterizations, it is not possible to conclusively determine whether or not cratonic mantle is neutrally buoyant from peridotite composition data alone. However, if it is assumed that continents are essentially in isostatic equilibrium with respect to the surrounding oceanic mantle, as is implied by the lack of strong free-air gravity anomalies over continents, then certain constraints follow from the different parameterizations. If cratonic peridotites are positively buoyant, as implied by Jordan’s parameterization, a missing lithology with significantly higher densities is needed to maintain a neutrally buoyant cratonic mantle. For example, using Jordan’s Mg\#-density parameterization (for an asthenospheric mantle with Mg\# between 88 and 89), the high Mg\# peridotites (93) need to be balanced by a denser lithology, such as eclogite. Assuming a minimum eclogite density of $\sim 3.52$ g cm$^{-3}$ based on a garnet/clinopyroxene ratio of $\sim 1$ and garnet and clinopyroxene densities similar to peridotite garnets and clinopyroxenes (see Table 1), the cratonic mantle at $\sim 4$ GPa would consist of a minimum of 28% (by volume) of eclogite in order to maintain neutrally buoyant conditions. On the other hand, using the Mg\#-density parameterization of this study, the cratonic mantle at $\sim 4$ GPa could have 0–10% of this eclogite and still maintain neutral buoyancy conditions. On the basis of the revised results presented here, there is no need to call upon on large contributions of eclogitic material in the cratonic lithosphere.

7. Conclusions

[46] Densities and elastic properties of 133 natural spinel- and garnet-peridotite samples at standard temperature (25°C) and pressure (1 atm, 1 atm = 101,325 Pa) conditions (STP) were calculated for compositions ranging from Mg\# ($100 \times Mg/(Mg + Fe)$) $\sim 86–94$ in order to investigate how small compositional variations in upper mantle peridotite control density and seismic velocity. The following conclusions are made:

[47] 1. $V_S$ correlates positively with Mg\# but not with olivine or orthopyroxene mode.

[48] 2. $V_p$ does not correlate with Mg\# but does exhibit a weak positive correlation with olivine mode. However, this correlation with olivine mode is likely to become even smaller at elevated pressures and temperature due to the higher-pressure derivative of orthopyroxene bulk modulus compared to olivine.

[49] 3. The ratio $V_p/V_S$ correlates negatively with Mg\# and appears to be more sensitive to variations in Mg\# than in temperature.

[50] The third conclusion implies that the ratio $V_p/V_S$ may be useful in “seeing through” the effects of temperature variation on seismic velocities as determined from seismic tomographic studies of the upper mantle. Finally, a new density versus Mg\# parameterization is derived here from natural peridotites. Combined with a compilation of the Mg\#s of peridotite xenoliths derived from cratonic lithospheric mantle, it can be shown that the higher Mg\#s and corresponding low intrinsic densities of cratonic peridotites approximately balances the negative thermal buoyancy of cratonic lithosphere imposed by its cooler thermal state relative to upwelling asthenospheric mantle.

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C.-T. A. Lee, Department of Earth Science, Rice University, MS 126, P.O. Box 1892, Houston, TX 77251, USA. (ctlee@rice.edu)