Currency Choice in Contracts*

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Abstract

We study the general equilibrium of an economy with credit chains in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. Denominating contracts in local currency helps mitigate fundamental risk, while denominating in a foreign currency minimizes risks due to the government's choice of monetary policy. In the aggregate, the equilibrium currency denomination calls for a coordination of currencies in bilateral contracts within a chain to avoid costly default due to currency mismatch. This implies that the incentives to denominate contracts in a foreign currency might persist even after government risk has been significantly reduced. Our model can help explain the observed hysteresis of dollarization that occurred in several Latin American countries. We show that the socially optimal allocation would call for even more dollarization than is privately optimal in order to constrain the government's choices ex-post. We also study the role of the credit network structure in determining whether the equilibrium currency choices matter and if so whether they are unique or not.

*Preliminary and incomplete.

1 Introduction

One of the central roles of currency is to serve as a unit of account in credit contracts. In several economies, this role is fulfilled by multiple currencies. The coexistence of multiple currencies in denominating contracts is specially relevant in emerging economies, which are often subject to high levels of government policy risk. In this paper, we address two related questions on the role of currencies as units of account. First, what determines the currency choice of contracts among private agents? Second, how do the collective private choices on the currency of denomination affect the government's choice of monetary policy?

To answer these questions, we study a general equilibrium model with credit chains in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. Denominating contracts in local currency helps mitigate fundamental risk, while denominating contracts in a foreign currency (dollar) minimizes risks due to the government's choice of monetary policy. In the aggregate, the equilibrium currency of denomination calls for coordination of currencies in bilateral contracts within a chain to avoid costly default due to currency mismatch. This implies that the incentives to denominate contracts in a foreign currency might persist, even after government risk has been significantly reduced. Our model can help explain the observed hysteresis of dollarization that occurred in several Latin American countries. Next, we show that the socially optimal allocation would call for even more dollarization than is privately optimal in order to constrain the government's choices ex-post. We also study the role of the credit network structure in determining whether the equilibrium currency choices matter and, if so, whether they are unique or not.

Our theory relies on three important features. The first is the presence of credit chains. Agents meet bilaterally and write credit contracts to exploit gains from trade. Agents that act as creditors in some contracts act as debtors in other contracts, which gives rise to credit chains. The presence of credit chains gives rise, in turn, to the risk of mismatch between an agent's assets and liabilities. The second feature is that defaulting on contracts is costly, and so there is a benefit from minimizing risks in agents' balance-sheets. We build on the work of Doepke and Schneider (2017) to incorporate these two features. The third and more novel feature is the presence of endogenous government monetary policy that depends on the distribution of contracts and their currency of denomination. By introducing this element and analyzing the economy in general equilibrium, we are able to analyze how the use of local currency in contracts responds to government policy risk.

We begin by characterizing the optimal bilateral credit contract. Agents engage in credit contracts to exploit gains from trade of a good with different valuations. Credit contracts stipulate the amount of a special good that is provided at the date the contract is signed, in exchange for an amount of local and/or foreign currency to be paid in the future. In our baseline model, we assume that the cost of default is infinite, and relax this assumption later. Currencies serve only as units of account, since the actual payment in the future is made in terms of a general good that is traded in a centralized market. At the time of signing a contract, buyers, or debtors, might have existing claims on local and foreign currency and also know that they will receive a stochastic amount of the centralized good in the future. The optimal contract calls for matching initial claims in both currencies as liabilities and setting additional debt, to be paid using the endowment of the centralized good, in the currency with the lowest price risk.

The intuition is best illustrated with the following example. A buyer arrives at a meeting with claims on 10 units of local and foreign currency, and has a future endowment of 1 unit of the centralized good. Suppose that the expected price of the centralized good is 1.5 in both currencies, but that the minimum valuation of that good is 1 in foreign currency and 0.5 in local currency. In this case, the optimal contract calls for promising 10 units of local currency debt and 11 units of foreign currency debt. This combination of debt in both currencies allows the buyer to obtain as much of the good that the seller provides, with default-free debt. The optimal contract has two features. First, there is gains from matching the currency of initial claims as liabilities in the same currency. The benefit of doing so is that this avoids any repayment risk. Second, any additional debt that is incurred using the future endowment of the centralized good as repayment is made in the currency that has the lowest price risk.

We embed the bilateral contract problem in an economy with credit chains to analyze the distribution of contracts that emerges in general equilibrium. We first do so by taking the government policy as given. The advantage of matching currencies on both sides of agents' balance-sheets generates a benefit of coordinating on the use of a currency as a unit of account within a credit chain. The aggregate choice of the currency used depends on its price risk relative to the domestic good that is used to settle debts. The price of foreign currency relative to the domestic good fluctuates due to aggregate fundamental risk that determines fluctuations in the real exchange rate. The price of local currency is determined by the government through the choice of inflation. If real exchange rate risk is higher (lower) than inflation risk, then contracts are set in local (foreign) currency. If these risks are the same, then the equilibrium features a coexistence of both currencies serving as units of account.

Next, we analyze the optimal government policy given a distribution of contracts and currency choices in the economy. The government cares about the welfare of all agents in the economy and dislikes consumption inequality. This implies that the government would like to redistribute from sellers to buyers ex-post. When choosing the price level of the local currency, the government trades-off the losses associated with inflation with these redistributive benefits. These benefits depend on the predominance of local currency as a unit of account in private contracts. When some private contracts are denominated in local currency, higher levels of inflation reduce the real burden of debt for buyers and redistributes resources from sellers to buyers. We show that a higher predominance of local currency in contracts leads to higher inflation risk. This is because there is uncertainty about the government's costs of inflating at the time of setting contracts. Therefore, while currency choices are complementary within credit chains, they become substitutes across chains. If local currency is used in some credit chains, the government is more willing to use inflation for redistribution, which generates inflation risk and makes the use of local currency less attractive in contracts signed within other credit chains. We argue that under fairly general conditions, a unique equilibrium emerges with a coexistence of local and foreign currency serving as units of account in private contracts.

When the uncertainty about government policy is higher, the share of contracts in local currency is lower. This model prediction is particularly useful to understand the cross-country heterogeneity in the use of foreign currency as a unit of account. We show that countries with higher inflation volatility have higher degrees of dollarization. This pattern holds both when we measure the incidence of dollar denominated financial contracts and the incidence of dollar invoicing in international trade contracts. Understanding the source of this cross-sectional heterogeneity is useful for thinking about the strong predominance of the dollar in the global economy. Our model predictions are also supported by micro evidence. Data from a household survey in various Eastern European economies point to the stability of the local currency and the trust in government as relevant factors in determining the individual decisions of saving in local currency.

We then use our model to shed light on the observed hysteresis in the share of foreign currency-denominated contracts. This pattern is most striking in various Latin American economies that still exhibit high levels of financial dollarization in spite of continued success in controlling inflation and inflation risk in the last decade. In our model, the currency of contracts exhibits hysteresis due to the fact that contracts are set sequentially and that there are benefits of matching the currency of denomination within credit chains. We illustrate this with an example in which at some point in time all agents learn that the government is more likely to perceive higher costs of inflation, thereby lowering expected levels of inflation. If prior to the arrival of this information the level of government policy risk was higher, then contracts display a high degree of dollarization. In response to the arrival of this information, the equilibrium level of dollarization only decreases gradually. The reason is that it is still optimal to match the currency of contracts set before the arrival of information regardless of the fact that inflation risk is lower.

One striking feature of our model is that the equilibrium is unique. This is not usually the case in models in which there is room for coordination on the use of currencies. We argue that the uniqueness result relies on the structure of the credit network. For open credit chains (i.e. chains that have a beginning and an end), the equilibrium currency choice is unique. This is because in open credit chains, the currency of the first contract is determined by comparing price risk only, and then all the subsequent contracts match currencies. We then consider the case of closed chains. These are best represented by agents located in a circle and trading with the agent to the right. Under this structure of credit chains, we show that there are multiple equilibria with various choices of currencies associated with the same real allocations. This result emerges because in closed chains with ex-ante homogeneous agents, the equilibrium features zero net assets and liabilities for every agent and the role of contracts is just to facilitate decentralized trading and exploiting static gains from trade.

The use of foreign currency in denominating private contracts has been a long standing source of concern for policy makers. In fact, several economies impose heavy regulations on the ability of agents to set contracts in foreign currency, with the perception that a strong predominance of foreign currency contracts may make the economy more vulnerable to exchange rate shocks.[1] Motivated by these policy initiatives, we investigate whether the currency choice of contracts that emerge in the competitive equilibrium are efficient or not. To do this, we solve for the optimal contract that a planner would choose behind the veil of ignorance taking into account that the government type, and hence the choice of monetary policy, is realized in the future. We argue that the level of dollarization of contracts in the competitive equilibrium is inefficiently low, for two reasons. First, agents do not internalize that a larger share of local currency contracts leads the government to engage in costly inflation ex-post. Second, contracting in local currency induces redistribution through inflation, which reduces the expected repayment from buyers and thus limits the gains from trade ex-ante.

Finally, we analyze a variant of the model in which we relax the assumption that contracts are default-free. The choice of the scale of the bilateral contract in this case trades-off the gains from trading the good with differential valuations, with the costs incurred in case of default. On the one hand, larger gains from trade imply a larger optimal contract scale. On the other hand, the higher default costs imply a smaller contract scale. The optimal choice of currency is given by the currency with lower price risk, relative to the good that is used for repayment. We also analyze the social planner's problem and find that the same negative externalities associated with the choice of local currency that are

[1] Examples from Latin America include Brazil and Colombia, that pose severe restrictions on the easiness to set up bank deposits in foreign currency. Argentina implemented a forced conversion of foreign currency loans and deposits to local currency in 2001. In Eastern Europe similar policies have been implemented. In Croatia, Hungary and Poland governments facilitated conversion of foreign currency mortgages. See Kolasa (2016) for further examples of regulation to dissuade foreign currency borrowing in Eastern European countries.

present in the baseline model are also present in this model. However, in this model a new externality arises which points in the direction of increasing the share of contracts in local currency. A higher share of contracts in local currency induces higher inflation which in turn helps reduce the incidence of costly defaults.

Related literature. Our paper contributes to the literature that studies the coexistence of currencies in fulfilling the roles of money, and is closely related to the papers that study the use of foreign currency as a unit of account in debt contracts.[2] Ize and Levy-Yeyati (2003) and Rappoport (2009) study models to characterize equilibrium levels of financial dollarization. Other papers study the role of currency denomination of debt in models with financial frictions (see Caballero and Krishnamurthy (2003), Schneider and Tornell (2004) and Bocola and Lorenzoni (2017)).[3] These papers stress the use of both currencies in debt contracts given their differential hedging properties associated with exchange rate fluctuations. In this paper, we build on the framework developed by Doepke and Schneider (2017), who study the general properties of the optimal unit of account, and contribute to this literature in two dimensions. First, we stress the role of coordination in the choice of currency in private contracts and its implications for hysteresis of dollarization and multiplicity of equilibria. Second, in contrast to Doepke and Schneider (2017), in our model prices are determined endogenously and depend on government policy risk. This enables us to study the interaction between government incentives and the distribution of contracts by currency, and assess the efficiency of the competitive equilibrium.

Our paper is also related to a literature the explores the benefits of a common currency/monetary policy to overcome commitment issues by governments. Examples include Chari et al. (2015), Arellano and Heathcote (2010) and Drenik and Perez (2017). While our model also has risky governments who choose policy ex-post, one important difference is that in our model the choice to denominate in a foreign currency might persist even after policy risk has been stabilized.

Finally, our paper contributes to a growing literature on the global role of the dollar (see, for example, Gopinath (2015), Gopinath and Stein (2017) and Chahrour and Valchev (2017)). We contribute to this literature by focusing on the relationship between the use of foreign currency (mostly dollars) and the risk associated to government policy. This

[2]Other strands of the literature have focused on the use of currencies for other purposes. Matsuyama et al. (1993) and Uribe (1997) study the use of a foreign currency as a means of payment. Other papers study the implications of full dollarization (for example, Alesina and Barro (2002), Gale and Vives (2002) and Arellano and Heathcote (2010)) or currency areas (for example, Chari et al. (2015), Aguiar et al. (2015)). A large literature has studied the effects of the currency of denomination of prices. Some examples include Engel (2006) and Gopinath et al. (2010) in the case of international prices and Drenik and Perez (2017) in domestic prices.

[3]Other papers study the optimal choice of currency for corporate debt (see, for example, Aguiar (2005) and Salomao and Varela (2017)) and sovereign debt (see, for example, Ottonello and Perez (2016)).

cross-country heterogeneity is relevant when explaining the predominance of the dollar and our model is able to rationalize it.

The rest of the paper is organized as follows. Section 2 presents motivating evidence. Section 3 presents the model and analyzes the properties of the competitive equilibrium. Section 4 analyzes the efficiency of the competitive equilibrium. Section 5 discusses the case of closed credit chains and the model with small default costs. Finally, Section 6 concludes.

2 Motivating Evidence

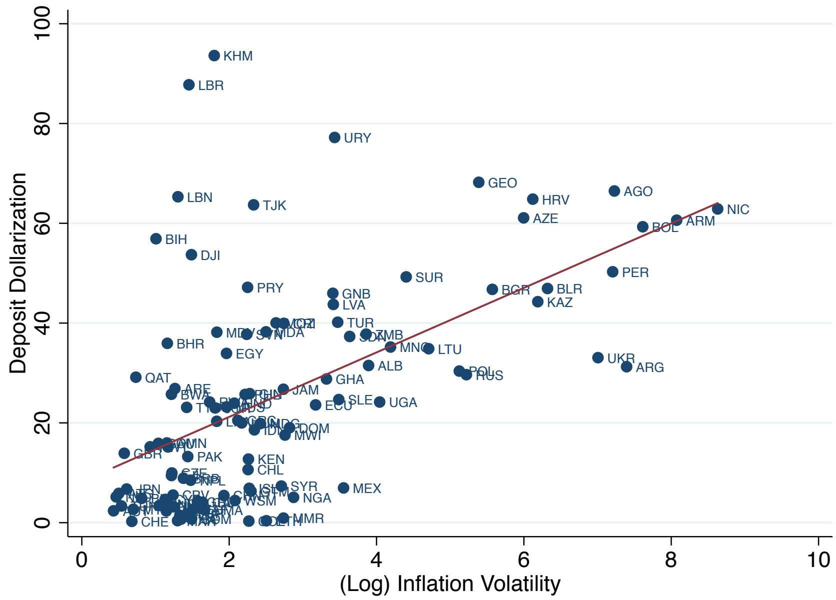
In this section, we briefly present some empirical evidence that motivates our research question. There has been an increasing role of the US dollar over time and across markets. Goldberg and Tille (2008) and Gopinath (2015) document that a significant fraction of international trade is invoiced in dollars, even when the US is not one of the trading partners in the transaction. Similarly, the US dollar has been the predominant currency used as the currency of denomination of international reserves, international debt securities and cross-border loans (ECB (2017)). The global willingness to hold US dollars is also documented in the capital market by Maggiori et al. (2018). However, this aggregate picture is not homogeneous across countries.

Figure 1 shows different measures of dollarization against the volatility of inflation of a country. In Panel 1a, we plot a country's deposit dollarization as a measure of financial dollarization, which is obtained from Levy-Yeyati (2006) and defined as a country's foreign currency deposits over total deposits in local deposit money banks. In Panel 1b, we plot the share of a country's imports invoiced in US dollars (obtained from Gopinath (2015)) net of the import share with the US. In both cases, we observe a positive relationship between the degree of dollarization and inflation volatility. Although a significant fraction of countries rely on the US dollar as a currency of denomination, dollarization is more prevalent in economies with more volatile monetary policy.

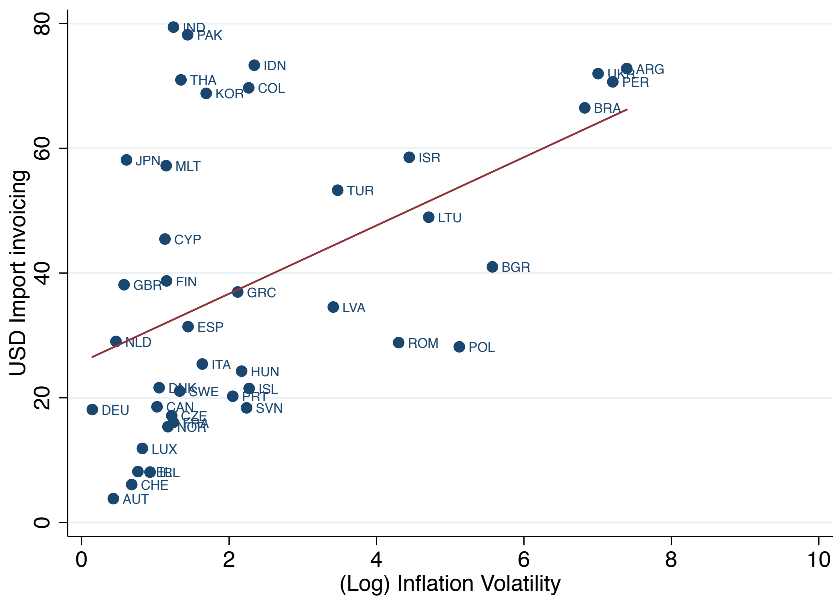
Previous literature has empirically emphasized the relationship between the degree of policy risk and the use of foreign currency as a unit of account. Ize and Levy-Yeyati (2003) develop a portfolio choice model of financial dollarization, in which the currency composition of the minimum-variance portfolio is a function of the relative volatility of inflation versus the volatility of the real exchange rate. They find that their measure of financial dollarization approximates actual dollarization closely. Another piece of evidence documenting the fact that inflation volatility matters for dollarization is presented by Lin and Ye (2013), who show that countries that adopted an inflation-targeting regime expe-

Figure 1: Dollarization and Monetary Instability

(a) Financial Dollarization



(b) Dollarization in Trade



Sources: Levy-Yeyati (2006), Gopinath (2015) and IFS.

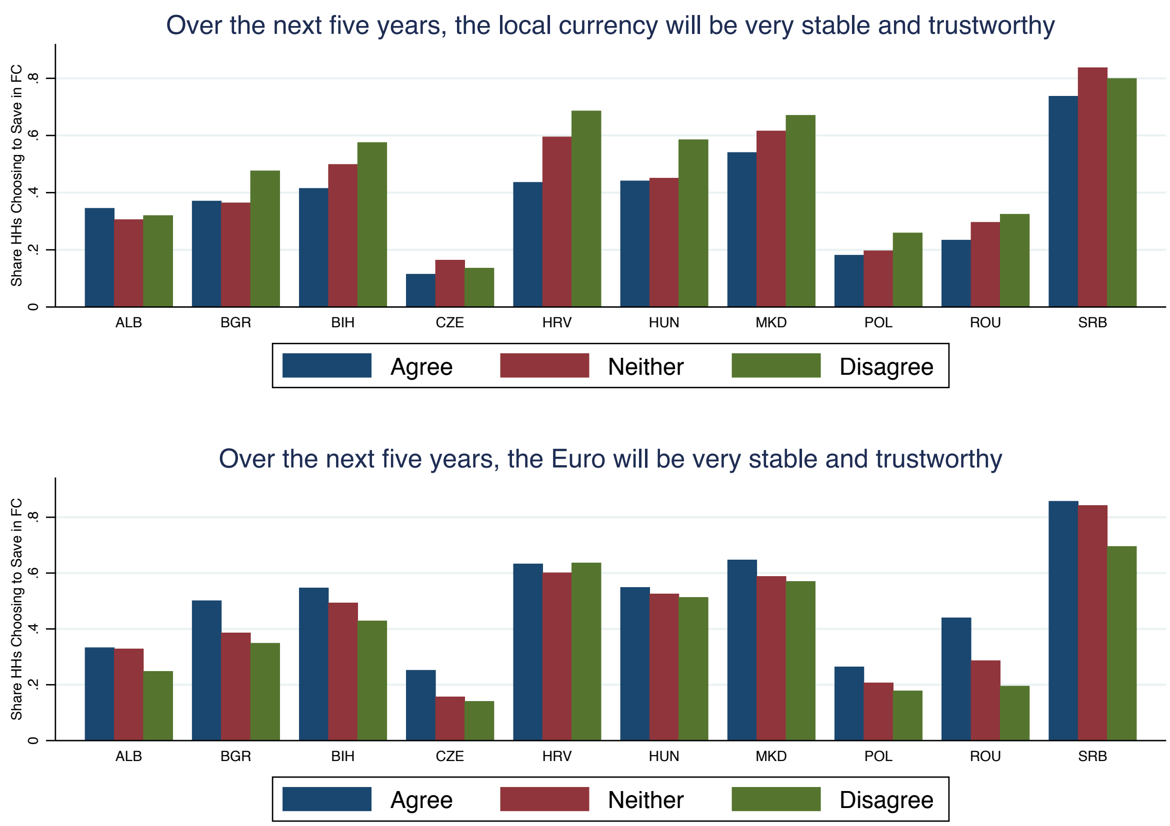
rienced a decline of both actual financial dollarization and the minimum variance portfolio dollarization. More evidence comes from Kamil (2012) who uses firm-level micro data and finds that after countries switch from pegged to floating exchange rate regimes, firms reduce their exposure to foreign currency. In addition, firms' currency-matching of foreign currency liabilities with foreign currency assets increases after the adoption of a flexible exchange rate regime.

The role of relative currency risk at the household level is documented in Figure 2 for a number of Eastern European countries. The source of micro data is the Euro Survey carried out by the Austrian Central Bank (OeNB) and covers 10 European countries (see Brown and Stix (2014) for a description of the survey). The survey asks households about their expectation about the stability of the local currency and the Euro, which is elicited from the degree of agreement between the following two statements: "Over the next five years, the [LOCAL CURRENCY] will be very stable and trustworthy" and "Over the next five years, the Euro will be very stable and trustworthy". In addition, households are asked about their preferences for saving in local or foreign currency.[4] Figure 2 plots, for each country, the share of households with a preference for savings in foreign currency against their expectations about the future stability of the local and foreign currencies. Broadly, the decision to save in foreign currency is positively related with households' expectations of an unstable local currency and negatively related to expectations of an unstable Euro. Brown and Stix (2014) presents regression analysis of this relationship, showing that both relationships are both statistically and economically significant. In addition, the paper shows that government policies and institutions matter for the degree of Euroization of deposits, captured by the fact that preferences for savings in foreign currency are positively associated with past crisis experiences and distrust towards the government.

Finally, we present evidence on the degree of hysteresis in financial markets: the empirical observation that financial dollarization tends to persist even after the motive that precipitated the increase in dollarization vanished. Figure 3 plots the evolution of deposit dollarization and annual inflation (capped at 100% per annum) for 4 developing countries from 1980 to 2007: Argentina, Bolivia, Peru and Uruguay. All economies went through episodes of rapid increases in the inflation rate, followed by a rapid increase in the fraction of deposits in US dollars. Surprisingly, even though inflation became fully stabilized, financial dollarization remained high and stable.

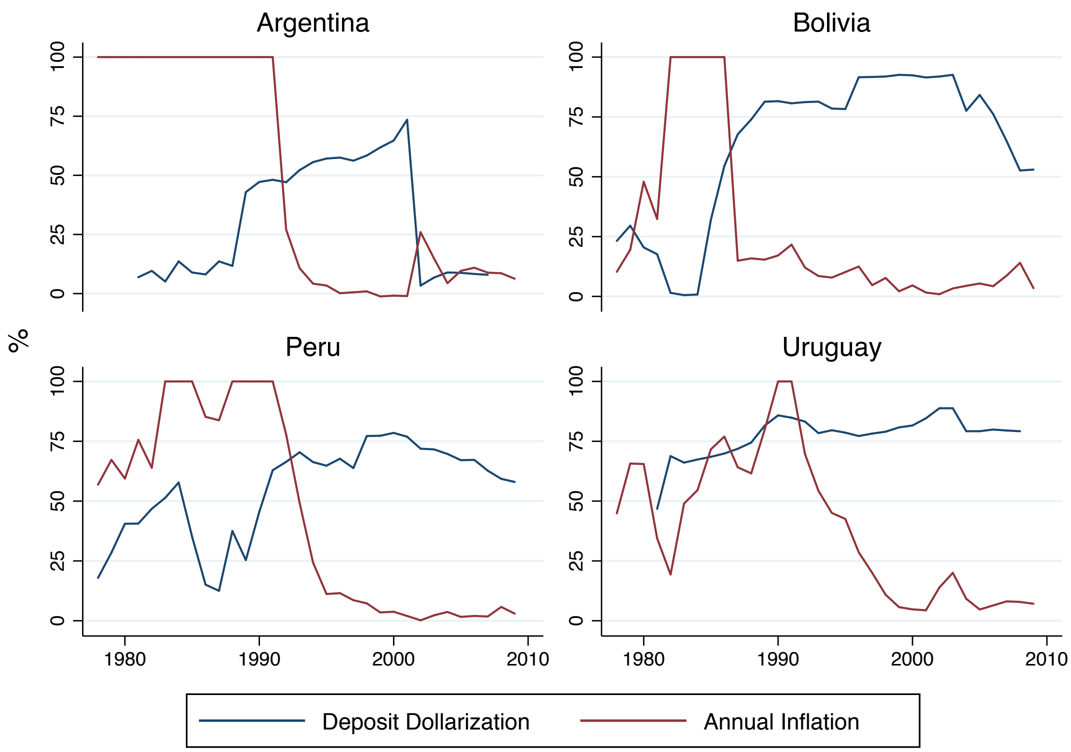
[4] Respondents are presented with the following hypothetical scenario: *'Suppose you had [two average monthly salaries in local currency] to deposit in a savings account. Would you choose to deposit this amount in (a) local currency, (b) Euro, (c) US dollar, (d) other foreign currency?'*

Figure 2: Currency Risk and Households' Choice of Unit of Account



Source: Euro survey conducted by OeNB.

Figure 3: Persistence of Financial Dollarization



*Sources:*Levy-Yeyati (2006) and IFS.

3 Model

3.1 General Environment

There are two periods $1, 2$. Consider an environment consisting of two regions- domestic and foreign. The domestic economy is populated by two types of agents: citizens and a government. Citizens are further divided into one of I publicly observable sub-types $\mathcal{I} \in \{1, 2, ..., I\}$ with continuum of each.

A citizen of type i has preferences over a special good produced by type $i+1$ and produces a special good valued by type $i-1$. All types also value the consumption of a composite good which takes place at the end of period 2. Preferences for the representative citizen type i are given by

$$u_i = (1+\lambda)\ {}_i x_{i+1} - {}_{i-1}x_i + \mathbb{E}[c_i]$$

where ${}_i x_{i+1}$ is the special good produced by a citizen of type $i+1$ for a citizen of type i and ${}_{i-1}x_i$ is the special good produced by a citizen of type i for a citizen of type $i-1$. The composite good c_i is an aggregate of tradable and non-tradable consumption

$$c_i = c_{iT}^{\alpha} c_{iN}^{1-\alpha}.$$

We assume that $\lambda > 0$ and $0 < \alpha < 1$. We also assume that ${}_0x_1 = {}_I x_{I+1} = 0$ so that type 1 does not produce a special good for any other type and type I does not consume a special good.

The timing of the model is as follows:

1. The first period $t = 1$ is divided into $I - 1$ sub-periods in which trade takes place sequentially:
   (a) In sub-period 1, citizens of type 2 produces a special good for citizens of type 1 in exchange for the promise of payment in period 2.
   (b) Similarly, in sub-period i, citizens of type $i+1$ produce a special good for citizens of type i in exchange for the promise of payment in period 2
2. The second period $t = 2$ is divided into three sub-periods:
   (a) In sub-period 1, the type of the domestic government is realized and it chooses its policy which is the aggregate price level
   (b) In sub-period 2, endowments for all citizens are realized
   (c) In sub-period 3, all signed contracts are executed in the order in which they were signed and finally, consumption of the composite good takes place.

All citizens are endowed with $y = (y_T, y_N) \in \mathbf{Y}$ units of tradable and non-tradable goods, respectively. We assume that y_T is random variable with cdf $F(y_T)$ and support $\left[\underline{y}_T, \bar{y}_T\right]$ and y_N is deterministic[5] and normalized to 1. Note that y_T is an aggregate shock common to all agents of each sub-type. Both tradable and non-tradable goods are traded in a centralized and competitive market in period 2. Unlike tradable goods (that can be traded internationally), non-tradable goods can only be traded within the domestic economy. Next, we formally define a contract and discuss its properties.

3.2 Bilateral Contracts

A contract between two parties consists of a provision of the special good in exchange for the promise of future payment. We impose three important assumptions on the contracting environment. The first is that payments are non-contingent and in particular, cannot depend on the realization of the aggregate state. The second is that payments cannot be made directly in terms of the composite, tradable or non-tradable goods. Instead, payments can only be made in two possible "units of account", which we will call *currencies*. We will denote the two possible currencies by l (local) and f (foreign). A payment b_l in currency l yields $\frac{b_l}{p}$ units of the domestic composite good in period 2, while a payment b_f in currency f yields $\frac{b_f}{\tilde{p}}$ units of the domestic composite good in period 2. In general, p and $\tilde{p}$ are random variables that are unknown at the time of the contract being signed. One interpretation of this assumption is that goods are observable but *unverifiable,* as is commonly assumed in the incomplete contracts literature. The third assumption that contracts must be *default-free*. In other words, promised payments must be less than or equal to the endowment in each state of the world. We study the implications of relaxing this assumption in Section 5.

Contracts involve trades between a buyer and seller. In a contract signed in sub-period i of period 1, the citizen of type i who consumes the special good is the buyer and the citizen of type $i+1$ who produces the special good is the seller. Formally, a bilateral contract signed in sub-period i is a the tuple (x, b_l, b_f), where x indicates the units of special good provided to the borrower and (b_l, b_f) are the units of local and foreign currency promised to be paid to the lender at date 2, respectively.

When a contract is signed in sub-period i, types i will typically be owed some payments from types $i-1$, from the contract signed in sub-period $i-1$. Denote $(\hat{b}_l, \hat{b}_f)$ as the claims on local and foreign currency that a buyer has from the previous bilateral contract. Additionally, let e be the nominal exchange rate, defined as units of local currency per unit of foreign currency, and let (p_T, p_N) be the local currency price of tradable and non-tradable goods, respectively. In order to ensure repayment in every state, contracts

[5] The assumption is not crucial since all we need is the relative endowment $\frac{y_T}{y_N}$ to be stochastic.

must satisfy the following condition

$$b_l + e b_f \leqslant p_T y_T + p_N y_N + \hat{b}_l + e \hat{b}_f \quad \text{for all } (y_T, y_N) \in \mathbf{Y} \text{ and } (e, p_T, p_N) \in \mathbf{P}, \tag{1}$$

where $\mathbf{P} \subset \mathbb{R}^3_+$ is the compact set of possible price realizations. This inequality states that for all possible price and endowment realizations, the income of the buyer should not exceed the promised repayment. Agents are exposed to risk from uncertainty about their endowments and aggregate prices. Prices in this economy are endogenous and citizens take them as given. Notice also that we implicitly restrict consumption of the composite good in period 2 to be non-negative.

Without loss of generality, we assume that in each bilateral meeting the buyer makes a take-it-or-leave-it offer to the seller. The seller is willing to participate in the contract as long as

$$\mathbb{E}\left[\frac{b_l + e b_f}{p}\right] - x \geqslant 0, \tag{2}$$

where p is the local currency price of the composite good and x is the disutility from providing the special good. We refer to p as the price level of the domestic economy. Also note that a claim on a unit of foreign currency entitles the seller to $\frac{1}{\bar{p}} \equiv \frac{e}{p}$ units of the domestic composite good. This inequality states that the expected utility of using income repayment to consume the composite good needs to exceed the disutility of exerting labor to provide the special good. The optimal contract for the buyer solves

$$\max_{x, b_l, b_f} \mathbb{E}\left[(1+\lambda)x + c\right]$$

subject to

$$pc = p_T y_T + p_N y_N - \left(\hat{b}_f - b_f\right) - e\left(\hat{b}_f - b_f\right), \tag{3}$$

(1) and (2) and the non-negativity constraints $b_l, b_f \geqslant 0$. Equation (3) denotes the budget constraint of the borrower at date 2 after all signed contracts have been executed.

3.3 Prices and Units of Account

At date 2, endowments are realized and relative prices are such that all centralized markets clear. Solving for the individual demands for tradable and non-tradable goods, and imposing market clearing in the non-tradable goods market yields the following expressions for the prices of tradable and non-tradable goods in local currency

$$p_T = \alpha p \left(\frac{y_N}{y_T}\right)^{1-\alpha} \quad \text{and} \quad p_N = (1-\alpha) p \left(\frac{y_T}{y_N}\right)^{\alpha}. \tag{4}$$

We assume that there is a foreign economy that is symmetric to the domestic economy at date 2. Assuming the law of one price holds for tradable goods, we obtain an expression for the nominal exchange rate

$$e = \frac{p}{p^*} \left(\frac{y_N}{y_T} \frac{y_T^*}{y_N^*}\right)^{1-\alpha}, \tag{5}$$

where p^* is the aggregate price level of the foreign economy, and (y_T^*, y_N^*) are the aggregate endowments of tradable and non-tradable goods. Since we are interested in the role of domestic risk in affecting currency choices of contracts we normalize $p^* = y_T^* = y_N^* = 1$.

3.4 Competitive Equilibrium given Government Policy

Local and foreign currency constitute two different units of account that have associated price risk relative to the domestic composite good. The local currency is subject to endogenous inflation risk associated with government policy. From the perspective of citizens, the price level p is a random variable with cdf $G(p)$ and support $[\underline{p}, \bar{p}]$. The price risk of foreign currency comes from real exchange rate risk.

When setting contracts, citizens face uncertainty in the value of the local and foreign currency (expressed in the domestic composite good), and in the value of their endowment. Citizens anticipate the equilibrium relationship of prices and acknowledge that these sources of risk reduce to only two: price risk of the local currency (which in equilibrium will be linked to government risk and uncorrelated to fundamental risk), and fundamental risk of the aggregate endowment of tradable goods. This is because both the price risk of foreign currency $\frac{e}{p} = y_T^{\alpha-1}$, and the value of their endowment $p_T y_T + p_N y_N = y_T^{\alpha}$. Note that citizens understand that there is a correlation between the value of foreign currency and the value of their income: when y_T is low, the value of their income is low and the value of foreign currency (expressed in the domestic composite good) is high.[6]

Given these expressions, we can now characterize the optimal bilateral contract between any two sub-types, given an exogenous distribution of y_T and taking the distribution of p as given.

[6] This equilibrium negative correlation between the level of economic activity and the real exchange rate is in line with the negative correlation observed in most emerging economies, where exchange rates tend to depreciate in bad times.

Proposition 1. *Suppose that* $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} > \mathbb{E}\left[\frac{\underline{p}}{p}\right]$. *Then,*

$$b_l = \hat{b}_l$$

$$b_f = \hat{b}_f + \underline{y}_T$$

Next, suppose that $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} \leqslant \mathbb{E}\left[\frac{\underline{p}}{p}\right]$, *then:*

$$b_f = \hat{b}_f - \frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)} \tag{6}$$

$$b_l = \hat{b}_l + \underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)}$$

providing that $\hat{b}_f - \frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)} \geqslant 0.$

All proofs are included in the Appendix. The terms $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}$ and $\mathbb{E}\left[\frac{\underline{p}}{p}\right]$ refer to relevant measures of the degree of real exchange rate and inflation risk, respectively. In particular, a high value of $\mathbb{E}\left[\frac{\underline{p}}{p}\right]$ $\left(\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}\right)$ implies a low level of inflation risk (real exchange rate risk). Notice that it is the minimal values of p and y_T that determine the levels of real exchange rate and inflation risk due to our assumption that contracts must be default free. As a result, budget constraints must continue to hold even for the lowest values of the random variables. The proposition characterizes the currency choice that allows for the maximal amount of the special good to be provided since it has associated gains of trade.

The first part of the proposition characterizes the case in which inflation risk is high compared to real exchange rate risk, or local currency is more risky than foreign currency, measured in terms of the composite good. In this case, the optimal contract calls for issuing liabilities in both currencies to match the initial claims of the buyer. Additionally, the buyer also issues additional liabilities in foreign currency for as much as can be credibly promised to be repaid in every state of the world using the future endowment. Notice that it is never optimal to issue all claims in the foreign currency since currency mismatch is costly. Engaging in currency mismatch is costly since it exposes the buyers to exchange rate risk.

Next, consider the second part of the proposition which characterizes the contract in the case in which inflation risk is lower than real exchange rate risk. While a logic similar to the previous case determines the optimal contract, there is an additional incentive to reduce foreign currency payments in order to hedge against real exchange rate risk. This is because the buyer can gain from being a net creditor in foreign currency to hedge against the bad states in which the value of her endowment is low.

Now we characterize the aggregate distribution of contracts taking as given local currency price risk.

Corollary 1. *In equilibrium,*

1. *If* $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} > \mathbb{E}\left[\frac{\underline{p}}{p}\right]$*, then*
$$B_{li} = 0$$
$$B_{fi} = i\underline{y}_T$$

2. *If* $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} < \mathbb{E}\left[\frac{\underline{p}}{p}\right]$*, then*
$$B_{fi} = 0 \tag{7}$$
$$B_{li} = i\underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)} \tag{8}$$

3. *If* $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} = \mathbb{E}\left[\frac{\underline{p}}{p}\right]$*, then, there exist a continuum of equilibria in which agents randomize. In particular,*
$$B_{li} = i\left(1-\gamma\right)\underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)} \tag{9}$$
$$B_{fi} = i\max\left\{\gamma\underline{y}_T - (1-\gamma)\left(\frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}\right), 0\right\} \tag{10}$$
is an equilibrium for any $\gamma \in [0,1]$.

Two features are worth mentioning. First, the scale of payments in a particular currency increases in each subsequent contract. This is because gains of trade are linear in the provision of the special good and each buyer uses their claims and the value of her future endowment to issue liabilities and obtain more of the special good. Second, the advantage of matching currencies on both sides of agents' balance-sheets generates a benefit of coordinating on the use of a currency as a unit of account within a credit chain. In particular, when real exchange rate risk is higher (lower) than inflation risk, then contracts are set in local (foreign) currency. In the case in which $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} = \mathbb{E}\left[\frac{\underline{p}}{p}\right]$, then randomization across chains is an equilibrium.

3.5 Government

The government controls monetary policy and chooses the price level of the domestic economy p. The government's willingness to create inflation (choose higher p) depends on its preferences towards redistribution and the cost of inflation. The government assigns Pareto weights of $\Theta = (\theta, 1, ..., 1)$ for sub-types, respectively, where $\theta > 1$. This implies that the government has strong redistribution preferences, since sub-type 1 are net debtors at date 2.

We assume that there is a utility loss for the government of engaging in inflation, captured by a loss function $\psi l\left(\frac{1}{p}\right)$ that satisfies $l'\left(\frac{1}{\underline{p}}\right) = 0$, $l''\left(\frac{1}{p}\right) > 0$ and $l'''\left(\frac{1}{p}\right) \geqslant 0$ for all p. The government type is indexed by ψ, which governs how costly is inflation. We assume that ψ is drawn in period 2 and at period 1 is a random variable with cdf $H(\psi)$ and support $\left[\underline{\psi}, \infty\right]$. Notice that higher ψ increases the marginal cost of inflation and therefore makes the government less likely to redistribute using inflation.[7] Denote $\mathbf{C} = (C_1, ..., C_I)$ to be the aggregate consumption of the composite good of each sub-type respectively. The objective function of the government is given by

$$\mathbb{E}\left(\Theta \cdot \mathbf{C}\right) - \psi l\left(\frac{1}{p}\right),$$

where

$$C_i = \frac{p_T y_T + p_N - \left(B_{li} - \hat{B}_{li}\right) - e\left(B_{fi} - \hat{B}_{fi}\right)}{p}$$

and (B_{li}, B_{fi}) denotes the aggregate levels of (b_l, b_f) in contracts signed between sub-types $i+1$ and i and $\left(\hat{B}_{li}, \hat{B}_{fi}\right) = (B_{li-1}, B_{fi-1})$. The first order condition of the government's problem is

$$-\theta B_{l1} - \sum_{i=2}^{I-1}\left(B_{li} - \hat{B}_{li}\right) + \hat{B}_{lI} = \psi l'\left(\frac{1}{p}\right)$$

or

$$(1-\theta)\, B_{l1} = -\psi l'\left(\frac{1}{p}\right).$$

We can define the optimal government choice as

$$p^*\left(\psi, B_{l1}\right) = \frac{1}{g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)}, \tag{11}$$

where $g = (l')^{-1}$ is an increasing function. Therefore, given B_{l1}, the distribution over ψ

[7] Alternatively, we could assume that θ is a random variable.

induces a distribution over p with support $\left[\underline{p}, \bar{p}\left(B_{l1}\right)\right]$ where

$$\bar{p}\left(B_{l1}\right) = \frac{1}{g\left(-\frac{(1-\theta)B_{l1}}{\underline{\psi}}\right)}.$$

We now define a competitive equilibrium in which the government chooses its policy optimally.

Definition 1. A competitive equilibrium is a tuple $(\mathbf{B}, p^*)$ where $\mathbf{B} = (B_{li}, B_{fi})_{i \in I}$ such that given p^*, $\mathbf{B}$ is determined using Corollary 1 and given $\mathbf{B}$, p^* is given by (11).

Proposition 2. *Let B^* be the fixed point of the system defined by (8) and (11). Then, if $\mathbb{E}\left(\frac{y_T}{y_T}\right)^{1-\alpha} > \mathbb{E}\left[\frac{p}{p^*(\psi, B^*)}\right]$, there exists a unique competitive equilibrium in which $\mathbf{B}$ is given by (9) and (10) and γ is chosen such that $\mathbb{E}\left(\frac{y_T}{y_T}\right)^{1-\alpha} = \mathbb{E}\left[\frac{p}{p^*(\psi, B_{l1})}\right]$. On the other hand, if $\mathbb{E}\left(\frac{y_T}{y_T}\right)^{1-\alpha} < \mathbb{E}\left[\frac{p}{p^*(\psi, B^*)}\right]$, then $\mathbf{B}$ is given by (8) and (7).*

Our definition of a competitive equilibrium implies that the competitive equilibrium can be computed as a solution to the fixed point of a system of equations. This proposition says that only two types of competitive equilibria can exist. In the first, at the fixed point of the system defined by (8) and (11), the implied inflation risk is too high to sustain an equilibrium in which all claims are made in local currency and, as a result, the unique equilibrium in this case is one in which agents randomize between issuing claims in the foreign and local currency. The second type of equilibrium is one in which the solution of the fixed point implies lower inflation risk, which in turn justifies a corner solution in which all claims are made in local currency. Notice that an equilibrium in which all claims made in foreign currency cannot exist since in this case the implied inflation risk would be zero, and thus agents will want to issue some claims in local currency.

3.6 Equilibrium Properties

We first consider the effect of an increase in government risk on the equilibrium level of currency choices. In particular, consider a small perturbation to the support of ψ in which each possible ψ is increased by ε. The distribution over the new support $\tilde{H}$ is such that $\tilde{h}(\psi) = h(\psi - \varepsilon)$.

Lemma 1. *Suppose that the equilibrium is interior with randomization level $\gamma(0) \in (0, 1)$. Then $\gamma(\varepsilon) < \gamma(0)$ for ε small.*

This model prediction is particularly useful to understand the cross-country heterogeneity in the use of foreign currency as a unit of account, shown in Section 2.

Next, we study the equilibrium implications in the case in which information about inflation risk arrives gradually over time. We will show that our model can generate outcomes consistent with the observed hysteresis of dollarization in Latin American countries, even after inflation risk subsided. To do this, consider our model with a slightly different information structure. At the beginning of $t = 1$, all agents believe that the support of ψ is given by $[\underline{\psi}, \infty]$. In some sub-period $i^* > 1$ (where i^* is known at $t = 1$) there is a public signal that determines whether ψ is in $\left[\underline{\psi}_1, \infty\right]$ or $\left[\underline{\psi}_2, \infty\right]$ for some $\underline{\psi}_1 < \underline{\psi} < \underline{\psi}_2$. And so in sub-period i^*, all agents receive either good ($\psi \in \left[\underline{\psi}_2, \infty\right]$) or bad news ($\psi \in \left[\underline{\psi}_1, \infty\right]$) about inflation risk.

Lemma 2. *Suppose that $t = 1$, we are at an interior equilibrium with $0 < \gamma < 1$. Now suppose good news arrives in sub-period i^*. Then, the rate of de-accumulation of foreign currency is bounded above by $\frac{\alpha}{1-\alpha}\left(\frac{1}{\mathbb{E}[y_T^{\alpha-1}]}\right)^{1/(1-\alpha)}$.*

The reason is that it is still optimal to match the currency of contracts set before the arrival of information regardless of the fact that government and inflation risk is lower.

4 Efficient Dollarization of Contracts

We now consider the problem of a social planner who chooses the allocation and inflation rate and is subject to the same constraints as private agents. For this section, we assume that $I = 2$. All our results generalize to the case in which $I > 2$. The social planner's problem is

$$\max_{x, B_l, B_f, p} \mathbb{E}\left[\theta u_1 + u_2 - \psi l\left(\frac{1}{p}\right)\right]$$

subject to (2), the no-default constraint (1), budget constraints (3) for each sub-type, non-negativity constraints, and the first-order condition (11). This problem can be written as

$$\max_{B_l, B_f} \mathbb{E}\left[\theta\left((1+\lambda)\left[\frac{B_l + eB_f}{p^*(\psi, B_l)}\right] + \frac{p_T y_T + p_N - B_l - eB_{fi}}{p^*(\psi, B_l)}\right) + \left(-\left[\frac{B_l + eB_f}{p^*(\psi, B_l)}\right] + \frac{p_T y_T + p_N + B_l + eB_f}{p^*(\psi, B_l)}\right)\right]$$

subject to

$$\frac{B_l}{\underline{p}} + B_f\left(\frac{1}{\underline{y}_T}\right)^{1-\alpha} = \underline{y}_T^{\alpha}$$

and so the problem is

$$\max_{B_l, B_f} \underline{p}\mathbb{E}\left[\theta\lambda\left[\left[\frac{\underline{p}}{p^*(\psi, B_l)} - \left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}\right] B_l + \underline{p}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} \underline{y}_T^{\alpha}\right] - \underline{p}l\left(\frac{1}{p^*(\psi, B_l)}\right)\right]$$

The foc w.r.t B_l is

$$\mathbb{E}\left[\theta\lambda\left[\frac{\underline{p}}{p^*(\psi,B_l)}-\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}\right]-\left(\theta\lambda B_l-l'\left(\frac{1}{p^*(\psi,B_l)}\right)\right)\frac{\underline{p}}{(p^*(\psi,B_l))^2}p^*_{B_l}(\psi,B_l)\right]$$

The first term in the parenthesis corresponds the first order condition of the competitive equilibrium. The second term corresponds to the externality that the social planner internalizes, but not private agents. In particular the solution to the above problem implies that that denominating a smaller share of payments in the local currency is welfare increases for two reasons. The first is through the direct costs of inflation $l'\left(\frac{1}{p^*(\psi,B_l)}\right)$ while the second is through the reduction in the provision of the special good as a consequence of higher inflation risk.

Proposition 3. *The solution the social planner's problem has* $B_l^{sp} < B_l^{ce}$*, where* B_l^{ce} *corresponds to the local currency choice in the competitive equilibrium allocation.*

5 Extensions

In this section, we analyze the implications of considering alternative network structures and relaxing the assumption of default-free contracts.

5.1 Closed Chains

In the previous section, we characterized competitive equilibria assuming a particular network structure, namely that all citizens were located in a line (open chains). Here, we show how our results might change if we modify the network structure. We assume a structure identical to the one before, except that citizen 1 produces a special good for citizen I. We also assume an upper bound ${}_i\bar{x}_{i+1}$ on the special good that each citizen can produce. The optimal bilateral contract between a buyer and seller is similar to the one above except for the constraint ${}_ix_{i+1} \leqslant {}_i\bar{x}_{i+1}$. We show that if ${}_i\bar{x}_{i+1} = \bar{x}$, then in any symmetric equilibrium, the currency choice is irrelevant.

Lemma 3. *Any contract* $(B_l, B_f, \overline{x})$ *with*

$$\mathbb{E}\left[\frac{B_l}{p}+\left(\frac{1}{y_T}\right)^{1-\alpha}B_f\right]=\overline{x}$$

is a competitive equilibrium, with the same real allocations (x, c).

The intuition behind this result is that contracts do not involve any credit to be repaid in the future, since all citizens are simply passing on the previously signed contract forward to the next bilateral contract. Thus, currency choice becomes irrelevant.

5.2 Model with Small Default Costs

In this section, we discuss the implications of relaxing the assumption of contracts being default-free. We consider a model in which default entails a utility loss and in which the choice of currency is discrete (the contract is set either in local or foreign currency). We outline the details of this variant of the model and its solution in Appendix B.

The choice of the scale of the bilateral contract in this case trades-off the gains from trade associated with the special good and the costs of incurring in default. On the one hand, the higher are the gains of trade involved in trading the special good, the larger is the optimal scale of the contract. On the other hand, the higher are the cost of default, the smaller is the optimal scale of the contract. The optimal choice of currency is given by the currency with lower price risk, relative to the composite good, which is the good that is used for repayment. The reason is that higher price risk increases the likelihood of costly default. Therefore, when real exchange rate risk is larger (smaller) than inflation risk, the optimal contract is set in local (foreign) currency.

The government's optimal choice of inflation is increasing in the share of contracts set in local currency. With a higher share of contracts set in local currency, the government has an additional motive to increase inflation in order to reduce the likelihood of costly defaults. In general equilibrium, the share of contracts in local currency is interior and such that the endogenous inflation risk is the same as the exogenous real exchange rate risk. As in the baseline model, an increase in the costs of inflation increases the share of contracts in local currency.

Finally, we analyze the social planner's problem. We find that the same negative externalities associated with the incidence of local currency that are present in the baseline model are also in place in this model. In particular, unlike private agents, the social planner internalizes that a higher share of contracts in local currency leads to higher inflation, which is socially costly and also limits the gains of trade that can be attained with credit contracts. However, in this model a new externality arises which points in the direction of increasing the share of contracts in local currency. A higher share of contracts in local currency induces higher inflation which in turn helps reduce the incidence of costly defaults.

6 Conclusion

This paper develops a framework to study the optimal choice of currency in the denomination of private credit contracts in credit chains. We characterize the unique equilibrium in the presence of real exchange rate risk and a benevolent government who chooses the inflation rate ex-post. We show that optimal contracts avoid mismatch so that even when good news about inflation risk arrives, there is only gradual de-accumulation of the foreign currency. This feature is consistent with the experience of several Latin American countries who experience hysteresis in dollarization. We also show that the equilibrium choice of local currency is inefficient in the sense that a planner confronted with the same frictions would choose to denominate a larger fraction of contracts in the foreign currency. Finally, we discuss the role of network structure in our results and consider the case in which default costs are not infinite.

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A Omitted Proofs

Proof of Proposition 1: The optimal contract solves

$$\max_{\hat{b}_{li},\hat{b}_{fi}} \frac{\lambda}{1+\lambda}\mathbb{E}\left[\frac{\hat{b}_{lj}}{p} + \left(\frac{1}{y_T}\right)^{1-\alpha}\hat{b}_{fj}\right]$$

subject to

$$\frac{\hat{b}_{lj}}{p} + \hat{b}_{fj}\left(\frac{1}{y_T}\right)^{1-\alpha} \leqslant \frac{b_{lj}}{p} + b_{fj}\left(\frac{1}{y_T}\right)^{1-\alpha} + y_T^{\alpha} \quad \forall (b_{lj}, b_{fj}, y_T, y_N)$$

Let $(\tilde{p}, \tilde{y}_T)$ denote the variables with property that the above constraint holds in all states of the world. This constraint can be written as

$$\hat{b}_{lj} \leqslant b_{lj} + \tilde{p}\left[(b_{fj} - \hat{b}_{fj})\left(\frac{1}{\tilde{y}_T}\right)^{1-\alpha} + \tilde{y}_T^{\alpha}\right]$$

Substituting this into the objective function and taking the first order condition wrt $\hat{b}_{ji}$ yields

$$\mathbb{E}\left(\frac{\tilde{y}_T}{y_T}\right)^{1-\alpha} - \mathbb{E}\left[\frac{\tilde{p}}{p}\right]$$

Notice that if there a positive endowment of the good, there are three possibilities for the solution; 1. $(\hat{b}_{fi} > b_{fi}, \hat{b}_{li} \leqslant b_{li})$ 2. $(\hat{b}_{fi} \leqslant b_{fi}, \hat{b}_{li} > b_{li})$ or 3. $(\hat{b}_{fi} > b_{fi}, \hat{b}_{li} > b_{li})$. Suppose first that $\mathbb{E}\left(\frac{y_T}{y_T}\right)^{1-\alpha} > \mathbb{E}\left[\frac{p}{p}\right]$.

1. Under case 1, $\tilde{p} = \bar{p}$ and $\tilde{y}_T = \underline{y}_T$. So if $\hat{b}_{li} < b_{li}$ then the foc is

$$\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} - \mathbb{E}\left[\frac{\bar{p}}{p}\right] < 0$$

which cannot be optimal. Therefore, in this case a possible solution is

$$\hat{b}_{lj} = b_{lj}$$

$$\hat{b}_{fj} = b_{fj} + \underline{y}_T > b_{fj}$$

and the welfare associated with this contract is

$$\frac{\lambda}{1+\lambda}\mathbb{E}\left[\frac{b_{lj}}{p} + \left(\frac{1}{y_T}\right)^{1-\alpha}\left[b_{fj} + \underline{y}_T\right]\right]$$
$$= \frac{\lambda}{1+\lambda}\left[b_{lj}\mathbb{E}\left[p^{-1}\right] + b_{fj}\mathbb{E}\left[y_T^{\alpha-1}\right] + \underline{y}_T^{\alpha}\mathbb{E}\left[\frac{\underline{y}_T^{1-\alpha}}{y_T^{1-\alpha}}\right]\right]$$

2. Under case 2, $\tilde{p} = \underline{p}$ but in general $\tilde{y}_T$ might be interior. We are interested in the value of y_T that minimizes the function function $f(y) = ay^{\alpha-1} + y^{\alpha}$ where $a = \left(b_{fj} - \hat{b}_{fj}\right)$. We have

$$f'(y) = y^{\alpha-2}\left[a(\alpha-1) + \alpha y\right]$$

so that the local minimum is $y = \left(\frac{1-\alpha}{\alpha}\right)a$ and $f'(y) < 0$ iff

$$y < \left(\frac{1-\alpha}{\alpha}\right)a$$

Next, we have

$$f''(y) = -(1-\alpha)y^{\alpha-3}\left[-a(2-\alpha) + \alpha y\right]$$

so that $f''(y) > 0$ iff

$$y < \left(\frac{2-\alpha}{\alpha}\right)a$$

Therefore, the function behaves as follows: f is decreasing and convex from 0 to $\left(\frac{1-\alpha}{\alpha}\right)a$ and is increasing and convex from $\left(\frac{1-\alpha}{\alpha}\right)a$ to $\left(\frac{2-\alpha}{\alpha}\right)a$ and is increasing and concave after that. In particular, it implies that the local minimum $y = \left(\frac{1-\alpha}{\alpha}\right)a$ must be global as well, as long as it is interior. Suppose that it is interior. Then using the value and substituting the budget constraint into the objective function yields

$$\max_{\hat{b}_{fi}} \frac{\lambda}{1+\lambda}\mathbb{E}\left[\frac{b_{lj}+\underline{p}\left[(b_{fj}-\hat{b}_{fj})\left(\frac{(1-\alpha)(b_{fj}-\hat{b}_{fj})}{\alpha}\right)^{\alpha-1}+\left(\frac{(1-\alpha)(b_{fj}-\hat{b}_{fj})}{\alpha}\right)^{\alpha}\right]}{p}+y_T^{\alpha-1}\hat{b}_{fj}\right]$$

The first order condition of this problem along with the constraint yields

$$\hat{b}_{fj}=b_{fj}-\frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)} \tag{12}$$

$$\hat{b}_{lj}=b_{lj}+\underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)} \tag{13}$$

Given this we have that $a=\frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}$ so given our conjecture that this is interior it must be that

$$\underline{y}_T \leqslant \left(\frac{1-\alpha}{\alpha}\right)a=\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}$$

or

$$\mathbb{E}\left[\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}\right]\leqslant\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}$$

which is a contradiction. Therefore it must be that $\tilde{y}=\underline{y}_T$ and so the first order condition is

$$\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}-\mathbb{E}\left[\frac{\underline{p}}{p}\right]$$

which is positive if $\hat{b}_{fi}<b_{fi}$ and so the only possible solution in this case is $\hat{b}_{fj}=b_{fj}$ and $\hat{b}_{lj}=b_{lj}+\underline{p}\underline{y}_T^{\alpha}$. The welfare associated with this contract is

$$\begin{aligned}&\frac{\lambda}{1+\lambda}\mathbb{E}\left[\frac{b_{lj}+\underline{p}\underline{y}_T^{\alpha}}{p}+\left(\frac{1}{y_T}\right)^{1-\alpha}\left[b_{fj}\right]\right]\\ =&\frac{\lambda}{1+\lambda}\left[b_{lj}\mathbb{E}\left[p^{-1}\right]+b_{fj}\mathbb{E}\left[y_T^{\alpha-1}\right]+\mathbb{E}\left(\frac{\underline{p}}{p}\right)\underline{y}_T^{\alpha}\right]\end{aligned}$$

3. Notice that the third case can never be optimal since the foc in this case is $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}-$

$\mathbb{E}\left[\frac{\underline{p}}{p}\right] > 0.$

So clearly the welfare maximizing contract is either of type 1 or type 2. Taken the difference in welfare between 1 and 2 yields

$$\left(\mathbb{E}\left[\frac{\underline{y}_T^{1-\alpha}}{y_T^{1-\alpha}}\right] - \mathbb{E}\left[\frac{\underline{p}}{p}\right]\right) y_T^{\alpha} > 0$$

which implies the first part of the proposition.

Next, suppose that $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha} < \mathbb{E}\left[\frac{\underline{p}}{p}\right]$. Now, the welfare associated with case 1 is identical to the previous case. In case 2, it will now be true that $\underline{y}_T \leqslant \left(\frac{1-\alpha}{\alpha}\right) a$ and so we need to check the other bound, i.e.

$$\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)} \leqslant \bar{y}_T$$

or

$$\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right) \leqslant \mathbb{E}\left(\frac{\bar{y}_T}{y_T}\right)^{1-\alpha}$$

which is always satisfied. Therefore, $\left(\hat{b}_{fj}, \hat{b}_{lj}\right)$ are given by (12) and (13). The welfare associated with this interior solution is

$$\frac{\lambda}{1+\lambda}\left[\left[b_{lj} + \underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)}\right]\mathbb{E}\left[p^{-1}\right] + \left[b_{fj} - \frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}\right]\mathbb{E}\left[(y_T)^{\alpha-1}\right]\right]$$

Finally, we need to consider the contract $\hat{b}_{fj} = b_{fj}$ and $\hat{b}_{lj} = b_{lj} + \underline{p}\underline{y}_T^{\alpha}$. By, an argument similar to the previous case the welfare associated with this contract,

$$\frac{\lambda}{1+\lambda}\left[b_{lj}\mathbb{E}\left[p^{-1}\right] + b_{fj}\mathbb{E}\left[y_T^{\alpha-1}\right] + \mathbb{E}\left(\frac{\underline{p}}{p}\right)\underline{y}_T^{\alpha}\right]$$

dominates that of the case 1 contract. Therefore, we need to compare the welfare of this with the interior contract. Taking the difference yields

$$
\begin{aligned}
&\frac{\lambda}{1+\lambda}\left[\left[\underline{p}\frac{1}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)}\right]\mathbb{E}\left[p^{-1}\right]-\left[\frac{\alpha}{1-\alpha}\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{1/(1-\alpha)}\right]\mathbb{E}\left[(y_T)^{\alpha-1}\right]-\mathbb{E}\left(\frac{\underline{p}}{p}\right)\underline{y}\right.\\
=&\frac{\lambda}{1+\lambda}\left[\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[y_T^{\alpha-1}\right]}\right)^{\alpha/(1-\alpha)}\mathbb{E}\left[\frac{\underline{p}}{p}\right]-\mathbb{E}\left(\frac{\underline{p}}{p}\right)\underline{y}_T^{\alpha}\right]\\
=&\frac{\lambda}{1+\lambda}\left[\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[\frac{\underline{y}_T^{1-\alpha}}{y_T^{1-\alpha}}\right]}\underline{y}_T^{1-\alpha}\right)^{\alpha/(1-\alpha)}-\underline{y}_T^{\alpha}\right]\mathbb{E}\left(\frac{\underline{p}}{p}\right)\\
=&\frac{\lambda}{1+\lambda}\left[\left(\frac{\mathbb{E}\left[\frac{\underline{p}}{p}\right]}{\mathbb{E}\left[\frac{\underline{y}_T^{1-\alpha}}{y_T^{1-\alpha}}\right]}\right)^{\alpha/(1-\alpha)}-1\right]\underline{y}_T^{\alpha}\mathbb{E}\left(\frac{\underline{p}}{p}\right)\\
\geqslant&0
\end{aligned}
$$

which completes the proof. Note that in the case in which $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}=\mathbb{E}\left[\frac{\underline{p}}{p}\right]$, the welfare associated with the interior contract as and the extreme ones are identical by the previous arguments.

Proof of Lemma 1:Given the assumption that we are in an interior equilibrium, it must be that $\mathbb{E}\left(\frac{\underline{y}_T}{y_T}\right)^{1-\alpha}>\mathbb{E}\left[\frac{\underline{p}}{p^*(\psi,B^*)}\right]$. We will show that that $\frac{\partial}{\partial\varepsilon}\mathbb{E}\left[\frac{\underline{p}}{p^*(\psi+\varepsilon,B^*(\varepsilon))}\right]\Big|_{\varepsilon=0}>0$. To see this first notice that

$$
\begin{aligned}
&\frac{\partial}{\partial\varepsilon}p^*\left(\psi+\varepsilon,B_{l1}\left(\varepsilon\right)\right)\Big|_{\varepsilon=0}\\
&=\frac{\partial}{\partial\psi}\frac{1}{g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)}\\
&=-\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\left[-\frac{(1-\theta)B_{l1}'(\psi)}{\psi}+(1-\theta)B_{l1}\frac{1}{\psi^2}\right]\\
&=-\frac{(1-\theta)}{\psi}B_{l1}\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\frac{1}{\psi}\left[-\frac{B_{l1}'(\psi)}{B_{l1}}\psi+1\right]
\end{aligned}
$$

where we use the fact that g is an increasing function. Next, from (8) we have

$$\ln B_{li}(\psi) = \ln i\underline{p}\frac{1}{1-\alpha} + \frac{\alpha}{1-\alpha}\ln\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right) - \frac{\alpha}{1-\alpha}\ln\left(\mathbb{E}\left[y_T^{\alpha-1}\right]\right)$$

and therefore

$$\begin{aligned}\frac{B_{l1}'(\psi)}{B_{l1}} &= -\frac{\alpha}{1-\alpha}\frac{1}{\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right)}\mathbb{E}\left[\frac{\underline{p}}{p^2}\right]\frac{\partial}{\partial\psi}p^*(\psi, B_{l1}) \\ &= -\frac{\alpha}{1-\alpha}\frac{1}{\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right)}\mathbb{E}\left[\frac{\underline{p}}{p^2}\right]\left(\frac{(1-\theta)}{\psi}B_{l1}\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\frac{1}{\psi}\left[\frac{B_{l1}'(\psi)}{B_{l1}}\psi - 1\right]\right)\end{aligned}$$

which implies that

$$\frac{B_{l1}'(\psi)}{B_{l1}}\psi = \frac{\frac{\alpha}{1-\alpha}\frac{1}{\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right)}\mathbb{E}\left[\frac{\underline{p}}{p^2}\right]\left(\frac{(1-\theta)}{\psi}B_{l1}\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)}{\left(1+\frac{\alpha}{1-\alpha}\frac{1}{\left(\mathbb{E}\left[\frac{\underline{p}}{p}\right]\right)}\mathbb{E}\left[\frac{\underline{p}}{p^2}\right]\frac{(1-\theta)}{\psi}B_{l1}\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)} < 1$$

Therefore,

$$\begin{aligned}&\frac{\partial}{\partial\varepsilon}\mathbb{E}\left[\frac{\underline{p}}{p^*(\psi+\varepsilon, B^*(\varepsilon))}\right]\Bigg|_{\varepsilon=0} \\ &= -\mathbb{E}\left[\frac{\underline{p}}{p^2}\right]\left(\frac{(1-\theta)}{\psi}B_{l1}\frac{1}{\left(g\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\right)^2}g'\left(-\frac{(1-\theta)B_{l1}}{\psi}\right)\frac{1}{\psi}\left[\frac{B_{l1}'(\psi)}{B_{l1}}\psi - 1\right]\right) \\ &> 0\end{aligned}$$

which yields the desired result.

B Model with Small Default Cost

B.1 General Environment

This appendix analyzes a stylized version of the model with contracts that allow for default on equilibrium. Consider a similar economy with two sub-types of citizens $\mathcal{I} = \{1, 2\}$. We refer to agents of type 1 as buyers and agents of type 2 as sellers.

Buyers value a special good x that sellers can provide. Preferences for the representative buyer are given by

$$u_1 = (1+\lambda)\, x + \mathbb{E}\left[c_1 - \kappa \mathbb{I}\{\text{default}\}\right],$$

where c_1 is consumption of the composite good and $\lambda > 0$. We assume that default, which we describe below, has associated a utility loss of $\kappa > 0$.

Sellers can provide the special good x at a linear disutility and also value the consumption of the composite good. The preferences of the representative buyer are given by

$$u_2 = -x + \mathbb{E}\left[c_2\right],$$

where c_2 is the seller's consumption of the composite good.

At date 2, buyers are endowed with y units of the composite good. This endowment is idiosyncratic to each buyer and randomly drawn from a uniform distribution with support $[\underline{y}, \overline{y}]$. We denote the distribution of the endowment as $F_y(y)$.

B.2 Bilateral Contracts

Each buyer meets a seller and signs a contract according to which the seller provides the special good in exchange for a promise of payments in the composite good. We relax the assumption that payments are default-free. We depart from the baseline model and assume that promised payments can be expressed in one of the two currencies. In particular, contracts are now denoted by the tuple (x, c, b), where x indicates the units of the special good provided to the buyer, $c \in \{l, f\}$ indicates the choice of currency (local or foreign), and b denotes the units of currency c promised to be repaid to the seller at date 2, respectively. A payment b in currency c yields bR_c units of the domestic composite good in period 2. R_l, which is the price of local currency in terms of the composite good, corresponds to $\frac{1}{p}$ in the baseline model. Similarly, R_f, which is the price of foreign currency in terms of the composite good, corresponds to $\frac{e}{p}$ in the baseline model.

We assume that the buyer can default in the contract when its realized endowment is not enough to pay for the value of its promised payments, i.e. when $y < bR_c$. When a default occurs, the buyer pays its endowment y to the seller. Therefore, the actual payment, measured in the composite good, that the seller receives at date 2 is $\min\{y, bR_c\}$.

From the perspective of individual agents, R_c (for $c = l, f$) are random variables that are uniformly distributed with support $[\mu_c(1-\epsilon_c), \mu_c(1+\epsilon_c)]$. We denote their distribution as $F_c(R_c)$. The parameter ϵ_c determines the price risk associated with currency c. In equilibrium, R_l is chosen by the government and its distribution will emerge endogenously. R_f, which captures the real exchange rate risk, is assumed to be exogenous.[8] Since

[8]This source of randomness can come from shocks to the foreign price level or from shocks to the en-

endowment shocks are idiosyncratic, they are independent from the price shocks, which are aggregate.

As in the baseline model, we assume that in each bilateral meeting the buyer makes a take-it-or-leave-it offer to the seller. The participation constraint of the seller is

$$\int_{\underline{y}}^{\overline{y}} \left(\int_{\mu_c(1-\epsilon_c)}^{\frac{y}{b}} bR_c dF_C(R_c) + \int_{\frac{y}{b}}^{\mu_c(1+\epsilon_c)} y dF_C(R_c) \right) dF_y(y) - x \geqslant 0. \tag{14}$$

This inequality states that the expected utility of using income repayment to consume the composite good needs to exceed the disutility of exerting labor to provide the special good. Implicitly in the writing of the integrals we assume that R_c shocks are "wide enough" (i.e. for any possible y there exists R_{c1}, R_{c2} such that there is repayment under R_{c1} and default under R_{c2}. We provide sufficient conditions for this below. The optimal contract for the buyer solves

$$\max_{x,c\in\{l,f\},b} (1+\lambda)x + \int_{\underline{y}}^{\overline{y}} \left(\int_{\mu_c(1-\epsilon_c)}^{\frac{y}{b}} (y - bR_c) dF_C(R_c) - \int_{\frac{y}{b}}^{\mu_c(1+\epsilon_c)} \kappa dF_C(R_c) \right) dF_y(y)$$

subject to (14). Before analyzing the optimal contract we make the following assumptions.

Assumption 1. *κ and λ are such that*

$$\overline{y}^2 \left(\frac{1}{3} - \left(\frac{1-\epsilon_c}{1+\epsilon_c} \right)^2 \right) - \frac{\overline{y}\underline{y}}{3} - \underline{y}^2 \leqslant \frac{\kappa}{\lambda}(\overline{y} + \underline{y}) \leqslant \frac{\overline{y}^2}{3} + \frac{\overline{y}\underline{y}}{3} - \frac{2\underline{y}^2}{3}.$$

This parametric assumption ensures that default costs are low enough such that some default occurs in equilibrium, and high enough such that they do not occur with probability one. The following proposition characterizes the solution to the optimal bilateral contract.

Proposition 4. *Suppose $\frac{\kappa}{\lambda} > Var(y)$. Then, the optimal bilateral contract (x, c, b) offered by buyers satisfies:*

1. *the currency choice is given by $c = l$ if and only if $\epsilon_l \geqslant \epsilon_f$.*

2. *the scale of the promised payments is given by*

$$b = \frac{\left(\mathbb{E}\left[y^2\right] - 2\frac{\kappa}{\lambda}\mathbb{E}\left[y\right]\right)^{1/2}}{\mu_c(1-\epsilon_c)}. \tag{15}$$

3. *the amount of special good provided x is such that (14) is satisfied.*

dowments in the foreign economy (see equation 5).

Proof. First note that the objective function is increasing in x. This implies that x is chosen such that (14) is satisfied. We then solve for the optimal b conditional on a currency c and finally show the optimal choice of c. If we substitute x back into the objective function the problem reduces to

$$U_c = \max_b \int_{\underline{y}}^{\overline{y}} \left(y + \lambda \left(\int_{\mu_c(1-\epsilon_c)}^{\frac{y}{b}} bR_c dF_C(R_c) - \int_{\frac{y}{b}}^{\mu_c(1+\epsilon_c)} (y-\kappa) dF_C(R_c) \right) \right) dF_y(y)$$

The first order condition associated to this problem is given by

$$\int_{\underline{y}}^{\overline{y}} \left(\int_{\mu_c(1-\epsilon_c)}^{\frac{y}{b}} R_c dF_C(R_c) - \kappa \frac{y}{b^2} \right) dF_y(y) = 0$$

Solving for b yields $b = \frac{\zeta}{\mu_c(1-\epsilon_c)}$, where $\zeta = \left(\mathbb{E}\left[y^2\right] - 2\frac{\kappa}{\lambda}\mathbb{E}\left[y\right]\right)^{1/2}$. Plugging back the optimal b into the objective function we can express the utility associated to currency c

$$U_c = \frac{\lambda}{2\epsilon_c} \left(\begin{array}{c} \frac{1}{\epsilon_c}\left(\frac{1}{2}\left(\frac{\mathbb{E}[y^2]}{\zeta} - \zeta\right) + \left(\mathbb{E}\left[y\right] - \frac{\kappa}{\lambda}\right) - \frac{1}{\zeta}\left(\mathbb{E}\left[y^2\right] - \frac{\kappa}{\lambda}\mathbb{E}\left[y\right]\right)\right) \\ + \left(\left(\mathbb{E}\left[y^2\right] - \frac{\kappa}{\lambda}\mathbb{E}\left[y\right]\right)\frac{1}{\zeta} - \frac{1}{2}\left(\frac{\mathbb{E}[y^2]}{\zeta} - \zeta\right) + \left(\mathbb{E}\left[y\right] - \frac{\kappa}{\lambda}\right)\right) \end{array} \right)$$

Note that only the term in the first line is specific to the currency. It can be shown that that term is non-negative as long as $\frac{\kappa}{\lambda} > Var(y)$, which is assumed to be the case. Therefore, we showed that U_c is decreasing in ϵ_c. Given the symmetry of the utility under both currencies, it follows that the optimal choice of currency is the one with lower ϵ_c. □

As in the optimal contract in the baseline model, here the choice of currency is given by the one that has the lower risk in terms of its price relative to the composite good. The reason is that default is costly and choosing a riskier currency increases its likelihood. Now the relevant notion of risk is ϵ_c which parametrizes the variance of R_c since the buyer cares about the entire distribution, not just the minimum realization. The optimal scale of the contract is increasing in the expected value of the endowment and decreasing in the default cost. Finally, as in the baseline model, the buyer chooses the amount of special good x to be provided such that the seller is at its participation constraint.

Denote s_l the share of contracts in local currency and b_c the individual level of debt in currency c. The following corollary characterizes s_l, b_c for a given government policy.

Corollary 2. *In equilibrium,*

If $\epsilon_l > \epsilon_f$ *then* $s_l = 0$, $b_f = \frac{\left(\mathbb{E}[y^2] - 2\frac{\kappa}{\lambda}\mathbb{E}[y]\right)^{1/2}}{\mu_f(1-\epsilon_f)}$ *and* $b_l = 0$.

If $\epsilon_l < \epsilon_f$ *then* $s_l = 1$ $b_f = 0$ *and* $b_l = \frac{\left(\mathbb{E}[y^2] - 2\frac{\kappa}{\lambda}\mathbb{E}[y]\right)^{1/2}}{\mu_l(1-\epsilon_l)}$.

If $\epsilon_l = \epsilon_f$ *then any* $s_l \in [0,1]$ *is an equilibrium and* $b_f = \frac{\left(\mathbb{E}[y^2]-2\frac{\kappa}{\lambda}\mathbb{E}[y]\right)^{1/2}}{\mu_f(1-\epsilon_f)}$ *and* $b_l = \frac{\left(\mathbb{E}[y^2]-2\frac{\kappa}{\lambda}\mathbb{E}[y]\right)^{1/2}}{\mu_l(1-\epsilon_l)}$.

B.3 Government

The government chooses the price of the local currency in terms of the composite good R_l. The objective of the government is to maximize an equally weighted sum of utilities of buyers and sellers net of a utility loss associated with inflation. The loss of inflation depends on the type of the government ψ. In particular, we assume that the loss function is $l(R_l, \psi) = \frac{\psi}{2}\left(R_l - \overline{R}\right)^2$, where $\overline{R}$ correspond to the level of inflation that minimizes losses. The government type ψ is a random variable whose inverse ψ^{-1} is uniformly distributed with support $\left[0, \overline{\psi^{-1}}\right]$.

Denote s_c the fraction of contracts that are set in currency c and b_c the scale of the contract for those contracts set in currency c. Then the aggregate consumption of the composite good of buyers and sellers, C_1 and C_2, are given by

$$C_1 = \sum_{c=l,f} s_l \max\{y - R_c b_c, 0\}, \tag{16}$$

$$C_2 = \sum_{c=l,f} s_l \min\{R_c b_c, y\}. \tag{17}$$

The objective function of the government is given by

$$\mathbb{E}\left[C_1 + C_2 - \kappa\left(s_l \mathbb{I}\{y \leqslant bR_l\} + (1-s_l)\mathbb{I}\{y \leqslant bR_f\}\right)\right] - l\left(R_l, \psi\right),$$

where the expectation is taken with respect to the realization of the idiosyncratic endowments and R_f, since its choice of prices happens before the realization of these variables. Substituting for the values of consumption in (16) and (17), we can re-express the objective function of the government (up to a constant) as

$$-s_l \int_{\underline{y}}^{R_l b} \kappa dF(y) - l_1(R_l, \psi). \tag{18}$$

The problem of the government is to choose R_l to maximize (18). The simplification of the objective function arises since we assume the government places equal weights to buyers and sellers and hence all redistributive effects that inflation may have do not affect the utility of the government. However, the private losses associated with default appear in the objective function since these affect the utility of buyers and are socially costly. The trade-off that the government faces by choosing inflation is that of avoiding costly defaults by reducing the real burden of outstanding debts, at the expense of incurring in losses associated with inflation. The optimal choice of inflation is captured by the following first order condition

$$-s_l \kappa f(R_l b) b = \frac{\partial l_1(R_l, \psi)}{\partial R_l}.$$

At the margin, the marginal cost of inflation is equal to the benefits of inflation of reducing the share of defaults that occur in equilibrium, which are proportional to the share of contracts that are set in local currency. Using our functional form assumptions and solving for R_l we obtain

$$R_l(\psi) = \overline{R} - \frac{\psi^{-1} \kappa s_l b}{(\overline{y} - \underline{y})}. \tag{19}$$

This equation states that for higher aggregate levels of debt in local currency $s_l b$, the government chooses a lower R_l (or a higher level of inflation). Also note that given our assumption that ψ^{-1} is uniformly distributed, we get that R_l is also uniformly distributed as it is linear in ψ^{-1}. Additionally, we can express μ_l and ϵ_l as a function of the primitives of the distribution of ψ^{-1}:

$$\mu_l(1 - \epsilon_l) = \overline{R} - \frac{\overline{\psi^{-1}} \kappa s_l b}{(\overline{y} - \underline{y})},$$
$$\mu_l(1 + \epsilon_l) = \overline{R}.$$

This verifies our original distributional assumption that we made about R_l in the previous section. Finally, it is worth noting that higher levels of debt in local currency also give rise to higher risk of the local currency. This is because the lowest level of inflation $\overline{R}$ is independent of the levels of debt in local currency, since there is always a positive probability of some government facing prohibitive inflation costs such that it implements the level of inflation that minimizes costs. This prediction is important for the characterization of the competitive equilibrium.

We can now define and analyze the competitive equilibrium given the optimal government policy.

Definition 2. A competitive equilibrium is a tuple $(s_l, b_l, b_f, R_f(\psi))$ such that

1. (s_l, b_l, b_f) are determined using Corollary 2 given $R_f(\psi)$
2. $R_f(\psi)$ is given by (19).

Denote $\overline{\epsilon_c}$ the positive root to the following equation:

$$\left(\frac{1-\epsilon_l}{1+\epsilon_l}\right) - \overline{R}\left(\frac{1-\epsilon_l}{1+\epsilon_l}\right) + \frac{\overline{\psi^{-1}}\kappa\left(\mathbb{E}\left[y^2\right] - 2\frac{\kappa}{\lambda}\mathbb{E}\left[y\right]\right)^{1/2}}{(\overline{y} - \underline{y})} = 0$$

This is the level of risk in local currency if all contracts are denominated in local currency. The characterization of the competitive equilibrium is summarized in the following proposition.

Proposition 5. *If $\overline{\epsilon_l} > \epsilon_f$, there exists a unique competitive equilibrium in which $s_l \in [0,1]$, (b_l, b_f) are given by (15) and s_l is chosen such that $\epsilon_l = \epsilon_f$.*

This proposition states that for sufficiently high government risk, the equilibrium share of contracts in local currency is interior and buyers are indifferent between choosing either currencies. The share is such that in equilibrium the government chooses a level of prices in local currency that has associated the same level of risk as the foreign currency.

B.4 Social Planner

In this section we ask whether s_l, given b_l, b_f and the time-consistent choice of R_l, is constrained efficient. The planner's objective is

$$W = \mathbb{E}_{y,\psi^{-1}}\left[\sum_{c=l,f} s_c \left(\begin{array}{c}(1+\lambda)x + (y - R_c b_c)\mathbb{I}\{y \geqslant R_c b_c\} - \kappa\mathbb{I}\{y < R_c b_c\} \\ +R_c b_c \mathbb{I}\{y \geqslant R_c b_c\} + y\mathbb{I}\{y < R_c b_c\}\end{array}\right) - l(R_l, \psi^{-1})\right] \tag{20}$$

The restriction we impose is that the choice of R_l is optimal ex-post. Therefore, the social planner's problem involves choosing s_l to maximize W, subject to (18). From that equation we can define a function $R_l(s_l b_l, \psi^{-1})$ which is decreasing in both arguments. After substituting the optimal private choices of x the objective simplifies to:

$$W = \mathbb{E}_{y,\psi^{-1}}\left[\begin{array}{c} s_l\left(\lambda\left(R_l(s_l b_l,\psi^{-1})b_l\mathbb{I}\{y \geqslant R_l(s_l b_l,\psi^{-1})b_l\} + y\mathbb{I}\{y < R_l(s_l b_l,\psi^{-1})b_l\}\right) - \kappa\mathbb{I}\{y < R_l(s_l b_l,\psi^{-1})b_l\}\right) \\ (1-s_l)\left(\lambda\left(R_c b_c \mathbb{I}\{y \geqslant R_c b_c\} + y\mathbb{I}\{y < R_c b_c\}\right) - \kappa\mathbb{I}\{y < R_c b_c\}\right) - l(R_l,\psi^{-1})\end{array}\right]$$

Consider the default threshold for local currency $y \geqslant R_l(s_l b_l, \psi^{-1})b_l$ and define the implicit inflation cost as the function $\Psi^{-1}(s_l, y)$, which is such that the threshold is satisfied with equality. This function $\Psi^{-1}(s_l, y)$ is increasing in s_l. Then, we can re-write the objective function as

$$W = \int_{y\in Y}\left[y + \begin{array}{c} s_l\lambda\left(\int_{\Psi^{-1}(s_l,y)}^{\overline{\psi^{-1}}} R_l(s_l b_l,\psi^{-1})b_l dF(\psi^{-1}) + \int_{\underline{\psi^{-1}}}^{\Psi^{-1}(s_l,y)} y - \frac{\kappa}{\lambda}dF(\psi^{-1})\right) \\ (1-s_l)\lambda\left(\int_{\underline{R_f}}^{y/b_f} R_f b_f dF(R_f) + \int_{y/b_f}^{\overline{R_f}} y - \frac{\kappa}{\lambda}dF(R_f)\right) - \int_{\underline{\psi^{-1}}}^{\overline{\psi^{-1}}} l(R_l,\psi^{-1})dF(R_f)\end{array}\right]dF(y).$$

Denote U_l (U_f) the ex-ante utility of those buyers choosing local currency

$$U_l = \lambda \int_{y \in Y} \left(\int_{\Psi^{-1}(s_l b_l, y)}^{\overline{\psi^{-1}}} R_l(s_l b_l, \psi^{-1}) b_l dF(\psi^{-1}) + \int_{\underline{\psi^{-1}}}^{\Psi^{-1}(s_l b_l, y)} y - \frac{\kappa}{\lambda} dF(\psi^{-1}) \right) dF(y),$$

$$U_f = \lambda \int_{y \in Y} \left(\int_{\underline{R_f}}^{y/b_f} R_f b_f dF(R_f) + \int_{y/b_f}^{\overline{R_f}} y - \frac{\kappa}{\lambda} dF(R_f) \right) dF(y).$$

Then, we can write first order condition of the social planner problem as

$$U_l - U_f +$$

$$\int_{y \in Y} \left(\underbrace{-\kappa \Psi_1^{-1} b_l}_{\geqslant 0} + \int_{\underline{\psi^{-1}}}^{\Psi^{-1}(s_l b_l, y)} \underbrace{b_l R_{l,1} b_l}_{\leqslant 0} dF(\psi^{-1}) - \int_{\underline{\psi^{-1}}}^{\overline{\psi^{-1}}} \underbrace{l_1(R_l, \psi^{-1}) R_{l,1} b_l}_{\leqslant 0} dF(R_f) \right) dF(y).$$

The term in the first line, when evaluated at the share of contracts in local currency that emerges in equilibrium, is equal to zero. The term in the second row correspond to the externalities associated with the choice of currency in contracts. The negative terms correspond to the same externalities as in the baseline model. A higher incidence of local currency in contracts lead the government to engage in inflation, which is costly (third term). Additionally, this higher level of inflation also reduces the expected repayments that buyers can make and hence reduces the extent to which buyers and sellers can exploit the gains of trade in the first period (second term). The new term is the first one and goes in the opposite direction of the other two. This term refers to the fact that, when private agents set their contracts, they do not take into account that when they choose local currency they are increasing the incentives of the government to engage in inflation and avoid costly defaults.