
**APPENDIX B. OMITTED ANALYSIS AND PROOFS FROM SECTION 3**

The period-two incentive compatibility constraints (IC$_{12}$) and (IC$_{22}$) are “easy”: each is essentially a static incentive compatibility constraint, and can therefore be reduced to the usual envelope and monotonicity conditions. The proof follows standard techniques and is therefore omitted.

**LEMMA B.1.** The period-two incentive compatibility constraints (IC$_{12}$) for buyers contracting in period one are satisfied if, and only if, for all $\lambda \in \Lambda$,  
\[
\frac{\partial}{\partial v} U_{12}(v, \lambda) = q_1(v, \lambda) \text{ almost everywhere, and } q_1(v, \lambda) \text{ is nondecreasing in } v.  \quad (IC'_{12})
\]

The constraints (IC$_{22}$) for buyers contracting in period two are satisfied if, and only if,  
\[
U'_{22}(v) = q_2(v) \text{ almost everywhere, and } q_2(v) \text{ is nondecreasing in } v.  \quad (IC'_{22})
\]

Since the underlying mechanisms are deterministic, Lemma B.1 implies that the allocation rules must be cutoff policies: there exists a function $k_1 : \Lambda \to V$ and a constant $\alpha \in V$ such that  
\[
q_1(v, \lambda) = \begin{cases} 
0 & \text{if } v < k_1(\lambda), \\
1 & \text{if } v \geq k_1(\lambda); 
\end{cases} \quad \text{and } q_2(v) = \begin{cases} 
0 & \text{if } v < \alpha, \\
1 & \text{if } v \geq \alpha.
\end{cases}
\]

Lemma B.1 also immediately yields the observation that $\frac{\partial}{\partial v} \tilde{U}_{12}(v, \lambda) = q_2(v)$ for all $\lambda \in \Lambda$ whenever (IC$_{22}$) is satisfied, as $\tilde{U}_{12}(v, \lambda)$ directly inherits the properties of $U_{22}(v)$. We can use this result to characterize the period-two continuation payoff $V_{12}(v, \lambda)$ of a cohort-one buyer when we impose constraint (RC) and allow buyers recontract in period two.

**LEMMA B.2.** Define $\tilde{q}_{12}(v, \lambda) := x_2(v, \lambda)q_1(v, \lambda) + (1 - x_2(v, \lambda))q_2(v)$ for all $\lambda \in \Lambda$ and $v \in V$, and suppose (IC$_{12}$), (IC$_{22}$), and (RC) are satisfied. Then for all $\lambda \in \Lambda$,  
\[
\frac{\partial}{\partial v} V_{12}(v, \lambda) = \tilde{q}_{12}(v, \lambda) \text{ almost everywhere, and } \tilde{q}_{12}(v, \lambda) \text{ is nondecreasing in } v.
\]
In addition, for all \( \lambda \in \Lambda \), \( q_{12}(v, \lambda) = q_2(v) \) for almost all \( v \in V \) such that \( x_2(v, \lambda) \in (0,1) \). Finally, \( q_{12}(v, \lambda) \) corresponds to a cutoff policy with threshold \( \bar{k}_{12}(\lambda) \) for all \( \lambda \in \Lambda \).

**Proof.** Suppose that (RC) is satisfied. Then

\[
V_{12}(v, \lambda) = \max_{v'} \{ x_2(v', \lambda)[q_1(v', \lambda)v - p_{12}(v', \lambda)] + (1 - x_2(v', \lambda))[q_2(v')v - p_{22}(v') + \tilde{p}_{12}(\lambda)] \}.
\]

Applying the Envelope Theorem—see Milgrom and Segal (2002)—yields

\[
\frac{\partial}{\partial v} V_{12}(v, \lambda) = x_2(v, \lambda)q_1(v, \lambda) + (1 - x_2(v, \lambda))q_2(v) = q_{12}(v, \lambda).
\]

Moreover, note that we can rewrite the optimality condition above as

\[
V_{12}(v, \lambda) \geq V_{12}(v', \lambda) + q_{12}(v', \lambda)(v - v') \text{ for all } v, v' \in V.
\]

A similar inequality is derived by interchanging the role of \( v \) and \( v' \). Adding these two inequalities yields \( (q_{12}(v, \lambda) - q_{12}(v', \lambda))(v - v') \geq 0 \), so \( q_{12}(v, \lambda) \) nondecreasing in \( v \) for all \( \lambda \in \Lambda \).

Now fix any \( v \in V \) such that \( x_2(v, \lambda) \in (0,1) \) and \( v \) is a point of differentiability for \( V_{12}(\cdot, \lambda) \). Recall first that Lemma B.1 implies that \( q_1(\cdot, \lambda) \) and \( q_2 \) correspond to cutoff rules, with cutoffs \( k_1(\lambda) \) and \( \alpha \) respectively. So suppose that \( k_1(\lambda) > \alpha \), and note that we can write

\[
q_{12}(v, \lambda) = \begin{cases} 
  x_2(v, \lambda)(1) + (1 - x_2(v, \lambda))(1) = 1 = q_2(v) & \text{if } v > k_1(\lambda); \\
  x_2(v, \lambda)(0) + (1 - x_2(v, \lambda))(1) = 1 - x_2(v, \lambda) < q_2(v) & \text{if } v \in (\alpha, k_1(\lambda)); \\
  x_2(v, \lambda)(0) + (1 - x_2(v, \lambda))(0) = 0 = q_2(v) & \text{if } v < \alpha.
\end{cases}
\]

Thus, if \( v \in (\alpha, k_1(\lambda)) \), we have \( \frac{\partial}{\partial v} V_{12}(v, \lambda) \leq \frac{\partial}{\partial v} \bar{U}_{12}(v, \lambda) \), implying that (RC) is violated for some \( v' \in (v, v + \epsilon) \) for \( \epsilon > 0 \) sufficiently small. Similarly, suppose that \( k_1(\lambda) < \alpha \). Then

\[
q_{12}(v, \lambda) = \begin{cases} 
  x_2(v, \lambda)(1) + (1 - x_2(v, \lambda))(1) = 1 = q_2(v) & \text{if } v > \alpha; \\
  x_2(v, \lambda)(1) + (1 - x_2(v, \lambda))(0) = x_2(v, \lambda) > q_2(v) & \text{if } v \in (k_1(\lambda), \alpha); \\
  x_2(v, \lambda)(0) + (1 - x_2(v, \lambda))(0) = 0 = q_2(v) & \text{if } v < k_1(\lambda).
\end{cases}
\]

Thus, if \( v \in (k_1(\lambda), \alpha) \), we have \( \frac{\partial}{\partial v} V_{12}(v, \lambda) \geq \frac{\partial}{\partial v} \bar{U}_{12}(v, \lambda) \), implying that (RC) is violated for some \( v' \in (v - \epsilon, v) \) for \( \epsilon > 0 \) sufficiently small. Thus, \( q_{12}(v, \lambda) \in \{ q_1(v, \lambda), q_2(v) \} \) for almost all \( v \).

Since both \( q_1 \) and \( q_2 \) are deterministic, this implies that \( q_{12}(v, \lambda) \in \{0,1\} \) almost everywhere. Of course, since \( q_{12}(v, \lambda) \) is nondecreasing in \( v \) for all \( \lambda \), we can therefore treat it as a cutoff policy.

In order to simplify (IC\(_{11}\)), consider the “effective” allocation rule

\[
\tilde{q}_1(v, \lambda) := x_1(\lambda)x_2(v, \lambda)q_1(v, \lambda) + (1 - x_1(\lambda)x_2(v, \lambda))q_2(v) = x_1(\lambda)q_{12}(v, \lambda) + (1 - x_1(\lambda))q_2(v).
\]

Clearly, \( \tilde{q}_1 \) inherits monotonicity in \( v \) from \( q_{12}(v, \lambda) \) and \( q_2(v) \). An envelope argument combines with the stochastic order on \( \{G(\cdot | \lambda)\}_{\lambda \in \Lambda} \) to show that \( \tilde{q}_1 \) must also be nondecreasing in \( \lambda \).\(^1\)

\(^1\)The sufficiency of monotonicity for incentive compatibility follows from known results; see Pavan, Segal, and Toikka (2014) for the general formulation of this observation. Necessity, on the other hand, relies on the restriction (as in Krähmer and Strausz (2011)) to deterministic allocation rules.
**Lemma B.3.** Suppose that constraints (SD), (IC\(_{12}\)), and (IC\(_{22}\)) are satisfied. If the initial-period constraint (IC\(_{11}\)) is satisfied, then

\[
V_1'(\lambda) = -\int_{\lambda} q_1(v, \lambda) G_\lambda(v|\lambda) dv \text{ almost everywhere, and} \tag{IC\(_{11}\)'}
\]

\[\bar{q}_1(v, \lambda) \] is nondecreasing in \( v \) and \( \lambda \). \tag{MON\(_{11}\)'}

In addition, for almost all \( \lambda \in \Lambda \) with \( x_1(\lambda) \in (0, 1) \), \( q_1(v, \lambda) = q_2(v) \) for all \( v \in V \). Finally, \( q_1(v, \lambda) \) corresponds to a cutoff policy with threshold \( \bar{k}_1(\lambda) \) for all \( \lambda \in \Lambda \).

**Proof.** We begin by showing the necessity of (IC\(_{11}\)'). Note first that we can rewrite (IC\(_{11}\)) as

\[
U_{11}(\lambda) = \max_{\lambda'} \left\{ -p_{11}(\lambda') + \int_{\lambda} V_{12}(v, \lambda') dG(v|\lambda) \right\}.
\]

Applying the Envelope Theorem, integration by parts, and Lemma B.2 yields

\[
U_{11}'(\lambda) = \int_{\lambda} V_{12}(v, \lambda) \frac{\partial g(v|\lambda)}{\partial \lambda} dv = -\int_{\lambda} \frac{\partial V_{12}(v, \lambda)}{\partial \lambda} G_\lambda(v|\lambda) dv = -\int_{\lambda} \bar{q}_{12}(v, \lambda) G_\lambda(v|\lambda) dv.
\]

In addition, note that we can use Lemma B.1’s envelope formulation of (IC\(_{22}\)) to show that

\[
\bar{U}_{11}'(\lambda) = \int_{\lambda} U_{22}(v) \frac{\partial g(v|\lambda)}{\partial \lambda} dv = -\int_{\lambda} \frac{\partial U_{22}(v)}{\partial v} G_\lambda(v|\lambda) dv = -\int_{\lambda} \bar{q}_2(v) G_\lambda(v|\lambda) dv.
\]

Finally, notice that (SD) and (IC\(_{11}\)) jointly imply that

\[
V_{11}(\lambda) = \max_{\lambda'} \left\{ x_1(\lambda') \left( -p_{11}(\lambda') + \int_{\lambda} V_{12}(v, \lambda') dG(v|\lambda) \right) + (1 - x_1(\lambda')) \int_{\lambda} U_{22}(v) dG(v|\lambda) \right\}.
\]

Once again, the Envelope Theorem, along with the previous two observations, implies that

\[
V_{11}'(\lambda) = x_1(\lambda) U_{11}'(\lambda) + (1 - x_1(\lambda)) \bar{U}_{11}(\lambda) = -\int_{\lambda} \bar{q}_1(v, \lambda) G_\lambda(v|\lambda) dv.
\]

We now show that, for almost all \( \lambda \in \Lambda \) such that \( x_1(\lambda) \in (0, 1) \), \( q_1(v, \lambda) = q_2(v) \). Fix any \( \lambda \in (\Lambda, \bar{\Lambda}) \) such that \( x_1(\lambda) \in (0, 1) \) and \( V_{11} \) is differentiable at \( \lambda \). Then we can write

\[
V_{11}'(\lambda) = -\int_{\lambda} q_1(v, \lambda) G_\lambda(v|\lambda) dv = -\int_{\lambda} q_2(v) G_\lambda(v|\lambda) dv - x_1(\lambda) \int_{\lambda} [\bar{q}_{12}(v, \lambda) - q_2(v)] G_\lambda(v|\lambda) dv.
\]

Since \( q_2(\cdot) \) and \( \bar{q}_{12}(\cdot, \lambda) \) correspond to cutoffs \( \alpha \) and \( \bar{k}_{12} \), respectively, \( V_{11}'(\lambda) \) can be rewritten as

\[
V_{11}'(\lambda) = -\int_{\alpha} G_\lambda(v|\lambda) dv - x_1(\lambda) \int_{\bar{k}_{12}(\lambda)}^\alpha G_\lambda(v|\lambda) dv = \bar{U}_{11}'(\lambda) - x_1(\lambda) \int_{\bar{k}_{12}(\lambda)}^\alpha G_\lambda(v|\lambda) dv.
\]

Thus, if \( \bar{k}_{12}(\lambda) < \alpha \), we have \( V_{11}'(\lambda) > \bar{U}_{11}'(\lambda) \), implying (SD) is violated for some \( \lambda' \in (\lambda - \epsilon, \lambda) \) for \( \epsilon > 0 \) sufficiently small. On the other hand, if \( \bar{k}_{12}(\lambda) > \alpha \), then \( V_{11}'(\lambda) < \bar{U}_{11}'(\lambda) \) and (SD) is violated for some \( \lambda' \in (\lambda, \lambda + \epsilon) \) for sufficiently small \( \epsilon > 0 \). Thus, we must have \( \bar{k}_{12}(\lambda) = \alpha \), or equivalently, \( \bar{q}_{12}(v, \lambda) = q_2(v) \) for all \( v \in V \). Therefore, we can conclude that

\[
q_1(v, \lambda) = x_1(\lambda) \bar{q}_{12}(v, \lambda) + (1 - x_1(\lambda)) q_2(v) = x_1(\lambda) q_2(v) + (1 - x_1(\lambda)) q_2(v) = q_2(v).
\]

Finally, to see that (IC\(_{11}\)) and (SD) imply (MON\(_{11}\)), fix any \( \lambda, \lambda' \in \Lambda \). We can write

\[
V_{11}(\lambda) \geq x_1(\lambda') [-p_{11}(\lambda') + \int_{\lambda} V_{12}(v, \lambda') dG(v|\lambda)] + (1 - x_1(\lambda')) \bar{U}_{11}(\lambda)
\]
where we define \( \bar{\lambda} \) and \( \Lambda \), and the final equality is by integration by parts. Reversing the roles of \( \lambda \) and \( \lambda' \) and then adding the resulting inequality to the above yields
\[
\int_{\mathbb{V}} [\bar{q}_1(v, \lambda) - \bar{q}_1(v, \lambda')] [G(v|\lambda) - G(v|\lambda')] dv \leq 0. \tag{B.1}
\]
Now, note that for all \( \mu, \mu' \), we can write
\[
\int_{\mathbb{V}} \bar{q}_1(v, \mu) G(v|\mu') dv = \int_{\mathbb{V}} [x_1(\mu) \bar{q}_{12}(v, \mu) + (1 - x_1(\mu)) q_2(v)] G(v|\mu') dv
= x_1(\mu) \int_{\bar{\kappa}_{12}(\mu)}^{\theta} G(v|\mu') dv + (1 - x_1(\mu)) \int_{\theta}^{\bar{\kappa}_{12}(\mu)} G(v|\mu') dv,
\]
where \( \bar{\kappa}_{12}(\mu) \) and \( \theta \) are the cutoffs corresponding to \( \bar{q}_{12}(\cdot, \mu) \) and \( q_2 \), respectively. Recall from our previous result, however, that \( \bar{q}_{12}(\cdot, \lambda) = q_2(\cdot) \) when \( x_1(\lambda) \in (0, 1) \). Therefore, we can write
\[
\int_{\mathbb{V}} \bar{q}_1(v, \mu) G(v|\mu') dv = \int_{\bar{\kappa}_1(\mu)}^{\theta} G(v|\mu') dv,
\]
where we define \( \bar{\kappa}_1(\mu) := x_1(\mu) \bar{\kappa}_{12}(\mu) + (1 - x_1(\mu))\alpha \). We can now rewrite (B.1) as
\[
\int_{\mathbb{V}} \bar{q}_1(v, \mu) G(v|\mu') dv \leq 0.
\]
Since Assumption 1 orders \( \{G(\cdot|\mu)\}_{\mu \in \Lambda} \) by first-order stochastic dominance, this inequality holds only if \( \bar{\kappa}_1(\cdot) \) is (weakly) decreasing (implying the effective allocation rule \( \bar{q}_1 \) is nondecreasing).

Finally, we turn to the seller’s period-two problem (\( \widehat{\mathcal{SR}} \)) when cohort-one buyers are free to re-contract in period two. This problem is somewhat more subtle than (SR), as the seller’s choice of period-two mechanism influences—through constraint (\( \mathcal{RC} \)) and its impact on \( x_2(v, \lambda) \)—the set of buyers that choose to re-contract. We show, however, that it is inefficient to induce recontracting using a second-period subsidy: the seller can more cost-effectively induce the same ex post sorting by modifying the set of initial-period contracts. Therefore, the optimal period-two contract is simply a price with no additional subsidy. Furthermore, cohort-one buyers will choose to re-contract whenever this period-two price is lower than their already-contracted cutoff.

**Lemma B.4.** In an optimal contract, the seller’s period-two problem in (\( \widehat{\mathcal{SR}} \)) can be written as
\[
\max_{\alpha} \left\{ \int_{\Lambda} \left( x_1(\lambda) \pi_{\lambda}(\min\{k_1(\lambda), \alpha\}) - U_{12}(\bar{\gamma}, \lambda) \right) + (1 - x_1(\lambda)) \pi_{\lambda}(\alpha) d\Lambda + \gamma \pi_H(\alpha) \right\} \tag{\( \widehat{\mathcal{SR}} \')).
\]

**Proof.** Using Lemmas B.1 and B.2, the seller’s period-two problem (\( \mathcal{SR} \)) becomes one of choosing a cutoff \( \alpha \) and subsidy \( U_{22}(\bar{\gamma}) \) to solve
\[
\begin{align*}
\max_{a, \mu} & \left\{ \int_{\Lambda} x_1(\lambda)[\pi_{\lambda}(\bar{k}_{12}(\lambda)) - V_{12}(\bar{\nu}, \lambda)]dF(\lambda) \\
& \quad + \int_{\Lambda} (1 - x_1(\lambda))[\pi_{\lambda}(a) - U_{22}(\bar{\nu})]dF(\lambda) + \gamma[\pi_{\lambda}(a) - U_{22}(\bar{\nu})] \right\} \\
\text{subject to} \ (\text{IR}) \text{ and } (\text{RC}),
\end{align*}
\]

where \( \bar{k}_{12} \) is the effective cutoff associated with \( q_{12}(v, \lambda) = x_2(v, \lambda)q_1(v, \lambda) + (1 - x_2(v, \lambda))q_2(v) \).

Our first goal is to characterize, given a period-two cutoff \( \alpha \in \mathcal{V} \) and subsidy \( u := U_{22}(\bar{\nu}) \geq 0 \), the behavior of \( \bar{k}_{12}(\lambda) \) and determine which buyers recontract. In doing so, we rely on the observation that \( \bar{p}(\mu) = U_{12}(\bar{\nu}, \mu) \) for all \( \mu \in \Lambda \). (This follows from (IR) and the “bang-bang” nature of payments implementing a cutoff allocation.)

Fix an arbitrary \( \lambda \in \Lambda \) and observe that \( \bar{q}_{12}(v, \lambda) = 1 \) for all \( v \geq \max\{k_1(\lambda), \alpha\} \), and \( \bar{q}_{12}(v, \lambda) = 0 \) for all \( v < \min\{k_1(\lambda), \alpha\} \). So to determine the behavior of \( \bar{q}_{12}(v, \lambda) \) when \( \min\{k_1(\lambda), \alpha\} \leq v < \max\{k_1(\lambda), \alpha\} \), suppose first that \( k_1(\lambda) \geq \alpha \). Then for any \( v \in (\alpha, k_1(\lambda)) \), we have

\[
U_{12}(v, \lambda) - \bar{U}_{22}(v) = (U_{12}(\bar{\nu}, \lambda) + \max\{v - k_1(\lambda), 0\}) - (\bar{p}(\lambda) + u + \max\{v - \alpha, 0\})
= 0 - (v - \alpha + u) < 0.
\]

This implies that \( x_2(v, \lambda) = 0 \) and, therefore, \( \bar{q}_{12}(v, \lambda) = 1 \) for all \( v \in (\alpha, k_1(\lambda)) \). (If \( v = \alpha \), then the buyer is indifferent between the two contracts and so \( \bar{q}_{12}(v, \lambda) \) is indeterminate. Note that this is a zero-probability event, however, and can therefore be safely ignored.)

Now suppose instead that \( k_1(\lambda) \in [\alpha - u, \alpha) \). Then for any \( v \in [k_1(\lambda), \alpha) \), we have

\[
U_{12}(v, \lambda) - \bar{U}_{22}(v) = (U_{12}(\bar{\nu}, \lambda) + \max\{v - k_1(\lambda), 0\}) - (U_{12}(\bar{\nu}, \lambda) + u + \max\{v - \alpha, 0\})
= (v - k_1(\lambda)) - (0 + u) = v - (k_1(\lambda) + u) \leq v - \alpha < 0.
\]

This implies that \( x_2(v, \lambda) = 0 \) and, therefore, \( \bar{q}_{12}(v, \lambda) = 0 \) for all \( v \in [k_1(\lambda), \alpha) \).

Finally, suppose that \( k_1(\lambda) < \alpha - u \). Then for any \( v \in [k_1(\lambda), \alpha) \), we have

\[
U_{12}(v, \lambda) - \bar{U}_{22}(v) = (U_{12}(\bar{\nu}, \lambda) + \max\{v - k_1(\lambda), 0\}) - (U_{12}(\bar{\nu}, \lambda) + u + \max\{v - \alpha, 0\})
= (v - k_1(\lambda)) - (0 + u) = v - (k_1(\lambda) + u).
\]

This is strictly negative if \( v < k_1(\lambda) + u \), in which case \( x_2(v, \lambda) = 0 \) and \( \bar{q}_{12}(v, \lambda) = q_2(v) = 0 \); on the other hand, this expression is strictly positive if \( v > k_1(\lambda) + u \), in which case \( x_2(v, \lambda) = 1 \) and \( \bar{q}_{12}(v, \lambda) = q_1(v, \lambda) = 1 \). (If \( v = k_1(\lambda) + U_{22}(\bar{\nu}) \), then the buyer is indifferent and \( \bar{q}_{12}(v, \lambda) \) is indeterminate. Of course, this is a zero-probability event and so can be safely ignored.)

Thus, the cutoff associated with \( \bar{q}_{12}(v, \lambda) \) is \( \bar{k}_{12}(\lambda) := \min\{\alpha, k_1(\lambda) + u\} \), so (B.2) becomes

\[
\max_{a, \mu} \left\{ \int_{\Lambda} (1 - x_1(\lambda))\pi_{\lambda}(\alpha)dF(\lambda) + \gamma\pi_{\lambda}(\alpha) \\
+ \int_{\Lambda} x_1(\lambda)(\pi_{\lambda}(\min\{a, k_1(\lambda) + u\}) - U_{12}(\bar{\nu}, \lambda))dF(\lambda) - (1 + \gamma)u \right\} \\
\text{subject to } u \geq 0.
\]

Now fix a candidate optimal contract (denoted by a \( * \) superscript), and suppose that the solution \((a^*, u^*)\) to (B.3) is such that \( u^* > 0 \). Define a new contract (denoted by a \( ** \) superscript) by

\[
x_{1}^{**}(\lambda) := x_{1}^{*}(\lambda); \quad p_{11}^{**}(\lambda) := p_{11}^{*}(\lambda); \quad \text{and } k_{1}^{**}(\lambda) := k_{1}^{*}(\lambda) + u^* \text{ for all } \lambda \in \Lambda, \text{ and}
\]
Denoting the objective function in problem (B.3) by $\Pi$, a period-two contract (on the equilibrium path) corresponds to a price $\alpha$, thus, in an optimal contract, the seller’s problem (B.3) can be written as

$$\max_{x_1} \left\{ \int_{\Lambda} (x_1(\lambda) [\pi_\lambda(\min\{k_1(\lambda), a\}) - U_{12}(\lambda, \alpha)] + (1 - x_1(\lambda)) \pi_\lambda(a) ) dF(\lambda) + \gamma \pi_H(a) \right\}.$$  

for all $\epsilon, \delta \in \mathbb{R}$, where the inequality follows from the (assumed) optimality of $(\alpha^*, u^*)$. Thus, $(\alpha^{**}, u^{**}) = (\alpha^*, 0)$ solves problem (B.3) given the revised (**) period-one contract.

Since this new contract effectively reduces the utility of all buyers across both cohorts by $u^* > 0$ (and also reduces the value of delay by $u^*$) while keeping allocations unchanged, the new ** contract satisfies all the period-one constraints and yields greater profits than the original * contract, contradicting the latter’s optimality. Thus, we may conclude that in any optimal contract, the period-two contract (on the equilibrium path) corresponds to a price $\alpha$ with no additional subsidies. Thus, in an optimal contract, the seller’s problem (B.3) can be written as

$$\max_{\alpha} \left\{ \int_{\Lambda} (x_1(\lambda) [\pi_\lambda(\min\{k_1(\lambda), a\}) - U_{12}(\lambda, \alpha)] + (1 - x_1(\lambda)) \pi_\lambda(a) ) dF(\lambda) + \gamma \pi_H(a) \right\}.$$  

References

