Capital Reallocation and Private Firm Dynamics

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ABSTRACT

We develop a theory of firm dynamics and capital reallocation in private firms and use it to study the taxation of business income, capital, and capital gains. Intangible assets—such as customer bases and trade names—are created using owners' time and are infrequently traded in bilateral meetings. We discipline the model with U.S. administrative data, which report purchase prices and counterparties in asset transfers, allowing us to calibrate the investment technology and output elasticity for otherwise unobservable intangible capital. The equilibrium features dispersion in marginal product of capital, transferable share of firm value, and return on business wealth. Introducing taxation, we find that capital gains taxes are most distortionary, primarily by discouraging entry and reallocation of capital, whereas income taxes are least distortionary.

KEY WORDS: business transfers, capital allocation, firm dynamics, capital taxation

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1 Introduction

We develop a theory of firm dynamics and capital reallocation that treats intangibles—such as customer bases, trademarks, and going-concern value—as essential inputs in production. The capital we model is indivisible, not traded in centralized markets, yet accounts for most assets exchanged in private business sales in the United States. Existing theories have largely overlooked these assets because they are observed only when a business is sold. This omission is especially important for private firms, where owners actively manage and build this capital. Because private business transfers are infrequent and not publicly disclosed, little is known about these investments, even though private firms are central to studies of productivity, wealth inequality, and tax policy and generate over half of all U.S. business income. Our theory is informed by administrative tax data from the Internal Revenue Service (IRS), with a key innovation being information on valuations and counterparties in asset transfers. The model delivers theory-based estimates of the dispersion in marginal products of capital, returns, and valuations for ongoing businesses and is used to revisit the classic question of how to tax business income and wealth.

Our environment is neoclassical in the spirit of Lucas (1978) and Hopenhayn (1992)—modified to include features that make it appropriate for studying firm dynamics and capital allocation for private business. Goods and services are produced with *nontransferable* capital that cannot be bought or sold, *transferable* capital that can be bought or sold but not rented, and *external* factors that are rented on spot markets. The nontransferable capital represents business owners' productivity or ability, which evolves stochastically and is inalienable. The transferable capital stocks are intangible business assets such as customer bases and trade names. This type of capital will be the focus of our analysis, and henceforth we refer to it as "business capital" or simply "capital." The external factors include employee time and fixed assets such as office space and equipment. Firms in the model accumulate capital in two ways: through internal investment and through purchases of other businesses. Internal investment is costly and requires owner time as an input. Buying and selling take time to complete, occur in pairwise meetings, and entail a transfer of the entire business capital of the seller. We view time to trade, bilateralness, and indivisibility as salient features of how intangible business capital is reallocated across firms. Owners who sell can restart another business or work as employees.

The trading protocol results in a gradual reallocation of capital, with owners who have low marginal products of capital selling to those with higher marginal products. We show that the equilibrium allocation of capital in our model is efficient. Our assumptions on the trading technology imply that per-unit prices depend on the quantity of capital exchanged between each pair of owners, and generate dispersion in marginal products of capital and returns on business wealth.

We calibrate the model using data from U.S. administrative tax filings of S corporations. These are private, pass-through entities, and unlike C corporations or partnerships, their owners must be individuals. We construct longitudinal panels spanning business and owner life cycles by linking business and individual tax forms for each owner. While data of this kind have been used in the firm dynamics and capital allocation literature, a major shortcoming is that neither the stocks nor the investment expenditures in intangible business capital are recorded until the business is sold. A distinctive feature of our data is that it includes information on business asset acquisitions, including the price paid and the identities of the counterparties involved in each transaction. Importantly for us, these data also include the allocation of the purchase price across various asset categories, such as marketable securities, fixed assets, and intangible assets.[1] This last category constitutes roughly 70 percent of the transferred value in a typical sale.

The transaction data play a central role in how we discipline our theory. Traditional life-cycle data on firm age, employment, and shares of rentable factors help identify parameters governing the productivity process and the output elasticities of external factors. For fixed assets, the literature has relied on direct measurements of quantities to calibrate both the contribution of capital to production and the parameters governing the investment technology. Such approaches are not applicable to the type of business capital we study, since direct measurements are not available. Our model of the capital market generates predictions for two key moments: who trades with whom and the terms of trade. We map these predictions to their empirical counterparts in the transaction data and use them to identify the output elasticity and investment-cost parameters for intangible capital. Intuitively, purchase prices are informative about the costs of internal investment, while the relative size of buyers and sellers is informative about the returns to scale of business capital in production.

We examine the implications of our model of private firm dynamics for the allocation of capital across firms. As noted above, our competitive equilibrium is efficient, hence it features no misallocation. However, the model predicts substantial dispersion in marginal products of capital due to

[1] The data come from filings of Form 8594, which must be submitted by both buyers and sellers, and distinguish intangible assets in accordance with IRC Section 197. These include customer- and information-based intangibles, non-compete covenants, licenses and permits, franchises, trademarks, trade names, workforce in place, business books and records, processes, designs, patterns, as well as goodwill and going-concern value. This information is required to determine capital gains for sellers and the asset bases for amortization by buyers.

the nature of business technologies and transfers. In our baseline, the standard deviation of the log of the marginal product of capital is 40 percent. We explore and quantify the roles of time to trade and indivisibility in capital exchange by comparing our baseline estimate of marginal product of capital dispersion to alternatives across a wide range of trade frequencies and assumptions about capital divisibility. We find significant dispersion throughout the empirically relevant parameter space.

The model also makes predictions for the stock of and returns on business wealth. We explore two familiar but distinct measures of wealth. The first is the value of transferable wealth, often reported in surveys such as the Survey of Consumer Finances and particularly relevant for analyzing the taxation of capital gains. The second is the total value of the going concern, which reflects the flow of dividends to owners over the life of the business. This measure includes not only the intangible assets that can be transferred, but also the owner's productivity and future growth prospects. We estimate an aggregate total value of 2.66 times private sector value added, with an average share of 22 percent that is transferable. Finally, we document significant dispersion across owners in both the ratio of transferable to total wealth and in returns to business wealth. This dispersion cautions against making simple imputations of wealth inequality based on value-to-book or value-to-income multiples for publicly listed corporations.

Our analysis of capital trade, valuations, returns, and marginal products provides critical inputs for understanding how businesses should be taxed. The presence of dispersion in marginal products and business returns in our model allows us to reconsider existing proposals that favor taxing the capital stock of a business rather than its net income. By modeling intangible business capital, we depart from the classical settings of firm taxation based on the dichotomy between capital use and ownership. This creates a distinction between different types of returns to capital: business income and capital gains. We assess the effects of raising a fixed revenue each period using alternative tax instruments: a tax on business income, a tax on the transferable value of business capital, and a tax on capital gains realized when businesses are sold.

Three findings emerge. First, a tax on business income causes smaller wage losses and better capital allocation than either a capital value or capital gains tax: its broad base spreads the burden across many owners, especially highly productive ones, whose entry is less elastic. Second, a capital gains tax is the most distortionary: by taxing the option value of selling, it induces lock-in, thereby raising dispersion in the marginal product of capital. It also shifts the burden toward low-productivity firms that are most likely to sell, hence reducing investment and entry by marginal owners. Third, a tax on assessed capital value has intermediate effects: it is broad-based like an income tax but more discouraging of entry and investment. Taxing the capital value modestly reduces measured dispersion in the marginal product of capital by shifting resources from low-productivity owners with a lot of capital to high-productivity owners with little capital. However, the efficiency gains of capital reallocation are outweighed by entry and investment distortions.

1.1 Related Literature

Our paper relates to the extensive body of work studying entrepreneurship, firm dynamics, productivity, and the allocation, valuation, and taxation of business capital.

The model we develop provides a bridge between the entrepreneurship literature starting with Lucas (1978) and the firm dynamics literature starting with Hopenhayn (1992). On the theoretical side, we retain much of the neoclassical spirit of these earlier frameworks but introduce technological and market-specific features relevant for most business capital currently used in production. In particular, we model capital assets as indivisible and non-rentable and the sale of a business as a transfer of a group of assets. In considering the indivisible nature of the trade, we are also building on Holmes and Schmitz (1990) who focus on heterogeneity in ability to start firms and focus on empirical measures of serial entrepreneurship.

Our paper is related to the literature on capital measurement and misallocation. Much of this literature has focused on the role of regulatory, financial, and informational constraints and capital adjustment costs. See, among others, Ramey and Shapiro (2001), Hsieh and Klenow (2009), Asker et al. (2014), Restuccia and Rogerson (2017), David and Venkateswaran (2019), Sterk et al. (2021), Jaimovich et al. (2025); see Cooper and Haltiwanger (2006), Baley and Blanco (2021), Lippi and Oskolkov (2023) for papers with a specific focus on capital indivisibility. We differ from this literature in two ways. First, we study intangible business capital, which is the dominant form of capital owned by and used in U.S. private firms, rather than physical capital in manufacturing. Because investment in and stocks of intangible assets are not directly observed, the standard approaches used to estimate output elasticities and investment costs cannot be applied. We instead exploit IRS data on business sales combined with model-guided identification. Second, our framework produces estimates of dispersion in marginal products of capital for private businesses, reflecting features of business capital such as indivisibility, reallocation delays, and bilateral trade. Since private business capital is not observable, no empirical counterpart of our estimates exists. However, to put our results in context, we obtain a dispersion in marginal products that is about half as large as common estimates for physical capital in U.S. manufacturing firms.

Estimates of business value for traded and non-traded private firms have been inputs to the growing literature on measuring private business wealth. Most studies in this area rely on structural models of entrepreneurship guided by survey data (see Cagetti and De Nardi (2006)) or estimation methods to capitalize income and investment flows (see Saez and Zucman (2016) Crouzet and Eberly (2023), Smith et al. (2023), Sveikauskas et al. (2024), Gomez and Gouin-Bonenfant (2025), and He et al. (2025)). Our approach to measuring business value differs in that it uses model-implied valuation concepts, informed by detailed data on business sales and the income statements of the buyers and sellers of businesses.

We contribute to the public finance literature on the taxation of capital and its returns. In standard models of firm taxation with perfect capital markets, Atkinson and Stiglitz (1976) prescribe zero taxes on business income or value and instead recommend taxing distributions and capital gains.[2] This prescription is not optimal in our case with non-deductible investment and entry costs that include owner time, which motivates our analysis of the comparative distortive effects of alternative tax instruments. In environments without perfect financial markets, Guvenen et al. (2023) argue for taxing liquid business assets rather than income, thereby addressing misallocation arising from borrowing constraints and heterogeneous returns to capital. Our focus is on a different form of business capital that is partly illiquid, with dispersion in returns generated by technological features rather than financial frictions. Finally, Chari et al. (2003) and Cavalcanti and Erosa (2007) study the taxation of business transfers in the context of the Holmes and Schmitz (1990) framework. We share with them the focus on the lock-in effect of capital gains taxation. We also connect our quantitative predictions to the empirical public-finance evidence on capital-gains elasticities—for example, Gentry and Bakija (2014) and Agersnap and Zidar (2021).

Finally, our work is connected to the literature on mergers, acquisitions, and the sale of business capital that uses models of random search with bargaining or directed search with one-sided heterogeneity to ensure tractability. See, most notably, Jovanovic and Rousseau (2002) and David (2021) on mergers and acquisitions, Gavazza (2016) and Ottonello (forthcoming) on fixed asset reallocation, and more recently, Gaillard and Kankanamge (2020) and Guntin and Kochen (2024) on the role of financial frictions in buying and selling firms. Unlike this literature, we model business sales as transactions in a frictionless, decentralized market. Using tools from the matching

[2] In recent work, Aguiar et al. (2025) also reexamine the optimality of taxing financial wealth and capital gains, particularly when revaluations interact with redistribution and insurance considerations. These motives are absent from our analysis, which focuses on productive business capital.

literature (Galichon et al. (2019)), we solve for the equilibrium set of prices and demonstrate that it leads to an efficient allocation. We find this efficiency property appealing as it allows us to isolate the dispersion in marginal products generated solely by the indivisibility of capital in private businesses. Furthermore, our focus is different: we use our model to generate predictions for the distribution of marginal products of capital, valuations, and returns for private businesses and derive implications for tax policy.

The rest of the paper is organized as follows. Section 2 details the environment, including timing of events, descriptions of problems solved by business owners, and a definition of a recursive equilibrium. A characterization of the equilibrium and the solution algorithm are provided in Section 3. In Section 4, we document statistical properties of U.S. firm-level data that guide the calibration described in Section 5. In Section 6, we document the model's predictions for the dispersion of marginal products of capital and for business wealth. In Section 7, we assess the impacts of business taxation. Section 8 concludes.

2 Theory

The economy is populated by a unit measure of individuals that can choose to run a business or work in paid-employment. Business owners are endowed with a technology that produces a homogeneous consumption good. The production inputs differ in their divisibility and transferability. The first input is entrepreneurial productivity or skill, z, which is nontransferable and evolves stochastically. The second input is intangible business capital—or simply "capital"—which we denote by k. Capital is accumulated through costly investments, is transferable via stochastic access to a capital market, but is not divisible or rentable. The remaining inputs are external factors b and n that are perfectly divisible and rentable in a spot market. Factor b is fixed assets such as equipment and office space in commercial buildings. Factor n is labor. The decision to switch occupations is made continuously. Details of these actions are provided next, followed by the owner's dynamic program, and a definition of a stationary recursive equilibrium.

2.1 Environment

Production. Let $s \in \mathcal{S}$ denote a pair (z, k) and use $z(s)$ and $k(s)$ to denote the first and the second component of s, respectively. Output is produced using the technology

$$y(s, b, n) = z(s)k(s)^{\alpha} b^{\beta} n^{\gamma}. \tag{1}$$

Productivity process. Productivity z follows the exogenous stochastic process

$$dz = \mu(z)dt + \sigma(z)d\mathcal{W},$$

where $\mathcal{W}$ is a standard Wiener process. Importantly, z only changes while the individual is a business owner, and it is fixed while working.

Investment. The investment technology for capital is modeled as a cost function $c(i)$, where c' and c'' are strictly positive. An owner incurs cost $c(i)$ to invest i and accumulate additional business capital. Specifically, the change in capital over an interval of length dt is equal to

$$dk = (i - \delta_k k)dt,$$

where δ_k is the depreciation rate of capital.

Rental and capital markets. The rental markets for labor, n, and fixed assets, b, are perfectly competitive with unit costs w and r, respectively. We assume that the fixed assets are owned by a competitive mutual fund sector that can convert consumption goods to investment one-for-one and faces a depreciation rate of δ_b.

The market structure for capital departs from the neoclassical framework in three dimensions: time to trade, bilateralness, and indivisibility. An owner with state s accesses the capital market at Poisson rate η. We refer to this intermittent access as *time to trade*. Once in the market, an owner faces a price-quantity menu denoted by $\{p^m(s, \tilde{s})\}_{\tilde{s} \in \mathcal{S}}$ and $\{k^m(s, \tilde{s})\}_{\tilde{s} \in \mathcal{S}}$. Consider an owner with state s, who is deciding on a trade with another owner that has state $\tilde{s}$. Owner s would pay $p^m(s, \tilde{s})$ to the trading partner $\tilde{s}$ and exit the trading stage with capital level $k^m(s, \tilde{s})$. The functions $k^m : \mathcal{S}^2 \to K$ and $p^m : \mathcal{S}^2 \to \mathcal{R}$ are determined as part of an equilibrium that we define later. We refer to the ability to trade with only one partner at a time as *bilateralness*. For the allocation of capital within a match, we impose that an owner can either sell their entire capital stock, buy the entire capital stock of their trading partner, or trade no capital at all. This assumption amounts to the following restrictions on k^m. For all pairs $(s, \tilde{s}) \in \mathcal{S}^2$,

$$k^m(s, \tilde{s}) \in \{k(s) + k(\tilde{s}), k(s), 0\} \tag{2}$$

We refer to restriction (2) as *indivisibility*. This restriction captures a key feature of our model, namely, that the reallocation of capital across owners in bilateral trades occurs in a "lumpy" fashion.

Entry, exit, and occupational choice. Entry into and exit from the economy occur at Poisson rate ψ_e. Newborns draw a state $s \sim G(s)$ and decide whether to become workers or business owners. Workers and owners have an option to switch occupation at Poisson rate ψ_o. Entry into self-employment entails a cost of c_e in units of goods.

Preferences. Owners and workers are risk-neutral and discount the future at rate ρ.

Discussion of assumptions. Before proceeding, we discuss a set of our simplifying assumptions. We impose them intentionally to highlight the novel aspects of business capital accumulation and trade. These assumptions can be relaxed within our framework.

First, we assume that buyers' and sellers' capital stocks combine additively. In practice, mergers may generate synergies or diseconomies of scale that affect valuations and trade decisions. Since we only observe the price paid for capital when a trade occurs, our data cannot distinguish whether a higher (lower) price reflects more (less) capital or instead the presence of synergies (diseconomies). Incorporating post-trade income gains of buyers could help address this limitation and allow for richer forms of capital aggregation.

Second, workers earn a constant wage w and do not accumulate human capital. This simplification keeps the analysis focused on business ownership and capital reallocation. In richer environments, workers accumulate skills with experience, which could affect their productivity as future business owners. Worker heterogeneity would then influence the response of entry to business taxation.

Third, we assume that the owner's time is the sole input into the accumulation of business capital. While effort is undoubtedly central, other inputs such as hired labor and materials may also contribute to building firm value. Relaxing this assumption to allow substitution across inputs is straightforward, though empirically challenging because it requires distinguishing investment expenditures from current production costs.

Finally, we adopt linear preferences as a starting point. This assumption isolates the roles of trading frictions and indivisibilities in shaping capital allocation and efficiency. The framework can be extended to incorporate the economic consequences of imperfect financial markets, such as undiversifiable risk and borrowing constraints.

2.2 Recursive Formulation

Let $V : S \to \mathcal{R}^+$ denote the value of an owner. Let $\lambda : \mathcal{S} \to \Delta(\mathcal{S})$ be a measure over the set $\mathcal{S}$ that describes the probability that type s choses to trade with an owner of type $\tilde{s}$. Let $V_{trade}(;\lambda) : \mathcal{S} \to \mathcal{R}^+$ be the owner's gains from trade for a given λ. Let $W \in R^+$ be the value of being a worker. Given functions $\{p^m, k^m\}$, the owner value solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned}(\rho + \psi_e)V(s) = \max_{b,n,i,\lambda} \, & y(s,b,n) - rb - wn + \partial_k V(s)(i - \delta_k k) - c(i) \\ & + \partial_z V(s)\mu(z) + \frac{1}{2}\partial_{zz} V(s)\sigma(z)^2 + \psi_o\{W - V(s)\}^+ + \eta V_{trade}(s;\lambda),\end{aligned} \tag{3}$$

where $y(s,b,n)$ is defined in (1). The term on the left-hand side is the annuitized value of being an owner of type s. The right-hand side includes flow output net of the rental cost of fixed assets and the wage bill, the gain from capital investment net of the cost of investment, the changes in value induced by the evolution of productivity z, the option value of exiting, and the expected gains from trade from accessing the market for capital, V_{trade}.

The last term of the HJB equation (3) is absent from traditional firm dynamics models, and we discuss it next. Consider a given price-quantity menus $\{p^m(s,\tilde{s}), k^m(s,\tilde{s})\}_{\tilde{s}\in S}$. Define $v(s,\tilde{s})$ as the value for firm type s after trade with $\tilde{s}$:

$$v(s,\tilde{s}) \equiv V(z(s), k^m(s,\tilde{s})) - p^m(s,\tilde{s}).$$

Note that by allowing λ to be a measure over $\mathcal{S}$, we allow the owner to mix over the set of trading partners. The gains from trade are then given by

$$V_{trade}(s) = \max_{\lambda(s,\cdot):\int \lambda(s,\tilde{s})d\tilde{s}=1} V_{trade}(s;\lambda), \tag{4}$$

where

$$V_{trade}(s;\lambda) \equiv \int \{v(s,\tilde{s}) - V(s)\}\lambda(s,\tilde{s})d\tilde{s}.$$

We assume that the productivity as owner z does not change unless actively running a business. An immediate consequence is that once a particular individual decides to be a worker, the choice is never overturned. Therefore, the value of being a worker, W, is the present value of wages until the exogenous stochastic time of death and given by

$$W = \frac{w}{\rho + \psi_e}. \tag{5}$$

The entry and exit decisions into business ownership are given by

$$\iota^{\text{in}}(s) = \{V(s) - W - c_e > 0\}, \tag{6}$$

$$\iota^{\text{out}}(s) = \{W - V(s) > 0\}, \tag{7}$$

where c_e is the cost of entry.

2.3 Equilibrium

We next describe the law of motion for the measure of owners induced by the policy functions and then define an equilibrium. Let $\phi \in \Delta(\mathcal{S})$ be the measure over owner types, and let operator $\mathcal{A}_{(i,\lambda)}$ be the infinitesimal generator associated with the Bellman equation (3) and $\mathcal{A}^*_{(i,\lambda)}$ be its conjugate operator. The law of motion of ϕ is described by

$$\dot{\phi}(s) = (\mathcal{A}^*_{(i,\lambda)}\phi)(s) + \psi_e \iota^{\text{in}}(s)dG(s) \tag{8}$$

The first term on the right hand side of equation (8) describes the evolution of the distribution of owners due to investment, productivity shocks, trades, and exit. The second term describes the entry of new owners into the economy.

Furthermore, we can compute the implied mass of business owners, m by integrating (8) over

s. This mass evolves according to:

$$\dot{m} = \psi_e \int \iota^{\text{in}}(s)dG(s) - m(\psi_e + \psi_o \int \iota^{\text{out}}(s)\phi(s)ds). \tag{9}$$

The first term is the entry flow due to the choice of becoming an owner upon entering the economy. The second term is the exit flow either from the economy or from the entrepreneurial sector and into the labor market.

We are now ready to define an equilibrium.

Definition 1. A *stationary recursive equilibrium* is given by (*i*) value functions $V : \mathcal{S} \to \mathcal{R}^+$ and $W \in R^+$; (*ii*) policy functions $n : \mathcal{S} \to \mathcal{R}^+$, $b : \mathcal{S} \to \mathcal{R}^+$, $i : \mathcal{S} \to \mathcal{R}^+$, $\lambda : \mathcal{S} \to \Delta(\mathcal{S})$, $\iota^{\text{in}} : \mathcal{S} \to \{0,1\}$, $\iota^{\text{out}} : \mathcal{S} \to \{0,1\}$; (*iii*) wage, w; (*iv*) price-quantity menus for capital $p^m : \mathcal{S}^2 \to \mathcal{R}$ and $k^m : \mathcal{S}^2 \to K$; (*v*) a measure over owner types $\phi \in \Delta(\mathcal{S})$ such that:

- Given $\{k^m(\cdot), p^m(\cdot)\}$ and wage w, the value function for owners and workers $\{V(\cdot), W\}$ solve the Bellman equations (3) and (5).
- The policy functions for investment, trade, and rentable inputs solve the maximization problem in equation (3). The entry and exit decisions for an owner satisfy (6) and (7).
- The rental rate for fixed assets satisfies $r = \delta_b + \rho$ and the labor market clears:

$$\int (1 + n(s))\, \phi(s)ds = 1. \tag{10}$$

- The trading arrangements are feasible, stable, and consistent. That is, for all pairs $(s, \tilde{s}) \in \mathcal{S}^2$ the function k^m satisfies (2); there does not exist a feasible trade for pair $(s, \tilde{s})$ that makes the pair strictly better off; and the trading policies satisfy:

$$\lambda(s, \tilde{s})\phi(s) = \lambda(\tilde{s}, s)\phi(\tilde{s}). \tag{11}$$

- The measure over owners is stationary

$$\dot{\phi} = 0. \tag{12}$$

3 Characterizing the Equilibrium

In this section, we provide a characterization of the equilibrium and discuss its properties. We compute the equilibrium in two steps. First, we take the value function V and the equilibrium measure of firms ϕ as given and characterize prices p^m, the allocation—that is, the choices of trading partners λ and capital k^m conditional on trading—and the gains from trade V_{trade} that are consistent with market clearing. Second, we solve for (V, ϕ) such that individuals optimize given the menu of prices and terms of trades and ϕ is in turn, consistent with their decisions.

3.1 Characterizing prices and allocations given (ϕ, V)

As a first step, we show that the equilibrium prices and allocation of capital can be characterized with an *assignment* problem that maximizes the total surplus—which is measured using V—by assigning capital subject to preserving the measure ϕ.

A simple example. Before formalizing this problem, we consider an example in order to provide some intuition about who trades with whom and how the terms of trade are determined. For the example, we use a simple production function, namely, $y = zk$, and assume that there are 20 owners with $z = 1$, 10 owners with $z = 0$, and all 30 have $k = 1$. Consider an allocation that is achieved by the low-z types selling their businesses to any 10 of the high-z types so that trading probabilities are $\lambda(s_L, s_H) = 1$, $\lambda(s_H, s_L) = 1/2$ and prices are $p^m(s_H, s_L) = 1$, $p^m(s_L, s_H) = -1$ and $p^m(s_H, s_H) = p^m(s_L, s_L) = 0$. It is easy to check that these allocations and prices implement trading arrangements that satisfy feasibility, stability, consistency, and private optimality.

We make a few observations about the trading outcome. First, the allocation of capital maximizes pairwise surplus. Second, the terms of the trade are such that the surplus split is determined by the short side of the market. In particular, the prices are such that the high-z types are indifferent between trading and not trading. We now formalize these observations to the general case.

Trades as an assignment problem. Define the surplus from matching for a pair $(s, \tilde{s})$ as follows:

$$X(s, \tilde{s}) = \max \left\{ V(z, k + \tilde{k}) + V(\tilde{z}, 0), V(s) + V(\tilde{s}), V(z, 0) + V(\tilde{z}, k + \tilde{k}) \right\} - (V(s) + V(\tilde{s})).$$

The three arguments are possible outcomes in a match, namely, type s buys the capital from type $\tilde{s}$, no trade, and type s sells the capital to $\tilde{s}$. We split the measure ϕ into two measures ϕ^a and ϕ^b

such that for $s \in \mathcal{S}$ we have

$$\phi^a(s) = \phi^b(s) = \frac{\phi(s)}{2}.$$

For measures $\{\phi^a, \phi^b\}$, an assignment $\pi : \mathcal{S}^2 \to \mathcal{R}_+$ that maximizes surplus, solves the following maximization problem:

$$Q(\phi, V) = \max_{\pi \geq 0} \int X(s, \tilde{s})\pi(s, \tilde{s}) ds d\tilde{s} \tag{13}$$

such that for $s \in \mathcal{S}$

$$\int \pi(s, \tilde{s}) d\tilde{s} = \phi^a(s) \tag{14}$$

$$\int \pi(\tilde{s}, s) d\tilde{s} = \phi^b(s). \tag{15}$$

We label this problem as *P1*. The next theorem shows that we can back out (p^m, k^m, λ) from the solution of *P1* using standard results from the matching literature.[3]

Theorem 1. *Let μ^a and μ^b be the Lagrange multipliers on* (14) *and* (15)*, respectively, in problem P1. Let π be the optimal assignment in problem P1. The functions*

$$k^m(s, \tilde{s}) \in \arg\max_{\tilde{k}} \{V(z, k + \tilde{k}), V(s) + V(\tilde{s}), V(\tilde{z}, k + \tilde{k})\} \tag{16}$$

$$p^m(s, \tilde{s}) = V(z, k^m(s, \tilde{s})) - V(s) - \mu^a(s) \tag{17}$$

$$p^m(\tilde{s}, s) = V(\tilde{z}, k^m(\tilde{s}, s)) - V(\tilde{s}) - \mu^b(\tilde{s}) \tag{18}$$

$$V_{trade}(s) = \mu^a(s) = \mu^b(s) \tag{19}$$

and measures for all $s, \tilde{s} \subseteq \mathcal{S}$

$$\lambda(s, \tilde{s}) = \frac{\pi(s, \tilde{s}) + \pi(\tilde{s}, s)}{\phi(s)} \tag{20}$$

constitute equilibrium trading arrangements and the gains from trade.

Theorem 1 states that the assignment from problem *P1* gives us all the information we need to figure out who trades with whom and at what prices. Analogous to the welfare theorems in standard settings, the solution to the planner's problem *P1* recovers the allocation, and the multipliers on constraints (14) and (15) recover the prices.

We can review the intuition in our context. The envelope theorem applied to problem *P1* implies the social gains from having more owners of type s in the market for capital are given

[3] See Galichon (2016) for details on the Monge-Kantorovich transportation problem.

by $\mu^a/2$+$\mu^b/2$. Given the symmetry of the assignment problem, it is easy to show that $\mu^a = \mu^b$ and each is equal to the social gains from trade. Equations (17) and (18) pin down the prices that implement those gains. The right-hand sides of equations (16) and (20) characterize outcomes of each potential meeting and trading frequency. These conditions yield trading arrangements that satisfy feasibility, stability, consistent as well as private optimality.

3.2 Characterizing (ϕ, V) given $(p^m, k^m, \lambda, V_{trade})$

In the second step, we use the outcomes of the first step to update value functions, V, and the invariant measure, ϕ. The characterization in the first step gives us a handy way of solving the Bellman equation. Given the value of $V_{trade}(s)$ from the solution of problem *P1*, the HJB can be solved using standard methods (for example, finite differences as in Achdou et al. (2021)). The policy functions for investment (i), trades (λ), and entry or exit (ι^{in}, ι^{out}) govern the law of motion of the distribution for which we find a stationary point (given by condition (12)). Together the two steps characterize the recursive competitive equilibrium as a fixed point. This characterization naturally lends itself to a computational algorithm where we iterate between the steps until convergence.

3.3 Properties of the Equilibrium

The next corollary further sharpens the characterization of the price function p^m.

Corollary 1. *There exists a function $\mathcal{P} : K \to R_+$ such that*

$$p^m(s, \tilde{s}) = \mathcal{P}(k(s)) \quad \textit{for all } k^m(s, \tilde{s}) = 0.$$

This corollary says that the pairwise prices only depend on the quantity sold. The intuition for this result is straightforward. The seller's value from trade is equal to the price he extracts from the buyer plus the value of starting anew with zero capital and the current level of productivity. The second component is independent of the trading partner. Thus, conditional on selling to different buyers, a seller who maximizes the value from trading must necessarily charge the same price to all buyers. A similar argument from the perspective of the buyer shows that the prices will not depend on the seller's productivity. While our general formulation of pairwise terms of trade allows for arbitrary gains from matching with any owner of type $\tilde{s}$, our assumption that productivity z is non-transferable delivers an equivalence between our trading protocol for capital and a competitive market with unit demand over differentiated products, which is indexed by the indivisible size of the capital sold. As such, the prices are only a function of the quantity traded. We summarize this dependence using the function $\mathcal{P}(\cdot)$.[4]

We conclude this section by discussing the efficiency properties of our competitive equilibrium. Given ϕ_0, consider a planner that solves the following maximization problem

$$P(\phi_0) = \max_{\{n_t, b_t, i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t, k_t^m\}} \int_0^\infty e^{-\rho t} \int \left[y(s, b_t, n_t) - r b_t(s) - c(i_t(s)) - c_e \psi_e \int \iota_t^{\text{in}}(s) \frac{dG(s)}{\phi_t(s)} \right] \phi_t(s) ds \text{dt}$$

such that the time-dependent analogues of the law of motion for the distribution of owners (8), labor market clearing (10), the feasibility condition (2) and the consistency of meeting probabilities (11), are all satisfied. We label this problem as *P2*. Given linear preferences, maximizing discounted welfare is the same as maximizing discounted net output. We denote a solution to *P2* as stationary if $\phi_t = \phi_0$ for all t.

In the next theorem, we show that a stationary recursive equilibrium is efficient.

Theorem 2. *A stationary recursive equilibrium as defined in Definition 1 with the stationary measure ϕ achieves $P(\phi)$ in problem P2. Furthermore, any stationary solution to P2 constitutes a stationary recursive equilibrium.*

The forces towards efficiency were foreshadowed in the formulation of the static problem *P1*. Given (ϕ, V), the optimal assignment maximizes value from trade. Beyond the static assignment, there are two additional features in problem *P2*—entry and investment—that need to be addressed. In the appendix, we show that the value of becoming an owner as well as the value of a new unit of capital to the planner coincides with the private value. Thus, private optimality with respect to entry and investment implies the competitive equilibrium allocation to be efficient.

We conclude our discussion of the efficiency properties of equilibrium by emphasizing that our framework is flexible enough to incorporate extensions likely to be relevant for private firms that may generate inefficiencies. For example, consider the case where business capital is rival. Investment in customer capital or supplier relationships may not only improve matches but also entail business stealing from other firms. A natural way to capture this feature in our setting would be to let the depreciation rate of business capital, δ_k, depend on aggregate investment. In-

[4]Our model can easily accommodate extensions in which the gains from purchasing capital depend on the productivity of the seller. Among other things, this feature could capture instances when the sale consideration includes a consulting contract with the seller in order to facilitate the transition to new ownership. See Bhandari and McGrattan (2021) for details.

tuitively, when many firms invest heavily, the returns to any one firm's past investments erode more quickly because competitors' efforts reduce their value. As a second example, consider a setting in which firms produce differentiated goods and compete with market power. Our framework can be extended to allow for size-dependent markups that give larger firms a competitive advantage. In this case, acquisitions could alter efficiency both directly, by changing the distribution of firm sizes, and indirectly, by shifting the extent of product-market power. We view both extensions as important and believe these mechanisms could be disciplined with appropriate data and quantified in future work.

Next, we describe the firm-level data used to parameterize the model.

4 Data

In this section, we describe the main datasets based on IRS administrative tax records that we use to calibrate the model.

4.1 IRS Samples

There are two main firm-level databases that we use at the IRS. The first database includes transcribed items from business tax filings for the universe of businesses that file Form 1120 as Subchapter C corporations, Form 1120-S as Subchapter S corporations, and Form 1065 as partnerships. We complement this database with the universe of electronically-filed returns that contains information on business sales recorded on IRS Form 8594 (*Asset Acquisition Statement Under Section 1060*) when there is a transfer of business assets that make up a trade or business for either the seller or the buyer.[5]

When a business is sold, the IRS must be informed about the allocation of the purchase price across different asset categories. In many cases, the buyer's basis in particular assets is determined only by the amount paid at the time of the sale. For example, values of Section 197 intangible assets such as customer bases, trade names, and goodwill are typically determined when acquired in a sale. The allocation of price across assets is relevant for the seller who must report capital gains and the buyer who may choose to amortize or depreciate the acquired assets. The seller and the buyer have opposite incentives for how to classify the assets. To minimize taxes, the seller prefers to allocate more of the purchase price to intangible assets that generate long-term capital

[5] See `https://www.irs.gov/forms-pubs` for details on these tax form.

Table 1: IRS SAMPLES

BUSINESS SAMPLES	COUNTS
S corporation population	3,167,266
S corporation sellers	105,162
Sales to S corporations	46,708
to Partnerships	33,462
to C corporations	35,792
Seller-buyer pairs	51,286
S Corporation − S Corporation	28,078
−Partnership	14,040
− C Corporation	9,168

Notes: The 'S corporation population' is drawn from the universe of S corporation filings over the period 1996–2022 and excludes any firms with wage bill under $10,000 or insufficient data for constructing a three-year growth rate in the wage bill. This set of firms is our *full sample*. The 'S corporation sellers' is drawn from asset sales recorded on Form 8594 and e-filed with the IRS. By law, both sellers and buyers are required to attach this form to their income tax returns, but the counts reported here avoid double counting if forms from both are e-filed or amended by either party. The 'Sales' counts are based on the number of sales found for the S corporation sellers and are listed by legal form of the buyer. The 'Seller-buyer pairs' are S corporate sellers found on e-filed Form 8594 that have available data on their wage bill in the year prior to the sale and buyer counterparties that have available data on their wage bill in the year after the sale. These firm pairs are our *trading sample*.

gains, while the buyer prefers to allocate more of the purchase price to tangible assets that can be depreciated or amortized over shorter periods. This suggests that the allocation of the purchase price across asset categories on Form 8594 is reliable.

In our quantitative analysis, we focus on corporate businesses that elect Subchapter S status for tax purposes and their counterparties across legal form status in business sales. Under Subchapter S, profits and losses flow through the corporation untaxed and are taxed directly as income to the owner on their individual tax forms. S corporations are now the most prevalent type of corporation in the United States, accounting for more than three-quarters of all corporate tax filings. Unlike other corporate forms, S corporations must have fewer than 100 owners—and typically have only 1 or 2—and the owners must be U.S. citizens or permanent residents. The fact that S corporations are owned by individuals is relevant for our analysis as we are interested in capital transfers between owners that actively manage their businesses. In contrast, most owners of C corporations are passive investors that rely on oversight from boards of directors. Subchapter C corporations and partnerships do not have the same ownership restrictions: C-corporate shareholders or partners can be businesses. However, we record information for these business types if they are counterparties in a sale of a S corporation.[6]

In Table 1, we record counts for three IRS data samples. The first sample is the universe of S corporations over the period 1996–2022 with $10,000 or more in wage bill. We also require that the firms in this sample have sufficient data to construct growth rates of the wage bill over a 3-year period—these rates use information for the current tax year and three years prior. We use wage bill to measure firm size because other measures such as income and profits can be easily manipulated using tax evasion strategies. As a consequence, we focus on employer firms using the cutoff of $10,000 in wage bill. With these restrictions, we have panel data for a total of 3.2 million S corporations. We refer to this set of firms as the *full sample*.

Using the full sample, we construct a second IRS sample of S corporation sellers. These are unique employment identification numbers (EINs) that can be linked to e-filers of Form 8594 that indicate they have sold a group of assets comprising a business. The database of e-filed Forms 8594 is available over the period 2005–2022. In Table 1, we record counts of sales by legal form of the buyer in which these sellers are the counterparty.

The final sample in Table 1 is our *trading sample*. The trading sample includes pairs of S corporations from the full sample and buyers found on the e-filed Forms 8594, where we restrict attention to those with available data on their business tax filings to construct measures of relative size. More specifically, we construct a sample of seller-buyer pairs for which we have information on the seller's wage bill in the year prior to the sale and the buyer's wage bill in the year after. We restrict attention to sales between employer firms and therefore also include a restriction that the buyer has a wage bill over $10,000. The trading sample constitutes only a subset of all S corporation asset sales, but contains essential moments for our calibration of production and investment technologies.[7]

Auxiliary data from brokered business sales and IRS published sources are used to inform some model moments that are not easily identified with the samples described above. In the case of brokered sales, we have a sample of 6,858 transactions by legal form from Pratt's Stats (currently DealStats) for the period 1994–2017. These data include the purchase price allocations that appear on IRS Form 8594, some pre-sale financial statistics, the business age, and details about the listing.

[6] Ideally, we would include sole proprietorships in our analysis of sellers. However, the electronically-filed database that contains Form 8594 does not include sole proprietorship filings.

[7] Not all parties that are involved in a business sale file a Form 8594. Partial information on business sales for which we have no Form 8594 should appear for the selling owners as a capital gain on Schedule D, but detailed information on the assets is not available.

Table 2: Intangible Intensities
U.S. S Corporation Trading Sample

Intangible Intensities	Percentiles 25th	50th	75th
Sales to S corporations	29.2	66.8	87.0
Partnerships	33.3	71.0	90.9
C corporations	39.3	73.4	92.7
All sales	32.1	69.3	89.3

Notes: To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table. Statistics are constructed from the subsample of firms in the trading sample drawn from asset sales recorded on e-filed Forms 8594 and linked to income tax filings.

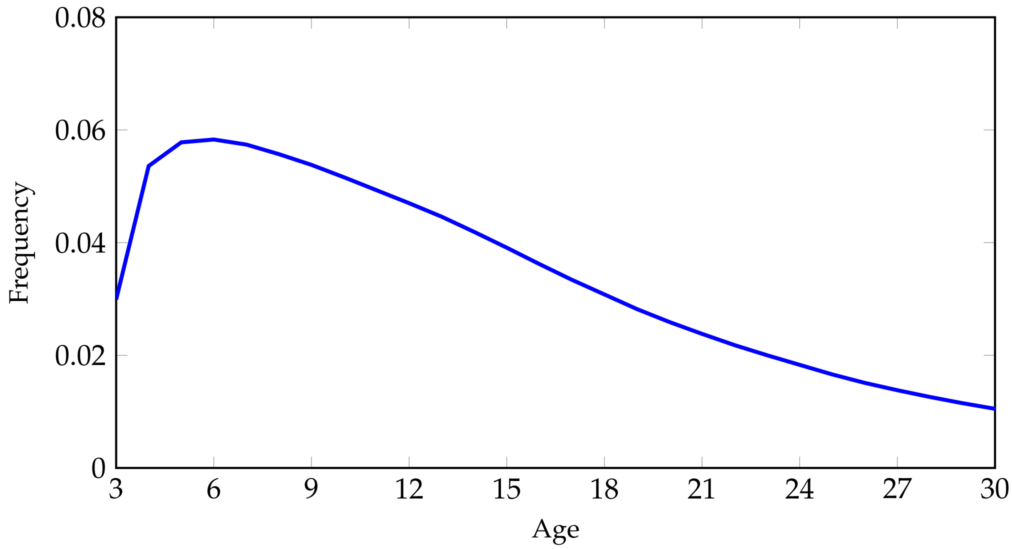
Important information in the Pratt's Stats dataset—not available in IRS filings—includes the date of listing and the date of sale. These dates are used to inform the model's time to trade. In the case of IRS published sources, we use business entity data provided by the Statistics of Income (SOI) program to measure cost shares in value added for S corporations. We use individual-level data on business owners from Bhandari et al. (forthcoming) to measure earnings of owners who switch occupations and compare them to earnings of those who do not.

4.2 Capital Measures

The literature on investment, adjustment costs, and misallocation has largely centered on physical capital in U.S. firms. This capital is relatively easy to measure using aggregate data from the fixed asset tables produced by the BEA, capital expenditures from the Annual Survey of Manufactures, and accounting data from annual 10-K filings for listed firms. In this section, we compare the relative magnitudes of two types of capital in S corporations: tangible assets such as buildings, equipment, and inventories and intangible assets such as customer relationships and goodwill.

For every tax year, the IRS's SOI program publishes aggregate data in the *Corporation Income Tax Returns Complete Report*. These data reports detailed information from the balance sheet and income statements of corporations, including S corporations that file Form 1120S. (See U.S. Internal Revenue Service (various years).) The balance sheet contains values for inventories and depreciable assets—namely, buildings and equipment—as well as the accumulated depreciation, which allow construction of net book values (historical cost less accumulated depreciation). The

Figure 1: Age Distribution
Sample of U.S. S Corporations



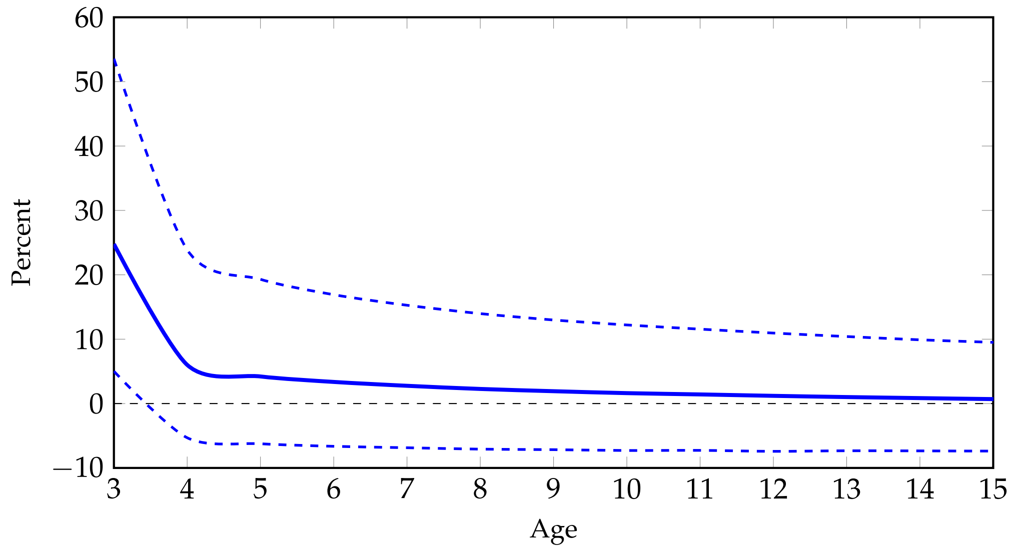
Notes: The age distribution is constructed using the universe of S corporation filings over the period 1996–2022 and excludes any firms with wage bill under $10,000 or insufficient data for constructing a three-year growth rate in the wage bill.

SOI income statements include values for business receipts and the cost of goods sold, which can be subtracted from receipts in order to estimate corporate value added. The ratio of book value to value added provides a measure of the capital-output ratio.

Using the latest available data (for year 2021), we find the ratio to be 0.36 including inventories and 0.21 excluding inventories.[8] To put this in perspective, one could compare this estimate to that of Cooley and Prescott (1995), who estimate a capital-output ratio of 3.3—the value now typically cited in the macro literature. Because Cooley and Prescott (1995) use the current-cost net stock of capital, their estimate of capital would be higher than book value from tax returns, but not by that much. To show this, we revalue historical-cost balance-sheet stocks using BEA price indexes and depreciation—to make them comparable to BEA current-cost measures—and find a current estimate of the S-corporate capital-output ratio of 0.47 (or 0.32 without inventories)—one order of magnitude lower than 3.3 or any subsequent capital-output estimates typically reported in the macro literature. We conclude that physical capital owned by S corporations, while easy to measure, is very small.

[8] If we compute this ratio using only manufacturing firms—which are the focus of many firm-level studies—we find the estimate is 0.79 including inventories and 0.38 excluding inventories.

Figure 2: DISTRIBUTION OF ANNUALIZED 3-YEAR GROWTH BY AGE
SAMPLE OF U.S. S CORPORATIONS



Notes: The growth distribution is constructed using the universe of S corporation filings over the period 1996–2022 and excludes any with wage bill under $10,000 or insufficient data for constructing a three-year growth rate in the wage bill.

Next we move to the key novelty of our data. We use transaction prices between seller-buyer pairs in our trading sample from Table 1, and construct the share allocated to Section 197 intangibles and goodwill (categorized by the IRS as Class VI and VII assets).[9] We call this share the *intangible intensity*. It includes the following intangible assets: workforce in place; business books and records, operating systems, or any other information base, process, design, pattern, know-how, formula, or similar item; any customer-based intangible; any supplier-based intangible; any license, permit, or other right granted by a government unit; any covenant not to compete entered in connection with the acquisition of an interest in a trade or a business; any franchise, trademark, or trade name; and any goodwill or going concern value.

In Table 2, we report moments of the intangible intensity distribution.[10] The median intangible share of the sale price is 69 percent of we consider all sales, but is hardly different across groups of buyers categorized by legal form.[11] We should note that the remaining assets could well be

[9] When computing the intangible share, we exclude Classes I through III that include cash and marketable securities and include only Classes IV through VII that include inventory, fixed assets, real estate, intangibles, and goodwill.

[10] Here and below, we compute percentiles as an average of observations in a window around corresponding percentile values listed in the table. This computation ensures that no confidential taxpayer information is disclosed.

[11] Bhandari and McGrattan (2021) find similar results using the Pratt's Stats database. See their appendix for more details and analysis of different subpopulations in the data.

custom capital and not easily divisible or rentable, making the shares in Table 2 lower bounds for business capital with the properties we model. For example, many fixed assets are customized for a business but appear among "fixed assets" (Class V) on Form 8594. If we were to include customized fixed assets—for example, computers with custom chips, delivery trucks with company logos, buildings with leasehold improvements, specialized kitchens—along with Section 197 intangibles, then the intangible intensities in Table 2 would be even higher. We conclude that intangible assets, while rarely observed outside of transactions, constitute most of the capital in S corporations. These assets are the main focus of our theory.

4.3 Empirical Moments

We next describe the data moments that will guide our model calibration. These moments include information on business age and growth as well as estimates of business valuation and relative size of buyers and sellers.

Using dates of business establishment, we compute a business age for each S corporation in our IRS full sample. In Figure 1, we plot the age distribution for these corporations, which starts at age three given our sample construction and peaks around age six. In Figure 2, we plot the distribution of annualized 3-year growth rates for the IRS full sample, which are constructed from wage bills. Here, we plot the 25^{th}, 50^{th}, and 75^{th} percentiles at each age. As the figure shows, the median growth is 25 percent initially and falls to 6 percent the following year. This declining age-growth profile is also documented in other studies of firm-level data (see, for example, Haltiwanger et al. (2013)). At young ages, the interquartile range of wage bill growth is roughly twice the median; it narrows as firms mature.

In Table 3, we report summary statistics for our full sample. The first rows are population moments for the data plotted in Figures 1 and 2. The interquartile range of business ages for this sample is 8 to 21. The interquartile wage growth is -7 to 12. We also report these percentiles for the log of the wage bill in the case of young firms—age-3 firms in the IRS full sample. Comparing the interquartile ranges, we do not find large differences between the statistics for the age-3 firms and those for all ages of the population. We see in Figure Figure 2 that the growth is rapid only in the few years following the establishment of the business, and thus we should not be surprised to find small differences between young firms and the remaining population.

In Table 4, we report key statistics from our IRS trading sample. In the top panel of the table, we report valuation multiples, which are computed as the ratio of the total price paid for a group of

Table 3: SUMMARY STATISTICS
SAMPLE OF U.S. S CORPORATIONS

STATISTIC	PERCENTILES 25th	50th	75th
Business Age	8.0	13.0	21.0
Wage Growth	−6.8	1.4	11.6
Log Wage Bill: Entrants	11.0	11.7	12.5
Population	11.1	11.9	12.8

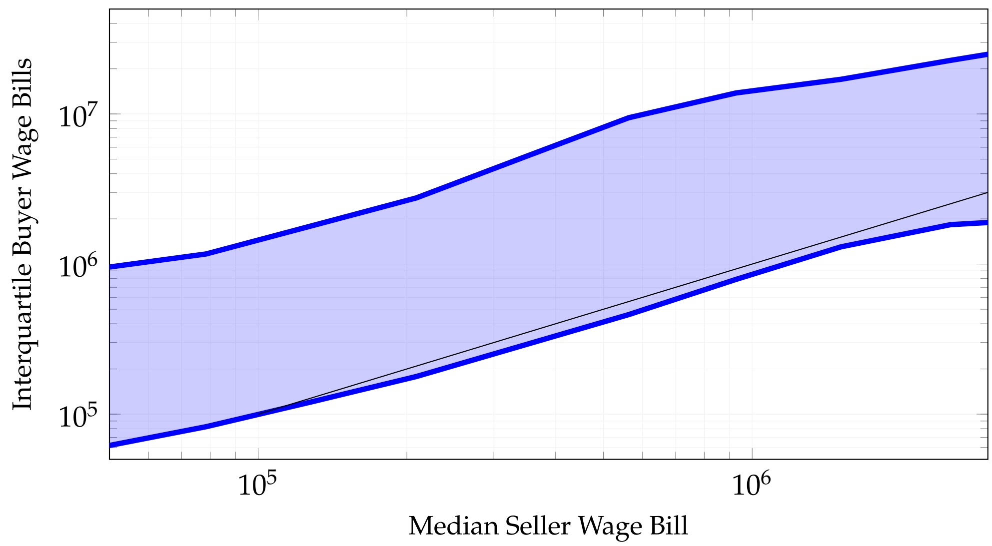
Notes: These statistics are based on the universe of S corporation filings over the period 1996–2022 that excludes any firm with a wage bill under $10,000 or insufficient data for constructing a three-year growth rate in the wage bill. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table.

Table 4: SUMMARY STATISTICS
SAMPLE OF U.S. S CORPORATION SELLERS

STATISTIC	PERCENTILES 25th	50th	75th
Valuation Multiples			
Sales to S corporations	1.0	2.4	5.2
Partnerships	1.4	3.5	8.6
C corporations	1.5	4.0	9.9
All sales	1.2	2.9	6.7
Relative Wage Bill Sizes			
Sales to S corporations	0.7	1.4	5.6
Partnerships	1.0	2.8	17.4
C corporations	2.2	14.9	130.7
All sales	0.9	2.1	13.5

Notes: Statistics are constructed from the subsample of firms drawn from asset sales recorded on e-filed Forms 8594. The "valuation multiple" is the ratio of firm sale price to the seller's wage bill in the year prior to the sale. The "relative wage bill size" is the ratio of the buyer's wage bill in the year after the sale to the seller's wage bill in the year prior to the sale. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the table.

Figure 3: Buyer and Seller Wage Bills by Seller Size



Notes: The sellers are S corporations in the IRS trading sample. Their wage bills one year prior to the sale of the business are assigned to 10 bins and medians of each bin are used for the figure's x-axis. The y-axis is the interquartile range of wage bills in the year after the sale for buyers who were counterparties on Form 8594 to sellers assigned to the bin. To ensure that no confidential information is disclosed, reported percentiles are computed as an average of observations around the value listed in the figure.

business assets divided by the wage bill of the seller in the year before the sale. We report sales to S corporations, C corporations, and partnerships separately since C corporations and partnerships owned by other business entities tend to be larger in size. Roughly half of the counterparties in sales involving S corporations are S corporations themselves. For the median sales across the three categories of exchange, we find a range of valuation multiples of 2.4 to 4 times the seller's wage bill.

In the bottom panel of Table 4, we report statistics for a measure of relative size: the ratio of the wage bill of the buyer a year after the sale to the wage bill of the seller a year before the sale. As compared to valuation multiples, we find much more heterogeneity in relative sizes. For our sample of S corporations, the median size ratios across the different legal form categories range from 1.4 where the buyers are S corporations to 15 where the buyers are larger C corporations.[12]

Because there is a wide range of values for the relative wage bill size by legal form, we also report information on this key statistic by the size of the S corporation seller. More specifically, we

[12]When parameterizing the model, we use a value-weighted estimate for sales to S corporations and partnerships rather than the equal-weighted "all sales" measure to avoid having the estimates dominated by relatively small sales of large publicly-traded companies.

take the log of the wage bills for all sellers in the IRS trading sample and assign them to 10 bins. For each bin, we compute the median wage bill of the seller and the interquartile wage bills of the buyers who are counterparties to the sales reported on Form 8594. In Figure 3, we plot these results on a log scale. The lower bound of the shaded region is the 25^{th} percentile for the buyer wage bills, which is very close to the 45-degree line. This means that most buyers are larger in size than the sellers. The 75^{th} percentile is about one order of magnitude higher—or 10 times. A clear pattern emerges in that the ratio is nearly constant across the seller's size distribution.

5 Calibration

In this section, we parameterize the model using the data described in Section 4 and show that the model fits the data well.

5.1 Parameter Estimates

In Table 5, we report the model parameters. The reporting period for tax filing is annual and therefore we choose a discount rate ρ of 5 percent. The rate of new births and deaths is ψ_e, which is set equal to $1/40$ to represent an average working life of 40 years. The parameter ψ_o governing the option to switch occupations is set to 1, so workers and owners can switch occupations once per year on average. Entry into self-employment comes at cost c_e, which is roughly 2.5 times the annual wage and less than a year of average profits to the owner. This cost estimate is informed by the fraction of individuals that choose self-employment versus paid-employment.

Parameters governing the entrant distribution $G(s)$ and the stochastic process for z are listed next in Table 5. On entry, $k = 0$ and productivity is drawn from a shifted log normal distribution with mean and standard deviation equal to -0.5 and 0.5, respectively. For the post-entry productivity process, we again work with the logarithm of z that we denote using $\hat{z}$. We assume the process is given by:

$$d\hat{z} = \theta(\hat{z}^* - \hat{z})dt + \sigma d\mathcal{W},$$

with θ and σ both equal to 0.1 and $\hat{z}^*$ normalized to one. As is standard in models of firm dynamics, the moments that motivate our choices are the age distribution of firms, the annualized three-year growth rate of firm wage bills, and the distribution of the log wage bills for young firms and the whole population.

Table 5: MODEL PARAMETERS

PARAMETER	EXPRESSION	VALUE
Discount rate	ρ	0.05
Entry and exit		
Birth and death rate	ψ_e	0.025
Occupation choice rate	ψ_o	1.0
Entry cost	c_e	4.0
Initial productivity mean	μ_0	-0.5
Initial productivity standard deviation	σ_0	0.5
Non-transferable productivity		
Mean	$\hat{z}^*$	1.0
Mean reversion	θ	0.1
Standard deviation	σ	0.1
Transferable capital trading rate	η	2.17
Transferable capital investment		
Scale of investment cost	A	1833
Elasticity of investment cost	χ	1.6
Production shares		
Transferable capital share	α	0.125
Rentable capital share	β	0.35
Labor share	γ	0.35
Depreciation rates		
Transferable business capital	δ_k	0.1
Rentable capital	δ_b	0.1

Notes: The production function is given by $y = zk^{\alpha}b^{\beta}n^{\gamma}$. The non-transferable productivity process is given by $d\log z = \theta(z^* - z)dt + \sigma d\mathcal{W}$. The cost function for capital investment is $c(i) = Ai^{1+\chi}/(1+\chi)$. The capital accumulation of transferable capital is governed by $dk = (i - \delta_k k)dt$. The initial productivity is drawn from a shifted lognormal distribution $LN(\mu_0, \sigma_0)$.

The next set of parameters in Table 5 relate to transferable capital trade and investment. In the baseline, we set $\eta = 2.17$, which is consistent with a trading opportunity occurring roughly twice per year. This estimate is based on the median time reported by Pratt's Stats between the date the business is listed and the date of the sale. This is a conservative estimate if owners require any additional time to prepare for the listing. Since this is an important parameter for firm dynamics, we also analyze cases with η equal to zero—corresponding to the no-trade case—and higher and lower frequencies, monthly and annual.

The nature of intangible assets poses a challenge for calibrating the capital elasticity of output,

α, and the investment cost function, which we parameterize as $c(i) = Ai^{1+\chi}/(1+\chi)$. The standard approach to estimating α uses production function methods that relate value added per worker to capital per worker (see Olley and Pakes (1996)). Estimation of investment technologies, in turn, relies on moments of the investment process to identify adjustment cost parameters (see Cooper and Haltiwanger (2006) and subsequent work). Both approaches require detailed measures of capital stocks and investment expenditures, which are unavailable for most forms of capital in private businesses. These methods are therefore not applicable in our context.

We propose an alternative identification strategy that combines our modeling of indivisible asset sales in bilateral meetings with key observations from IRS data: the sales price and the wage bills of buyers and sellers, which we use as proxies for their size. In our framework, higher investment costs and higher output elasticities both imply higher prices per unit of capital. Since we do not observe capital quantities directly, we construct a proxy for the per-unit price using a valuation multiple, defined as the total sales price relative to the seller's wage bill in the year prior to the sale. Intuitively, if internal investment is more costly, buyers are willing to pay more to expand through capital acquisitions. Likewise, when the capital elasticity of output is higher, buyers can deploy capital more productively, raising their willingness to pay. The model also predicts a systematic link between the relative size of buyers and sellers and the capital elasticity of output. When α is low, the gains from reallocating capital are concentrated in matches between large and small firms, so buyers tend to be much larger than sellers. As α rises and production approaches constant returns, even small differences in owner productivity generate large gains from trade. This narrows the buyer-seller size gap. We exploit this prediction by mapping relative size in the model to its empirical counterpart observed in our trading sample.

The remaining parameters in Table 5 are production shares for external factors and depreciation rates. Values for production shares are informed by revenue shares reported in U.S. Internal Revenue Service (various years) separately for S corporations. (See, for example, Table 6.1 in the most recent issue.) Values for depreciation rates are informed by capital obsolescence studies conducted by the BEA and service lives of intangibles reported by U.S. General Accounting Office (1991).

5.2 Model Fit and Validation

In Table 6, we compare the key moments from the data to counterparts in our model. The model does well in accounting for the fraction of owners, mean and median business age, annualized

Table 6: FIT OF THE MODEL

MOMENT	MODEL	DATA
Fraction of owners	0.16	0.18
Business age		
Median	8.9	10.0
Mean	11.8	12.0
Annualized growth rates		
Age 3, Median	0.27	0.25
Age 3, IQR	0.48	0.49
Age 10, Median	0.03	0.02
Age 10, IQR	0.27	0.20
Log wage bill		
Population, IQR	1.60	1.70
Entrants, IQR	1.14	1.50
Valuation multiples		
25^{th} percentile	3.86	1.27
50^{th}	4.25	3.13
75^{th}	4.75	7.47
Relative size		
25^{th} percentile	1.84	0.90
50^{th}	2.68	2.33
75^{th}	3.84	13.5

Notes: Growth rates are annualized over three years. The IQR is the interquartile range for the statistic noted. The U.S. valuation multiple and relative size estimates are based on value-weighted statistics for S corporation sales to counterparties that are S corporations or partnerships.

growth rates across the age distribution, and the dispersion in the log of the wage bill of the population of businesses. Dispersion in the log of the wage bill of the entrants, on the other hand, is lower in the model despite the fact that we do well in accounting for wage growth across ages. Part of the difference may be due to the fact that we start all entrants with $k = 0$. There is also more heterogeneity in valuation multiples and relative sizes than in the model. For these statistics, we targeted the median ratios and avoided adding any extra shocks or differences across firms that can potentially fit the data without changing key mechanisms in the model.

Beyond the moments we target in Table 6, the model delivers several features about trading patterns that are consistent with the data. First, a key implication of our theory is selection into selling. To validate this prediction, we compare the earnings of owners who exit relative to those who continue operating. The median ratio in our baseline is 52 percent—an estimate we can validate against data on owners in Bhandari et al. (forthcoming) who switch out of self-employment.

Second, the model predicts that indivisible trades generate systematic patterns in prices and counterparties. Larger businesses are more likely to be sold at a discount and purchased by larger buyers. In the model, this appears as a per-unit price, $\mathcal{P}(k)/k$, that declines with the size of the capital being traded (see Figure 6 in Appendix C), while the relative size of buyers and sellers remains roughly constant conditional on seller size. Empirically, we observe the same patterns: valuation-to-wage-bill multiples decline with firm size, while the relative size of buyers and sellers is approximately constant across seller size in our transaction data.

More broadly, our parsimonious investment and trading technology delivers a rich set of firm-level capital dynamics. In particular, it generates all major types of capital adjustments: large negative adjustments through sales, small negative adjustments through depreciation, small positive adjustments through incremental investment, and large positive adjustments through purchases. That such a diverse range of behaviors emerges from a relatively simple structure highlights the flexibility of our framework, even though some of these adjustment patterns cannot be directly observed for the type of business capital we study.

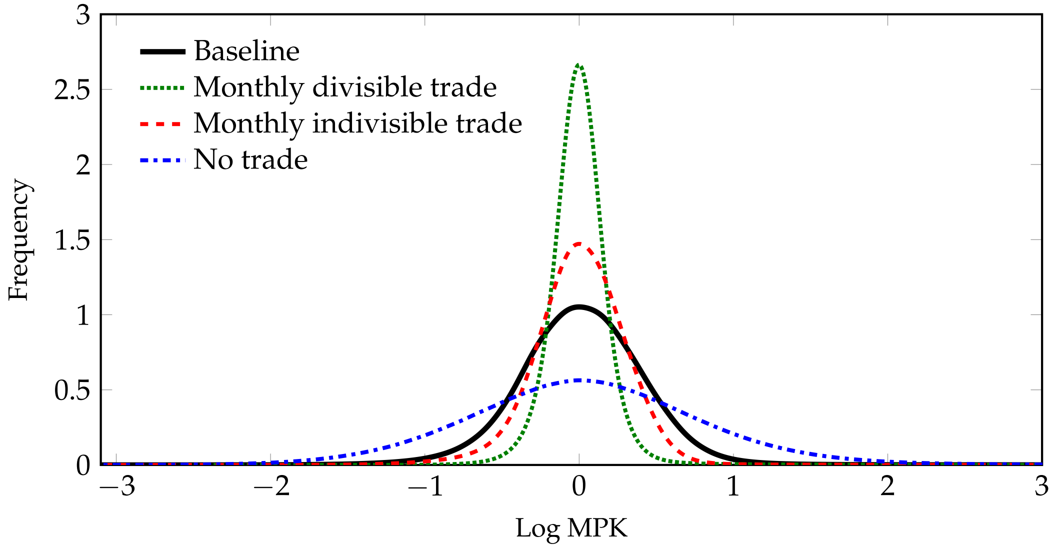
6 Model Predictions

In this section, we report on key model predictions that have no counterpart in U.S. data on private businesses but are relevant for our policy analysis. The first is the dispersion in the marginal product of capital, which is a central statistic in the literature on capital misallocation. We compute this dispersion in our baseline and show how it changes as we vary the time to trade and allow for perfect divisibility of capital. The second set of statistics relate to private business wealth, on which partial information is typically available only when transferred, but central in the literature on wealth inequality. We report the model's predictions for income yields, the share of value that is transferable, and estimates of total private value to output in private business.

6.1 Dispersion in Marginal Product of Capital

In a neoclassical benchmark economy with continuously accessible rental markets and perfectly divisible capital, the marginal products of capital, $\alpha y(s)/k(s)$, are equalized across firms. Because we depart from those features, our model will generate dispersion in marginal products. We plot

Figure 4: PREDICTED DISTRIBUTIONS OF LOG MPK
VARYING DIVISIBILITY AND TIME TO TRADE



Notes: MPK is the marginal product of capital given by $\alpha y(s)/k(s)$. The distributions of log MPK are constructed using the universe of model corporations with sufficient data to construct a three-year growth rate in wage bill. In the baseline model, we assume indivisible trade with $\eta = 2.2$. Monthly trade assumes $\eta = 12$. No trade assumes $\eta = 0$.

the distribution of model-generated $\alpha y(s)/k(s)$ in Figure 4 for the baseline parameterization and for alternative cases to elicit the role of our departures from the neoclassical benchmark.

Starting with the baseline parameterization (the solid black line in Figure 4), we find significant heterogeneity across firms. The distribution has a standard deviation in logs of 40 percent. There are two key features of the model driving our measures of dispersion: time to trade and indivisibility in capital exchange. The first limits the speed at which firms can access the capital market, while indivisibility constrains the set of feasible capital allocations within each trading pair.

In Figure 4, we plot results for alternative economies to highlight the role of trading frequency and indivisibility. Consider the trade frequency first. In the baseline calibration, we set η equal to 2.2 to replicate a trading frequency of 168 days based on the Pratt's Stats listings. To see how the dispersion changes as we vary this statistic, we consider two alternatives: an average trading frequency of once per month ($\eta = 12$) and no trade at all ($\eta = 0$). The results are shown alongside the baseline in Figure 4 (and labeled "Monthly indivisible trade" and "No trade," respectively). Without trade, the standard deviation is higher—roughly 77 percent. Somewhat more surprising is that with very frequent trade, in this case monthly, the standard deviation is still high—roughly 30 percent.[13]

Next, we investigate the role of indivisibility by considering a version of the model in which firms can trade any amount of capital with each other given an opportunity to trade. In practice, this implies that the marginal value of capital is equalized within each trading pair. The rest of the trading protocol is left unchanged, so that trading is still bilateral and at stochastic intervals. We find that indivisibility does not significantly alter capital allocation at the baseline trading frequencies. Having a better allocation of capital within the pair is not important when trade is rare. This point is starkest at $\eta = 0$, in which case divisibility is irrelevant. If trading opportunities are frequent, then divisibility becomes salient: shrinking firms are able to downsize their capital holdings smoothly and expanding firms are less concerned with hedging against the risk of future negative productivity shocks.

The result for the version of the model with divisible capital and monthly trade ($\eta = 12$) is shown in Figure 4 (and labeled "Monthly divisible trade"). In this case, the standard deviation of the log of the marginal product of capital is roughly 18 percent. The hallmark of such a reduction in marginal product dispersion is the emergence of the law of one price (per unit of capital). We document this reduction using histograms of per-unit prices for the baseline parameterization and the divisible capital case in Figure 5 in the Appendix C.

Our findings on the dispersion in the marginal product of capital cannot be directly compared to studies in the literature—as no such empirical measure exists—but should serve as a theory-guided reference in the case of non-rentable, indivisible capital in private businesses. The literature has primarily analyzed data on plant and equipment from Annual Survey of Manufactures (see, most notably, Cooper and Haltiwanger (2006) and Hsieh and Klenow (2009)) and from accounting data of publicly-traded firms (see, most notably, David and Venkateswaran (2019) and David et al. (2022)). The dispersion estimates in these studies are typically larger than ours—generally ranging between 60 and 100 percent, depending on the time frame, firm sample, and type of capital—prompting policy debates aimed at addressing capital misallocation.[14] Our findings offer a more benign view of the dispersion in marginal product of capital, and suggest caution in the design of such policies.

[13] The standard deviation of the log of the marginal product of capital with annual trading is 48 percent.

[14] In their study of plant and equipment in U.S. manufacturing plants, Asker et al. (2014) claim that adjustment costs are sufficient to account for observed dispersion in marginal products of capital. We view our contribution as providing a theory-guided counterpart for the question posed by Asker et al. (2014) in the case of private business capital adjustment.

Table 7: PREDICTED INCOME YIELDS AND TRANSFERABLE SHARES

STATISTIC	INCOME YIELD	TRANSFERABLE SHARE
Percentiles: 5^{th}	2.1	6.8
10^{th}	2.4	7.4
25^{th}	3.5	9.8
50^{th}	5.4	14.1
75^{th}	8.1	19.7
90^{th}	11.8	29.6
95^{th}	14.8	39.1
Average	6.4	16.8
Aggregate	11.3	21.7

Notes: The income yield is the ratio of owner income, $y(s) - wn(s) - rb(s) - c(i(s))$, to business value $V(s)$. The transferable share is the ratio of the transferable value $\mathcal{P}(k(s))$ to the total value $V(s)$. Statistics related to the distribution are are reported as well as the ratios of economy-wide aggregates.

6.2 Dispersion in Returns to Business Wealth

The model we work with has two concepts of business wealth. The first is the present discounted value of owner dividends, $V(s)$, which captures returns to both transferable capital k and non-transferable capital z. More familiarly, this value can be interpreted as the private-business counterpart of a stock price for shares of a publicly-traded corporation. We use these values to estimate variation in business returns. The second measure of business wealth is often reported in surveys of consumer finances that ask respondents to estimate the price of their business if it were sold today. This measure in our model is the price of transferable capital, $\mathcal{P}(k(s))$. We use these values to estimate variation in transferable shares of private business wealth and later as inputs when comparing the effects of taxing businesses.

In Table 7, we report distributional statistics for income yields and transferable shares. The *income yield* is a common measure of the return to business and is given by the ratio of owner income $(1 - \beta - \gamma)y(s) - c(i(s))$ to business value $V(s)$. As in the case of marginal products of capital, we find significant heterogeneity in income yields, with estimates ranging from 2.1 percent at the 5^{th} percentile of the distribution to 14.8 percent at the 95^{th}. Comparing these results to U.S. publicly-traded companies, we find similar median and mean yields, but our estimates indicate that there is much less dispersion in private business yields than in those of publicly-traded firms. For example, Bhandari et al. (2020) compute yields ranging from -5.3 at the 25$^{\text{th}}$ to 10.4 at the 75$^{\text{th}}$ for publicly-traded businesses in the CRSP-Compustat database.[15]

The second column of Table 7 shows the *transferable share*, $\mathcal{P}(k(s))/V(s)$. As with income yields, we find significant heterogeneity in shares. At the 5$^{\text{th}}$ percentile, the value of transferable capital is equal to 6.8 percent times the total business value, and at the 95$^{\text{th}}$ percentile, the ratio is 39.1 percent. If we compute the equal- or value-weighted shares, we find 16.8 percent and 21.7 percent, respectively.

Some studies impute business wealth by capitalizing incomes (see, for example, Piketty et al. (2018)). The Flow of Funds, in contrast, imputes the value of closely-held businesses by scaling their SOI book values or revenues using publicly-traded "comparables" and applying a 25 percent discount for illiquidity. Neither approach is well suited to our context since our model generates substantial heterogeneity in income yields and in the measures of transferable shares as shown in Table 7. We therefore construct model-analogous valuations that are internally consistent and yield aggregate measures of business wealth relative to private output, which we compare to existing estimates.

For transferable capital, the total value is given by $\int \mathcal{P}(k(s))\phi(s)ds$, which we estimate to be 0.58 times private output. It follows from the aggregate value in Table 7 that the total private business wealth, $\int V(s)\phi(s)ds$, is estimated at 2.66 times private business output. We can compare this estimate to publicly-listed companies. To do that, we construct the market value of listed firms using the CRSP-Compustat database. To measure the corresponding value added of these firms, we start from their reported sales and apply industry- and year-specific ratios of value added to gross output from the BEA. This procedure imputes a BEA-consistent notion of value added for the publicly-listed sector. Comparing equity market value to this BEA-adjusted value added, we find that the ratio ranges between 1.69 in 2008 and 4.48 in 2020, with an average over the 2000–2020 period of 2.73—only slightly higher than our estimate for private businesses.

[15] Fagereng et al. (2020) and Boar et al. (2022) attempt to measure returns to private businesses using firm-level capital stocks available in their datasets. These stocks are book values and do not include the self-created intangible assets that constitute most of the value of the firms.

7 Tax Policy Analysis

Our analysis of capital trade, valuations, returns, and marginal products provides key inputs to the active public debate on how to tax businesses, particularly regarding whether to tax business income, wealth, or capital gains. In this section, we situate our model in the context of the literature on capital taxation and report the implications of different tax instruments.

In standard models with perfect financial markets, the classical result of Atkinson and Stiglitz (1976) on uniform commodity taxation implies a zero tax rate on the value of capital or its returns. Taxing capital introduces an intertemporal wedge, effectively taxing consumption at different dates by different rates. In the context of business taxation, this insight underpins proposals to tax only distributions—business income net of investment costs—as a way to raise revenue without creating distortions. In our framework, applying the Atkinson-Stiglitz logic would require allowing deductions for both entry costs and investment costs. This is impractical because part of these costs reflects the opportunity cost of the owner's time. This limitation motivates our exploration of alternative, second-best approaches to taxing businesses; the most natural candidates are taxes on business income, business value, and capital gains.

The empirical evidence on dispersion in returns to wealth and in marginal products of capital has motivated a growing literature on firm heterogeneity and imperfect financial markets. In such environments, Guvenen et al. (2023) emphasize the distinction between taxing financial wealth and taxing the return on financial wealth, and argue in favor of taxing stocks rather than flows. As discussed in Section 6, our model generates persistent heterogeneity in capital returns but from very different mechanisms. This allows us to revisit the Guvenen et al. (2023) insights in the context of business taxation and the relative merits of taxing capital values versus capital returns.

Because there has been little theoretical work on capital gains taxation in the context of business capital transfers, the implications are not well understood.[16] The standard treatments in public finance textbooks focus on corporate equity in settings where ownership shares are freely tradable (see, for example, Atkinson and Stiglitz (2015)). In that environment, taxes on distributions or capital gains affect valuations but leave investment decisions unchanged, reflecting the dichotomy between capital use and capital ownership. Our framework departs from this view: when capital is indivisible and its value is tied to the productivity of its owner, this dichotomy breaks down and capital gains taxation directly distorts investment. This motivates studying cap-

[16] As discussed in Section 1.1, the limited theoretical work that does address transfers—such as Chari et al. (2003) and Cavalcanti and Erosa (2007) within the Holmes and Schmitz (1990) framework—provides a starting point.

Table 8: PREDICTED TAX POLICY CHANGES

STATISTIC	BUSINESS INCOME	CAPITAL VALUE	CAPITAL GAINS
Mass of firms	1.8	−2.0	−6.5
Fraction traded	0.7	−1.3	−48.1
Average investment	0.3	0.3	−3.2
Dispersion in MPK	−0.4	−0.4	22.0
Per-unit price	0.2	1.0	7.7
Wage	−0.4	−2.9	−3.0

Notes: The table reports percent changes in equilibrium values in response to the tax change. The mass of firms is the stationary value of m in equation (9). The fraction traded is the amount of capital $k(s)$ transferred in the period relative to economy-wide capital stock. The average investment is the average value for $i(s)$. The dispersion in marginal product of capital is the standard deviation of the log of the marginal product of capital, $\alpha y(s)/k(s)$. The per-unit price is the average of $\mathcal{P}(k(s))/k(s)$.

ital gains taxation in our setup. We then compare the elasticities implied by the model with those estimated in the empirical public finance literature.

7.1 Comparing Tax Instruments

In this section, we compare predictions of the model when taxing business income, capital values, and capital gains. More specifically, we consider the problem of a government that wants to raise a certain amount of revenue using either linear taxes on owner income ($y(s) - wn(s) - rb(s)$), the value of the transferable capital ($\mathcal{P}(k(s))$) each period—assuming the government is able to assess the value of the business assets—or the realized gains after a business sells.[17]

For all three tax experiments, we raise revenue equal to 1.2 percent of output in the baseline economy and compare aggregate outcomes relative to a no-tax baseline. The tax rates on income, capital, and capital gains needed to raise this sum are 4.1 percent, 2.4 percent, and 20 percent, respectively. The impacts of interest are the changes in firm entry; amounts of capital traded; own investment in the business; dispersion in marginal products of capital; per-unit prices; output, and wages.

[17] For the tax on capital value, we assume that the IRS could use financial data and valuations ("comps") from recent sales of businesses that are in the same industry and of similar size. In principle, one could also consider taxing the total value of the business $V(s)$, but we view the implementation challenges for such a tax to be so severe that we focus on the more feasible case of taxing the transferable value. For the tax on capital gains, we assume that the basis is zero, which is in line with the U.S. tax treatment of self-created intangible assets.

All taxes distort entry, investment, and capital reallocation, but they differ in whom they primarily affect. Starting with impacts on entry shown in the first row of Table 8, we see that taxes on owner net income or capital value have more modest effects on the mass of firms than the tax on capital gains, which triggers a relatively large drop in entry equal to -6.5 percent. The tax on business income does not deter entry, as the high-productivity owners affected by this tax are less elastic in their entry decision. This contrasts with capital gains taxes which affect marginal owners for whom the option of selling is a significant part of the value of entry. Taxes on capital do deter entry, and the incidence falls mainly on medium- to low-productivity owners.

In the second and third rows of Table 8, we report results for the fraction of capital traded and capital investment. We find a dramatic decline in the fraction of capital traded when we impose a capital gains tax: on the order of -48.1 percent. This decline is driven by a "lock-in" effect: capital remains with less productive owners due to discouraged trade.

Turning to investment outcomes, we find that both types of capital taxation reduce total investment. However, average investment is little changed under a tax on capital value or income, whereas it falls by -3.2 percent under a capital gains tax. In contrast to capital gains, the tax base for business income or capital value is broader, so the burden is spread across many owners and the distortions are smaller. A further offsetting general equilibrium channel is also present: when capital prices rise, productive owners substitute toward investment, partially mitigating the decline.

As with investment, the dispersion in the marginal product of capital—shown in the fourth row of Table 8—is most notably impacted by the tax on capital gains. Dispersion is higher by 22.0 percent. A higher tax rate on capital gains leads to a collapse in the trading market and, as we showed in Section 6.1, when trade is shut down, the standard deviation of the log of the marginal product of capital rises. In response to fewer trades, pre-tax prices of capital rise by 7.7 percent. This implies that sellers face 60 percent of the economic incidence of the tax and the rest is borne by buyers.

The final row of Table 8 reports the impacts on wages. The wage falls in all cases, although there is a stark difference between taxes on income and taxes on capital. The wage loss with an increase in the income tax rate is -0.4 percent as compared to -3.0 percent for capital gains and similarly for capital value. These results concisely summarize the distortive nature of taxing capital in a rich model environment with entry, investment, and trading decisions on the part of business owners. Given convex investment costs, it is relatively efficient to tax income and avoid taxing entry and the investment of marginal entrants who build small businesses, which are eventually sold to productive owners who can buy their way to optimal scale.

Interestingly, these results stand in contrast to Guvenen et al. (2023), who argue for taxes on business wealth rather than business income. In their environment, fiscal policy is used to redistribute wealth from owners with low productivity to owners with high productivity. This redistribution ameliorates the misallocation of capital in their economy with imperfect financial markets. Here, any rectification of "mismatched" z and k across owners is achieved through business transfers because it would be impossible for the government to move customer bases and trademarks from the less productive owners to their more productive peers.

7.2 Comparing Tax Elasticities on Capital Gains

Given the distortive effects of capital gains taxes, it is natural to ask whether the model-implied elasticities are in line with empirical estimates. While direct data on the elasticity of business transfers in response to changes in capital gains taxes is unavailable, we can compare our model-implied elasticity estimates to empirical findings on the responsiveness of all capital gains to changes in capital gains tax rates. Using state-level variation in capital gains tax rates, Gentry and Bakija (2014) and Agersnap and Zidar (2021) estimate these elasticities to be in the range of -0.3 to -0.66 depending on the horizon of the reform.[18] The Joint Committee on Taxation (JCT) and the U.S. Treasury use higher elasticity estimates of -0.7 and -1, respectively. In our baseline calibration, we compute a long-run elasticity of -0.38, which aligns well with the estimate of -0.41 of Agersnap and Zidar (2021) in the case of reform horizons that are between 6 and 10 years.

8 Conclusion

We develop a theory of firm dynamics in light of new evidence that significant value in the business sector derives from owner-created intangible assets such as customer bases, trademarks, and going-concern value. We use the theory to study firm dynamics, business wealth, and business taxation. With parameters calibrated to IRS administrative tax data, the model predicts substantial dispersion in marginal products of capital, returns to business wealth, and heterogeneity in

[18] Summers et al. (2022) use the same elasticities when making the case to increase U.S. tax rates on capital gains, but focus only on the revenue-raising potential of such a reform.

transferable capital shares. Comparisons of business income taxes with alternative wealth-based taxes reveal a clear ranking: income taxation is preferred to wealth or capital gains taxation.

We made simplifying assumptions to keep the theory and measurement transparent; these can be relaxed in future work. Our analysis focuses on the United States because of the rich data available on business transfers, but we doubt that U.S. lawyers, doctors, contractors, and other private business owners are unique in their ability to build value in their businesses. Much as models of firm dynamics have shaped the study of productivity and capital allocation in manufacturing, our framework—augmented with features relevant for developing economies—offers a natural tool for analyzing the role of private businesses in economic development beyond manufacturing.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," *Review of Economic Studies*, 2021, *89* (1), 45–86.

Agersnap, Ole and Owen Zidar, "The Tax Elasticity of Capital Gains and Revenue-Maximizing Rates," *American Economic Review: Insights*, 2021, *3* (4), 399–416.

Aguiar, Mark, Benjamin Moll, and Florian Scheuer, "Putting the 'Finance' into 'Public Finance': A Theory of Capital Gains Taxation," 2025. Working Paper, Princeton University.

Asker, John, Allan Collard-Wexler, and Jan De Loecker, "Dynamic Inputs and Resource (Mis)allocation," *Journal of Political Economy*, 2014, *122* (5), 1013–1063.

Atkinson, Anthony B. and Joseph E. Stiglitz, "The Design of Tax Structure: Direct versus Indirect Taxation," *Journal of Public Economics*, 1976, *6* (1-2), 55–75.

— **and** — , *Lectures on Public Economics: Updated Edition*, Princeton, NJ: Princeton University Press, 2015.

Baley, Isaac and Andres Blanco, "Aggregate Dynamics in Lumpy Economies," *Econometrica*, 2021, *89* (3), 1235–1264.

Bhandari, Anmol and Ellen R. McGrattan, "Sweat Equity in US Private Business," *Quarterly Journal of Economics*, 2021, *136* (2), 727–781.

— **, Serdar Birinci, Ellen McGrattan, and Kurt See**, "What Do Survey Data Tell Us about U.S. Businesses?," *American Economic Review: Insights*, 2020, *2* (4), 443–458.

— **, Tobey Kass, Thomas May, Ellen McGrattan, and Evan Schulz**, "On the Nature of Entrepreneurship," *Journal of Political Economy*, forthcoming.

Boar, Corina, Denis Gorea, and Virgiliu Midrigan, "Why Are Returns to Private Business Wealth So Dispersed?," 2022. NBER Working Paper, 29705.

Cagetti, Marco and Mariacristina De Nardi, "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, *114* (5), 835–870.

Cavalcanti, Ricardo and Andres Erosa, "A Theory of Capital Gains Taxation and Business Turnover," *Economic Theory*, 2007, *32*, 477–496.

Chari, V.V., Mikhail Golosov, and Aleh Tsyvinski, "Business Start-ups, the Lock-in Effect, and Capital Gains Taxation," 2003. Working Paper, University of Minnesota.

Cooley, Thomas F. and Edward C. Prescott, "Economic Growth and Business Cycles," in T.F. Cooley, ed., *Frontiers of Business Cycle Research*, Princeton, NJ: Princeton University Press, 1995, pp. 1–38.

Cooper, Russell and John Haltiwanger, "On the Nature of Capital Adjustment Costs," *Review of Economic Studies*, 2006, *73* (3), 611–633.

Crouzet, Nicolas and Janice Eberly, "Rents and Intangible Capital: A Q+ Framework," *Journal of Finance*, 2023, *78* (4), 1873–1916.

David, Joel, Lukas Schmid, and David Zeke, "Risk-Adjusted Capital Allocation and Misallocation," *Journal of Financial Economics*, 2022, *145* (3), 684–705.

David, Joel M., "The Aggregate Implications of Mergers and Acquisitions," *Review of Economic Studies*, 2021, *88* (4), 1796–1830.

_ **and Venky Venkateswaran**, "The Sources of Capital Misallocation," *American Economic Review*, 2019, *109* (7), 2531–2567.

Fagereng, Andreas, Luigi Guiso, Davide Malacrino, and Luigi Pistaferri, "Heterogeneity and Persistence in Returns to Wealth," *Econometrica*, 2020, *88* (1), 115–170.

Gaillard, Alexandre and Sumudu Kankanamge, "Buying and Selling Entrepreneurial Assets," 2020. Working Paper, Brown University.

Galichon, Alfred, *Optimal Transport Methods in Economics*, Princeton University Press, 2016.

_ **, Scott Duke Kominers, and Simon Weber**, "Costly Concessions: An Empirical Framework for Matching with Imperfectly Transferable Utility," *Journal of Political Economy*, 2019, *127* (6), 2875–2925.

Gavazza, Alessandro, "An Empirical Equilibrium Model of a Decentralized Asset Market," *Econometrica*, 2016, *84* (5), 1755–1798.

Gentry, William M. and Jon M. Bakija, "Capital Gains Taxes and Realizations: Evidence from a Long Panel of State-Level Data," 2014. Working Paper, Williams College.

Gomez, Matthieu and Emilien Gouin-Bonenfant, "Inelastic Capital in Intangible Economies," 2025. Working paper, Columbia University.

Guntin, Rafael and Federico Kochen, "Financial Frictions and the Market for Firms," 2024. Working Paper, University of Rochester.

Guvenen, Fatih, Gueorgui Kambourov, Burhan Kuruscu, Sergio Ocampo, and Daphne Chen, "Use It or Lose It: Efficiency and Redistributional Effects of Wealth Taxation," *The Quarterly Journal of Economics*, 2023, *138* (2), 835–894.

Haltiwanger, John, Ron S. Jarmin, and Javier Miranda, "Who Creates Jobs? Small versus Large versus Young," *The Review of Economics and Statistics*, 2013, *95* (2), 347–361.

He, Bianca, Lauren Mostrom, and Amir Sufi, "Investing in Customer Capital," 2025. NBER Working Paper, 33171.

Holmes, Thomas J. and James A. Schmitz, "A Theory of Entrepreneurship and its Application to the Study of Business Transfers," *Journal of Political Economy*, 1990, *98* (2), 265–294.

Hopenhayn, Hugo A., "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 1992, *60* (5), 1127–1150.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *Quarterly journal of economics*, 2009, *124* (4), 1403–1448.

Jaimovich, Nir, Stephen J. Terry, and Nicolas Vincent, "The Empirical Distribution of Firm Dynamics and Its Macro Implications," 2025. Working Paper, University of California, San Diego.

Jovanovic, Boyan and Peter L. Rousseau, "The Q-Theory of Mergers," *American Economic Review, Papers and Proceedings*, 2002, *92* (2), 198–204.

Lippi, Francesco and Aleksei Oskolkov, "A Structural Model of Asymmetric Lumpy Investment," 2023. Working paper, Luiss University.

Lucas, Robert E., "On the Size Distribution of Business Firms," *Bell Journal of Economics*, 1978, *9* (2), 508–523.

Olley, G. Steven and Ariel Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 1996, *64* (6), 1263–1297.

Ottonello, Pablo, "Capital Unemployment," *Review of Economic Studies*, forthcoming.

Piketty, Thomas, Emmanuel Saez, and Gabriel Zucman, "Distributional National Accounts: Methods and Estimates for the United States," *The Quarterly Journal of Economics*, 2018, *133* (2), 553–609.

Ramey, Valerie A. and Matthew D. Shapiro, "Displaced Capital: A Study of Aerospace Plant Closings," *Journal of Political Economy*, 2001, *109* (5), 958–992.

Restuccia, Diego and Richard Rogerson, "The Causes and Costs of Misallocation," *Journal of Economic Perspectives*, 2017, *31* (3), 151–74.

Saez, Emmanuel and Gabriel Zucman, "Wealth Inequality in the United States Since 1913: Evidence from Capitalized Income Tax Data," *Quarterly Journal of Economics*, 2016, *131* (2), 519–578.

Smith, Matthew, Owen Zidar, and Eric Zwick, "Top Wealth in the America: New estimates and Heterogeneous Returns," *Quarterly Journal of Economics*, 2023, *138* (1), 515–573.

Sterk, Vincent, Petr Sedláček, and Benjamin Pugsley, "The Nature of Firm Growth," *American Economic Review*, 2021, *111* (2), 547–79.

Summers, Lawrence H., Natasha Sarin, Owen Zidar, and Eric Zwick, "Taxing Wealth and Capital Income: New Insights and Policy Implications," in R. Moffitt, ed., *Tax Policy and the Economy 36*, University of Chicago Press, 2022, pp. 1–33.

Sveikauskas, Leo, Rachel Soloveichik, Corby Garner, Peter B. Meyer, James Bessen, and Matthew Russell, "Marketing, Other Intangibles, and Output Growth in 61 United States Industries," *Review of Income and Wealth*, 2024, *70* (4), 1190–1215.

U.S. General Accounting Office, "Tax Policy: Issues and Policy Proposals Regarding Tax Treatment of Intangible Assets," GAO-91-88, 1991.

U.S. Internal Revenue Service, *Statistics of Income: Corporation Income Tax Reports Complete Report*, U.S. Department of Treasury, Publication 16, various years.

Appendix

For Theorem 1 and Theorem 2, we assume a discrete type space, $\mathcal{S} = \{s_1, ..., s_N\}$. We do so to keep notation simple, but the result extends naturally to a continuum of types with appropiate measure-theoretic arguments.

A Proof of Theorem 1

We prove the theorem by using duality to cast the Monge-Kantorovich problem in a form that highlights the properties of the allocation, in particular the feasibility of capital and prices and the stability of the equilibrium. Consider problem $P1$, [19]

$$\begin{aligned} \max_{\pi \geq 0} & \sum_{s,\tilde{s}} X(s,\tilde{s})\pi(s,\tilde{s}) \\ \text{s.t.} & \sum_{\tilde{s}} \pi(s,\tilde{s}) = \phi(s)/2 \\ & \sum_{s} \pi(s,\tilde{s}) = \phi(\tilde{s})/2. \end{aligned}$$

Formulate the Lagrangian as follows:

$$\begin{aligned} & \max_{\pi \geq 0} \sum_{s,\tilde{s}} X(s,\tilde{s})\pi(s,\tilde{s}) \\ & + \min_{\mu^a,\mu^b} \sum_{s} \mu^a(s)[\phi(s)/2 - \sum_{\tilde{s}} \pi(s,\tilde{s})] + \sum_{\tilde{s}} \mu^b(\tilde{s})[\phi(\tilde{s})/2 - \sum_{s} \pi(s,\tilde{s})] \end{aligned}$$

and apply the minimax theorem to get:

$$\min_{\mu^a,\mu^b} \quad \sum_{s} \mu^a(s)\phi(s)/2 + \sum_{\tilde{s}} \mu^b(\tilde{s})\phi(\tilde{s})/2 + \max_{\pi \geq 0} \quad \sum_{s,\tilde{s}} [X(s,\tilde{s}) - \mu^a(s) - \mu^b(\tilde{s})]\pi(s,\tilde{s})$$

or equivalently,

$$\begin{aligned} \min_{\mu^a,\mu^b} & \sum_{s} \mu^a(s)\phi(s)/2 + \sum_{\tilde{s}} \mu^b(\tilde{s})\phi(\tilde{s})/2 \\ \text{s.t.} & \ \mu^a(s) + \mu^b(\tilde{s}) \geq X(s,\tilde{s}). \end{aligned}$$

[19]To avoid unnecessary notation, we are omitting the option of being unmatched. This is without loss of generality in a monopartite matching problem like ours. If a given type of owner was worse off from being in a given match compared with being unmatched, then two such owners of the same type could form a match without exchanging capital nor paying each other any price and both be strictly better off, hence violating stability.

The latter problem is the dual of $P1$. Observe that the dual problem is invariant to swapping the labels a and b on μ, which implies that at an optimal solution $\mu^a = \mu^b$. We conclude the proof of the theorem in two steps.

First, it is easy to see that conditional on a match, the choice of capital is feasible by definition of X. Hence conditions (2) is satisfied. In addition, for matches that are formed in equilibrium, that is, $\pi(s,\tilde{s}) > 0$, $\mu^a(s) + \mu^b(\tilde{s}) = X(s,\tilde{s})$. This result follows from complementary slackness of the dual problem. It is immediate to verify that (20) satisfies (11), guarantees that $\lambda(s,\tilde{s}) > 0$ only for matches that are formed in equilibrium, and is such that $\sum_{\tilde{s}} \lambda(s,\tilde{s}) = 1$ (simply sum both sides of the constraints on problem P1).

Second, we show that the pair (p^m, k^m) satisfies pairwise stability given V, and that (19) holds. Suppose, by contradiction, that it is not the case. That is, there exists a pair $(s,\tilde{s})$, feasible capital allocation $\hat{k}^m(s,\tilde{s})$ and prices $\hat{p}$, such that

$$V(z,\hat{k}^m(s,\tilde{s})) - \hat{p}(s,\tilde{s}) - V(s) \geq \mu(s)$$
$$V(\tilde{z},\hat{k}^m(s,\tilde{s})) - \hat{p}(\tilde{s},s) - V(\tilde{s}) \geq \mu(\tilde{s}),$$

with at least one inequality being strict, and

$$\hat{p}(s,\tilde{s}) + \hat{p}(\tilde{s},s) \geq 0.$$

Without loss of generality, we consider the capital allocation that would maximize the sum of the values of the deviating pair. Summing up the values from deviating we get

$$X(s,\tilde{s}) - (\hat{p}(s,\tilde{s}) + \hat{p}(\tilde{s},s)) > \mu(s) + \mu(\tilde{s}).$$

Using the first constraint in the dual problem,

$$\mu(s) + \mu(\tilde{s}) \geq X(s,\tilde{s}),$$

which implies $\hat{p}(s,\tilde{s}) + \hat{p}(\tilde{s},s) < 0$, a contradiction. Since we established that p^m is a set of equilibrium prices, (19) follows from the definition of gains from trade. Q.E.D.

B Proof of Theorem 2

In this section, we assume that the rental rate on fixed assets is exogenous and equal to r. This is without loss of generality given the linear technology and competitive mutual fund assumption in the text. Let $g(s)$ be the probability mass function of entrants of type s. Consider a planner that solves the following optimization problem.

$$\begin{aligned} P(\phi_0) = \max_{\{n_t, b_t, i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t, k_t^m\}} \int_0^\infty e^{-\rho t} \sum_s \left[y(s, b_t, n_t) - r b_t(s) - c(i_t(s)) - c_e \psi_e \sum_s \iota_t^{\text{in}}(s) \frac{g(s)}{\phi_t(s)} \right] \phi_t(s) \text{d}t \\ \text{s.t.} \quad \dot{\phi}_t(s) = \Gamma(s, \phi_t; i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t, k_t^m) \\ 1 = \sum_s (1 + n_t(s)) \phi_t(s) \qquad (21) \end{aligned}$$

and feasibility of k_t^m and feasibility and consistency of λ_t. The multiplier on equation (21) is ξ_t.

Set-up

The recursive formulation of the planner's problem is

$$\begin{aligned} \rho P(\phi_t) = \max_{\{n_t, b_t, i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t, k_t^m\}} & \sum_s [y(s, b_t, n_t) - r b_t(s) - c(i_t(s))] \phi_t(s) - c_e \psi_e \sum_s \iota_t^{\text{in}}(s) g(s) \\ & + \sum_s \frac{\partial P(\phi_t)}{\partial \phi_t(s)} \dot{\phi}_t(s) + \xi_t \left[1 - \sum_s (1 + n_t(s)) \phi_t(s) \right]. \end{aligned}$$

Let $\Gamma(s)$ denote $\Gamma(s, \phi_t; i_t, \iota_t^{\text{in}}, \iota_t^{\text{out}}, \lambda_t, k_t^m)$ with arguments other than s omitted. The optimality conditions for the planner's problem with respect to $i_t(s)$, $\iota_t^{\text{in}}(s)$, $\iota_t^{\text{out}}(s)$, $n_t(s)$, and b_t are

$$\begin{aligned} c'(i_t(s)) \phi_t(s) &= \sum_{\hat{s}} \frac{\partial P(\phi_t)}{\partial \phi_t(\hat{s})} \frac{\partial \Gamma(\hat{s})}{\partial i_t(s)} \\ \iota_t^{\text{in}}(s) &= \left\{ \sum_{\hat{s}} \frac{\partial P(\phi_t)}{\partial \phi_t(\hat{s})} \frac{\partial \Gamma(\hat{s})}{\partial \iota_t^{\text{in}}(s)} - c_e \psi_e g(s) \right\}^+ \\ \iota_t^{\text{out}}(s) &= \left\{ \sum_{\hat{s}} \frac{\partial P(\phi_t)}{\partial \phi_t(\hat{s})} \frac{\partial \Gamma(\hat{s})}{\partial \iota_t^{\text{out}}(s)} \right\}^+ \\ \gamma z(s) k(s)^\alpha b^\beta n^{\gamma - 1} &= \xi_t \\ \beta z(s) k(s)^\alpha b^{\beta - 1} n^\gamma &= r. \end{aligned}$$

By the envelope theorem,

$$\rho \frac{\partial P(\phi_t)}{\partial \phi_t(s)} = [y(s, b_t, n_t) - rb_t(s) - c(i_t(s))] - \xi_t(1 + n_t(s))$$
$$+ \sum_{\hat{s}} \frac{\partial P(\phi_t)}{\partial \phi_t(\hat{s})} \frac{\partial \Gamma(\hat{s})}{\partial \phi_t(s)} + \sum_{\hat{s}} \frac{\partial^2 P(\phi_t)}{\partial \phi_t(s) \partial \phi_t(\hat{s})} \Gamma(\hat{s}).$$

If we define the marginal value to the planner of additional mass at type s at time t as

$$\tilde{H}(s; \phi_t) \equiv \frac{\partial P(\phi_t)}{\partial \phi_t(s)},$$

then we can formulate the envelope condition as

$$\rho \tilde{H}(s; \phi_t) = y(s, b_t, n_t) - rb_t(s) - c(i_t(s)) - \xi_t(1 + n_t(s))$$
$$+ \sum_{\hat{s}} \tilde{H}(\hat{s}; \phi_t) \frac{\partial \Gamma(\hat{s})}{\partial \phi(s)} + \sum_{\hat{s}} \frac{\partial \tilde{H}(s; \phi_t)}{\partial \phi(\hat{s})} \Gamma(\hat{s}).$$

We define the marginal value along the optimal trajectory as

$$H_t(s) \equiv \tilde{H}(s; \phi_t)$$

and obtain its time-derivative

$$\frac{\partial H_t(s)}{\partial t} = \sum_{\hat{s}} \frac{\partial \tilde{H}(s, \phi_t)}{\partial \phi_t(\hat{s})} \Gamma(\hat{s}).$$

Using this, the envelope condition can be simplified to

$$\rho H_t(s) = y(s, b_t, n_t) - rb_t(s) - \xi_t n_t(s) - c(i_t(s)) - \xi_t + \sum_{\hat{s}} H_t(\hat{s}) \frac{\partial \Gamma(\hat{s})}{\partial \phi(s)} + \frac{\partial H_t(s)}{\partial t}. \tag{22}$$

We focus on a stationary planner's problem, which allows us to drop the time subscript and the time derivative from the problem above. Express the stationary marginal value

$$H(s) = \hat{V}(s) - \hat{W}$$

for some function $\hat{V}$ and constant $\hat{W}$.

We will show later that that $\hat{V}$ and $\hat{W}$ correspond to the value function of an owner and the value of a worker, respectively, in the equilibrium of our model. Since $\hat{W}$ is a constant and does not depend on s, changes in H induced by changes in the state of owners depend on $\hat{V}(s)$ only.

Using the expression for Γ, we get

$$
\begin{aligned}
(\rho+\psi_e)\hat{V}(s) = y(s,b,n) - rb(s) - \xi n(s) - c(i(s)) - \xi \\
+ \partial_k \hat{V}(s)(i-\delta_k) - c(i(s)) + \partial_z \hat{V}(s)\mu(z) + \tfrac{1}{2}\partial_{zz}\hat{V}(s)\sigma(z)^2 \\
+ \sum_{\hat{s}} \hat{V}(\hat{s}) \frac{\partial \Gamma_\lambda(\hat{s})}{\partial \phi(s)}.
\end{aligned} \tag{23}
$$

where $\Gamma_\lambda(\hat{s})$ is the component of $\Gamma(\hat{s})$ that is induced by the trading policy λ.

The optimality conditions for the planner stated above become

$$
\begin{aligned}
c'(i(s))\phi(s) &= \sum_{\hat{s}} H(\hat{s}) \frac{\partial \Gamma(\hat{s})}{\partial i(s)} = \partial_k \hat{V}(z,k)\phi(s) \\
\iota_t^{\text{in}}(s) &= \left\{ \sum_{\hat{s}} H(\hat{s}) \frac{\partial \Gamma(\hat{s})}{\partial \iota^{\text{in}}(s)} - c_e \psi_e g(s) \right\}^+ = \left\{ \hat{V}(s) - \hat{W} - c_e \right\}^+ \\
\iota^{\text{out}}(s) &= \left\{ \sum_{\hat{s}} H(\hat{s}) \frac{\partial \Gamma(\hat{s})}{\partial \iota^{\text{out}}(s)} \right\}^+ = \{ \hat{W} - \hat{V}(s) \}^+ \\
\gamma z(s) k(s)^\alpha b^\beta n^{\gamma-1} &= \xi \\
\beta z(s) k(s)^\alpha b^{\beta-1} n^\gamma &= r.
\end{aligned}
$$

Next, we turn to a linear programming problem in which we solve for the optimal set of matches and capital allocations (λ, k^m). We also show that the last term in (23) is equal to the multiplier associated to the constraints of the same linear programming problem.

Optimal Matching

We set up the following linear problem

$$
\begin{aligned}
\max_{\lambda \geq 0, k^m} & \sum_s \hat{V}(s)\Gamma_\lambda(s) \\
\text{s.t.} & \sum_{\tilde{s}} \lambda(s,\tilde{s}) = 1 \quad \forall s \\
& \sum_s \lambda(s,\tilde{s})\phi(s) = \phi(\tilde{s}) \quad \forall \tilde{s}
\end{aligned}
$$

Rearrange the objective function using the definition of Γ_λ as follows:

$$
\sum_s \hat{V}(s) \left[\sum_{s',s''} \lambda(s',s'') \mathbb{I}\{k^m(s',s'') = k(s), z(s') = z(s)\} \phi(s') - \sum_{s',s''} \lambda(s,s'')\phi(s') \right]
$$

$$= \sum_{s',s''} \frac{\lambda(s',s'')}{2} \phi(s') \sum_{s} \hat{V}(s) \left[\mathbb{I}\{k^m(s',s'') = k(s), z(s') = z(s)\} - 1 \right]$$

$$+ \frac{\lambda(s'',s')}{2} \phi(s'') \sum_{s} \hat{V}(s) \left[\mathbb{I}\{k^m(s',s'') = k(s), z(s'') = z(s)\} - 1 \right].$$

Imposing feasibility of k^m amounts to restricting the indicators above to be such that either s' is a buyer, or s'' is, or neither. The optimal choice of k^m is equivalent to solving

$$\begin{aligned} X(s',s'') = & \max \{ \hat{V}(z',k'+k'') + \hat{V}(z'',0),\ \hat{V}(s') + \hat{V}(s''),\ \hat{V}(z',0) + \hat{V}(z'',k'+k'') \} \\ & - (\hat{V}(s') + \hat{V}(s'')). \end{aligned}$$

The objective function thus simplifies to

$$\sum_{s',s''} \frac{\lambda(s',s'')}{2} \phi(s') X(s',s'').$$

Let $\pi(s,\tilde{s}) = \frac{\lambda(s,\tilde{s})}{2} \phi(s)$. We label the value of the matching problem as Q.

$$\begin{aligned} Q(\phi) = & \max_{\pi \geq 0} \sum_{s,\tilde{s}} \pi(s,\tilde{s}) X(s,\tilde{s}) \\ & \text{s.t.} \ \sum_{\tilde{s}} \pi(s,\tilde{s}) = \frac{\phi(s)}{2} \\ & \quad \sum_{s} \pi(s,\tilde{s}) = \frac{\phi(\tilde{s})}{2}. \end{aligned} \tag{24}$$

Notice that this formulation of the matching problem is analogous to the one in the competitive equilibrium. Let $\mu^a(s)$ and $\mu^b(s)$ be the multipliers attached to the constraints of (24). From the envelope theorem,

$$\frac{\partial Q}{\partial \phi(s)} = \frac{\mu^a(s) + \mu^b(s)}{2}$$

and by the symmetry of $X(\cdot,\cdot)$, $\mu^a(s) = \mu^b(s) \equiv \mu(s)$. Since at the solution,

$$Q(\phi) = \sum_{s} \hat{V}(s) \Gamma_\lambda(s)$$

is satisfied for all ϕ, we differentiate both sides to obtain

$$\sum_{\hat{s}} \hat{V}(\hat{s}) \frac{\partial \Gamma_\lambda(\hat{s})}{\partial \phi(s)} = \mu(s).$$

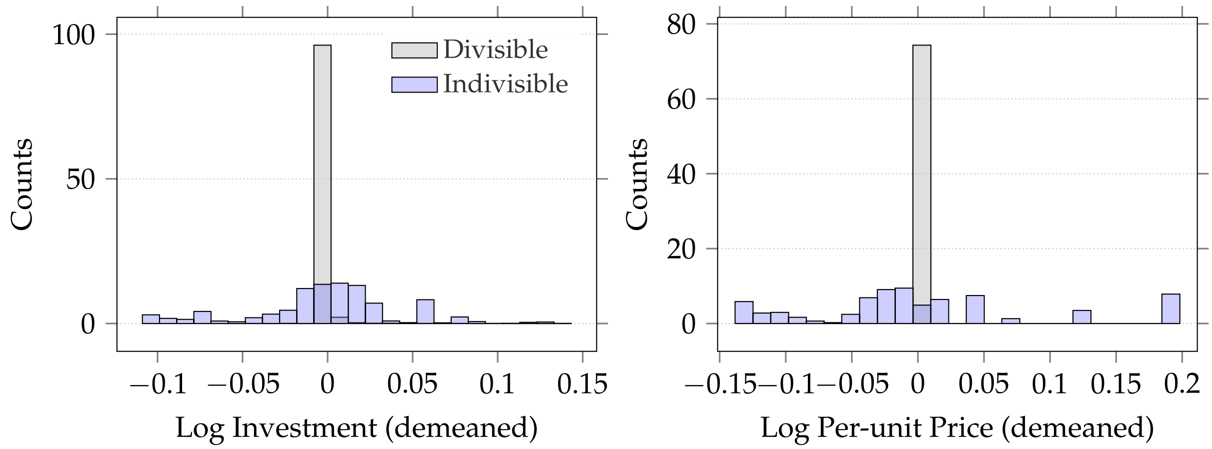
Wrap up

When the equilibrium wage w equals ξ, the equilibrium value of the onwer net of the value of being a worker, that is, $V(s) - W$ satisfies the Bellman equation (23) for the planner's marginal value. Thus $V = \hat{V}$ and $W = \hat{W} \equiv \xi/(\psi_e + \rho)$. Given the value functions match, the optimality conditions for the owner and the planner are identical. While we focused on a stationary planner's allocation for the sake of consistency with our equilibrium notion, we note that nothing in our proof relies on stationarity. Therefore, our equilibrium path would coincide to the planner's even outside of steady state. Q.E.D.

C Additional Figures

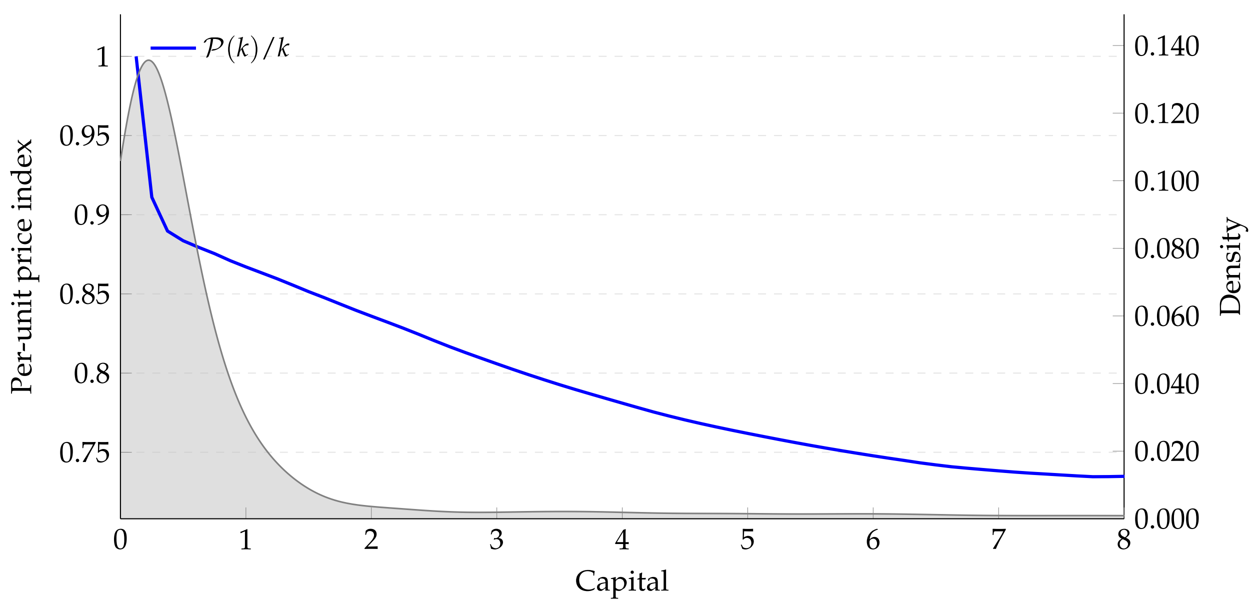
In this section, we include additional results related to the dispersion of marginal products of capital discussed in Section 6.1 In Figure 5, we plot histograms for two key variables, namely the log of investment and the log of the per-unit prices for economies with the same trading frequency ($\eta = 12$) but different assumptions about the divisibility of capital when businesses are sold. We can see clearly from these graphs that the histograms for investment and price are clustered in the divisible case, whereas there is significant dispersion in the indivisible case. As a point of reference, we plot the per-unit price schedule and capital density for our baseline model in Figure 6, which shows how much the price varies across small and large sales.

Figure 5: Histograms of Demeaned Log Investment and Price per Unit of Capital



Notes: Histograms of log investment (left) and log price per unit of capital (right) are both demeaned by their respective means. The gray bars represent the model with divisible capital, while the blue bars represent the model with indivisible capital. The trading frequency is set to $\eta = 12$ (monthly) in both models.

Figure 6: Price per Unit with Capital Density



Notes: The price per-unit schedule has been normalized to start at 1 (left scale) and the distribution over capital has been smoothed (right scale). These are results for the baseline model with trading frequency set to $\eta = 2.2$ and indivisible capital exchange.