COLORADO MATHEMATICS TEACHER - WINTER 2013

The Official Publication of Colorado Council of Teachers of Mathematics

*Mathematical Practice Standard 1:
Making sense of problems and persevering in solving them.

CONTENTS & FEATURES

The CMT presents a variety of viewpoints. Unless otherwise noted, these views should not be interpreted as official positions of CCTM or CMT.

From the Editor's Desk

Sandie Gilliam, Editor

"I can't do word problems"

is frequently uttered by middle and high school students, as well as both experienced and student teachers in elementary classrooms. Why is that? With the Common Core Mathematical Practice Standard 1: *Make sense of problems and persevere in solving them* in place, how do we reverse that *can't do* attitude?

Let's first look to the past. For those of you that can remember back to your own high school math experience, you may recall that each chapter in most textbooks (then and even now) always ended with a word problem section, or within each lesson there were *oral exercises*, followed by *written exercises*, and then *problems (see page 3)*. Of course, those problems were meant to put the information from THAT section into a context, so students could see how the math would be applied. Not as simple as giving the students a procedure with numbers and equations and having them follow the same steps with different numbers! With mostly words and a few numbers, many students found these problems too tough to be solved, and after a few minutes gave up, further solidifying their can't do attitude. When taking the final exam with no clear reference to which procedure from which chapter might be applied to which word problem, students often scored poorly on problem solving.

Fast forward to a few examples from today:

- Last summer, my class of experienced elementary teachers told me that they always thought there was only one correct way to do each type of word problem, and if one couldn't remember that one way, they wouldn't be able to find a solution. **What happened in their educational experience that MOST of them believed this?**
- Secondly, I had a student teacher who was working on a 5th grade problem about people getting on and off a train. She emphatically told me there just had to be an equation for all train problems, and I must have forgotten to give it to her. "Without the equation," she said, "one can't find a solution!" **Is there a correlation between her thoughts and the aforementioned word problems at the end of each section?**
- Next, I know of high school teachers that completely skip doing ANY word problems with their students, because of the time needed to both explain the procedure and go over the problems the next day—let alone the student frustration and vocal can't do attitude. **How do we get students beyond the *can't do*—to persevere?**
- Lastly, one of my methods students stated, "The vast majority of our sixty-minute 5th grade math classes are taught directly from the book. I cannot think of a single example of our students making their own sense of a problem and persevering to solve them. There is simply not enough time in the day/ quarter/ year to let them "struggle" and persevere to their own solutions using problem solving. We try to present a concrete example for them to connect to, but then we present the information, have a small discussion and are lucky to have ten minutes to practice it before the class is over. **What are the benefits and unintended consequences of "I do, we do, you do" when it comes to word problems?**

In fact, how do we help teachers understand the difference between the word problems of the past, and those that the Common Core envisions and PARCC will assess?

The starred articles in this issue of the CMT represent the focus of this issue: *Make sense of problems and persevere in solving them*. I hope these specific articles will enable you to glean additional insight into this Standard for Mathematical Practice and better prepare your students to be problem-solvers.

To multiply a polynomial by a monomial, use the distributive property: multiply each term of the polynomial by the monomial, and then add the products.

ORAL EXERCISES

Read the product of each indicated multiplication.

2. $15(2x - 1)$
4. $-.3(2a + b)$
6. $9(5m - 2n)$
7. $a^2(a - .3)$
9. $-3x^4(-2 + x)$
13. $-3a(-2a - 4)$
15. $(2n - 4)(-1)$
20.

$$\begin{array}{l} -y^2 + 2y + 5 \\ -4y \\ \hline \end{array}$$

WRITTEN EXERCISES

Find each of the following products.

1. $-5(x^2 - 3x + 7)$
3. $a(a^2 - 2ab + b^2)$
5. $b^2(-a + b + c)$
7. $-x^2(5x^2 - x + 2)$
9. $(2 - 3v - 4v^2)(-2v^4)$
11. $3x^2y(5 - 2xy^4 + 3x^2y^3 - y^5)$

Solve each of the following equations.

15. $3x + 5(x - 3) = 9$
19. $(5x + 3) - 3(x - 2) = 1$
21. $12(x + 1) + 3(4x - 2) = 18$

B

23. $2x - 2(x + 21) = 3x - 3(7 - x)$
27. $5 - 3[2n - 2(5 - 3n)] = 4(2 - 3n)$

PROBLEMS

Give each answer as a polynomial or as a directed number.

1. A man travels for 2 hours at (45 + x) miles per hour, then at (200 + x) miles per hour for 2 hours. Represent the total distance traveled.

4. Mr. Macy drove 303 miles one day. During the first x hours, he averaged 34 miles per hour, but during the next (x + 3) hours, his average speed was 47 miles per hour. How long did the trip take?

6. Mr. Bernard's and Mr. Cayne's houses stand on rectangular lots of equal depth d. Mr. Bernard's lot is 30 feet wider than Mr. Cayne's. Find the depth of the lots if their areas differ by 3600 square feet.

7. Charles earned d dollars on each of two days and (d - 1) dollars on each of the next three days. What were his daily earnings if he earned $19.50 during the five days?

8. Mr. Reynolds traveled for 2 hours on a jet airplane and for one-half hour more on a piston-driven aircraft. The average speed of the jet was 4.6 miles per minute more than that of the piston-driven craft. If Mr. Reynolds traveled 1467 miles on this trip, find each plane's speed.

from: Dolciani, M.P., Berman, S.L., & Freilich, J. (1962). *Modern Algebra: Structure and Method, Book 1*. Boston, MA: Houghton Mifflin Company.

President's Message

Catherine Martin, CCTM President

As we continue to support our students' progress toward college and career readiness, our attention to the Standards for Mathematical Practice is essential to that support. This issue of the Colorado Mathematics Teacher continues the focus on the Standards for Mathematical Practice and highlights Practice 1: Make sense of problems and persevere in solving them. This Practice describes an essential disposition to be developed and reinforced throughout all students' K–12 mathematical experiences.

Fundamental to this Practice is a clear understanding of what problem solving entails. According to the National Council of Teachers of Mathematics, problem solving means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings (p. 52).

Thus, a problem is such that students do not always initially see a productive pathway and often will need to try multiple approaches before finding success. Therefore, the teacher's role in supporting students in problem solving is not about teaching specific problems or problem types. Rather, it is about helping students understand how to use their knowledge of and skill in mathematics and how to select and use resources and tools to support their endeavors.

The first essential element in supporting students in making sense of and persevering in solving problems is the selection of rich tasks for classroom instruction. Such tasks allow for multiple approaches, require students to make decisions about the mathematics to use, and necessitate students' connection of multiple ideas and connecting these ideas with the underlying concepts. These tasks may also have multiple solutions, require students to make assumptions as they begin solving, and necessitate students' justification of the reasonableness of their solution within the context of the problem.

A second essential element in support of this Practice is support for students' development of a problem-solving disposition. We know that many students become quickly frustrated when solving mathematics problems, especially when they struggle to know how to begin. Students, too, are concerned that they might get stuck in the process, pursue an idea that doesn't work, and/or arrive at an answer that is wrong. Our role, then, is to help students develop the recognition that confusion is part of the learning process, that persistence is vital to this process, and that self-monitoring supports their success in problem solving with modification in the process as needed.

How, then, do we shift classroom practices to support students in making sense of and persevering in solving rich tasks? Consider the following classroom shifts described in the table below.

As you continue to support your students in developing the ability to make sense of problems and persevere in solving them, be sure to peruse the rich tasks and resources for the practice standards at the Illustrative Mathematics website: http://www.illustrativemathematics.org.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

Classroom Practice	From	To
Classroom Environment	Emphasis on speed and correctness of responses	Emphasis on persistence, with acknowledgement that confusion is part of the learning process, and celebration of multiple, reasonable approaches
Task Selection	List of problems that focus on procedures and that can be solved quickly with little thought	One or two problems that press students to apply their understanding of mathematical ideas and require deep engagement with the mathematics
Questioning	Questions that focus on the answer and can be quickly answered	Questions that focus on the mathematics in the task and require students to think deeply about the task and explain their thinking
Classroom Discussions	Discussions are cursory and often include the use of unreliable methods to determine a strategy (e.g., key words) with a focus on how students "did" the problem	Discussions focus on students' thinking about how they interpreted the task, how they chose a particular strategy, and how they can justify their solution
Feedback to Students	Focuses on correctness of responses	Focuses on effort and persistence with suggestions for new thinking

Colorado Department of Education (CDE) Corner

Mary Pittman, Mathematics Content Specialist, Colorado Department of Education

Welcome to the CDE Corner where updates from the Colorado Department of Education relevant to mathematics educators are shared. In this issue, I will share updates on Colorado's District Sample Curriculum Project and the Presidential Awards for Excellence in Mathematics and Science Teaching.

Colorado's District Sample Curriculum Project

Thirteen years ago I moved to Colorado to attend graduate school and never quite left. Over the years, I have traveled to various parts of the state but never really off the beaten path. This fall I had the opportunity and privilege to visit some amazing small towns and best-kept secrets of Colorado. I have worked with teachers in Meeker, Norwood, Collbran, Burlington, Clear Creek, Leadville, and Oak Creek, and these teachers have impressed and honored me with their time and commitment to the children—not only in their districts but also the entire state of Colorado.

For three days, teachers in each of these towns came together to build-out one unit of Colorado's District Sample Curriculum. Both content teachers and specialists (special education, English language development, talented and gifted) worked together to create a unit which would include learning experiences, resources suggestions, differentiation options, and assessment ideas. I am excited by the work that has been completed thus far and hope all of you will visit the CDE website at the start of March to see these built-out units from the sample curriculum.

Presidential Awards for Excellence in Mathematics and Science Teaching

The Presidential Awards for Excellence in Mathematics and Science Teaching is the highest recognition that a kindergarten through 12th-grade mathematics or science teacher may receive for outstanding teaching in the U.S. The 2012 and 2013 finalists for Colorado were honored at the CCTM conference in October of this year. The 2012 finalists for elementary mathematics are Melanie Dolifka from Falcon Elementary School of Technology in Peyton and Joan Standefer from Heatherwood Elementary in Boulder. The 2013 finalists for secondary mathematics are Jean Elizabeth Allen-Hatcher from

From left to right: *Jean Elizabeth Allen-Hatcher, Kimberly Race, Joan Standefer, Melanie B Dolifka, and Kirstin Oseth*

Bruce Randolph High School in Denver, Kimberly Race from Castle View High School in Douglas, and Kirstin Oseth from Cheyenne Mountain Junior High School in Cheyenne Mountain. We wish the two finalists from 2012 and three finalists from 2013 luck as they move forward in the process. The 2014 nominations are now open for elementary mathematics teachers; please nominate a teacher today! https://www.paemst.org/

If you have any questions or comments please feel free to email me at: pittman_m@cde.state.co.us

EQUITY

Maintaining our focus in the transition to the Common Core State Standards

Floyd Cobb II, Executive Director of Curriculum and Instruction, Cherry Creek School District

As an educator charged with supporting teachers in their developmental readiness toward the implementation of the Common Core State Standards (CCSS), I've noticed a frequently occurring theme. While most educators wholeheartedly agree that the transition to the CCSS will produce positive outcomes for students, many remain consumed—solely focused on the concrete and tangible elements of this transition. Whether it's questions about Common Core aligned textbooks, updated pacing guides or computers for testing, many fixate on the technical requirements associated with this change—instead of the improved instruction that this transition will clearly necessitate.

While these material elements are certainly necessary to complete this transition, we must remain conscious of the fact that their significance is marginal relative to the greater purposes that encouraged this shift. We certainly recognize that the true purpose of this shift is to ensure that all students be "...prepared to enter first-year courses in college, without remediation, or to enter workplace training programs for careers that offer competitive salaries" (Rothman, 2011, p. 10). Therefore, we must not lose sight of the fact that this need was not born out of a demand for physical products but out of a necessity to create thorough opportunities for college and workforce readiness for all students. This enhanced expectation cannot be understated, since it requires the difficult work of examining whether our belief systems and subsequent instructional practice in fact align with this new expectation.

While most educators would sincerely assert that all students have the ability to learn and do so at high levels, many would also contend in the very same breath that not all have the ability or perhaps even need to acquire school-based knowledge that is consistent with postsecondary preparedness. The advocacy for the latter position is highly logical when that need is supported by an academic system that permits high school graduation without the assurance of student learning. Students can routinely be relegated to a myriad of academic experiences below the grade level standard and be rewarded with a diploma that carries the same value as their peers who only engaged in college preparatory experiences.

However, a proper adoption of the CCSS will disrupt this paradigm and force us all to confront the brutal fact (Collins, 2001) that America's public schools are not and have never been equitable spaces. While a bevy of Supreme Court decisions and federal laws have forced equitable access to schools for a number of historically disenfranchised groups (i.e. racial minorities, students with disabilities, English learners, etc.), all have stopped short of requiring public school students equitable access to the curricula that is known to be requisite for college and postsecondary preparedness (Darling-Hammond, 2010). It is this obvious and very legitimate tension that will make the intended transition to the CCSS so challenging because it forces us to confront our longstanding practices of academic exclusivity. More importantly this transition requires that we reevaluate and perhaps even reimagine long-held definitions of student success.

While the CCSS exists for both English Language Arts and Mathematics, this transition will prove much more challenging for the latter. With its traditions of tracking and prerequisite structure for entry into upper level courses, the mathematics curriculum epitomizes the exclusive structure on which our educational system was comprised. Designed largely out of a need to meet the instructional needs of a wide range of students, many of our current mathematics pathways are specifically designed to sort and select students based upon perceived needs. However, the adoption of the Common Core State Standards in Mathematics creates an academic floor of Algebra II by grade 11, forcing educators to reconsider pathways to access content that is necessary for successful entry into the Armed Forces and most career and technical programs (Adleman, 2006). For schools in Colorado, there is considerable work to be done.

According to ACT (2013), 36% of Colorado students in the high school graduating class of 2012 failed to complete a mathematics course sequence that included coursework beyond the now mandatory Algebra II. Moreover, 22% of all the students in this cohort failed to experience coursework that even included any elements of Algebra II, leaving a large percentage of students arguably incapable of answering questions in the now obligatory 11th grade end-of-course exam in that subject. However, the now required inclusion of these students who've historically been excluded from this experience makes this transition to the CCSS so monumental. While reforms to public education are not new, none have so deliberately forced curricular equity—making this reform not only an examination of our instructional abilities but a test of our collective will to truly meet the needs of all students

We know undoubtedly that several students in our schools will not be prepared to immediately meet the increased demands of these new expectations. However, our response to this seemingly monumental task will be determined by our belief systems about the academic needs and intellectual capacity of these underprepared students. Will we disregard the new mathematics standards and question their necessity, or will we be brave enough to accept the challenge to prioritize student learning and actually leave no child behind? Will we seek to blame prior educators, parents and societal factors for interfering with the academic development of these students, or will we embrace this critical moment and give ourselves permission to imagine curricular and instructional solutions that are incomprehensible in most of our academic structures?

Our beliefs relative to these important questions must be understood prior to investing in the attractive tangible elements. Whether the request is for curriculum maps, pacing guides, or common core aligned textbooks, we must not forget that our instructional practices and the beliefs that support them will determine how these highly sought after items will be implemented. This difficult and oftentimes avoided work must be addressed first, to enable us to uncover innovative ways for them to be applied to enable all students to meet these new minimal expectations. If we fail to interrogate and make plain our fundamental beliefs about our student success, then these tools will become nothing more than another gimmick attempting to improve student outcomes. Therefore we must ask ourselves first if we are truly committed to doing whatever it takes to ensure that students learn. Those of us fearless enough to say yes will ignore the distractions of these items of lesser importance and keep our focus on the most important product of them all, our students.

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IN THE FIELD

*In the High School Classroom: Making sense of problems and persevering in solving them

Cathy Martin, Denver Public Schools

As teachers support students in mathematical problem solving, it's essential that we have a common definition of a mathematical problem. Unlike many word problems in textbooks, a mathematical problem is typically a non-routine and nontraditional problem where the student does not have a readily accessible strategy. It often involves students grappling with the problem to try to understand it, trying a solution strategy, reflecting on their work, and making multiple attempts to solve the problem.

So what would such a problem look like? Consider the following problem:

Two siblings—a brother and a sister—attend the same school. Walking at constant rates, the brother takes 40 minutes to walk home from school, while the sister takes only 30 minutes on the same route. If she leaves 6 minutes after her brother, how many minutes has he traveled before she catches up to him? [Task Source: Adapted from Krutetski, V. A. (1976). The Psychology of Mathematics Abilities in School in School Children. Chicago: University of Chicago Press.]

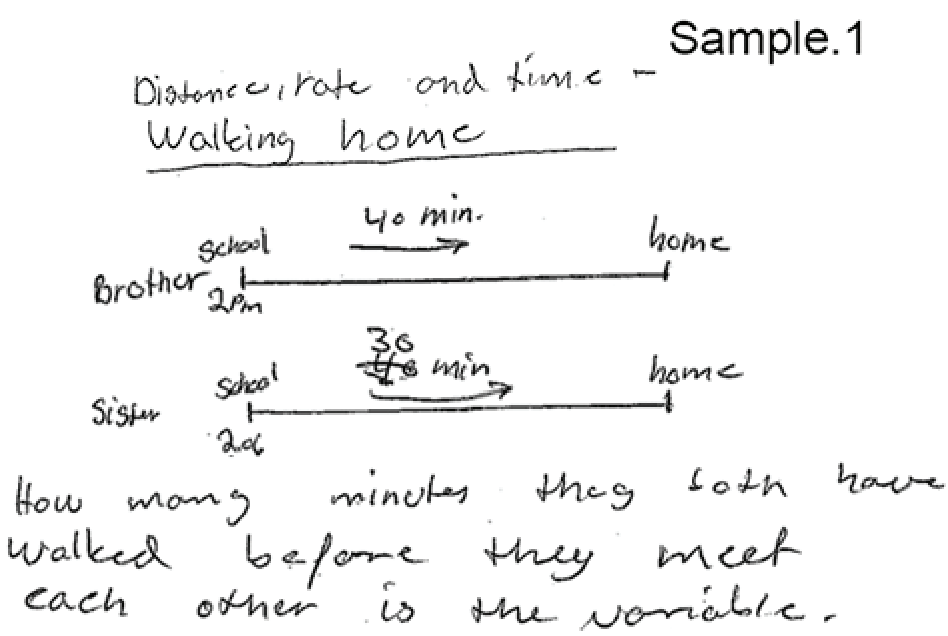
Suggested Activity for Using the Task

Begin by solving the task. As you do so, make note of the strategies and ideas and any questions you surface as you work. Try to solve the task in more ways to understand the depth of the task. Think about the Standards for Mathematical Practice you engage in as you solve the task. Try the task with your students and help them to make sense of the problem and persevere in solving it.

Then examine the samples of student work. As you do so, look for evidence of how each student was in his or her own way trying to make sense of the problem. Look for evidence of the student persevering in solving the problem. Ask yourself: What does this student know about the mathematics involved in solving the task? What are the incomplete understandings and/or misconceptions the student has?

Continue to provide students with rich tasks where they are active participants in the sense-making process. Such problem solving is central to the kind of mathematics learning called for in the Common Core State Standards.

Student Work



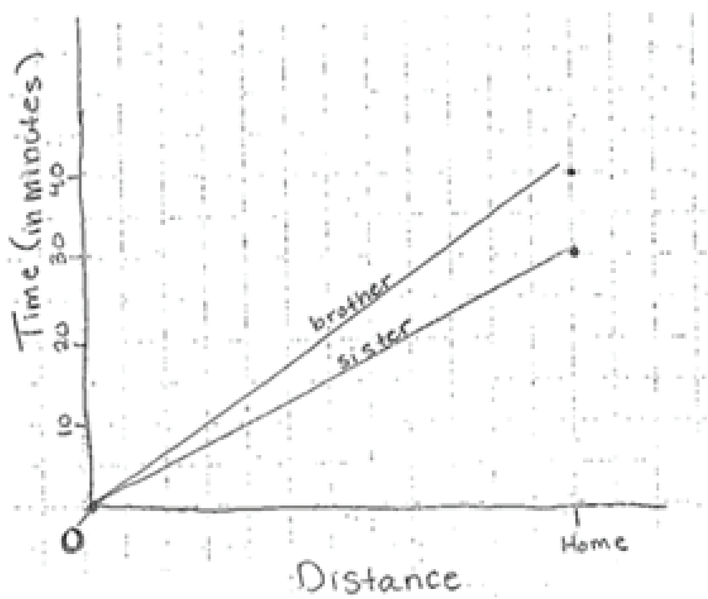
$40x = 30x + 6$

$40x - 30x = 6$

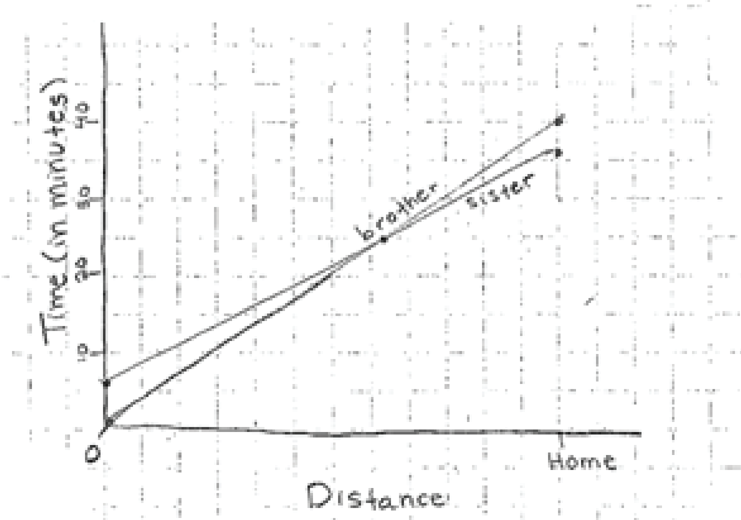
$\frac{10x}{10} = \frac{6}{10}$

They meet at $\frac{6}{10}$ of the way home

$\boxed{\begin{array}{l} \frac{6}{10} \text{ of } 40 \text{ min. is: } 24 \text{ min} \\ \frac{6}{10} \text{ of } 30 \text{ min is: } 18 \text{ min} \end{array}}$



Sample.2



He has traveled approximately 25 minutes before his sister catches up to him.

Sample.3

$\text{Rate} = \frac{\text{Distance}}{\text{Time}}$

If she leaves 6 minutes later than he has only 34 minutes left on his route home.

40 mins
30 mins

4 more minutes before she catches up, and then she would arrive home six minutes earlier than him.

6 min delay

34 min

30

1st Attempt

Brother-40 Sister=30

$\frac{30}{40} = \frac{x}{6}$

$\frac{40x}{40} = \frac{180}{40}$

x = 4.5 minutes

Sister Brother
4.5 min = 6 min
5.25 = 7 min
6 = 8 min
9 = 12 min
12 = 16 min
15 = 20 min
19 = 24 min

$\frac{30}{40} = \frac{x}{1}$

$\frac{30}{40} = \frac{40x}{40}$

$0.75 - \frac{3}{4} = x$

It will take 24 minutes for the sister to catch up

$\overline{}$
40

$\frac{24}{40} =$

Sample.4

Brother: 40 mins/route
Sister: 30 mins/route

Sister travels the same distane as Brother, In 3/4 of the time.

Brother

Sister

X

$\frac{3}{4}x$

Point of intersection

The six minute headstart by Brother

$$\underbrace{\frac{3}{4}x}_{\text{Sister's travel time to Point of Intersection}} = \underbrace{x}_{\text{Brother's travel time to point of intersection.}} - 6$$

$$-\frac{3}{4}x + 6 + \frac{3}{4}x = x - 6 + 6 - \frac{3}{4}x$$

$$6 = \frac{1}{4}x$$

$$\boxed{x = 24}$$

After 24 minutes, Sister will catch Brother.

Sample.5

brother = 40 min
sister = 30 min

$d = t \cdot r$
$r = d \div t$

set distance to 5 miles

$r_b = \frac{d}{40} \rightarrow r_b = \frac{5}{40} \rightarrow$ brother rate = .125 mi/m
$r_s = \frac{d}{30} \qquad r_s = \frac{5}{30}$ sister rate = .166 mi/m

check correct speed
brother = t = d/r → 5/.125 = 40 minutes (to go 5 mi)
sister = t = d/r → 5/.166 = 30 minutes (to go 5 mi)

test it
brother = .125 mi/m • 6 min = .75 miles in 6 minutes
sister = .75 mi / .166 mi/m = 4.5 minutes

distance travelled

minutes	B	S
1	.125	0
2	.25	0
3	.375	0
4	.5	0
5	.625	0
6	.75	0
7	.875	.166
8	1.00	.332
9	1.125	.498
10	1.25	.664
11	1.375	.83
12	1.5	.996
13	1.625	1.162
14	1.75	1.328
15	1.875	1.494
16	2.0	1.66
17	2.125	1.826
18	2.25	1.992
19	2.375	2.158
20	2.5	2.324
21	2.625	2.49

	B	S
22	2.75	2.656
23	2.875	2.822
24	3.0	3.0

MINUTES BEFORE SISTER CATCHES UP TO BROTHER: 24

What I did:
first I found the brother's & sister's speed per minute by inputting a random distance (5 miles). Then I made a table using their speed per minute and found their overlap at 24 minutes.

IN THE FIELD

*In the Middle School Classroom: Making sense of problems and persevering in solving

Jennifer Yacoubian, Denver Public Schools

A colleague and I work with a group of middle school math teachers on implementing the Standards for Mathematical Practice in their classrooms. Our teachers engage in open-ended performance tasks meant to address both the CCSS mathematics content standards and the Standards for Mathematical Practice (SMPs). Great discussion about mathematics always accompanies these tasks, because even in a room full of professional teachers, there are always different entry points to the problems and different representations of the solution. The focus of these tasks is not based solely upon the accuracy of the answer, but also on the thinking used to get to that answer. Our teachers are eager to take these tasks back to the classroom and try them with their middle school students. One teacher even noted that some of these tasks are rich enough to be used at multiple grade levels to bring out various competencies. These performance tasks allow students to grapple with and make sense of the mathematics without being told what to do or being given an algorithm to use. After teachers use these tasks in their classrooms, we reconvene to analyze the student work.

The following is an example of a task that our middle school teachers took back to their students:

Suppose the post office only sold five-cent stamps and seven-cent stamps. Some amounts of postage can be made with just those two kinds of stamps. For example, 1 five-cent and 2 seven-cent stamps make 19 cents in postage, and 2 five-cent stamps makes 10 cents in postage. Which amount of postage is it impossible to make using only five-cent and seven-cent stamps?

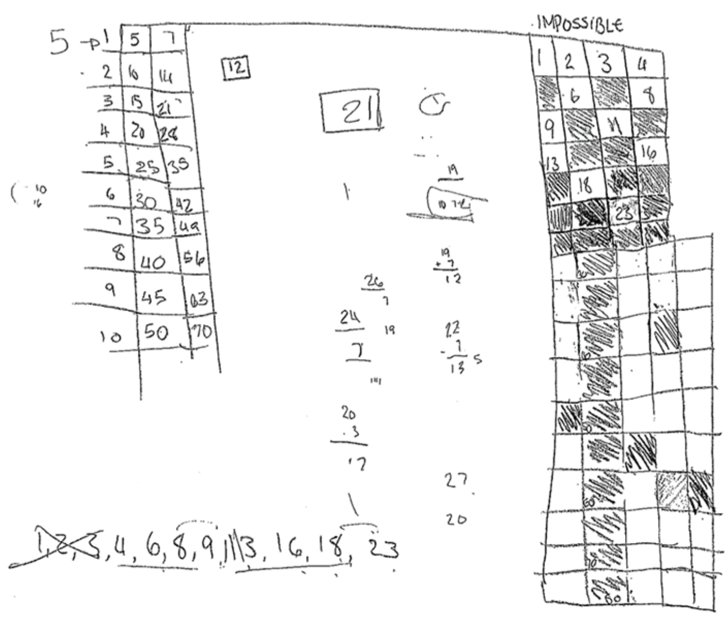
Suggested Activity for Using the Task

The only instructions given are work on the mathematics task shown, first individually and then in pairs. (Note: This individual think time is crucial for students, particularly our English Language Learners, so that they can have time to develop their own ideas.) Keep track of ideas, strategies, and questions that you pursue as you work on the task.

Below are student work samples that teachers brought from their own classrooms to use in analyzing evidence of student engagement in the SMPs. As you read the samples, look for the evidence of this engagement with the SMPs. We encourage you to try this task in your own classroom.

Student Work

MS Students Sample .1



MS Students Sample .2

The amounts of postage it is possible to make is 5, 7, 10, 12, 14, 15, 17, 19, 20, 21, 24, 25, 26, 27 28, 30

The amounts of postage it is impossible to make is 1, 2, 3, 4, 6, 8, 9, 11, 13 16, 18, 22, 23,

Once a number becomes possible you can always add 10 to it and it will always be possible.

$5+5+5+5+5+5=30$

$12 - 5 = 7$, $7 - 5 = 2$

$x = 1¢$
$x = 2¢$
$x = 3¢$
$x = 4¢$
$v = 5¢$

$7+7+7+7=28$

$7+7+7+5=26$

$5+5+5+7=27$

$5+5+7=17$

$5+5+5+5=20$ $7+5=12$

$7+7+7+7=21$

$5+5=10$ $7+7=14$ $5+5+5+5+5=25$.

$7+7+5=19$

$5+5+5=15$

MS Students Sample .3

$1\times5=5$. $1\times10=10=5+10=15$
$2\times5=10$ $1\times10=10=10+10=20$.
$2\times5=10$ $2\times10=20=10+20=30$

$5+7=12$,
$5+5+7=17$
$5+5+5+7=22$
$5+5+5+5+7=27$
$5+5+5+5+5+7=32$
$5+5+5+5+5+5+7=37$
$5+5+5+5+5+5+5+7=42$
$5+5+5+5+5+5+5+5+7=47$

possible.
5, 10, 15, 20, 25,
30, 35, 40, 45, 50,
55, 60, 65, 70, 75,
80, 85, 90, 95, 100
12, 17, 22, 27, 32,
37, 42, 47, 52, 57,
62, 67, 72, 77, 82,
87, 92, 97, 102, 107,
112, 117, 122, 127, 132
137, 142, 147, 152, 157

Impossible Solutions.
1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, 26,
27, 28, 29, 31, 32, 33, 34, 36, 37, 38, 39, 41, 42, 43, 44, 46, 47,
48, 49, 51, 52, 53, 54, 56, 57, 58, 59, 61, 62, 63, 64,
66, 67, 68, 69, 71, 72, 73, 74, 76, 77, 78, 79, 81, 82, 83
84, 86, 87, 88, 89,

Impossible:
1, 2, 3, 4, 6, 8, 9, 11, 13, 16, 18, 23

23 is the last number ~~it is~~ because 23 is the last/biggest multiple, therefore the easier numbers are going to go into the number

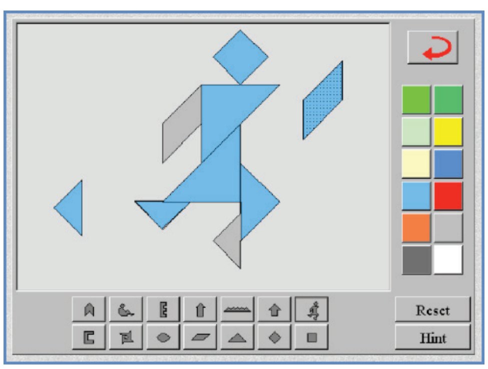
YOUR TECHNOLOGY CORNER

*Persevering using virtual manipulatives

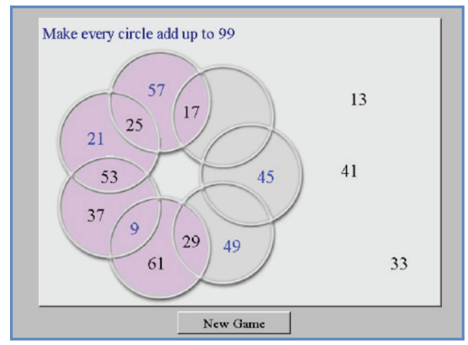
Beth Vinson & Dawn Bauer, Carson Elementary, Denver Public Schools

This month we would like to highlight a few technology experiences that facilitate Mathematical Practice 1: *Make sense of problems and persevere in solving them.* Many of us have experienced the gratifying struggle of playing a challenging video game and using perseverance to "get to the next level." We know many of our students spend their free time playing video games. How can we offer some similar experiences in the classroom, where students can enjoy the challenge of rigorous problem solving while engaging with technology? Below are a few of the experiences from the Library of Virtual Manipulatives (http://nlvm.usu.edu/) we have tried with our students.

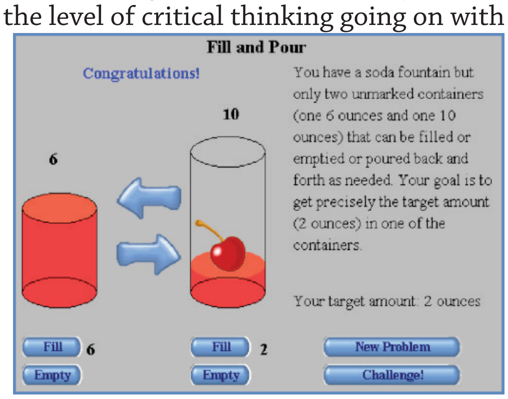
An old favorite, **Tangrams**, allows students to meet the standards of spatial reasoning and problem solving. This activity is puzzle-like, and students persevere as they decide how to compose the geometric shapes—a thread through the new geometry standards. While students are busy working on the puzzle, they have the opportunity to think about a variety of geometric shapes and relationships. Our students have used this activity during center rotations. Upon completion, they feel so proud and can't wait to show their friends!



The second activity, **Circle 99**, is geared more towards upper elementary classes but also proves to be a challenge, even for older students and adults. In this game, students are given a circular puzzle that looks like a Venn diagram the entire way around. Some of the parts are filled in with numbers, and there are other values listed alongside the puzzle. The object is to get every circle (composed of 3 different numbers) to equal 99. Students must use their addition and subtraction skills efficiently and accurately so they can work their way around the circle. If there were no given values to the side, this activity would be much less of a challenge, but because the values are there, it limits students on the possible numbers that can go in each section. Many times a pair of students will get three quarters of the way around the circle with 99s and then realize that there is no way to complete the puzzle with the values they have left. That's when their perseverance really has to kick in. The idea of leaving a puzzle undone is never fun for anyone, so this game may require a bit of extra time to complete. However, the satisfaction of seeing all the circles light up at the end of a successful round is sure to bring out some very proud students! There are other versions of this game involving sums to 0, 3, and 21 using both positive and negative integers, which makes it great for differentiation!



The third and final activity for this month, **Fill and Pour**, is part of the Measurement section of the 3rd–5th grade activities in the National Library of Virtual Manipulatives. In this challenging game, you are given two containers of various sizes and a target amount of liquid to get into one of the cups. The idea is to fill up, pour, and/or empty the containers until you have reached the target amount. This often takes multiple moves and a fluid sense of mental addition and subtraction. Once the target has been reached, you get immediate feedback, with an added cherry or other tasty fruit in the cup. To add to the level of perseverance needed, there is also a challenge mode that gives impossible scenarios and asks students to figure out why the target number cannot be reached. This mode will certainly up the rigor of any mathematical discussion! Try this activity as a station for partners or a whole class game, and we're sure you'll be impressed at the level of critical thinking going on with all of your students.



ASSESSMENT

Update from PARCC Educator Leader Cadre

Cathy Martin (CCTM President), **Joanie Funderburk** (Cherry Creek Schools), **& Ann Hirsch** (Littleton Public Schools).

PARCC is on schedule for delivering the Performance Based Assessment (PBA) and the End of the Year Assessment (EOY), both of which are required for spring 2015. In addition, PARCC is developing a Mid-Year Assessment (optional, scheduled to be ready in fall 2014), K–1 formative assessments (optional, scheduled for fall 2015) and Grades 2–8 Diagnostic Assessment (optional, scheduled for fall 2015).

Field testing for the PBA and EOY will occur in spring 2014 with many schools across the state and in other states participating. PARCC will provide a practice test, available to all, in spring 2014. This test will operate on the same platform (test NAV 8) as the Colorado Measures of Academic Success for Science and Social Studies.

An extensive FAQ on the PARCC assessment is available at http://www.parcconline.org/sites/parcc/files/PARCCFAQ_9-18-2013.pdf.

Performance Level Descriptors

The final versions of the Performance Level Descriptors (PLDs) were released in July 2013. The PLDs describe in broad terms the mathematical knowledge, skills, and practices students in a given performance level are able to demonstrate. The PLDs serve multiple purposes, including communicating to teachers, students and parents the expected performances for students to be considered college- and career-ready or on track to be, serving as the basis for standard setting in 2015, providing information for use in developing curricular and instructional materials, and informing item and rubric development for the PARCC assessments.

The PLDs are written for each grade level, 3–8, and for high school courses Algebra 1, Geometry, Algebra 2, and Math I, Math II, Math III. Information is organized by the five sub-claims PARCC is using for design and reporting purposes. Within sub-claims A (Major content of the grade or course with connections to the math practices) and B (Supporting and Additional content of the grade or course with connections to the math practices), the level descriptors are organized by "big ideas," the mathematical concepts highlighted in that sub-claim. Sub-claims C (Reasoning and Argument) and D (Modeling) connect grade or course content with the Standards for Mathematical Practice. For grades 3–6, sub-claim E assesses Fluency as outlined in the Common Core State Standards (CCSS).

Within the level descriptor statements (from Level 2, *partial command*, to Level 5, *distinguished command*), bolded text indicates the increase in performance and understanding that differentiates each level from the previous. A student performing at a Level 4, *strong command,* is considered on track or college- and career-ready. The coding in the far left column aligns to the Evidence Statements for each grade level or course, often the same as the standards. Some Evidence Statements integrate multiple standards (indicated by an "Int" label), and those in sub-claims C and D refer to specific content scopes. All standards for a grade or course are included within the PLD document.

The PLDs provide teachers and opportunity to evaluate their curricular resources and classroom activities for alignment to the PARCC assessment. They serve as a guide for providing rich and varied opportunities for students to demonstrate their understanding, and may help teachers make adjustments as they differentiate instruction for varied student needs. Additionally, the PLDs can provide clarity around helping each student progress to at least the *strong* level with the mathematics of each grade level or course. A teacher can use the PLD to assess a student's current level and then use the descriptor of the next level as a target for the student's increased learning.

The PLDs and additional information about them can be found at http://www.parcconline.org/math-plds.

Item Review Process

For more than a year, ETS and Pearson have been developing potential test questions and tasks for the PARCC assessment. A thorough process has been developed for reviewing potential math items, ensuring that the items on the assessment are of the highest quality. Reviewers come from PARCC governing and participating states, and include K–16 educators, department of education staff, and mathematics experts. Reviews happen through different groups, including PARCC leadership, bias and sensitivity experts, and volunteer teachers.

In reviewing an item, the following questions are asked:

- Does the task measure the intended evidence statement(s)?
- Does the task measure the intended mathematical practice(s)?
- Is the task mathematically correct and free from errors?
- Is the wording of the task clear, concise, and grade-level appropriate?
- Are the graphics/stimuli in the task clear, accurate, appropriate for the task, and appropriate for the grade?
- Do each prompt and all associated graphics/stimuli contribute to the quality of the task?
- Is the scoring guide/rubric clear, correct, and aligned with the expectations for performance that are expressed in the task?

The bias and sensitivity review includes considering whether an item: disadvantages any population (gender, race, ethnicity, etc.) for non-educationally relevant reasons; contains controversial or emotionally charged subject matter that is not supported by the Common Core State Standards; is potentially offensive, demeaning, insensitive, or negative toward any population; or depicts any population in a stereotypical manner.

Each reviewing group can accept items as they are, suggest revisions or edits, or reject an item. The PARCC Operational Working Group (OWG) reconciles items after each group review. Assessment items that are approved through all stages of the review process are available for field testing and become a part of the test item bank for the PARCC assessments, which go live in spring of 2015.

Sample Items

PARCC released sample items in August 2013. These new sample items are first presented in PDF format to emphasize a focus on the content of the items. In November–December 2013, the new sample items will be re-released as technology-based items to be used by students, educators, and parents to better understand the content and the technologies PARCC will use. The sample items represent the current state of PARCC item development and provide users a snapshot of what the 2014-2015 assessments will look like. These items can be found at: http://www.parcconline.org/samples/math.

PARCC

Performance Level Descriptors – Grade 6 Mathematics

	Grade 6 Math: Sub-Claim A The student solves problems involving the Major Content for grade/course with connections to the Standards for Mathematical Practice.			
	Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**
Expressions and Inequalities 6.EE.1-1 6.EE.1-2 6.EE.2a 6.EE.2b 6.EE.2c-1 6.EE.2c-2 6.EE.4	Writes, reads and evaluates numerical and algebraic expressions, including those that contain whole number exponents. Identifies parts of an algebraic or numerical expression using mathematical terms and views one or more parts of an expression as a single entity. Identifies equivalent expressions using properties of operations.	Writes, reads and evaluates numerical and algebraic expressions, including those that contain whole number exponents. Identifies parts of an algebraic or numerical expression using mathematical terms. Identifies equivalent expressions using properties of operations.	Reads and evaluates numerical and algebraic expressions, including those that contain whole number exponents. Identifies parts of an algebraic or numerical expression using mathematical terms. Identifies equivalent expressions using properties of operations. Writes numerical expressions and some algebraic expressions, including those that contain whole number exponents.	Reads numerical and algebraic expressions including those that contain whole number exponents. Identifies parts of an algebraic or numerical expression using mathematical terms.

CCTM AND YOU

CCTM and Colorado's Teacher Quality Standards

Juli Lenzotti, Denver Public Schools

As the state of Colorado moves toward full implementation of the Colorado State Model Evaluation System for Teachers (Senate Bill 10-191), your Colorado Council of Teachers of Mathematics (CCTM) membership is even more important than ever before. "The Teacher Quality Standards outline the knowledge and skills required of an effective teacher," [1]and CCTM—your professional association—provides opportunities for you to enhance your knowledge and skills related to the Teacher Quality Standards, specifically Quality Standard I Element C, Quality Standard III, and Quality Standard V.

Quality Standard I: Teachers demonstrate mastery of and pedagogical expertise in the content they teach. The elementary teacher is an expert in literacy and mathematics and is knowledgeable in all other content that he or she teaches (e.g., science, social studies, arts, physical education, or world languages). The secondary teacher has knowledge of literacy and mathematics and is an expert in his or her content endorsement area(s). Element C: Teachers demonstrate knowledge of mathematics and understand how to promote student development in numbers and operations, algebra, geometry, and measurement and data analysis and probability. **Quality Standard III:** Teachers plan and deliver effective instruction and create an environment that facilitates learning for their students.	At the 2013 CCTM Conference, educators throughout Colorado and surrounding states enjoyed opportunities to deepen their understanding of current best practices for mathematics instruction by attending a variety of interactive sessions facilitated by national and local mathematics educators. At regional workshops co-sponsored by CCTM and Colorado Department of Education (CDE), members explored a variety of topics: Creating the 21st Century Mathematics Classroom Using Colorado's New Standards; Enhancing Understanding of Ratio and Proportion; Addition and Subtraction: Whole Numbers to Complex Numbers; What Does Algebra (ECE-12) Look Like in the Colorado Academic Standards? The CCTM-published journal, Colorado Math Teacher, is an amazing resource, helping educators align instruction with academic standards and student assessment results.[2]

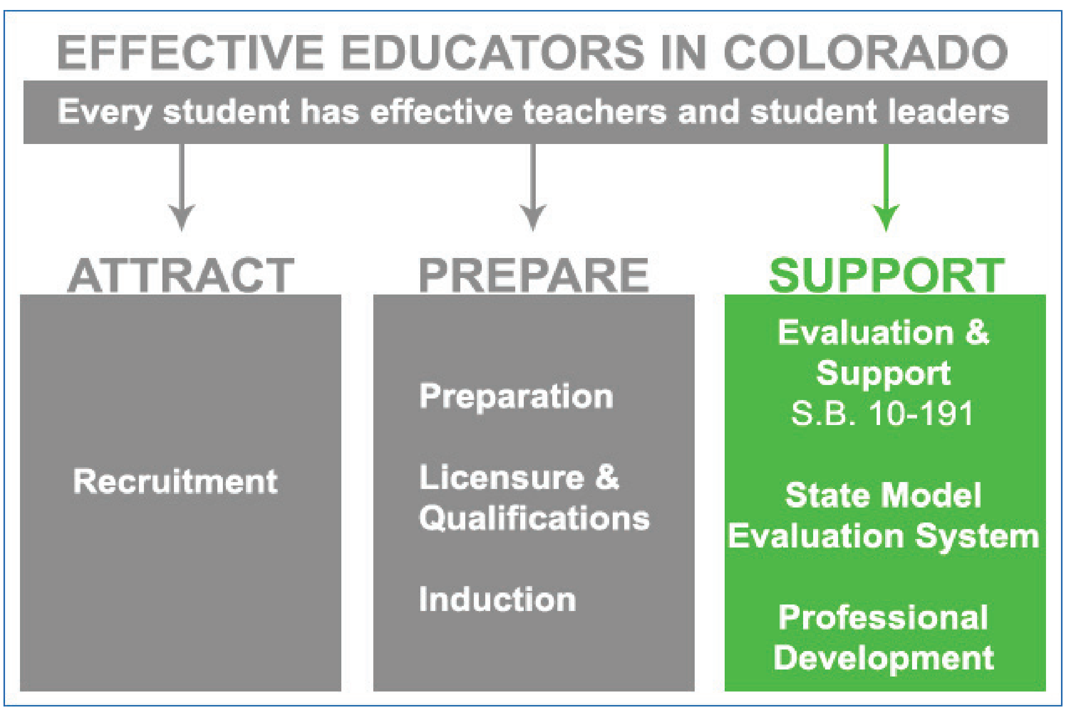
Quality Standard V: Teachers demonstrate leadership.	CCTM is an affiliate of the National Council of Teachers of Mathematics (NCTM), the largest professional organization of mathematics teachers in the country. Through their participation in CCTM- and NCTM-sponsored professional development opportunities, CCTM members initiate and lead collaborative

Not only will your membership in CCTM expand your knowledge of mathematics education as well as your professional relationships and networking opportunities, it will support you in becoming an even more effective mathematics educator.

[1] These are from the Rules issues on November 9, 2011 and approved on February 15, 2012. The Colorado Teacher Quality Standards and Their Related Elements and Artifacts, p. 44. Colorado State Model Evaluation System for Teachers

[2] Quality Standard III, p. 57. Colorado State Model Evaluation System for Teachers.

	activities, sharing ideas to improve teaching and learning and support struggling students.[3] They participate in district-wide decision-making processes that impact the school community, including families.[4] At the recent 2013 CCTM Conference, CCTM members experienced a variety of networking and professional development opportunities. School and district administrators learned about the Colorado Academic Standards (Common Core State Standards) and PARCC assessment and collaborated and planned to lead schools with the new standards and Senate Bill 10-191. Led by the Colorado PARCC Educator Leader Cadre, the conference Pre-Session, "Teacher Resources for the PARCC Assessment," allowed CCTM members to engage with the resources available from PARCC—Model Content Frameworks, Performance Level Descriptors, and released tasks and prototype items—to guide instruction for new standards implementation.



[3] Quality Standard V, Element A, p. 61. Colorado State Model Evaluation System for Teachers.
[4] Quality Standard V, Element B, p. 61. Colorado State Model Evaluation System for Teachers.

CONFERENCE AND PROFESSIONAL DEVELOPMENT

Conference Report

Joanie Funderburk, Conference Chair

CCTM's 2013 Annual Conference, "Navigating New Standards, Charting New Practices" was held on October 10–11, 2013 at the Denver Mart. Two pre-sessions were held on Thursday, followed by a reception and the Colorado Teacher Awards ceremony. Over 100 breakout sessions were offered on Friday, and Ellen Whitesides, who works with the Institute for Mathematics and Education at the University of Arizona, gave a very informative keynote talk, "A Common Core Compass Toward Student Understanding, Teacher Collaboration, and Future Possibilities for America." (Note: See the highlights of her talk in the next article.)

Nearly 50 Colorado district- and school-level administrators and teacher leaders attended the pre-session "Leading Math in the New Standards Era." During this three-hour interactive session led by Sharon Kallus (Principal, Broadway Elementary and Grand River Virtual Academy) and Lanny Hass (Principal, Thompson High School), participants gained a deeper understanding of the role of the Standards for Mathematical Practice in aligning classroom practice to the new standards. They identified where the math practices are evident in the Colorado Teacher Quality Standards rubric, and learned about resources to support teachers as they implement the math practices in their classrooms. Participants also learned about the upcoming PARCC assessment, and about resources available to educators as they prepare for the new assessment in spring 2015. Those who attended the administrator pre-session shared their appreciation for math-specific information, and the time to share and collaborate with others across the state.

Approximately 120 Colorado educators attended the teacher pre-session, "Leveraging PARCC Resources for Teaching and Learning." This session provided the opportunity to take a closer look at three, free, online resources available from PARCC. Participants got an overview about the assessment, then explored the Model Content Frameworks, which provide a frame for the PARCC assessments and also serve as a valuable tool for teachers, providing "Examples of Key Advances from the Previous Grade," "Examples of Major Within-Grade Dependencies," and "Examples of Opportunities for Connecting Mathematical Content and Mathematical Practices." Next, attendees learned about the process by which the Performance Level Descriptors were created, and looked more closely at these PLDs for their course or grade-level. This activity provided the opportunity to have conversations with other teachers of the same course or grade level from across the state. Finally, attendees reviewed several of the PARCC sample items and task prototypes, and compared them to TCAP questions to help illustrate the significant differences between the PARCC items and previous state

math assessments. Attendees enjoyed the chance to collaborate with other teachers, and gained enough familiarity with the resources that they would be comfortable further exploring on their own. (Find these resources and others at www.parcconline.org.)

Friday's sessions were well attended and enjoyed by teachers from the Denver metro area, as well as the western slope, northern Colorado, as well as southern Colorado. Highlights included the morning session "Structures and Strategies: The Keys to Active Engagement," where Carolyn Merritt, from Falcon School District 49, shared ways to gets student moving and participating in math class. Cassie Gannett and Angela Hunter from District 2 had an afternoon session on "Reasoning and Sense-Making," helping participants focus on Math Practices 2 and 3 with practical classroom ideas. Other popular sessions included those focused on "flipped" instructional models and a "Magnificent Mathemagician" who shared mathematical magic tricks connected to problem solving and proof.

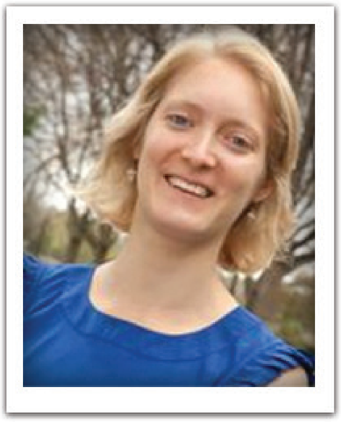
Over the lunch hour, participants enjoyed shorter "Blast" sessions, which allowed vendors to share specifics about their products in a small group format, and allowed first-time presenters a more formal opportunity to share ideas. Shawna Reger from Poudre School District shared a "Family Math Night Make and Take" session, and Dana Plewka from the Boulder Daily Camera helped participants see how they could use non-fiction to bring math lessons to life.

Those who attended the conference this year are encouraged to complete the conference survey at https://docs.google.com/forms/d/1o_pu52f21eWxKp7fIHIrbT-d6CD5EzaD7fuozPoXzZN4/viewform, and provide us feedback to help make next year's conference even better!

CONFERENCE AND PROFESSIONAL DEVELOPMENT

A Common Core Compass: Ellen Whitesides' CCTM Keynote Presentation

Joanie Funderburk, Conference Chair



The 2013 Annual CCTM conference welcomed Ellen Whitesides as our keynote speaker. Ellen works closely with the Institute for Mathematics and Education at the University of Arizona, and is the Director of Community Building for Illustrative Mathematics, an online resource providing "guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards" (www.illustrativemathematics.org). Ellen shared relevant and meaningful information and perspective to approximately 600 conference attendees.

Ellen's talk, titled "A Common Core Compass Towards Student Understanding, Teacher Collaboration and Future Possibilities for America," focused on the three areas in the title: student understanding, teacher collaboration, and future possibilities for America.

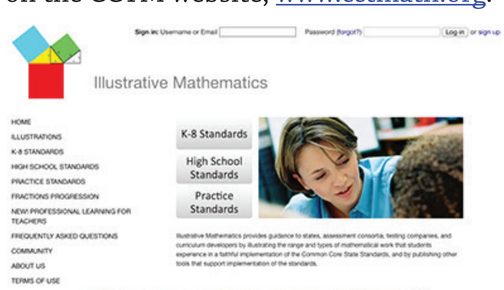
The Common Core State Standards (CCSS) are written specifically to allow for student understanding of mathematical concepts. Ellen shared how the "coherence" in the CCSS creates a progression of mathematical concepts for students that connect to form a firm foundation for high school mathematics. For instance, when students understand fractions as numbers and their relative size dependent on the whole, they are better able to connect fraction operations to operations with whole numbers. Similarly, students' introductory work with expressions and equations in middle school can be explicitly connected to the properties of operations learned in elementary school, and set the stage for more complex expressions and equations they will study in high school. The coherence built into the standards allows students to engage in math practice 7: *Look for and make use of structure.* Students will utilize structure when they write expressions in equivalent forms to solve problems, or highlight certain properties of the expression.

Teacher collaboration will be an important component of successful implementation of the CCSS. Ellen shared that, according to research, teacher collegiality and collaboration lead to increased changes in teacher practices and increased learning for students. Her work with Illustrative Mathematics provides resources for teachers to use in their collaborative planning, and opportunities for teachers to collaborate with others across the country in their "task talks." Teachers are encouraged to explore and participate in these opportunities at www.illustrativemathematics.org .

Successful implementation of the CCSS will contribute to improvements in the achievement and futures of Colorado students. Ellen cited Colorado's average on the 8th grade NAEP test; while above the national average, we are still below the level considered "proficient." Preparing students with stronger math backgrounds is timely in our state, where there are approximately 1.5 jobs per applicant in STEM fields as opposed to 1 job per 3 applicants in non-STEM fields. Ellen's talk ended with results of recent surveys of educators regarding the implementation of new standards:

- About 73% of those surveyed agree or strongly agree that implementing CCSS is or will be challenging,
- 76% believe the new standards will positively impact students' ability to think critically and reason, and
- 73% of teachers across all grade levels are enthusiastic about implementing CCSS in their classrooms.

CCTM members have access to Ellen's Keynote PowerPoint on the CCTM website, www.cctmath.org.



FROM COLLEGE TO THE CLASSROOM

*Sense Making and Persevering in Problem Solving

Gulden Karakok, Assistant Professor, School of Mathematical Sciences, University of Northern ColoradoDistrict

One of the mathematical practices in the *Common Core State Standards is "Make sense of problems and persevere in solving them."* This practice highlights the importance of starting a problem by understanding the givens, constraints, and relationships among them. It is essential for students to make conjectures and devise a plan to evaluate these conjectures in their problem-solving process. Engaging in making connections to similar problems, trying to solve simpler problems, and observing special cases help students gain insight about the solution of a given problem. Using available resources and various representations provide further aids to understand the problems and possible solutions. These essential aspects of problem-solving processes help students practice sense-making of a problem situation. Furthermore, practicing these aspects gives them opportunities to see the importance of the process, rather than "jumping into a solution attempt" without understanding the problem.

This particular mathematical practice also highlights the importance of the process of evaluating solutions, and identifying similarities and differences among different solutions. Through such processes, students would recognize that some problems would not be solved in a short period of time and thus require perseverance and more attempts.

Similar to how students develop other mathematical skills, they need more opportunities to practice problem-solving processes more frequently. As mathematics educators, we should provide opportunities for students to engage in problems with multiple entry points, and foster their engagement in a supportive environment. However, it is not always an easy task to implement these ideas in classrooms. We need to find more ways to engage in such problem-solving processes and activities, and discussion on possible ways of implementing them in our classrooms.

As pre-service teachers complete their program, they usually take courses to foster aforementioned skills in problem solving. For in-service teachers, there are many professional development opportunities to further strengthen their problem-solving processes and ways in which these practices could be implemented in their classroom. The Math Teachers' Circle is one of these professional development opportunities for teachers to solve different problems and engage in discussion on implementation of these problems in their classroom. (For more information on Math Teachers' Circle visit: http://www.mathteacherscircle.org/). In Colorado, there are two active Math Teachers' Circles (MTC) providing free professional development centered around problem solving: Rocky Mountain MTC (for more information visit: http://rmmtc.ucdenver.edu/) and Northern Colorado MTC (for more information visit: http://www.unco.edu/nhs/mathsci/mtc/).

Participants of the MTCs usually engage in solving math problems geared towards the level of teachers, rather than the level of their students. Providing opportunities for teachers to collaboratively solve math problems helps them strengthen their problem-solving skills that are highlighted in the mathematical practice of "Mak[ing] sense of problems and persevere in solving them." For example, some of the participants of the Northern Colorado MTC workshop mentioned that it was beneficial for them to experience some of these skills again:

> *"I learned to use pictures a lot more when solving problems. I learned not to focus on the answers, but to have fun with the steps to get to an answer."*
>
> *"Believe it or not, I have used breaking things into a smaller problem for the first time!"*
>
> *"The idea of parity is new to me. I didn't think about problem solving in this way previously. I definitely had to put into practice making an easier problem to solve many problems this week."*
>
> *"Making the problem simpler to collect data at each level was a strategy that I knew but did not use as often. Was great to need to use it."*

Further discussions of solutions, various aspects of problems, and ways in which the problems could be implemented in classrooms rejuvenate participants' approach to teaching problem-solving in their classrooms. The following excerpts from participants of the Northern Colorado MTC highlights some of the implementation aspects:

"I will make my explanations of the problems simpler and shorter to allow more time for students to have fun solving problems."

"I like the fact that a question would be posed to us without giving the answer soon after. I would like to try this and see how it works with my students, so they know they can think about the problem longer."

"I have more math ability than I give myself credit for. Working with mathematicians this week helped me be more aware of this. Students have more math ability than they think they do too, and this has reminded me that I need to bring that to their conscience."

The following problem was presented to MTC participants at the CCTM 2013 conference:

Take a three-digit number, reverse its digits, subtract the smaller from the larger. Reverse the digits of the result and add it to the original result. For example, 123 becomes 321, and 321-123=198. 198 becomes 891, and 198+891=1089. Try this process with several numbers. What happens? Why?

Participants worked on the problem, shared their approaches and discussed how they plan to proceed in their solutions. A couple of participants used algebraic equations to communicate their solutions, whereas others used base-10 manipulatives to share their initial observation about the problem. As participants tried to connect the observations from base-10 manipulatives to the algebraic solutions, they recognized some special cases that needed further investigation. After achieving a solution, participants discussed how this particular problem could be used to reinforce student understanding of addition and subtraction operations in base 10 and meaning of *trading in* and *regrouping*. Discussions of the solutions and the implementation of this particular problem were limited by the conference session time, however the problem was still suitable for allowing the participants to approach it in multiple ways, and make connections between their various approaches. Some problems posed in MTC sessions require more time to solve, and allow participants to experience persevering.

Discussions of how participants felt during the problem solving process also helped them re-think how to create an environment that supports problem solving. Participants raised important aspects of such environments and teaching as "Finding the balance between letting students struggle and not wanting them to quit," and highlighting that "It is ok to fail at times. Solving these problems is like everyday life."

Overall, it is important for our students to understand that some problems need more time to solve and perseverance in problem solving is important. As math educators, it is our job to provide environments that foster problem-solving processes and support as students persevere in this process.

This problem is taken from the book *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10* by Mark Driscoll.

THE CCTM BOOK CLUB

*Insights into Mathematical Thought: Excursions with Distributivity

Reviewed by **Christy Pruitt and Cassie Gannett**, CCTM Regional Representatives

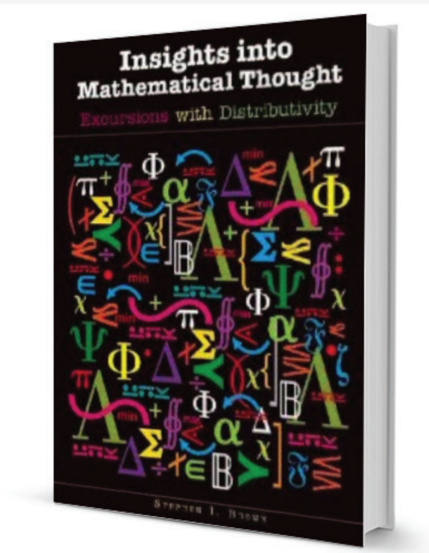
For our fellow math nerds who like playing with numbers and seeing things in new ways, *Insights into Mathematical Thought: Excursions with Distributivity* by Steven I. Brown is the book for you. The hub of the book revolves around using the distributive property as a vehicle to search for mathematical connections many people are innately aware of. Engaging the reader in cerebral works, both algebraic and geometric, the reader's mathematical thinking expands as patterns are examined and answers are discovered.

For math teachers who are missing the thought provoking experiences of their college Math Theory course, this book allows you to revisit the philosophical journey of mathematical proof. The author explores procedures and tricks people use for quick mental calculations. For instance, a simple version for squaring multiples of five in your head is developed. In addition, Brown also looks at the conceptual foundation in the number systems. One example is through defining imaginary numbers and how they relate to the real number system.

This recent publication from NCTM inspires connections and ways of thinking about mathematics. Brown provides ideas that would push the thinking of teachers, as well as gifted and talented and advanced students, and challenges us to think beyond the procedures and solutions often central to mathematical experiences in the classroom.

As our book club questions, we have chosen to use some of the thought provoking questions that Brown has used throughout the book. For those of you interested in discussing this book with others, visit [Colorado Mathematics Teachers Network](). We look forward to hearing your thoughts.

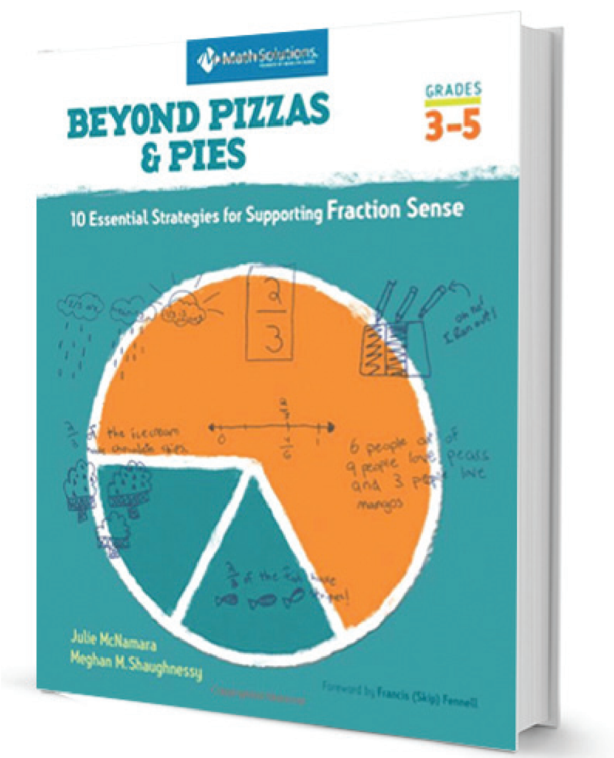
- How might some of the ideas in this book bring to life the Common Core State Standards for Mathematical Practice?
- When you consider our current climate of high stakes testing, how do you feel about this statement suggested by Brown to "view problems as something more than an invitation for solution" (p. 102).
- "An observation from a nine-year-old inspired another new algorithm useful for division; she originally did not want to presume her observations because "it's not the one the teacher taught the class. If I show it to her, she'll tell me...that maybe when I grow up and understand about such numbers, I can do division that way (Brown, 1981, p. 12). What do you see as the pros and cons of that statement from an educational point of view?" (p. 102)



FOR YOUR BOOKSHELF

Beyond Pizzas & Pies: 10 Essential Strategies for Supporting Fraction Sense

***Reviewed by* Rachael Risley,** Elementary Math Instructional Coordinator, Aurora Public Schools



The Common Core Standards have expanded expectations for student sense-making and understanding of fractions. *Beyond Pizzas & Pies: 10 Essential Strategies for Supporting Fraction Sense* by Julie McNamara and Meghan M. Shaughnessy is a strong resource to support upper elementary teachers in expanding their knowledge and practice for teaching fractions.

This research-based, teacher-friendly book is well organized into eight chapters, each of which addresses a subtopic within fractions. The chapters are cross-referenced with the related Common Core standards, and each begins with a Classroom Scenario that illustrates student conceptions and misconceptions around fractions in an authentic classroom setting. *What's the Math*? follows each of the classroom scenarios. These sections help teachers better understand the mathematics of each subtopic and are helpful for understanding the big ideas for the subtopics within fractions. *What's the Research*? sections provide a summary of the research around the teaching strategies suggested for the chapter's subtopic. Finally, the authors provide Classroom Activities, high quality tasks that can be used to supplement teachers' current resources for teaching fractions with understanding.

Chapter 2, for example, Top or Bottom: Which one Matters deals with fraction equivalence and ordering, and supports student reasoning about numerators and denominators, cornerstones of the fourth grade Common Core Standards. In this chapter, the:

- *Classroom Scenario* invites the reader into a classroom where the teacher has posed the question, "Which is bigger, one fourth or one eighth?" The children explore their initial answer, "the smaller the denominator, the larger the fraction," as their teacher poses the next question, which is bigger, ½ or ¾? This new challenge question spurs the class into a deeper discussion.
- *What's the Math?* offers the following insight into comparing fractions, "When comparing fractions, students need to consider the size of the wholes and interpret each fraction as a single number defined by the relationship between the numerator and denominator" (p. 18). The authors provide models and illustrations of strategies that children might use to compare fractions.
- *What's the Research?* provides research-based confirmation that often children attend to only the numerator or only the denominator when comparing fractions and therefore lose sight of the value of the fraction.
- *Classroom Activities* consist of a number line activity that utilizes Cuisenaire rods and an activity that supports children to examine their current "rules" by asking, "Is it always true?" These tasks are both designed to help students attend to the quantity represented by the fraction—rather than the numerator or denominator in isolation.

This valuable book can be used in its entirety, or the chapters can stand alone to support particular classroom needs. Either way, this text is a strong support for teachers who are seeking to help students develop a strong understanding of fractions.

Fast Connections

Check out the CCTM website, where you will find an expanded version of the announcements or important information listed below.

CCTM AWARDS 2013 **Kate Canine, Rachel Risley, & Diane Weaver**	SPRING ELECTIONS 2013 **Stacy Larson**, Elections Chair

CCTM Awards 2013

Kate Canine, Rachel Risley, & Diane Weaver

At our fall conference, CCTM recognized the following mathematics leader, and teachers in each region, for having displayed excellence in their craft:

Jerry Overmyer – Leadership
Sarah Smith – Region 1
Deborah Maruyama – Region 2
Anita Venohr-Madden – Region 2
Natalie Johnson – Region 3
Jen Smart – Region 4
Dora Davis – Region 5
Timi Terry – Region 5
Brooke Caster – Region 6
Jessica Guerra – Region 7
Tiffiny Nierderhofer Vaughn – Region 7

Congratulations all! More information about each of the awardees can be found at: http://www.cctmath.org/page//awardees2013.htm

Call for Nominations for the Spring 2014 Elections

Stacy Larson, Elections Chair

Nominations are now open for **seven** positions with the Colorado Council of Teachers of Mathematics (CCTM): president-elect, vice-president, secretary, NCTM representative, and representatives for regions 1, 4, and 7. This makes 2014 a very important year. Please take this opportunity to nominate someone who is willing to work for this professional organization and give something back to the Colorado Mathematics community. You may nominate yourself as well. This is your chance to be involved and influence the future path of CCTM. Elections will be held in the spring with all terms of office beginning June 1, 2014.

For more information on each position, as well as a nomination form, go to our website at http://www.cctmath.org/page//elections2014.htm

Nominations close **January 10, 2014**.