COLORADO MATHEMATICS TEACHER - FALL 2015

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THE COLORADO MATHEMATICS TEACHER

Starting Off the New School Year: Strategies, Opportunities, and Resources for Teachers and Parents

CONTENTS & FEATURES

From The Editor's Desk

Sandie Gilliam, CCTM Editor

Students are getting ready: Back to school sales, back to school supplies,... and back to school clothing haul?

Teachers are getting ready: Decorating their classrooms, attending professional development workshops, and perhaps enjoying that last little bit of vacation!

What are you doing to prepare yourself for teaching math? This FULL issue of the *Colorado Mathematics Teacher* presents tools that you can use to do just that. From learning more about math goals, thinking through the math rules you may want to amend (or just let expire), to a handout for parents that may guide them in helping their child with math AND MUCH MORE—it's in this edition! For ease in navigating this issue, click on the title or page number to go directly to that article!

All too often in my former schools, those opening staff development sessions offered little, if any, practical ideas that easily transferred to math. I remember the session on "Reading in the Content Area." After patiently waiting for the presenters to connect their workshop to math, we finally asked. "Sorry, we don't have any suggestions for you on math" was their response. So, if you are looking for great math professional development—or your administrators could use some—check out the information within on the CCTM Fall Conference, register, and bring along a colleague.

Are you part of a PLC or teachers' book club? Start the year by reading *Principles to Action: Ensuring Mathematics Success for All* OR *The Common Core Mathematics Companion: The Standards Decoded, K–2*.

Think back on 2014–2015. What were your accomplishments? Build on them. Reflect on any summer work you engaged in. For me, teaching 23 middle school students at a summer math camp at Stanford University was the highlight of my summer. I learned so much about growth mindset working with Jo Boaler. While I currently work primarily with pre-service teachers, getting back to my roots of teaching secondary students (after eight years filled my heart and inspired me to continue to research and implement growth versus fixed mindset messages with all ages. I enjoy re-watching the summer math camp kids and seeing their joy with math: https://www.youcubed.org/youcubed-summer-math-camp-2015/

Life-long learning is my goal. Have a great start to your school year!

President's Message

Joanie Funderburk, CCTM President

In the five years since Colorado adopted the Common Core State Standards, many resources have been developed to support teachers in implementing them, and one of our favorites is *Principles to Actions: Ensuring Mathematics Success for All* (NCTM, 2014). Each of the next eight issues of our Colorado Mathematics Teacher journal will focus on the eight Mathematics Teaching Practices outlined in *Principles to Actions*. We hope you will read the book along with us, and use the journal issues to support your understanding, collaborative conversations, and reflections regarding the great ideas in *Principles to Actions*.

The first Mathematics Teaching Practice is to "Establish mathematics goals to focus learning." *Principles to Actions* notes that, "effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions." It may seem rather obvious that our lessons should be focused on goals for student learning; however, this teaching practice is likely the first one for a very good reason. Not only does establishing clear goals for student learning help us as teachers to determine appropriate learning activities, drive formative assessment practices, and allow us to help observers understand our lesson, but it also anchors students in the mathematics of the day, allows them to track their learning, and focuses them on what is most meaningful and relevant during the class period, series of lessons, or unit. Research data indicate that student learning is enhanced when the goals for learning are clear and explicit.

In addition to sharing goals with students before and during the lesson, it is important that teachers situate these goals within a larger mathematical learning progression. Mathematics is a cohesive,

Establish mathematics goals to focus learning
Teacher and student actions

What are *teachers* doing?	What are *students* doing?
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

From *Principles to Actions*, pg 16

connected discipline, and one of the shifts of the Common Core State Standards is to attend to the coherence built into the standards and present in the nature of mathematics. I taught Algebra 1 for probably 16 of my 25 years as a math educator (all of them *before* the new standards were in place!), and never once did I explicitly help my students understand that the ideas we were studying were just extensions of what they had studied in elementary and middle school! When solving equations, they were using the properties of operations, just like they had in learning their addition and subtraction "fact families." With the coherence of the standards, each year's learning builds on and extends the learning of previous years. When teacher's learning goals are firmly situated in these learning progressions, students see their knowledge developing, growing deeper and more sophisticated, and building upon what is already familiar, rather than seeing each math topic as distinct and disconnected from previous learning. By anchoring learning goals in progressions, teachers can also help students understand why a single learning goal may be the focus of learning over several days or weeks. Phil Daro, one of the lead writers of the Common Core math standards, reminds us that "mathematical ideas do not always break down into lesson-sized chunks." (http://serpmedia.org/daro-talks/). Clear understanding of the mathematical goal for the day and how it points to the bigger mathematical idea within adds clarity for both teachers and students.

Finally, it is important to separate goals for mathematics learning from lists of classroom activities, such as group work, exit tickets, etc. Knowing the activities won't necessarily help students understand what mathematics is important in the lesson. How does it connect to what they've learned before and the larger scope of mathematical understanding? How is the work of the class purposeful and connected to their learning? As Yogi Berra once said, "If you don't know where you're going, you might end up somewhere else!" When students know where the lesson is going, and what the important mathematics is for the day, they are better able to attend to and engage in their own learning.

I take a group fitness class at my gym, and a couple of weeks ago our trainer was showing us a new exercise using straps attached to a high bar. He wanted us to squat while holding the straps above our heads. At first, I found the move awkward, and I wasn't diligent about keeping the straps taut throughout the up and down motion of my squat. When my trainer explained that this move was preparing me to do an overhead squat with a barbell, I understood why we were doing it, and was better able to self-correct and attend to the proper form without his constant supervision.

Similarly, a clear and explicit goal for learning helps students to engage strategically in their own learning, and both students and teachers can be more focused on attaining the goal. The learning goal serves as a beacon for students and teachers as the lesson progresses. When a goal is perceived as challenging but attainable, students are motivated even further to persevere in their learning and find mathematical success.

As this chapter of *Principles to Actions* reminds us, taking the time to establish clear goals for the mathematics that students are learning, situating those goals within learning progressions, and using them to guide instructional decisions is a high-yield teaching practice, one that will go far in supporting all students in their mathematics learning journey. How will you improve your learning goals to benefit your students?

IN THE FIELD

13 Rules that Expire

Karen Karp, Sarah Bush & Barbara Dougherty

13 RULES *That Expire*

Overgeneralizing commonly accepted strategies, using imprecise vocabulary, and relying on tips and tricks that do not promote conceptual mathematical understanding can lead to misunderstanding later in students' math careers.

By Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty


FUSE/THINKSTOCK

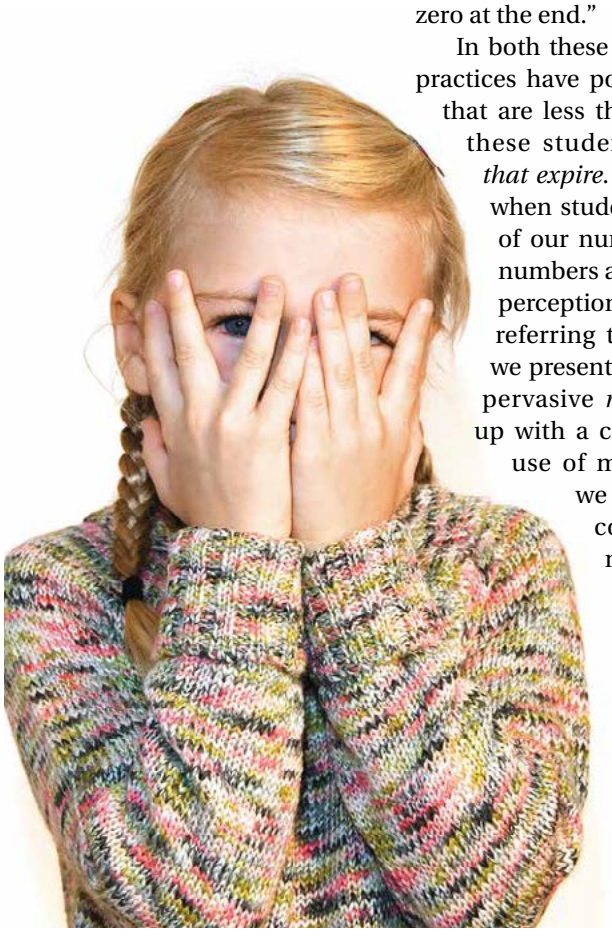
Imagine the following scenario: A primary teacher presents to her students the following set of number sentences:

$$3 + 5 = \square$$
$$\square + 2 = 7$$
$$8 = \square + 3$$
$$2 + 4 = \square + 5.$$

Stop for a moment to think about which of these number sentences a student in your class would solve first or find easiest. What might they say about the others? In our work with young children, we have found that students feel comfortable solving the first equation because it "looks right" and students can interpret the equal sign as *find the answer*. However, students tend to hesitate at the remaining number sentences because they have yet to interpret and understand the equal sign as a symbol indicating a relationship between two quantities (or amounts) (Mann 2004).

In another scenario, an intermediate student is presented with the problem 43.5 × 10. Immediately, he responds, "That's easy; it is 43.50 because my teacher said that when you multiply any number times ten, you just add a zero at the end."

In both these situations, hints or repeated practices have pointed students in directions that are less than helpful. We suggest that these students are experiencing *rules that expire*. Many of these rules "expire" when students expand their knowledge of our number systems beyond whole numbers and are forced to change their perception of what can be included in referring to *a number*. In this article, we present what we believe are thirteen pervasive *rules that expire*. We follow up with a conversation about incorrect use of mathematical language, and we present alternatives to help counteract common student misunderstandings.



ALINA555/THINKSTOCK

The Common Core State Standards (CCSS) for Mathematical Practice advocate for students to become problem solvers who can reason, apply, justify, and effectively use appropriate mathematics vocabulary to demonstrate their understanding of mathematics concepts (CCSSI 2010). This, in fact, is quite opposite of the classroom in which the teacher does most of the talking and students are encouraged to memorize facts, "tricks," and tips to make the mathematics "easy." The latter classroom can leave students with a collection of explicit, yet arbitrary, rules that do not link to reasoned judgment (Hersh 1997) but instead to learning without thought (Boaler 2008). The purpose of this article is to outline common rules and vocabulary that teachers share and elementary school students tend to overgeneralize—tips and tricks that do not promote conceptual understanding, rules that "expire" later in students' mathematics careers, or vocabulary that is not precise. As a whole, this article aligns to Standard of Mathematical Practice (SMP) 6: *Attend to precision*, which states that mathematically proficient students "…try to communicate precisely to others. …use clear definitions … and … carefully formulated explanations…" (CCSSI 2010, p. 7). Additionally, we emphasize two other mathematical practices: SMP 7: *Look for and make use of structure* when we take a look at properties of numbers; and SMP 2: *Reason abstractly and quantitatively* when we discuss rules about the meaning of the four operations.

"Always" rules that are not so "always"

In this section, we point out rules that seem to hold true at the moment, given the content the student is learning. However, students later find that these rules are not always true; in fact, these rules "expire." Such experiences can be frustrating and, in students' minds, can further the notion that mathematics is a mysterious series of tricks and tips to memorize rather than big concepts that relate to one another. For each rule that expires, we do the following:

1. State the rule that teachers share with students.
2. Explain the rule.
3. Discuss how students inappropriately overgeneralize it.
4. Provide counterexamples, noting when the rule is not true.

5. State the "expiration date" or the point when the rule begins to fall apart for many learners. We give the expiration date in terms of grade levels as well as CCSSM content standards in which the rule no longer "always" works.

Thirteen rules that expire

1. When you multiply a number by ten, just add a zero to the end of the number.

This "rule" is often taught when students are learning to multiply a whole number times ten. However, this directive is not true when multiplying decimals (e.g., 0.25 × 10 = 2.5, not 0.250). Although this statement may reflect a regular pattern that students identify with whole numbers, it is not generalizable to other types of numbers. Expiration date: Grade 5 (5.NBT.2).

2. Use keywords to solve word problems.

This approach is often taught throughout the elementary grades for a variety of word problems. Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside of the problem context (Clement and Bernhard 2005). Unfortunately, many keywords are common English words that can be used in many different ways. Yet, a list of keywords is often given so that word problems can be translated into a symbolic, computational form. Students are sometimes told that if they see the word *altogether* in the problem, they should always add the given numbers. If they see *left* in the problem, they should always subtract the numbers. But reducing the meaning of an entire problem to a simple scan for key words has inherent challenges. For example, consider this problem:

> John had 14 marbles in his left pocket. He had 37 marbles in his right pocket. How many marbles did John have?

If students use keywords as suggested above, they will subtract without realizing that the problem context requires addition to solve. Keywords become particularly troublesome when students begin to explore multistep word problems, because they must decide which keywords work with which component of the problem. Keywords can be informative but must be used in conjunction with all other words in the problem to grasp the full meaning. Expiration date: Grade 3 (3.OA.8).

3. You cannot take a bigger number from a smaller number.

Students might hear this phrase as they first learn to subtract whole numbers. When students are restricted to only the set of whole numbers, subtracting a larger number from a smaller one results in a negative number, an integer that is not in the set of whole numbers, so this rule is true. Later, when students encounter application or word problems involving contexts that include integers, students learn that this "rule" is not true for all problems. For example, a grocery store manager keeps the temperature of the produce section at 4 degrees Celsius, but this is 22 degrees too hot for the frozen food section. What must the temperature be in the frozen food section? In this case, the answer is a negative number, (4° – 22° = –18°). Expiration date: Grade 7 (7.NS.1).

4. Addition and multiplication make numbers bigger.

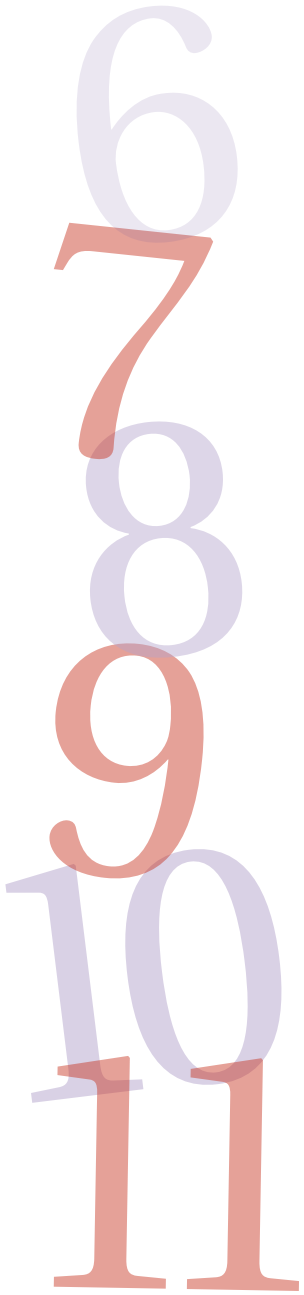
When students begin learning about the operations of addition and multiplication, they are often given this rule as a means to develop a generalization relative to operation sense. However, the rule has multiple counterexamples. Addition with zero does not create a sum larger than either addend. It is also untrue when adding two negative numbers (e.g., –3 + –2 = –5), because –5 is less than both addends. In the case of the equation below, the product is smaller than either factor.

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

This is also the case when one of the factors is a negative number and the other factor is positive, such as –3 × 8= –24. Expiration date: Grade 5 (5.NF.4 and 5.NBT.7) and again at Grade 7 (7.NS.1 and 7.NS.2).

5. Subtraction and division make numbers smaller.

This rule is commonly heard in grade 3: both subtraction and division will result in an answer that is smaller than at least one of the



numbers in the computation. When numbers are positive whole numbers, decimals, or fractions, subtracting will result in a number that is smaller than at least one of the numbers involved in the computation. However, if the subtraction involves two negative numbers, students may notice a contradiction (e.g., –5 – (–8) = 3). In division, the rule is true if the numbers are positive whole numbers, for example:

$$8 \div 4 = 2 \text{ or } 4 \div 8 = \frac{1}{2}$$

However, if the numbers you are dividing are fractions, the quotient may be larger:

$$\frac{1}{4} \div \frac{2}{5} = \frac{5}{8}$$

This is also the case when dividing two negative factors: (e.g., –9 ÷ –3 = 3). Expiration dates: Grade 6 (6.NS.1) and again at Grade 7 (7.NS.1 and 7.NS.2c).

6. You always divide the larger number by the smaller number.

This rule may be true when students begin to learn their basic facts for whole-number division and the computations are not contextually based. But, for example, if the problem states that Kate has 2 cookies to divide among herself and two friends, then the portion for each person is 2 ÷ 3. Similarly, it is possible to have a problem in which one number might be a fraction:

> Jayne has $\frac{1}{2}$ of a pizza and wants to share it with her brother. What portion of the whole pizza will each get?

In this case, the computation is as follows:

$$\frac{1}{2} \div 2 = \frac{1}{4}$$

Expiration date: Grade 5 (5.NF.3 and 5.NF.7).

7. Two negatives make a positive.

Typically taught when students learn about multiplication and division of integers, rule 7 is to help them determine the sign of the product or quotient. However, this rule does not always hold true for addition and subtraction of integers, such as in –5 + (–3) = –8. Expiration date: Grade 7 (7.NS.1).

8. Multiply everything inside the parentheses by the number outside the parentheses.

As students are developing the foundational skills linked to order of operations, they are often told to first perform multiplication on the numbers (terms) within the parentheses. This holds true only when the numbers or variables inside the parentheses are being added or subtracted, because the distributive property is being used, for example, 3(5 + 4) = 3 × 5 + 3 × 4. The rule is untrue when multiplication or division occurs in the parentheses, for example, 2 (4 × 9) ≠ 2 × 4 × 2 × 9. The 4 and the 9 are not two separate terms, because they are not separated by a plus or minus sign. This error may not emerge in situations when students encounter terms that do not involve the distributive property or when students use the distributive property without the element of terms. The confusion seems to be an interaction between students' partial understanding of terms and their partial understanding of the distributive property—which may not be revealed unless both are present. Expiration date: Grade 5 (5.OA.1).

9. Improper fractions should always be written as a mixed number.

When students are first learning about fractions, they are often taught to always change improper fractions to mixed numbers, perhaps so they can better visualize how many *wholes* and *parts* the number represents. This rule can certainly help students understand that positive mixed numbers can represent a value greater than one whole, but it can be troublesome when students are working within a specific mathematical context or real-world situation that requires them to use improper fractions. This frequently first occurs when students begin using improper fractions to compute and again when students later learn about the slope of a line and must represent the slope as the rise/run, which is sometimes appropriately and usefully expressed as an improper fraction. Expiration dates: Grade 5 (5.NF.1) and again in Grade 7 (7.RP.2).

10. The number you say first in counting is always less than the number that comes next.

In the early development of number, students are regularly encouraged to think that number

TABLE 1

Some commonly used language "expires " and should be replaced with more appropriate alternatives.

Expired mathematical language and suggested alternatives

What is stated	What should be stated
Using the words *borrowing or carrying* when subtracting or adding, respectively	Use *trading* or *regrouping* to indicate the actual action of trading or exchanging one place value unit for another unit.
Using the phrase ___ *out of* __ to describe a fraction, for example, one out of seven to describe $\frac{1}{7}$	Use the fraction and the attribute. For example, say *one-seventh of the length of the string.* The *out of* language often causes students to think a part is being subtracted from the whole amount (Philipp, Cabral, and Schappelle 2005).
Using the phrase *reducing fractions*	Use *simplifying fractions*. The language of *reducing* gives students the incorrect impression that the fraction is getting smaller or being reduced in size.
Asking how shapes are *similar* when children are comparing a set of shapes	Ask, *How are these shapes the same? How are the shapes different?* Using the word *similar* in these situations can eventually confuse students about the mathematical meaning of similar, which will be introduced in middle school and relates to geometric figures.
Reading the equal sign as *makes*, for example, saying, *Two plus two makes four* for 2 + 2 = 4	Read the equation 2 + 2 = 4 as *Two plus two equals or is the same as four.* The language *makes* encourages the misconception that the equal sign is an action or an operation rather than representative of a relationship.
Indicating that a number *divides evenly* into another number	Say that a number *divides* another number a whole number of times or that it *divides without a remainder.*
Plugging a number into an expression or equation	Use *substitute values* for an unknown.
Using *top number* and *bottom number* to describe the numerator and denominator of a fraction, respectively	Students should see a fraction as one number, not two separate numbers. Use the words *numerator* and *denominator* when discussing the different parts of a fraction.

relationships are fixed. For example, the relationship between 3 and 8 is always the same. To determine the relationship between two numbers, the numbers must implicitly represent a count made by using the same unit. But when units are different, these relationships change. For example, three dozen eggs is more than eight eggs, and three feet is more than eight inches. Expiration date: Grade 2 (2.MD.2).

11. The longer the number, the larger the number.

The length of a number, when working with whole numbers that differ in the number of digits, does indicate this relationship or magnitude. However, it is particularly troublesome to apply this rule to decimals (e.g., thinking that 0.273 is larger than 0.6), a misconception noted by Desmet, Grégoire, and Mussolin (2010). Expiration date: Grade 4 (4.NF.7).

12. Please Excuse My Dear Aunt Sally.

This phrase is typically taught when students begin solving numerical expressions involving multiple operations, with this mnemonic serving as a way of remembering the order of operations. Three issues arise with the application of this rule. First, students incorrectly believe that they should always do multiplication before division, and addition before subtraction, because of the order in which they appear in the mnemonic PEMDAS (Linchevski and Livneh 1999). Second, the order is not as strict as students are led to believe. For example, in the expression $3^2 - 4(2 + 7) + 8 \div 4$, students have options as to where they might start. In this case, they may first simplify the 2 + 7 in the grouping symbol, simplify 3^2, or divide before doing any other computation—all without affecting the outcome. Third, the *P* in PEMDAS suggests that parentheses are first, rather than

Other rules that expire

We invite *Teaching Children Mathematics* (*TCM*) readers to submit additional instances of "rules that expire" or "expired language" that this article does not address. If you would like to share an example, please use the format of the article, stating the rule to avoid, a case of how it expires, and when it expires in the Common Core State Standards for Mathematics. If you submit an illustration of expired language, include "What is stated" and "What should be stated" (see **table 1**).

Join us as we continue this conversation on *TCM*'s blog at **www.nctm.org/TCMblog/MathTasks** or send your suggestions and thoughts to **tcm@nctm.org**. We look forward to your input.

grouping symbols more generally, which would include brackets, braces, square root symbols, and the horizontal fraction bar. Expiration date: Grade 6 (6.EE.2).

13. The equal sign means *Find the answer* or *Write the answer*.

An equal sign is a relational symbol. It indicates that the two quantities on either side of it represent the same amount. It is not a signal prompting the answer through an announcement to "do something" (Falkner, Levi, and Carpenter 1999; Kieran 1981). In an equation, students may see an equal sign that expresses the relationship but cannot be interpreted as *Find the answer*. For example, in the equations below, the equal sign provides no indication of an answer. Expiration date: Grade 1 (1.OA.7).

$$6 = \square + 4$$
$$3 + x = 5 + 2x$$

Expired language

In addition to helping students avoid the thirteen rules that expire, we must also pay close attention to the mathematical language we use as teachers and that we allow our students to use. The language we use to discuss mathematics (see **table 1**) may carry with it connotations that result in misconceptions or misuses by students, many of which relate to the Thirteen Rules That Expire listed above. Using accurate and precise vocabulary (which aligns closely with SMP 6) is an important part of developing student understanding that supports student learning and withstands the need for complexity as students progress through the grades.

No expiration date

One characteristic of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) is to have fewer, but deeper, more rigorous standards at each grade—and to have less overlap and greater coherence as students progress from K–grade 12. We feel that by using consistent, accurate rules and precise vocabulary in the elementary grades, teachers can play a key role in building coherence as students move from into the middle grades and beyond. No one wants students to realize in the upper elementary grades or in middle school that their teachers taught "rules" that do not hold true.

With the implementation of CCSSM, now is an ideal time to highlight common instructional practices that teachers can tweak to better prepare students and allow them to have smoother transitions moving from grade to grade. Additionally, with the implementation of CCSSM, many teachers—even those teaching the same grade as they had previously—are being required to teach mathematics content that differs from what they taught in the past. As teachers are planning how to teach according to new standards, now is a critical point to think about the rules that should or should not be taught and the vocabulary that should or should not be used in an effort to teach in ways that do not "expire."

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Addendum to 13 Rules that Expire

Cathy Martin, Past President

After reading *13 Rules that Expire*, we challenge you to generate your own list of rules that expire or rules that students tend to over generalize. To get you started, we'd like to propose the following:

14. FOIL. This acronym is used to support students in multiplying two binomials. However, FOIL expires when students are asked to multiply a binomial and a trinomial. Teaching polynomial multiplication with the distributive property eliminates the need for FOIL. Let's foil FOIL!

15. It's not possible to take the square root of a negative number. This rule expires as soon as students are introduced to imaginary and complex numbers. Avoid this rule when teaching square roots by clarifying that the square root of a negative number is not a real number.

16. Anything to the 0 power is 1. This rule does not apply if 0 is raised to the 0 power. You can avoid this rule by defining: for any nonzero number a, $a^0=1$.

Bring your additional *rules that expire* to the registration desk at the CCTM Annual Meeting. The Board will select additional rules to feature in a future issue, and a *Ruler* will be chosen!

From Rules That Expire to Patterns that Change

Tessa Ziser, Academy International Elementary School

I love math, and I love teaching it! Working hard to teach math deeply and accurately, I want my students to enjoy the mysteries of math, the struggles to learn, and the joys of understanding. However, I was humbled while reading the article, "13 Rules That Expire." Now aware of these mistakes, I recognize I still have room to grow as a teacher of mathematics.

Thinking back on the times that I have taught these inconsistent rules and inaccurate vocabulary, I can identify two reasons. First is simple ignorance—not realizing that the vocabulary that I was using did not provide the whole picture, or was limiting in some way. Teaching second grade math, I did not think far enough ahead to understand that teaching one of these "rules" might cause future trouble. The second reason I have taught these inconsistencies and inaccuracies is because it is easier—it takes less time to teach second graders that you just add a zero when you multiply by ten than to allow them to discover the why behind the how. In addition, if you do strive to teach the underlying concept, and some students do not grasp it, it is easy to teach them the rule so that they "get it." (I know that this is lazy and far from helpful, however.) Having now spent more time in the classroom, I have worked to move away from these easy "fixes" to a deeper understanding for all. I have to remind myself daily that the time spent today will save hours of work, relearning, and frustration in the future.

I do not think we should completely throw out these "rules," however. Although the rules listed in this article should not be taught as rules, they are patterns in mathematics that should be identified. From the perspective of a second grader, multiplying by ten does mean that you simply add a zero. When one of my students discovers this while we are exploring the concept, it is a worthy topic to discuss. We can talk about how this is a *pattern* (a distinction from a *rule* because patterns can be broken), and *why* it is occurring. We all learn, especially children, by organizing what we learn into the framework and understandings that we already have. We then often have to reorganize when we get new and more nuanced information. As long as children understand that the patterns that they identify in math may at some point have to be "reorganized," learning these patterns can be a benefit, not a setback. As educators we must move our teaching and our students from the rigidity of rules that expire to the flexibility of patterns that change.

PARENTS AS PARTNERS

Helping Parents Understand What We're Doing

Steve Leinwand, American Institutes for Research

Ed. Note–The full title of this article is *Some thoughts on helping parents understand what we're doing in math and giving parents some practical ideas for really helping their children in math*

I AM OFTEN ASKED by teachers and administrators, "Yes, but what about the parents?" This is usually shorthand for things like "How can we convince them that the Common Core is best for their children?", "How can we better help them understand the math we are asking their children to do?", and "How can we recruit parents to help their children without them resorting to just telling their children *how* to get answers?"

I think that the answers are **a school-wide written vision of the teaching and learning of mathematics, ongoing communication, and practical ideas for helping parents make mathematics commonplace in their children's lives**. Let's look at each of these in turn.

Vision. I am often surprised by the fact that nearly all schools and districts have vision and/or philosophy statements, but very few schools or districts have vision or philosophy statements about the teaching and learning of mathematics. Such statements, when collaboratively crafted, help to focus everyone—students, teachers, administrators *and* parents—on the direction the school or district is taking for mathematics and the reasons why. Here is the introduction to one such vision statement:

It is a core mission of this school that every student be prepared to be a confident user of mathematics, a powerful quantitative thinker, and a productive problem solver. This mission can only be achieved within a mathematics program that balances mathematical skills, concepts, and applications with instruction that emphasizes explanation, justification, and number sense. That is, our school is committed to a mathematics program built on teaching and learning that actively engages students in experiences that stimulate curiosity, inquiry, joy, and deep understanding of the mathematics outlined in the Common Core.

While an effective mathematics program must be *guided* by a clear set of content standards, it must be *grounded* by a shared vision of teaching and learning that is evident in the ongoing interactions among students, teachers, and the mathematics found in every mathematics lesson.

The adults must consistently expect and support our students to:

- Persevere with solving interesting problems,
- Reason abstractly and quantitatively,
- Construct viable arguments,
- Critique the reasoning of others, and
- Model with mathematics.

Thus, teachers must consistently and expertly:

- Respond to most student answers with "Why?", "How do you know that?", or "Can you explain your thinking?";
- Craft instruction around powerful tasks that promote reasoning and problem solving and promote productive struggle;
- Elicit and celebrate alternative approaches to solving mathematics problems, conveying to students that we value understanding and not just memorizing the right procedure to get the one right answer;
- Use and connect multiple representations—for example, models, diagrams, number lines, tables and graphs, as well as symbols—of all mathematical work to support the visualization of skills and concepts;
- Take every opportunity to develop number sense by asking for, and justifying, estimates, mental calculations, and equivalent forms of numbers;
- Create language-rich classrooms that emphasize vocabulary, explanations, and solutions in the context of meaningful discourse among students;
- Embed the mathematical content students are

learning in real world contexts; and
- Devote the last portion of every lesson to formative assessment, for example, an exit slip, to assess student understanding.

When parents ask teachers why they are doing this or that, teachers and administrators can proudly and confidently turn to the vision and explain that these decisions are not only in alignment with the school's vision, but are in the best interests of students.

Communication. Often when we teach math to children in exactly the same way it was taught to their parents, there is delight from the parents for whom math worked ("It worked for me, so obviously it is what will work for my children") and dismay from the parents for whom math did not work ("I hated it, and unless it is taught a lot better, my children are likely to hate it"). Conversely, when we teach math in rather different ways from how parents were taught, the delight and dismay simply switch audiences. That is why is it so essential that we are clear about what we are doing and why we are doing it. I find it helpful to:

- Ask parents to compare and contrast the math that they did in school to the math they do as citizens, consumers, and workers. Even the hard core engineers end up admitting that they would be in deep trouble without the technology they use and their understanding of what button to press when. These discussions help parents understand that the world has changed dramatically and the math their children need has changed accordingly;
- Provide weekly problems for parents to tackle with their children that present the side of mathematics that requires perseverance, reasoning, and making arguments; and
- Send monthly letters home to parents explaining and describing the mathematics they can expect their children to be doing during the next month. Such letters help parents understand how and why we are using pictures and other representations and how and why we are focusing on a range of alternative approaches to doing mathematics.

Parents who feel in the loop and parents who grow to trust what their children's teachers are doing are far less likely to protest when things don't look the same as they did twenty and thirty years ago.

Making math commonplace. Finally, we need to curb parents' natural inclination to want to help their children and "explain" how to do the math. Unfortunately, as we all know, this is often a recipe for confusion or disaster. It is also why, instead of asking parents to help their children, I find it most useful to ask parents to simply do everything they can—at the restaurant, the gas pump, the dinner table, the kitchen, the grocery store, etc.—to make the mathematics of daily life commonplace for their children.

Here are some examples of ideas that parents can use to best support the mathematical development of their children by making mathematics commonplace and fun whenever and wherever it occurs naturally in our daily lives. Questions can be simple and informal, and answers can be responded to with "How did you get that?" or "Share with us your thinking?" or "Could you solve that in a different way?" For example:

At McDonalds or any other fast food restaurant – in line or at the drive-thru:

- About how much do you think your order will cost?
- What's the least and most expensive reasonable meal we could order?
- Can we all get a meal and spend less than $20? How?
- About how much should the tax be?
- Is a Happy Meal a good deal?
- Does it make sense to order a large soda if there are free refills?

At the Restaurant:

- You can spend up to $8 without going over. What could you order?
- So about how much do you think the bill will be?
- What's the most expensive reasonable meal we could order?
- How much should we tip?

At the gas station and on a trip:

- So about how many miles per gallon are we getting?
- So if gas is $4.09/gallon, about how much will

we spend?
- If we only have $30, how many gallons of gas can we get?
- About how much longer should it take us to get there (looking at mileage signs)?
- If it's 1:30 now, when do you think we'll get there?

At the grocery store:
- How much do you think we just spent (looking at a full shopping cart)?
- What fractional part of the items is taxable (looking at the register tape)?
- About how much do we pay per item in the cart?
- What should the scale say if we order 1½ pounds of cheese?
- About how much will three-quarters of a pound of ham cost?
- What's the unit price? Which is the best buy?
- What does it say on the nutrition label?

At the bank:
- What's a withdrawal? What's a deposit? Which is addition and which is subtraction?
- If I have $_____, how much will I have after a deposit/withdrawal of $_____?

In the kitchen:
- Recipes (Can you measure that much out? How much more or less? Suppose we doubled/halved the recipe?)
- Measuring cups and spoons
- Ounces, cups, pints, quarts and gallons; ounces and pounds

From the newspaper:
- Graphs and tables
- Sports statistics
- Scavenger hunts (for percents, for numbers greater than 1000, etc.)

Just for fun anytime:
- About how big is that? (height, width, weight, capacity)
- About how many would fit? (For example: How many dogs could be fit in the car? How many McNuggets boxes could fit in the trunk? About how many people could fit in this room?

Powerful games:
- Yahtzee
- Cribbage
- Card games (for example, Uno, 24, and Set)

Those are some brief thoughts on how we can help our students' parents, and how they can best help their children and us.

Editor's Note: Join me in welcoming Steve Leinwand, the Keynote Speaker at the CCTM Annual Conference. Steve's talk will be, "Teaching Math: Insights and Reflections on More Than 1000 Observations." Be prepared to start your day with energy and laughter!

Math at Home

Reviewed by Charlee Passig Archuleta, Rudy Elementary School

As the end of summer is arriving for students and teachers alike, it time for me to once again reflect upon the past school year. Which activities were most engaging? Which were in need of fine-tuning? Which interventions were most likely successful? How can I help parents best support their child to be successful in the upcoming year, especially in light of the change in demand for mathematical thinking in the Colorado Academic Standards (CAS) and on the PARRC test?

After reading *Math at Home* by Paul Giganti, Jr. (see pp. 18–48 in this issue), I have decided that it is the most comprehensive tool I have found to date to answer this final question. When parents meet with me at the beginning of each year, one question that is high on their list is, "How can I help my child at home with this 'new' math?" The litany of information on the Internet can be overwhelming to parents. Much of the information presented there is in worksheet form or uses rote memorization. However, there is so much more to mathematics intervention and assistance.

While written to address issues faced in California, *Math at Home* provides succinct information that can guide Colorado parents to the needed tools that will help their children become next generation thinkers. These thinkers will need to persevere and persist through challenges, find creative and efficient ways to solve problems, see math all around them in the world, and explain their thinking to others in a collaborative fashion using appropriate tools.

Perseverance and persistence are the first measures of mathematics that my students face each year, not because this is a part of our Mathematical Practice embedded in the CAS, but because it is part of life. It has become much too easy for students to face a problem for which they lack basic skills, and then give up in the face of this deficit. Parents have questioned if this is an issue of rote memorization. While there is a need to have stored basic information, countless hours of memorizing flashcards is not the answer, nor is it addressed here. Parents are encouraged to help their student find the patterns in everyday tasks and activities. Once the child has connected personal meaning with the task, discussing these patterns makes even more sense across other scenarios. By doing this, parents become the advocate for mathematics instruction in the classroom. Students in my class whose parents actively take on this role are those who are able to think flexibly in mathematics. They persevere through almost any problem I give them and are able to lead collaborative mathematics conversations. Then they experience reduced stress when they reach Colorado assessments like PARRC and have to explain their thinking.

This was simply one example of how powerful a tool this handbook can be. Many interventions I suggest to parents, during drop in conversations, parent-teacher conferences, or even in emails concerning student practice work, can also be found here. Common Core Standards, which are already embedded in the CAS, are addressed in everyday terms, making sense to anyone who is interested. In the section "homework", parents are provided with sensible questions that they can ask their child to support learning and thinking. Many of the questions identified here are those that I use in my own classroom. Finally, *Math at Home* bridges all academic levels. This component is important, as one skill leads directly to the next and across all grade levels.

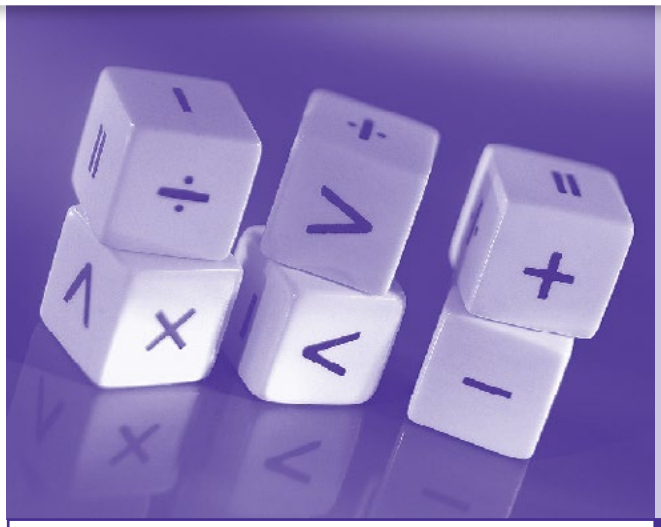
So, as I begin the end of summer ritual in preparation for next fall, I now know that I have one resource that I can easily take out and share with parents. I can now mark that off of my list!

Another Resource

A PDF copy of *Early Learning Math at Home* (Birth to Age 5) can be found both in English and Spanish at www.earlymathlearning.com

Math at Home

Paul Giganti, Jr., Math Consultant



Helping your children learn and enjoy mathematics

MATH at HOME

- Making math part of your family's life
- Giving your child a good start in math
- Discovering the math in your home
- Connecting math and reading
- Understanding common core math standards
- Helping with math homework
- Building a math tool-kit
- Engaging English learners in math
- Linking technology to math
- Meeting the algebra challenge
- Taking a look at high school math
- Preparing for college and career

A public service publication of the California Mathematics Council, the Sonoma County Office of Education, and the California Math Project: North Coast

About this Publication

Math at Home: Helping Your Children Learn and Enjoy Mathematics was first developed by the Sonoma County Office of Education in 2001, and updated in 2010. The text was written by Paul Giganti, Jr., math consultant and author of children's books (pgiganti@berkeley.edu). The art designs are Sue Schreiner and photographs by Patty Bernstein. This 2014 edition was made available through a partnership of the California Mathematics Council, the California Mathematics Project: North Bay, and the Office of Education Sonoma County.

California Mathematics Council | *www.cmc-math.org*

The California Mathematics Council (CMC) is the state's largest professional organization for pre-kindergarten through college mathematics educators. CMC believes that all students have the capacity to become mathematically competent and confident when provided a rigorous and challenging mathematical program supported by high expectations and quality teaching. CMC is committed to:

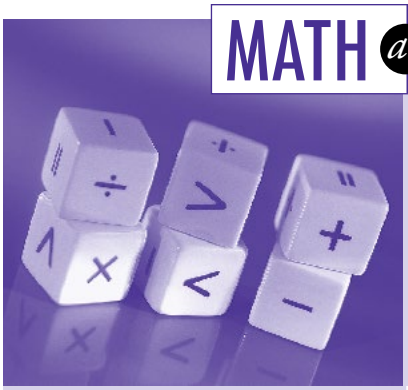
- Promoting professional activities that ensure continual improvement toward excellence in the teaching of mathematics;
- Communicating with educators, parents, the public, and legislative bodies concerning issues related to teaching rigorous, challenging mathematics; and
- Increasing the diversity of membership of the California Mathematics Council and the diversity of leadership in mathematics education at the local, state, and national levels.

Sonoma County Office of Education | *www.scoe.org*

The mission of the Sonoma County Office of Education (SCOE) is to foster student success through service to students, schools, and the community. As one of 58 county offices of education in California, SCOE functions as an educational service agency and strategic partner to the 40 school districts in Sonoma County and their 182 schools. The agency provides service and support to help districts meet legal mandates, operate cost-effectively, and raise student achievement. SCOE provides fiscal oversight to districts, operates schools for special education and alternative education students not enrolled at district sites, and coordinates career technical education (CTE) throughout the county. Under the direction of an elected county superintendent of schools, SCOE also leads and organizes efforts to bring increased educational resources to Sonoma County and initiates projects to engage parents and the community in the education of children.

California Math Project: North Coast | *www.cmpnorthcoast.org*

Established in 2001, the California Math Project: North Coast (formerly the North Bay Math Project) is one of 19 California Mathematics Project regional sites. Led by Sonoma State University mathematics faculty and K-12 mathematics educators, the California Math Project: North Coast (CMP:NC) provides professional learning opportunities in mathematics for K-12 teachers in Del Norte, Humboldt, Lake, Mendocino, and Sonoma counties. The Project's mission is to deepen teachers' understanding of mathematics and strengthen their ability to communicate mathematical concepts. Working with schools, districts, county offices of education, and efforts such as the Regional System of District and School Support, CMP:NC has become an integral part of mathematics education in North Coast public schools. The Project has hosted numerous professional learning opportunities and secured grants from the California Mathematics and Science Partnership and CPEC. Lesson Study has been used as the vehicle for transferring the information learned by teachers into the classroom. The Project has also taken advantage of a variety of technologies to enhance, record, and distribute its work.



A MESSAGE TO PARENTS

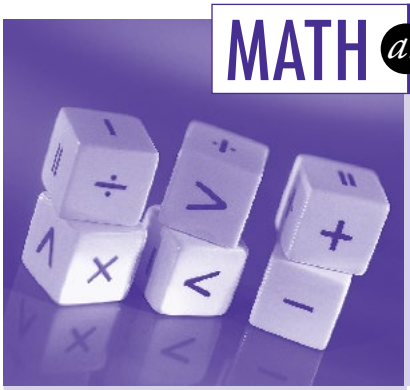
It's common knowledge that young children whose parents read to them have a tremendous advantage in school. But did you know that you can also help your children learn mathematics by doing and supporting math at home?

Today, mathematics is more critical to school success than ever before. The standards for mathematics and the state tests are very demanding because they reflect the mathematics that will be required for entrance into college and transition to a career. Modern occupations now require a firm foundation in mathematics—and that's true for almost any type of job your children will consider in the future.

How you encourage and promote your children's math learning, from preschool to high school, can be pivotal to their attitude toward mathematics and their achievement in this subject area. Even if you haven't studied mathematics in depth yourself, you *can* assist your children. Something as simple as expecting your children to be capable in math can make a difference in their mathematics learning.

This booklet is designed to give you ideas and resources to support your role in your child's math education. The information you'll find in the pages that follow comes from a variety of sources and represents today's best thinking about how to help children learn mathematics. The goal in publishing the booklet is two-fold: to encourage stronger, more informed parent support for math education and to increase the mathematics achievement of all our students.

The Sonoma County Office of Education, California Math Project: North Coast, and California Mathematics Council bring this booklet to you in the hope that you and your family will find it to be a valuable, informative, and useful resource.



MATH at HOME

HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

MAKING MATH PART OF YOUR FAMILY'S LIFE

Every child and adult needs to know and understand mathematics. It's part of our everyday life. We all "do math." We count money, measure things, sort from biggest to smallest, know how many miles it is into town and how long it takes to get there. At work, we may use spreadsheets, a calculator or computer, a cash register, or a precision measuring tool. The list goes on and on.

Children are taught mathematics in school, but research shows that families are an essential part of this learning process. In other words, by doing math with your children and supporting math learning at home, you can make a great difference.

There are many ways to make math part of your family's life. As you establish your own traditions for supporting your children's math learning, consider the following checklist of key ideas.

- **ALWAYS TALK ABOUT MATH IN POSITIVE WAYS**
  Regardless of your own mathematics background, let your children know that learning math is very important. Communicating a positive, can-do attitude about math is the single most important way for you to ensure that your children are successful in mathematics. Always be positive when you talk about math—never tell your children that math is too hard or that you hated it when you were in school. Let them know how critical math is by pointing out how people use math in everyday life. Encourage them to always do their very best in this subject area.

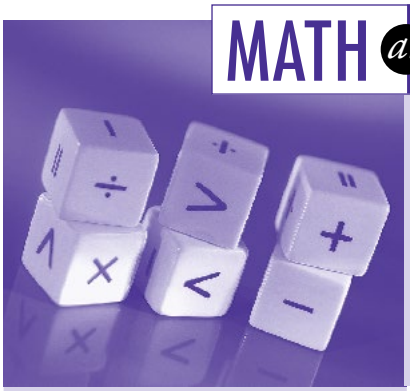
- **KNOW WHAT YOUR CHILDREN ARE STUDYING IN MATH**
  Be aware of the math your children are learning each year and know the standards they're required to meet. Ask them what they're studying in math class, regularly check in with them about math homework, and help them with school projects when it's appropriate. If your children experience difficulties in their math learning, work with them to overcome these trouble spots. (Some strategies for helping with math are provided on page 12.) Don't hesitate to talk with your child's teacher if you need more information or assistance.

- **Have high expectations for your children**
  Research shows that when you believe your children can learn challenging concepts, they will rise to the occasion—so expect a lot from them! Be confident that your children can learn mathematics and then actively support them as they do so. Seek out math-focused programs and activities for them. As they get older, encourage them to take as many advanced mathematics courses in high school as possible.

- **Encourage your children to use technology in math**
  Help your children use calculators, computers, and the Internet to do math at home. Mathematics and technology are great partners. Tasks such as long and complex calculations, charts, tables, graphs, and spreadsheets show the power of using mathematics and technology together. Doing tasks that involve math and technology helps prepare your children for the future.

- **Make math an everyday part of your family**
  Find math at home. (The information on page 6 provides some ideas on how to do that.) Spend time with your children on simple board games, puzzles, and activities that involve math. Involve your children in activities like shopping, cooking, and home fix-it projects to show them that math is practical and useful. Encourage your children to solve problems that involve math. Engage your children in conversations about what they're thinking about when they solve math problems. Find opportunities to explore math together.

- **Notice mathematics in the world**
  You can help your children see the usefulness of mathematics by pointing it out wherever you see it—not just in your home, but *everywhere*. Tell your children about the math you do in your job and why it's important. When you're outside your home, look hard for ways to point out math: What shape does that tree look like? How many more miles before we get there? How does mathematics figure in sports, music, car building, or the design of a Ferris wheel? If you start looking for math in the world, you'll find more and more of it—and so will your children.



MATH at HOME

HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

GIVING YOUR CHILD A GOOD START IN MATH

Young children begin learning math before they take their first step into a kindergarten classroom. When toddlers hold up three fingers and ask for "this many cookies," they are already doing math and ready to learn more.

As a parent, it may be tempting to think you don't need to worry about helping your child learn mathematics until elementary school, but the seeds of many important math concepts are planted when children are very young. In fact, early experiences can determine how your child looks at mathematics for the rest of his or her life. It's never too early to start learning—and liking math!

Children between the ages of two and four generally experience mathematics through simple counting. Counting is a basic and very important concept that helps children bring order to the world around them. Early counting and "how many" experiences introduce children to math concepts that become deeper and more complex in elementary school. For example, counting three dimes becomes a way of understanding 30 cents.

Photo by Paul Giganti, Jr.

The more opportunities young children have to count, the better they understand the meaning and use of numbers—and the more confidence they'll have with mathematics later on. By reading your children counting books, singing counting songs, and playing counting games, you're having fun with numbers and giving them a foundation for success in math.

TIPS FOR PARENTS OF YOUNG COUNTERS:

- Count frequently. Find things to count every day, everywhere, and in every way. Start slowly with just a few things. As your child's ability to count grows, find bigger and bigger collections of different things to count.

- Count real objects: cookies, coins, toys, etc. Children discover that counting is more than a sing-song repetition when they count real objects. Encourage

your child to say one number as he or she touches each object. Arrange objects in different ways for counting—for example, in piles, rows, and circles.

- Reinforce your child's counting. When your child finishes counting, you could say, "One, two, three cookies. You counted three cookies!" To correct a mistake, gently count again along with your child, holding a finger and touching each cookie as you say the number.

- Sometimes children forget which objects they've counted. If this happens, have your child move each object into a "counted" pile as he or she counts. If your child gets frustrated or continues repeating the same mistake, be patient. For the moment, you could simply stop counting and try it again another day.

- Don't worry if your child uses his or her fingers for counting. Fingers are the best mathematical tools children have! They're always handy and ready to use. You can also encourage your child to use other objects to keep track of their count: one bean for every letter in their name or one popsicle stick for each door in your home.

- Once your child has mastered basic counting, start practicing how to count by twos, fives, and even tens. This will give your child a great start for learning math in school.

THE COUNTING GAME: A FUN ANYTIME MATH ACTIVITY

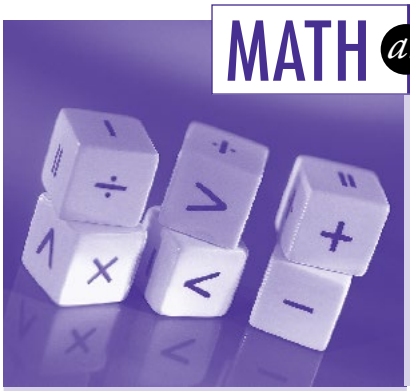
The Counting Game is so simple it can be played by young children, yet so mathematically intriguing it can challenge older children as well. It's a perfect "anywhere, anytime" game. Anyone who can count to 20 can play it. Here's how it goes:

1. Two people take turns counting aloud. The first person starts at ONE and the game is over when the counting reaches TWENTY.
2. Each player counts aloud by saying one, two, or three more numbers *in order* from where the other player stopped counting.
3. Whoever says the number TWENTY, by itself or in a group of numbers, is the winner.

For example:	*Player One*	*Player Two*
	1, 2, 3	4
	5, 6	7, 8
	9, 10, 11	12
	13, 14	15, 16
	17	18, 19, 20!

At first glance, the game seems like a simple way for young counters to practice—but there's actually more to it than that. If you play this game with an older child, you'll both start to see number patterns and discover that there's a strategy for winning. (Hint: it has to do with the multiples of 4.)

You can play four or five rounds of the Counting Game quickly—so play it often to give your child a chance to figure out the winning strategy. Later on, try modifying the rules of the game by having a different winning number, such as TWENTY-ONE, and see what happens.



MATH at HOME

Helping your children learn and enjoy mathematics

Discovering the Math in Your Home

Math is everywhere! It's in the world that surrounds us, it's in nature, and it's in your home, both inside and out. By pointing out the math in everyday life, you can help your young child learn some basic concepts and understand why math is so important.

If your child is in kindergarten or one of the early grades, you can really reinforce the math they're being taught in school with practice at home. Math at home doesn't have to happen sitting at a desk. During playtime, on a walk, while you're fixing dinner, or when your child is just looking for something to do—these are all great opportunities to suggest a math activity. Here are a few ideas that will help your children discover—and use—the math around them.

In a play area, your child can:

- Count blocks as he or she builds a tower.
- Sort toys by size, kind, or color.
- Put dolls, cars, or blocks in order from largest to smallest.
- Play "What am I thinking of?" by describing a toy's size and shape.
- Play make-believe "store" with toys and play money.

In the kitchen, your child can:

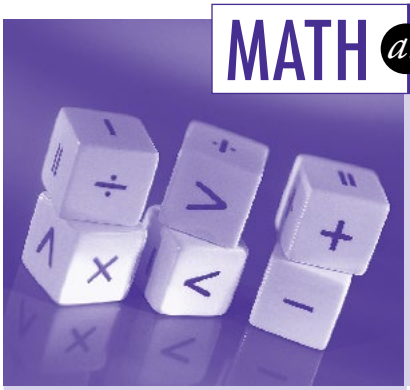
- Look for familiar two-dimensional shapes—circles, squares, triangles, etc.—like a round plate or square napkin.
- Put cans of food in order by size or type.
- Sort silverware from the dishwasher to the drawer.
- Count plates, utensils, cups, or even olives.
- Divide a plate of cookies evenly so that each family member gets an equal share and deside what to do if there are some left over.
- Find how many glasses of milk are in a full milk carton.
- Help you double a recipe, or cut one in half.

Around the house, your child can:

- Count the days on a calendar until a special event.
- Find the length and width of a room by pacing it off.
- Draw a diagram of how to rearrange furniture in a room.
- Make a "map" of the whole house.
- Create a family TV schedule and track the amount of time watched.

Outside the house, your child can:

- Set up and operate a lemonade stand.
- Plant a garden with rows and columns of seeds.
- Count the petals on different flowers.
- Measure a sunflower or bean plant daily, keeping track of how it grows.
- Count how many times he or she can jump rope or shoot baskets in a row.
- Keep a daily chart of the temperature.
- Find triangles, squares, circles, and rectangles around the neighborhood.



MATH at HOME

HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

CONNECTING MATH AND READING

All parents understand the importance of reading, but have you ever combined math and reading at the same time? It's possible to put math and reading together in a meaningful way and have fun doing it. Reading books with math themes will enhance your children's enjoyment and pique their interest of both subjects simultaneously.

Reading a math-focused children's book is especially effective when you take the time to encourage your child to think about the math in the story. This means you may need to modify your approach to reading together when you choose a book with a math theme.

For example, *The Doorbell Rang* by Pat Hutchins is a delightful story about sharing cookies. You and your child will enjoy reading it from cover to cover, but you can also use it as an opportunity to do mathematics together in a comfortable, relaxed setting. Take advantage of the math situations embedded in the story by following these suggestions.

- Before beginning the book, it might be fun to bake some cookies or have a plate of store-bought cookies nearby to help your child visualize the math problems you'll be reading about—just don't eat them until the end of the story!

- Read a few pages, then pause when you come to the first math situation in the book. Ask a math-related question that anticipates what happens next. For example, "How many children have to share the cookies now?"

- Once your child solves the problem, continue reading. Stop and ask other math-related questions as long as your child stays interested. (Don't stop too often or your child may lose the story line.)

- Sometimes it's wise to skip over information that's in the book. For instance, the first math situation in *The Doorbell Rang* occurs when two children must share 12 cookies. The book reads, "That's six each …." Instead of immediately reading this line, you might say, "If there are 12 cookies to begin with, how many cookies will each child get?"

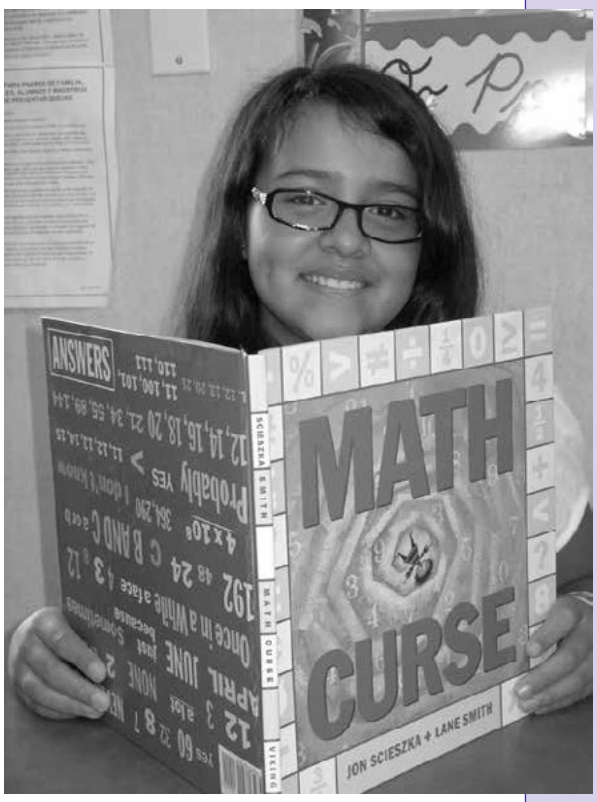
- Let your child do the math (or sort the cookies you have on hand) and answer the question. Be patient and allow time to solve the problem before continuing to read. "You're right! That's six each …."

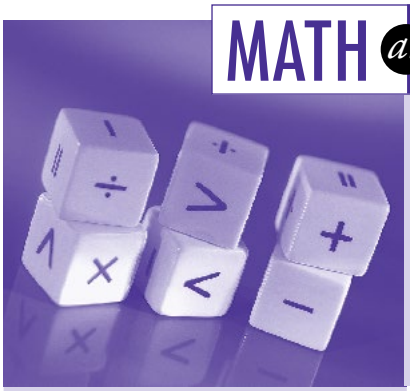
This is just one example of how to combine math and reading. Children's books with math themes will give you many opportunities to stop and do math.

CHILDREN'S LITERATURE WITH MATH THEMES

When children's books have math themes, reading becomes an opportunity to enjoy a good story and think about math at the same time. Take the opportunity to explore math concepts while reading together at home! The books listed here artfully combine reading and math. They are appropriate for kindergarten through fourth or fifth-grade readers.

12 Ways to Get to 11, by Eve Merriam
17 Kings and 42 Elephants, by Margaret Mahy
Alexander, Who Used to Be Rich Last Sunday, by Judith Viorst
Amanda Bean's Amazing Dream, by Cindy Neuschwander
Anno's Counting Book, by Mitsumasa Anno
Anno's Magic Seeds, by Mitsumasa Anno
Anno's Mysterious Multiplying Jar, by Masaichiro and Mitsumasa Anno
The Button Box, by Margarette S. Reid
A Cloak for the Dreamer, by Aileen Friedman
Counting on Frank, by Rod Clement
The Doorbell Rang, by Pat Hutchins
Each Orange Had Eight Slices, by Paul Giganti, Jr.
Frog and Toad are Friends, by Arnold Lobel
G is for Googol, by David M. Schwartz
A Grain of Rice, by Helena Clare Pittman
Grandfather Tang's Story, by Ann Tompert
The Greedy Triangle, by Marilyn Burns
How Big is a Foot?, by Rolf Myller
How Many Feet in the Bed?, by Diane Johnston Hamm
How Many Snails?, by Paul Giganti, Jr.
How Much is a Million?, by David M. Schwartz
If You Made a Million, by David M. Schwartz
Incredible Comparisons, by Russell Ash
The King's Chessboard, by David Birch
Math Curse, by Jon Scieszka and Lane Smith
One Grain of Rice, *A Mathematical Folktale*, by Demi
One Hundred Hungry Ants, by Elinor J. Pinczes
Only One, by Marc Harshman
The Phantom Tollbooth, by Norton Juster
A Remainder of One, by Elinor J. Pinczes
Rooster's Off to See the World, by Eric Carle
Round Trip, by Ann Jonas
Sir Cumference and the First Round Table, by Cindy Neuschwander
Ten Black Dots, by Donald Crews
The Twenty-One Balloons, by William Pene du Bois
The Very Hungry Caterpillar, by Eric Carle
What Comes in 2's, 3's & 4's?, by Suzanne Aker





UNDERSTANDING THE COMMON CORE MATH STANDARDS

California has adopted new curriculum and instruction standards called the California Common Core Standards for Mathematics. They represent national agreement on the mathematics that students must understand at each grade level in order to be career- and college-ready when they graduate from high school. The Common Core also lists eight Standards for Mathematical Practice, highlighted below, that describe how mathematically proficient students are expected to use and apply their mathematical knowledge.

1. Make sense of problems

Good students try hard to make sense of a problem, find a way to begin a new problem, and keep working even when a problem is difficult. When they believe they have solved a problem, they think about whether an answer makes sense. If other students did the problem in a different way, they listen to their solutions and try to understand their reasoning.

Photo by Ross Hause

2. Reason abstractly

Good students use numbers in real and abstract ways. They use numbers, math symbols, and equations to represent mathematical relationships in abstract and actual situations. They consider the size and meaning of numbers in different situations, and apply this "number sense" in their thinking and problem solving. Good math students consider whether their answer makes sense and solves the problem.

3. Construct arguments

Good students use all the information they have, and all they the math they know, to find answers. They make intelligent guesses and apply logical thinking to explore and test their ideas. They are able to use math tools such as models, diagrams, calculations, and technology, along with sound mathematical thinking, to explain their answers. They ask good questions and listen carefully to other students' ideas and solutions.

4. Model with mathematics
Good students solve math problems they find in school, at home, and in their daily life. Using mathematical modeling, they work with numbers to find real-world solutions. They make drawings, create diagrams, and build physical and computer models of the problems they encounter. When possible, they write equations that model situations.

5. Use tools appropriately
Good students consider all the math tools available to them for every problem-solving situation, including objects, paper and pencil, calculators, models, spreadsheets, and statistical software. They carefully choose the best tools for any given mathematical situation, and use those tools in the right way to solve the problem.

6. Attend to precision
Good students calculate accurately and efficiently, and share mathematical ideas with others by using the best vocabulary and math notation they know. They take care to make sure the mathematics they do is correct. When they find an error, they redo their work to get the best possible answer for each problem.

7. Look for and use structure
Good students try to discover and observe patterns, logical order, and structure in math situations. They use the order and patterns they discover to help them solve problems. Good students can step back and view the whole picture, while at the same time paying careful attention to individual facts and numbers.

8. Look for repeated reasoning
Good students decide whether to apply a traditional method, use a creative approach, or employ a shortcut in solving a problem. Good students apply what they've learned in similar problems and continually check their progress as they work. They use their experience and observation of patterns to solve similar problems efficiently.

The full text of the *Common Core Standards for Mathematical Practices* can be found at www.corestandards.org/Math/Practice

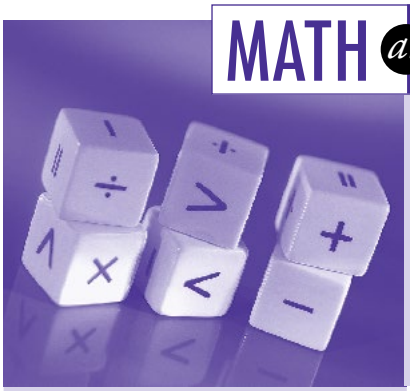
STANDARDS: MORE THAN ARITHMETIC

Studies show that students need to learn more than paper-and-pencil arithmetic to thrive in our increasingly complex and technology-rich world. That's why mathematics instruction is changing with the Common Core Standards. These standards focus on preparing students for college and career, so they take real-world applications into account. With the new standards, mathematics learning goes beyond the use of step-by-step procedures and engages students in solving problems in both traditional and creative ways.

Arithmetic skills, although still critical, are no longer enough for students who will graduate into a world marked by advances in science and technology and changing workplace expectations. In addition to learning arithmetic in math class, your children will now be asked to:

- Solve real-life problems;
- Explain their thinking to others;
- Identify and analyze trends from data;
- Create graphs, charts, and other representations of information;
- Use modern technology to solve mathematical problems.

Instead of math worksheets, your children may have homework that is related to real life—investigating patterns, mapping their neighborhood, or creating a personal budget. This kind of learning builds a deeper understanding of mathematics and its uses in the world.



HELPING WITH MATH HOMEWORK

Many parents worry about helping their children with math homework, especially as their children get older and the mathematics becomes more complex. If that's the case, here's something you'll be happy to learn: you don't have to be a mathematics expert to help with math homework.

Providing a well-lit table and comfortable chair is an important place to start, but remember, the best location for homework is not necessarily your son or daughter's bedroom. If your child studies at the kitchen or dining room table instead, you can help without having to sit down the entire time. You can assist when help is needed and still go about your own tasks. At the same time, you'll have the opportunity to keep homework time focused by giving your child support, encouragement, and gentle reminders.

A good strategy is to pass by your child's work area and periodically "check in." A quick glance will often tell you if it's time to stop and provide some extra support. When it's clear that your assistance is needed, sit down and give your child your full attention. Although it's sometimes difficult, maintaining a calm demeanor and being patient can really help your child when he or she is struggling with math.

Many parents worry about not knowing the math their children are studying. In this case, the way to provide homework help is actually quite simple: ask questions and practice careful listening. Simple generic questions can help your child gradually make sense of math, build confidence, and encourage mathematical thinking and communication. When given the opportunity to talk about math, children are often able to remember what they learned in class and see the solution themselves. A good question can open up your child's thinking about the problem at hand.

Here are some useful questions for parents to try. Remember that listening to your child's answers—and providing calm responses—is as important as the questions you ask.

When your child isn't sure how to begin a problem, ask:

Can you tell me what you know now? What math facts do you have?
What do you need to find out? Can you estimate the answer?
How might you begin? What can you try first?
Can you make a drawing or picture to get started?

While your child is working on a problem, ask:

How can you organize your information?
Will a list or table help?
What would happen if …? Show me what you did that didn't work.
Can you explain the strategy you're using to solve this? Why did you …?
What could you do next?
Do you see any patterns?

When your child finds an answer, ask:

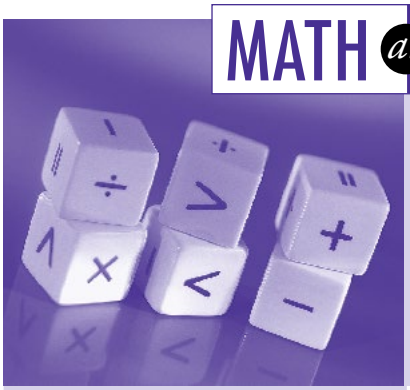
Does that answer make sense? Why do you think that?
How did you get your answer? Why do you think it's right?
Convince me that your solution makes sense. Explain it in a different way.

When questions alone just won't do, another strategy for helping your child is to identify a friend or relative who knows more mathematics than you do. Find out if that person would be willing to answer an occasional phone call from your son or daughter.

The Internet can also be a resource when your child needs homework assistance, although some sites charge a fee for this service. One free website that's worth exploring is Ask Dr. Math, www.mathforum.com/dr.math, which is a project of the Math Forum at Drexel University. This site has a large searchable archive of math questions and answers for students of all ages. It also invites students to submit questions if they aren't able to find the help they need.

PROBLEM-SOLVING STRATEGIES

1. Guess and check
2. Look for a pattern
3. Draw a picture or diagram
4. Act it out
5. Work backwards
6. Simplify the problem
7. Eliminate possibilities
8. Make a systematic list
9. Get advice or do research
10. Sleep on it!



MATH at HOME

Helping your children learn and enjoy mathematics

Building a Math Tool-Kit

When elementary and middle school students work on math, they sometimes need a little help getting organized. Math requires a few basic tools, and it can be frustrating when children are doing math homework and those tools aren't readily available.

You can encourage and support math learning at home by building a "math tool-kit" with your child. A tool-kit is a collection of just about every tool your child will need for math homework—all contained in a handy box.

To create the tool-kit, you'll want to find or purchase the following list of supplies. All of these items can be purchased cheaply at discount or drug stores.

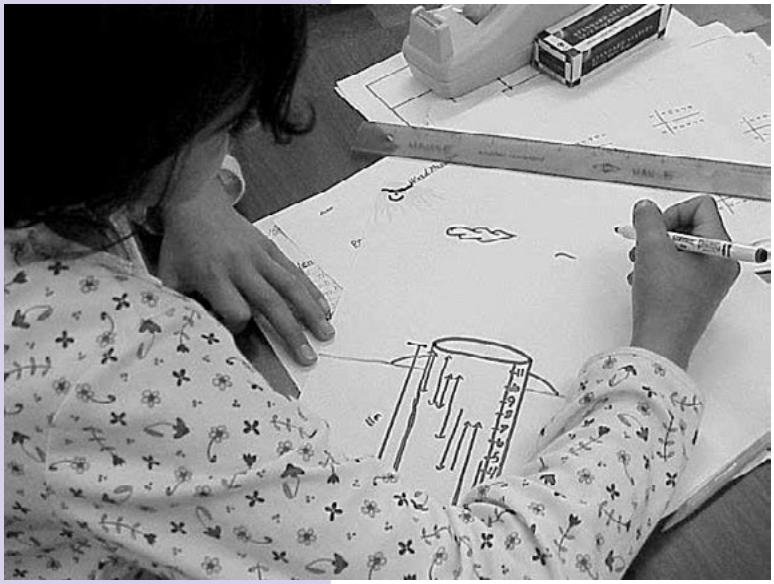


Photo by Ross Hause

- **A shoebox or plastic box that closes.** A box about a foot long will hold all the tool-kit items.
- **A couple sharpened pencils**—one never seems to be enough.
- **A small plastic pencil-sharpener.** It's amazing how long it can take to find a sharpener if there's not one in the box.
- **One large pink eraser.** In math, mistakes are part of the learning process and the small eraser on the end of a pencil just doesn't last long enough.
- **A small pad of scratch paper.** This is helpful for drawing pictures and writing down calculations that don't need to go on your child's homework paper.
- **A plastic or wooden ruler.** Make sure to get one that's marked in *both* inches and centimeters.
- **A pair of scissors.** The ones with plastic handles and metal blades are best.

- **A GLUE STICK**. You'll be surprised how often your child uses this for math projects.
- **A COMPASS FOR DRAWING CIRCLES**. Get the kind that holds a real pencil. This tool is used by students in grades 4 to 8.
- **A PLASTIC OR METAL PROTRACTOR** for measuring angles. This tool is also used by grade 4-8 students.
- **ABOUT 20 "COUNTERS" FOR SOLVING PROBLEMS.** Counters can be buttons, pennies, lima beans, or any other small objects.
- **A SOLAR-POWERED POCKET CALCULATOR** (no batteries needed) that adds, subtracts, multiplies, and divides. This is helpful for checking work and doing complex calculations. Your child will probably begin using a calculator in fourth grade.

The math tool-kit will be more meaningful if your child helps you find the tools on a "math shopping trip." Once home, your son or daughter can make the tool-kit their own by decorating it with stickers and cut-out pictures.

Another great idea is to suggest that your child occasionally use the math tool-kit *just for fun* to draw a math picture or write a math story—even when there's no assigned homework.

PICKING THE BEST MATH TOOL

Part of being "good" at math is choosing the right math tool for the job. What would be the best math tool (estimation, mental math, calculator, or pencil and paper) to solve each of these math problems?

1. 3256.98 ÷ 78
   ☐ Estimation ☐ Mental math
   ☐ Calculator ☐ Paper and pencil

2. 500 x 30
   ☐ Estimation ☐ Mental math
   ☐ Calculator ☐ Paper and pencil

3. Which is closer to 1,000?
   398 + 607 *or* 292 + 655
   ☐ Estimation ☐ Mental math
   ☐ Calculator ☐ Paper and pencil

4. 312 x 7
   ☐ Estimation ☐ Mental math
   ☐ Calculator ☐ Paper and pencil

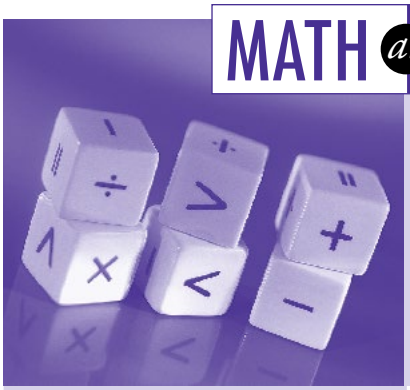
Answers:

1. A calculator is often the best tool for a complex division problem such as this, although upper grade students should be able to find the correct answer using paper and pencil.

2. Whenever a problem can be done quickly in your head, there should be no need for a calculator or pencil and paper. In this problem, basic knowledge of multiplication is all that's needed, so mental math is a good tool.

3. Since an exact calculation of these addition problems isn't required, this problem can be done using estimation. By "rounding off" the numbers, it's easy to tell that 398 + 607 is closer to 1,000.

4. For most people, pencil and paper is probably the best tool for this problem. The multiplication is not so difficult that a calculator is needed to get a correct answer quickly. Mental math would probably not be successful since most people can't hold this many numbers in their head. Finally, because an exact answer is required, estimation would not be appropriate.



MATH at HOME

HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

ENGAGING ENGLISH LEARNERS IN MATH

Teachers have discovered a number of techniques to supplement spoken and written math instruction in order to help English learners grasp mathematical concepts. These hands-on strategies work just as well at home as in the classroom and are great resources for parents looking to support their children's math learning. If you are the parent of English-language learners, encourage your children to use techniques like the ones described below. Remember, the more opportunities that your children have to hear, see, and respond to math situations and problems, the more likely they are to learn mathematics quickly and correctly.

Draw pictures and diagrams

Pictures and diagrams can build understanding while minimizing the language needed to communicate mathematical ideas. Many math concepts can be pictured clearly and easily in drawings that require few, if any, words. This makes using diagrams an excellent way for students with limited English to "see" math and share their mathematical thinking with others. Drawing diagrams also helps students organize their work and find solutions to math problems with greater ease.

Children don't automatically draw pictures to learn math, so it helps if adults suggest using this approach. You can also show them how to do this by drawing pictures yourself as you solve math problems. The concept of multiplication, for example, can be illustrated by drawing equal rows of circles. This drawing represents 4 x 6:

o o o o o o
o o o o o o
o o o o o o
o o o o o o

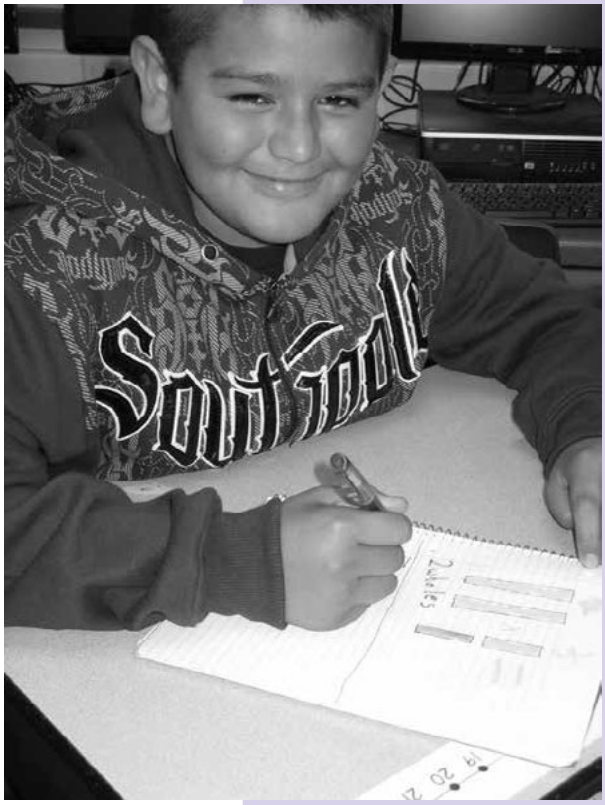
The number of rows (4) multiplied by the number of circles in each row (6) tells you the number of circles (24). Children can verify that 4 x 6 = 24 by actually counting all of the circles in the diagram.

Practice with blocks and beans

Objects that children can handle and sort can create hand-to-eye-to-brain connections that make a positive difference in learning math. In the classroom, teachers often use small blocks and plastic counters (known as manipulatives) to show students how math concepts work. At home, your children can practice simple addition, subtraction, multiplication, and

division problems using similar objects. Something as simple as dried beans can help your children learn a math concept at the kitchen table, giving them the opportunity to see what they didn't quite grasp when it was explained in words. Other objects, shapes, and puzzles can help them learn more advanced concepts, like those in geometry.

Ask your children's teachers to suggest how objects found at home can be used to reinforce what's being taught in school. You may find that your children enjoy mathematics more, and learn more, if both their hands and minds are involved.



Build meaning in real situations

For many students, mathematics seems too abstract. When it can be related to things they see and do in everyday life, the concepts become real and meaningful.

The interplay of money and mathematics provides a great example. Thinking in terms of dollars, dimes, and pennies can help students learn addition, subtraction, and the base 10 system as they're used in the real world. Changing 10 pennies into a dime and 10 dimes into a dollar teaches children about "regrouping."

Learning about geometry by taking a walk around town and looking for geometric shapes is another real-world math lesson. After all, traffic signs, buildings, clocks, automobiles, and playing fields all have geometric shapes.

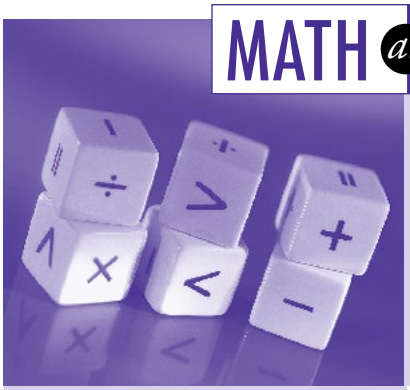
Talk and write about math

Putting math concepts into words is the most advanced math-learning strategy for students who are also learning English. Talking and writing about math may be difficult at first, but it can be rewarding! When students learn to express their mathematical ideas in words, it builds math and language skills at the same time.

Encourage your child to start slowly, then expand a little bit at a time. Here's one example, which can be done in English or your home language. Begin by asking your child draw a picture of a math situation or problem, such as "How many wheels do three tricycles have?" Then, ask your child to make up a title for the picture, like "3 Tricycles," and write it at the top of the page. If your child is young, start by simply talking about the drawing, doing the writing yourself, or taking turns writing. Gradually transition your child to doing all the explaining and writing by him or herself.

Next, identify each part of the drawing and say or write a sentence to explain it: "My drawing shows that there are 9 wheels on 3 tricycles." (Make sure to use a complete sentence.) If you do activities like this regularly, your child will soon be talking and writing about every math situation!

The secret to putting math into words is that the more often children do it, the more comfortable they become in expressing their mathematical thinking. This is a very important skill for success in math throughout all grade levels.



MATH at HOME

Helping your children learn and enjoy mathematics

Linking Technology to Math

Calculators and computers were invented to save time and allow us to solve challenging problems with greater ease. These tools are so good at this that they've become essential for doing mathematics at home and in the workplace. Technology gives us visual images of mathematical ideas, helps us organize and analyze data, and allows us to make calculations quickly, efficiently, and accurately.

Technology can also help students learn and understand mathematics. It can stimulate interest, increase problem-solving abilities, and—when used wisely—give all students increased access to math. With proper guidance, your children can use technology tools to solve difficult mathematical problems, build computational skills, and tackle real-life math problems like planning a family road trip, building a budget, or saving for a purchase.

But technology is not a replacement for learning and doing mathematics. As much as we might like technology to provide magic solutions to math problems, it doesn't do that. For technology to help us with math, we must still know how and when to add, subtract, multiply, and divide. We must understand numbers, know how to make calculations, and be familiar with problem-solving strategies. This is true for both children and adults.

A calculator won't help your children find the solution to a "story problem" if they don't know whether it requires multiplication or division. Technology can support and develop student learning of mathematics, but it does not replace basic skills or understanding of math concepts.

Here are some thoughts about how technology might figure in your children's mathematics learning.

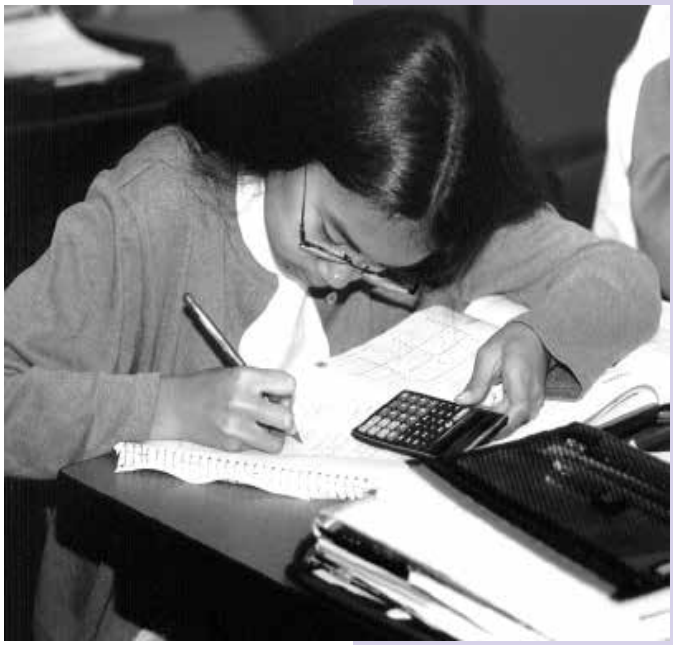
- Help your children decide which math tool is best for a given problem—technology, paper and pencil, mental math, or estimation. Help them

think and talk through the process of solving the problem. Writing the problem on paper first may make it easier to choose the best tool for solving it (see exercise on page 15).

- If you have a calculator, computer, or other math tools at home, guide your children in exploring the kinds of tasks they can perform. Help them experiment, then let them play! When the time comes to choose a tool for a real math task, they'll be more likely to select the best one.

- When using calculators and other technology, determining if the answer makes sense is critically important. Learning basic math facts, knowing how to estimate, practicing mental math, and understanding the math behind real-life situations will help your children do that.

- Sometimes a calculator or computer can help children focus on problem-solving procedures or see number patterns without getting bogged down by calculations. These tools and other specialized technologies can also assist students who have special needs or physical challenges become more engaged in math.

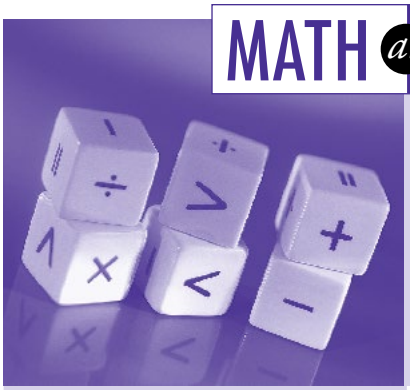


- For children struggling to learn math facts, software and online programs can provide extra drill and practice. Most of these programs are engaging, move at the student's pace, and give immediate feedback. They can be very helpful, but parents are cautioned not to put too much emphasis on math drills alone.

- High-tech tools can give older children opportunities to see visual representations of complex mathematical ideas. For example, spreadsheet software can help students organize statistical information, turn numbers into visual charts and graphs, identify patterns in science and math, and make predictions based on the information they compile.

- Graphing calculators are great tools for high school students, allowing them to pose "what if" questions and see what happens when a single variable is changed and everything else remains the same. Because these calculators can produce 20 graphs in the time it takes to plot just one with paper and pencil, it encourages students to thoroughly explore mathematics situations.

- Internet websites can provide students of all ages with math instruction, homework help, interactive math-focused games, interesting problems, and challenging puzzles. The Internet resources listed on page 27 are a good place to start looking for educational math websites.

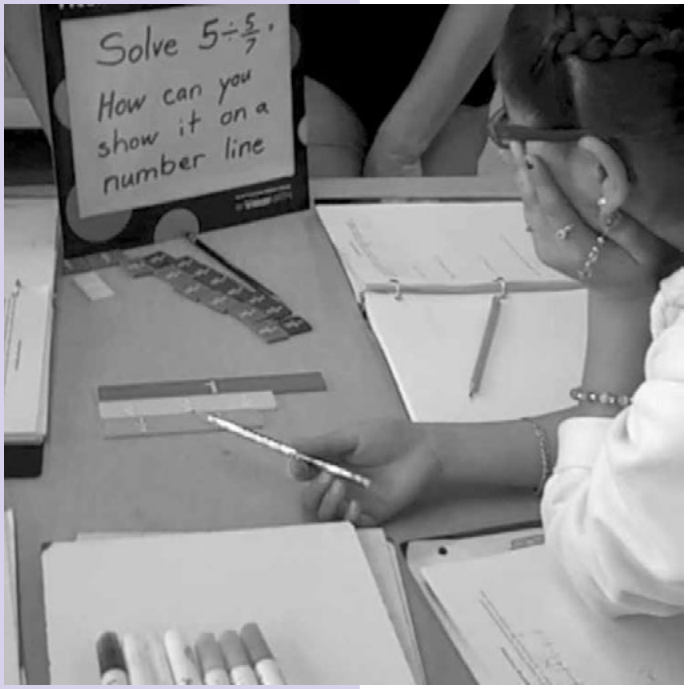


HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

MEETING THE ALGEBRA CHALLENGE

Algebra is important! It helps us investigate, describe, and understand our world. Algebra is mathematics that allows us to use letters and symbols to generalize relationships and analyze mathematical situations via formulas and equations. For students, algebra is also the gateway to success in college and careers!

Algebra represents a different way of thinking about and using math. Everyone needs algebra. It is useful in all areas of mathematics—measurement, statistics, probability, problem solving, and geometry—and it is now required in most professions, including those that depend on science, technology, engineering, and mathematics (STEM). To learn higher mathematics, all students need to master this abstract way of thinking as part of a solid mathematics foundation.



Because of increased international competition and a growing body of research about college and career readiness, our state's Common Core Mathematics Standards introduce "pre-algebra" ideas in the elementary grades, then add more sophisticated algebra concepts through middle school. Although algebra is only one of the mathematics subjects that students are required to study, it presents a unique challenge for many learners because it is a transition from concrete arithmetic and computation to the symbolic language and abstract thinking of advanced mathematics.

Parents and guardians can actively support their children as they learn algebra throughout the grades and smooth their transition to higher math. As a parent, you aren't expected to teach your children algebra yourself, but you can help by understanding algebra's importance, supporting your children as they learn pre-algebra skills in elementary and middle school, and giving encouragement and moral

support when they advance to higher mathematics in the traditional or integrated high school course sequences of the Common Core mathematics curriculum.

What do your children need to study in elementary and middle school to ensure later success in algebra? They will need to:

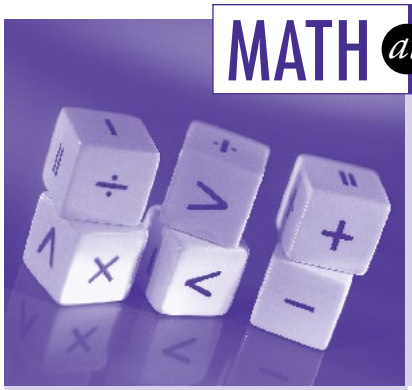
- **Build fluency with numbers:** A strong "number sense" allows students to understand the uses of mathematics beyond simple computation. An understanding of how numbers fit together in our number system is needed in order to make generalizations in algebra.
- **Develop proficiency with fractions:** The same concepts and skills that allow us to add, subtract, multiply, divide, convert, and compare equivelent and unlike fractions are also used in solving algebraic equations.
- **Discover and investigate patterns:** Patterns are everywhere in our world. Once students discover patterns, they also need to find the rules underlying those patterns. Algegra can be used to describe the "rules" of patterns mathematically.
- **generalize relationships:** A function is a relationship between two characteristics that vary, but are affected by each other. Algebra can be used to make mathematical generalizations about such relationships that are true for all cases.
- **Integrate ideas of geometry and equations:** The ability to analyze two- and three-dimensional geometric shapes, understand proportional relationships, and find unknown lengths, angles, and areas can all be described using algebraic equations, formulas, and graphs.

These broad topics alone do not lead to proficiency in algebra, but they are essential components. As your children are introduced to these concepts and study them in greater depth throughout the grade levels, seek assistance as soon possible if they struggle. Algebra builds on previous mathematical knowledge, so it is essential that students don't fall behind.

WHAT IS ALGEBRA?

Algebra generalizes mathematical ideas by using letters or symbols for numbers in equations. It is a language of variables, operations, and formulas.

Algebra is often used to state mathematical generalizations, such as the laws of physics that determine whether bridges and buildings stand or fall. Algebra allows us to discover important patterns in nature and express those patterns in equations that are universal and can be used in problem-solving situations.



HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

TAKING A LOOK AT HIGH SCHOOL MATH

Mathematics concepts increase in difficulty and complexity when students enter high school. All students should expect to encounter material that is more challenging than what they studied in elementary and middle school. Success in high school math courses is the result of hard work and perseverance.

You can help your high school students be successful in mathematics by understanding what they are required to learn, encouraging them to keep up with homework, and suggesting they ask questions in class. Tell your children you have confidence in their ability to succeed and actively support their learning at home and in school. Always speak positively about mathematics and remind your children how important math is for getting into collage.

To help you understand the requirements and challenges of high school mathematics, here is some key information.

THERE ARE STANDARDS FOR HIGH SCHOOL MATH

The high school mathematics curriculum is based on the Common Core Standards. These standards cover the body of conceptual understanding, skills, and applicatons that all students should learn to be "college and career ready." Courses that incorporate the Common Core Standards are more rigorous and demanding than mathematics classes offered in the past. They are designed to prepare students for the challenges they will face, no matter what they study in college or which career they choose.

THREE YEARS OF HIGH SCHOOL MATH IS THE GOAL

To graduate, California requires all high school students to successfully complete at least two years of mathematics including Algebra I or a combination of courses that meet the content standards for algebra. However, most colleges and universities now require three full years of high school mathematics for admission and many, including the University of California, now recommend that students who wish to major in science, technology, engineering, or math

take four years of high school mathematics, including calculus or statistics.

There are two high school mathematics course sequences

All college-prep high school mathematics courses now align with the Common Core Standards and offer students two course-of-study "pathways." Though the content has changed, schools may offer the traditional sequence of courses—Algebra I, Geometry, Algebra II—or they may offer classes that integrate these subjects each year in a Mathematics I, II, and III course sequence. Both sequences include the same mathematics concepts, but organize them differently.

Math learning is tested prior to graduation

Beginning in 2014-15, every eleventh-grade student in California must take the Smarter Balanced Assessment for high school mathematics, a rigorous test aligned to the Common Core Standards. This assessment includes open-response items that assess students' reasoning and problem solving, along with multiple-choice items that address basic procedural knowledge and understanding.

During the transition to the Common Core Standards, students will still be required to take and pass the California High School Exit Exam (CAHSEE), which is not directly aligned to the Common Core Standards. However, all of the mathematics on the Exit Exam is addressed in the Common Core Standards. Students who have mastered the standards will be prepared to do well on the High School Exit Exam.

AP courses provide additional challenges

If your child is successful in mathematics and enjoys academic challenges, he or she may have the opportunity to take Advanced Placement (AP) mathematics courses. These courses offer the highest level of mathematics study available in high school and are meant to count for college credit. Taking AP math courses can provide an advantage to college-bound students because colleges and universities often give special consideration to applicants who have completed these courses.

If your child is having difficulty in a high school math course, talk to the teacher, school counselor, or principal. They can recommend additional learning strategies and may provide extra resources, such as after-school tutoring or summer school, that can make a difference in your child's success.

More information about the Common Core Standards for high school mathematics is available at your local school or online at: www.corestandards.org/Math/

EXERCISES, PROBLEMS, AND INVESTIGATIONS

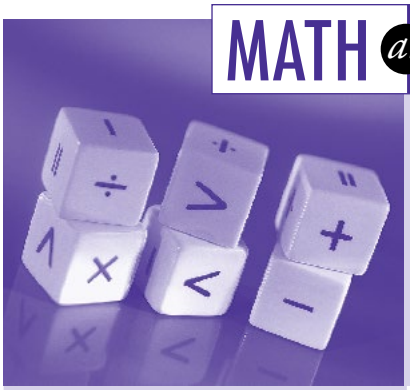
Students don't study math exclusively by completing worksheets filled with numbers anymore. Although great emphasis is placed on learning mathematical facts and procedures, schools are also teaching students to think and communicate mathematically.

Math exercises, problems, and investigations are examples of the kinds of work students are doing in school to foster mathematics learning. The samples below illustrate how each approach leads to a different type of learning.

- **A Math Exercise:** Find the area and perimeter of a rectangle with a length of 7.5 inches and a width of 4.75 inches.

- **A Math Problem:** The perimeter of a rectangle is 36 inches. What are all the possible whole number dimensions of this rectangle?

- **A Math Investigation:** What is the relationship between the area of a rectangle and its perimeter? For a rectangle with an area of 48 square feet, what are its possible dimensions—that is, what lengths, widths, and perimeters are possible? Do all rectangles with the same perimeter have the same area? Prepare a report describing your work and your findings. Provide any charts, tables, or graphs that help explain your thinking.



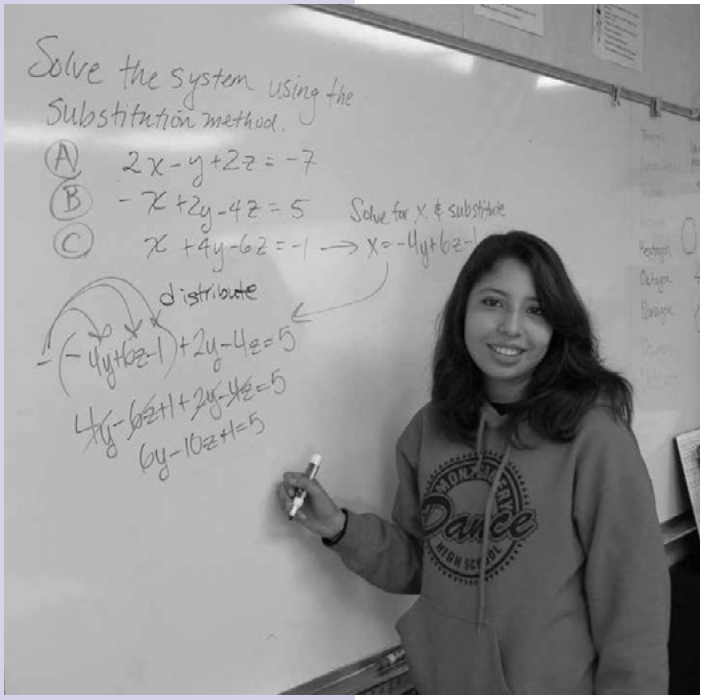
HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

PREPARING FOR COLLEGE AND CAREER

You play a role in your children's college plans! As a parent, you can advise them, encourage them, and support them in this decision. You can make sure they know the benefits of going to college.

One benefit has to do with your child's future earning potential. It is well known that people who graduate from college usually make more money than those with only a high school education. Beginning salaries can be twice as much for college graduates, so earning a four-year degree can be integral to your child's financial success in life. It is also generally true that the greatest potential for job growth comes in occupations that require a college diploma. Jobs requiring only a high school education are less likely to lead to future job growth and economic success.

The importance of a college education today can be compared to that of a high school diploma 40 years ago. Over a lifetime, students with a college degree—no matter what field—earn vastly more than those without a degree. The payoff is huge, and it's growing. More and more of today's jobs require a degree or credential. There are also more career choices for people who go to college, regardless of their major area of study. College is the gateway to more options and better opportunities for students.



However, college isn't only about preparing students to make a living. In addition to providing a pathway to a high-paying job, college helps young adults become intelligent and well-informed individuals. It provides an opportunity for students to spend time considering what they want to do in life, learn what they are truly passionate about, see more of the world, make new friends and other connections, learn financial responsibility, and mature in their decision-making. College stimulates students to think, ask questions, and explore new ideas—all of which are

great advantages in both personal and professional life. Though college doesn't ensure happiness or success, it opens doors and helps create opportunities for the future. And more! Studies have shown that when students go to college, their children are more likely to go to college as well.

Why is mathematics important for college? Students completing a sequence of college-prep high school math courses more than double their chances of successfully earning a four-year college degree.

Two-year colleges and vocational training also require rigorous math, so students planning to attend community college or vocational school, or go directly into an entry-level job, should consider taking three years of high school mathematics. If they later decide to go to a four-year college or university, they will be better prepared to transfer or enroll.

COLLEGE ADMISSIONS REQUIREMENTS

In order to attend a four-year college or university, your children must take specific required high school courses. These courses are designed to prepare high school students for advanced study at the college level regardless of their choice of major. Your children must complete each of these courses with a C or better in order to apply for most public or private colleges and universities in the United States.

University of California and California State University Requirements

History and Social Science
Two years of history/social science, including one year of U.S. history or one-half year of U.S. history and one-half year of civics or American government; and one year of world history, cultures, and geography.

English
Four years of college-preparatory English composition and literature.

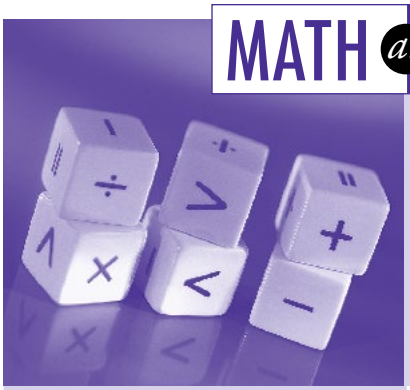
Mathematics
Three years, including Algebra, Geometry, and Algebra II, or Mathematics I, II, and III.
Four years of mathematics recommended.

Science
Two years with lab, chosen from biology, chemistry, and physics.Three years of science recommended.

Language other than English
Two years in same language required. Three years of a foreign language recommended.

Visual and Performing Arts
One year of art chosen from the following: dance, drama/theater, music, or visual art.

College Preparatory Elective
One additional year selected from the subjects listed above or another approved course.



MATH at HOME

HELPING YOUR CHILDREN LEARN AND ENJOY MATHEMATICS

RESOURCES TO HELP YOU AND YOUR CHILD WITH MATH

If you'd like more information about family math activities, mathematics education, or strategies for supporting your child's math learning, you'll find the following list of publications and Internet websites helpful.

PUBLICATIONS

Adding It Up: Helping Children Learn Mathematics (National Academies Press, 2001). This report from the National Research Council of the National Academies will be of interest to parents who want to explore current research about mathematics education in the United States.

Algebra To Go (Great Source Education Group, 2000). This reference book is designed to help students when they're not clear about a math topic and need someplace to look up definitions, procedures, explanations, and rules. The book uses lots of graphics and charts, and includes test-taking strategies, tips for using graphing calculators, and more.

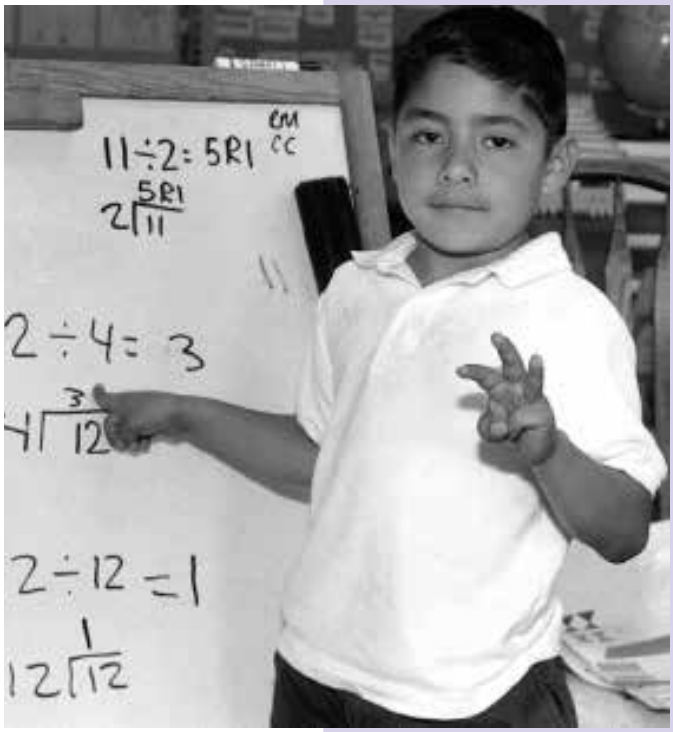
Family Math, by Jean Stenmark, Virginia Thompson, and Ruth Cossey (Lawrence Hall of Science, University of California, Berkeley Press, 1986). *Family Math* is a popular book with dozens of math activities that parents and children, age 8 to 12, can do together. Included are activities related to number sense, geometry, probability and statistics, and algebra. A Spanish version of the book, *Matemática Para La Familia*, is also available.

Family Math for Young Children, by Grace Dávila Coates and Jean Kerr Stenmark (Lawrence Hall of Science, University of California, Berkeley Press, 1997). A sequel to the first *Family Math* publication, this book was developed for families with children age 4 to 8.

Family Math—The Middle School Years, Virginia Thompson and Karen Mayfield-Ingram (Lawrence Hall of Science, University of California, Berkeley Press, 1998). The activities in this book cover algebraic reasoning and number sense and are appropriate for students in grades six, seven, and eight.

A Family's Guide: Fostering Your Child's Success in School Mathematics (National Council of Teachers of Mathematics, 2004). This guide summarizes what today's mathematics classroom is like, offers tips on how parents can help their children have a positive attitude about mathematics, and presents practical ways to discuss and do math at home together.

Helping Your Child Learn Mathematics (U.S. Department of Education, 2004). This publication, available in both English and Spanish, may be downloaded for free at www.ed.gov/parents/academic/help/hyc.html. It highlights activities that parents can do with children from preschool age through grade 5 to strengthen math skills and build strong, positive attitudes toward math.



Math On Call (Great Source Education Group, 2004). Short definitions, examples, and lessons on over 300 mathematics concepts studied in kindergarten through eighth grade are included in this small handbook for middle school students and parents.

Internet sites

Calculation Nation (calculationnation.nctm.org), developed by the National Council of Mathematics Teachers, uses interactive games organized around content from the upper elementary and middle grades math curriculum. Students must establish an account to play online math strategy games that promote learning about and practice with fractions, factors, multiples, and much more.

The *California Mathematics Council* (cmc-math.org/for-families) website includes a "For Families" section that offers free math education articles and activities for students from pre-kindergarten through high school. Use the pull-down menu to access the many resources that are available including free PDF's of this *Math at Home* booklet.

Figure This! (www.figurethis.org) includes a collection of math challenges for middle school students and their families. Each challenge comes with a hint and the complete solution, along with related information and questions to think about.

Illustrative Mathematics (www.illustrativemathematics.org) is an online resource of free mathematical tasks that illustrate every one of the Common Core Standards for each grade.

The Math Forum (www.mathforum.org) hosts "Ask Dr. Math" and has weekly/monthly math challenges, Internet math hunts, and math resources organized by grade level.

The *National Library of Virtual Manipulatives* (nlvm.usu.edu) promotes three key areas of mathematics: procedural skills, conceptual understanding, and problem solving. Over 100 virtual manipulatives are sorted by grade level, providing interactive tutorials that engage students in number sense, algebra, geometry, measurement, and probability learning. The site offers a free trial version; there is a modest charge for an individual license.

Thinkfinity (www.thinkfinity.org) is a project of the Verizon Foundation. The site has thousands of resources—including many math-focused ones—that have been screened by educators to ensure that content is accurate, up-to-date, and appropriate for students. Resources are grouped by grade and subject area.

FIGURE *it* OUT

A FEW MATH CHALLENGES FOR YOUR FAMILY

THIRTY-TWO PEOPLE ENTER A PING-PONG TOURNAMENT. When a person loses a game, he or she is eliminated from the contest. How many games must be played to figure out who is the best Ping-Pong player?

AT A FIRE SCENE, A FIREMAN STOOD ON THE MIDDLE STEP OF A LADDER to shoot water onto the flames. When the smoke cleared, he went up three steps. A sudden burst of flames forced him to go down five steps. A few minutes later, he climbed up seven steps and worked there until the fire was out. Then he climbed the seven remaining steps and entered the building. How many steps does the ladder have?

A RANCHER HAS 48 METERS OF FENCING TO BUILD A CORRAL FOR HIS COWS. Since his property is bordered by a river, what is the biggest rectangular area he can fence if he uses the river as one side of the corral?

AT A PARTY ATTENDED BY 12 FRIENDS, the activities begin with every person shaking every other person's hand once (and only once). How many handshakes take place?

Photo by Ross Hause

OSGOOD SMART GLUED TOGETHER 125 SMALL CUBES to make one big solid cube, then he painted all six sides of the big cube bright red. Later on, he broke the big cube back into small cubes and found that some cubes had three sides painted, some cubes had two sided painted, some cubes had one side painted, and some cubes had no paint on them at all. How many of each color variation did he have?

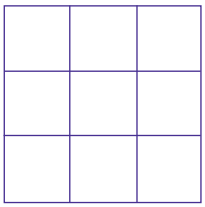
A 200-POUND MAN AND HIS TWO DAUGHTERS (each of whom weigh 100 pounds) are standing on the bank of a river teeming with piranhas. They want to get to the other side, but their canoe can hold no more than 200 pounds. How can they get across?

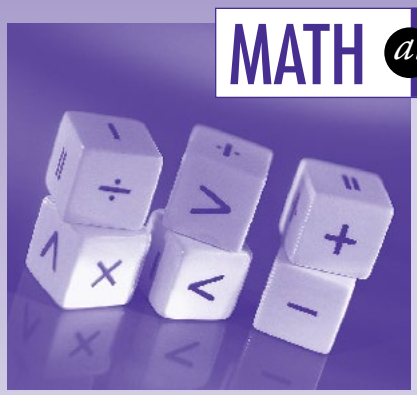
A GIANT HERO SANDWICH HAS BEEN CREATED THAT IS 30 FEET LONG. It has been divided into four parts so that each part is one foot longer than the previous one. What are the lengths of the four pieces?

IF YOU HAVE CHOCOLATE, STRAWBERRY, AND VANILLA ICE CREAM, how many *different* double-dip cones can you make? Remember, some people like to eat their strawberry *before* they eat their chocolate, and some like it the other way around.

IF A STATE'S LICENSE PLATES CONSIST OF ONE DIGIT followed by three letters, followed by three digits—such as 1ABC123—how many unique license plates can be issued?

HOW MANY DIFFERENT SHAPES, not counting reflections or rotations, can be cut from a 3-inch by 3-inch grid if cutting is allowed on the grid lines only?





MATH at HOME

HELPING
YOUR CHILDREN
LEARN AND ENJOY
MATHEMATICS

A PUBLIC SERVICE PUBLICATION FROM:

CALIFORNIA MATHEMATICS PROJECT
NORTH COAST

Sonoma County
Office of Education

CALIFORNIA MATHEMATICS COUNCIL

Printed copies of *Math at Home* are available.
To order more than 100 copies, visit www.cmc-math.org/for-families
To order less than 100 copies, visit www.scoe.org/math

CDE CORNER

Cha...Cha...Changes

Mary Pittman, Mathematics Content Specialist, Colorado Department of Education

As with the start of any new school year, there are many changes occurring. New principals, new teachers, new students...it can be very exciting and scary all at once. This year I am experiencing some of those same changes with all of you. The Commissioner of Education, Robert Hammond, retired this summer and several key members of his executive team moved on as well. Luckily Elliot Asp will be serving as Interim Commissioner while the State Board of Education works to find a permanent replacement. Additionally, my supervisor, Brian Sevier, left at the start of August to take a position at Metro State University as the Associate Dean of Teacher Education.

There are also important changes around our state mathematics assessment. At the end of the 2015 legislative session, House Bill 15-1323 passed and was signed into law. The focus of H.B. 15-1323 is state-wide assessments. I recommend reading an overview of the changes. I am going to highlight just a few that directly impact teachers of mathematics:

High School Tests: This bill included provisions to eliminate 11th- and 12th-grade PARCC tests and replace 10th grade PARCC tests with a college-preparatory exam. According to the legislation, students in 11th grade will continue to take a college entrance exam. The new 10th grade college-preparatory exam and the 11th grade college entrance exam will be procured through a competitive request for proposal (RFP) process. CDE does not know which college entrance exam and its associated 10th-grade exam will be selected for 2015–16 until that process is complete.

Course-based versus grade-based PARCC tests: PARCC assessments at the high school level are course-based rather than grade-based. House Bill 15-1323 states students must take a mathematics state assessment in grades 3–9. Students in grades 3–6 will continue to take the assessment of the grade level in which they are enrolled. CDE is currently working with the United States Department of Education to determine if we may continue to offer flexibility for students in grades 7 and 8 to take either the 7th and 8th grade tests or the high school course-based assessments. Students in grade 9 will be able to take any of the six available course-based assessments from PARCC (Mathematics I, II, III or Algebra I, Geometry, Algebra 2).

Paper/pencil versus online: While this year about 85% of the students in Colorado took the mathematics test online, H.B. 15-1323 allows districts to request paper/pencil formats of online state assessments.

Parent Exemption: Each district must also adopt a written policy and procedure allowing a student's parent to excuse a student from participating in one or more state assessments.

PARCC assessments are also changing! PARCC will have only one assessment window. The window for Colorado will be April 11–29, but if a district needs extra time due to the number of computers available, they may begin testing as early as March 14. This shorter window brings with it a shortened test. The testing time will be approximately **90 minutes *less*** for the combined mathematics and English language arts assessments. The one testing window and shortened test, however, still allow for performance tasks focused on reasoning and modeling.

Stay tuned to CoMath, CCTM and the CDE Office of Assessment to get important updates as they become available.

PAEMST Awardee and Finalists Selected

Mary Pittman, Colorado Department of Education

The Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) is the highest recognition that a kindergarten through 12th-grade mathematics or science (including computer science) teacher may receive for outstanding teaching in the United States. On July 1, 2015, President Obama named 108 mathematics and science teachers as recipients of the prestigious 2013 Presidential Award for Excellence in Mathematics and Science Teaching, and Kirstin Oseth from Colorado Springs was among the 108.

Kirstin has taught mathematics at Cheyenne Mountain Junior High School for 28 years. Through her teaching she fosters a student-centered classroom environment that empowers students to construct viable arguments and critique the reasoning of others. Students use formative assessment to self-assess, conduct error analysis, and communicate a variety of ways to solve problems. Kirstin also uses differentiated instruction to reach students both above and below grade level. She specializes in reaching students who have previously been at least one year below grade level.

Kirstin has presented at workshops on formative assessment, the Standards for Mathematical Practice, and the student-centered classroom. She has co-presented at state conferences on Response to Intervention and an Apprenticeship Model of Working with Student Teachers. She served on the district autism committee that resulted in district accreditation by the Colorado Department of Education.

Kirstin has a B.A. in mathematics from Augustana College and a M.A. in educational computing and technology from the University of Colorado. She is certified in secondary education.

I am honored and humbled to receive the Presidential Award. It is a tribute to the outstanding teachers, administrators, and students of Cheyenne Mountain School District with whom I work. I feel fortunate to be a representative of the thousands of teachers throughout our country who work passionately, creatively, and endlessly to push our students to be critical thinkers and problem solvers. I am proud to accept this award on behalf of my colleagues and students.

Presidential Awardees receive a certificate signed by the President of the United States, a trip to Washington D.C. to attend a series of recognition events and professional development opportunities, and a $10,000 award from the National Science Foundation.

"These teachers are shaping America's success through their passion for math and science," President Obama said. "Their leadership and commitment empower our children to think critically and creatively about science, technology, engineering, and math. The work these teachers are doing in our classrooms today will help ensure that America stays on the cutting edge tomorrow."

While the year 2013 Presidential Awardees were just announced by the White House, I am also pleased to announce a very strong group of year 2015 finalists. We will be sending forward five secondary mathematics teachers: Jean Elizabeth Allen-Hatcher (Bruce Randolph School, Denver), Jennifer Amos (Mesa Middle School, Castle Rock), Lisa Bejarano (Aspen Valley High School, Colorado Springs), Elissa Pitts (Poudre High School, Fort Collins), and Karl Remsen (Lake County High School, Leadville). Please join me in celebrating and congratulating these teachers at the Annual CCTM Awards Reception on September 24th.

The 2015–2016 school year focuses on outstanding *elementary* teachers. Please consider nominating someone today: www.paemst.org

CONFERENCES AND PROFESSIONAL DEVELOPMENT

2015 CCTM Conference

Leigh Ann Kudloff, Conference Chair

The 2015 CCTM Conference will feel like an NCTM regional conference, as three nationally known guests will be supporting our learning. "Putting the Pieces Together for Mathematical Success," will be held on September 24–25, 2015 at the Denver Mart.

Thursday, September 24th

The conference kicks off with an administrator and district leader workshop from 12:00–3:30 p.m. **Matt Larson**, President-Elect of the National Council of Teachers of Mathematics, will address administrators, district leaders, and math coaches/specialists. Dr. Larson is the K–12 curriculum specialist for mathematics in Lincoln Public Schools (Nebraska). He has taught mathematics at the elementary through college levels and has held an honorary appointment as a visiting associate professor of mathematics education at Teachers College, Columbia University. Matt was a member of the writing team for the NCTM publication, *Principles to Actions*, a coauthor of the IES What Works Clearing House Report: "Teaching Strategies for Improving Algebra Knowledge in Middle and High School," and also a coauthor of a new set of books co-published by NCTM and Solution Tree called *Mathematics in a PLC at Work*. The set includes guides for K–5, 6–8, 9–12, and leaders. Larson will share the material from the leader's guide with Colorado school and district leaders. His presentation, "Assessment and Instructional Team Actions Necessary to Improve Student Learning," will outline high-leverage actions for principals, including the most effective ways to prepare students for next-generation assessments. A frequent keynote speaker at national meetings, Dr. Larson's humorous presentations are well-known for their application of research findings to practice.

Phil Daro, one of the principal authors of the Common Core State Standards in Mathematics, will lead the teacher pre-session, from 4:00–6:30 p.m. with his talk "Using Progressions to Make Progress." Phil is the lead mathematics designer for the pad based Common Core System of Courses developed by Pearson Education. He also works in a partnership of the University of California, Stanford and others with the Oakland and San Francisco Unified School Districts for the Strategic Education Research Partnership (SERP), with a focus on mathematics and science learning. Previously, Daro was a Senior Fellow for Mathematics for America's Choice, and the executive director of the Public Forum on School Accountability. He directed the New Standards Project and managed research and development for the National Center on Education and the Economy. Phil has directed large-scale teacher professional development programs for the University of California, including the California Mathematics Project and the American Mathematics Project. We are grateful to Pearson for sponsoring this opportunity for Colorado math teachers to once again hear from Phil Daro.

The Colorado Mathematics Teaching Awards Ceremony immediately follows. We are so pleased that Texas Instruments will be sponsoring this celebration. All are welcome to honor our award winners and nominees while celebrating excellence in Colorado classrooms. A selection of hors d'oeuvres and a cash bar will add to the festivities.

Friday, September 25th

Conference activities begin with registration/check-in at 7:00 a.m. Interactive sessions for all audiences will be offered from 8:00 a.m. until 4:00 p.m.

We are very excited to have **Steve Leinwand** as our keynote speaker. In his presentation, "Teaching Math: Insights and Reflections on More Than 1000 Observations," Steve will reflect on what he has learned through these classroom visits and share what can be used in Colorado classrooms. Leinwand is a principal research analyst at American Institutes for Research (AIR), where he currently serves as mathematics expert on a wide range of projects that turn around schools, improve adult education, evaluate programs, develop assessments, and provide technical assistance. Steve has held over 35 years of leadership positions in mathematics education, including 22 years as Mathematics Consultant with

the Connecticut Department of Education. There he was responsible for the development and oversight of a broad statewide program of activities in K–12 mathematics education including the provision of technical assistance and professional development, the evaluation of Title I and K–12 mathematics programs, the assessment of student achievement and teacher competency, and the coordination of statewide mathematics programs and activities. Steve has served on the NCTM Board of Directors and has been president of the National Council of Supervisors of Mathematics. He is the author of several mathematics textbooks and has written numerous articles. Steve's publications include *Sensible Mathematics: A Guide for School Leaders* and *Accessible Mathematics: 10 Instructional Shifts that Raise Student Achievement*.

Matt Larson, Phil Daro, and Steve Leinwand will also present hour-long small group sessions on Friday, which will provide Colorado teachers with an intimate learning experience from some incredible experts!

The conference schedule will enable attendees to have dedicated time to visit the exhibits area. The registration fee includes a lunch voucher that can be spent among several on-site catering venues, so participants can make the most of their conference time. Friday will wrap up with an update from the Colorado Department of Education, and some fabulous door prizes.

The Denver Mart is located near I-25 and 58th Avenue in Denver. For those requiring an overnight stay close to the conference, the DoubleTree Westminster (located near the Boulder Turnpike at Sheridan Blvd.) offers a conference rate that includes a breakfast buffet and free shuttle to and from the Denver Mart.

Registration begins August 15, 2015 and can be accessed via the CCTM website (cctmath.org).

We look forward to seeing you there!

EQUITY: MATH FOR ALL

Culturally Responsive Teaching

Daniel Greenberg, Secondary Math Specialist, Boulder Valley School District

I can recall sitting on an interview committee for a middle level math position. One of the questions the committee asked each applicant was, "How do you make the instruction in your classroom culturally relevant to your students?" This question elicited a variety of responses that ranged from students needing to be able to connect math to their everyday lives, to explanations that there really was not a whole lot that is culturally relevant about math instruction for students.

Since those of us on the committee were evaluating candidates' responses, we had a discussion about what we were looking for in the answers to this question. Most of us were as equally stumped as our counterparts, and eventually we came to one conclusion: it is important for students to feel connected to their classroom community and that is what makes the instruction culturally relevant.

For many children it is that personal connection that must come first. The relationship students have with their teacher and their school community must be strong before they are driven to learn each day and try their best. This single component to classroom culture, when absent, can derail and render wholly ineffective even the most pedagogically advanced and creative classroom.

A positive classroom culture with strong relationships is only one aspect of a connection to classroom community. While imperative, it only gets us so far. A classroom that consists of students from a variety of backgrounds requires a variety of methods and modalities for delivering instruction, because how we teach and the ways in which we learn are informed significantly by much of our own cultural background (Nuri-Robbins, Lindsey, Lindsey, & Turrell, 2005). A common math instruction model is structured around the teacher who provides examples of how to solve a particular math problem or the use of a specific math strategy. What follows is student practice of these strategies with the teacher re-teaching one-on-one or in small groups for those who didn't get the procedure the first time. This instructional model is indicative of a cultural background that emphasizes linear thought with an emphasis on individual achievement. This form of instruction can be very effective for students who share this cultural background; many other students will struggle in such an environment.

Our students who fall into the achievement gap frequently come from cultural backgrounds that do not emphasize linear thought or individualism. Many cultures place a premium on interpersonal relationships and cooperation. Stories are not told in plot driven linear fashion with a distinct beginning, middle and end, but rather in modes and styles that many of us might initially consider disorganized or wandering. For example, the Paraprofessional Resource and Research Center (paracenter.cudenver.edu) refers to several structures they call "Communication Differences" such as Spanish/Romance languages that, while having a linear logic "… allow for a great deal of digression that would be considered superfluous in English." Asian and Native American languages "… have a circular logic [and] speakers of these languages consider being too direct to be rude." Our students who come from these backgrounds will learn best in classrooms that allow for thinking and knowledge building with these emphases in mind.

This also means that we must analyze our own teaching and communication styles and look for biases in ourselves of which we may not even be aware. Are we more accepting of one student's verbal reasoning than another's, not due to content but due to communication style? Do we deliver instruction in a style that precludes many from following our thinking and explanations simply

because they are raised in a culture that communicates using a different structure? Do we provide a context for the math, which may allow students to be more comfortable and articulate when called upon to explain and demonstrate their thinking?

What Does This Look Like?

How then, do we combine not only the needs of a multi-level class, but the needs of a multi-cultural one? Systems must be put in place that honor all learning styles. Pair/share activities contribute to a

culturally responsible lesson by allowing students to learn both collaboratively and culturally. This is one example, and it is important for us to seek out structures and activities that allow for cooperative thinking and problem solving. When students are given opportunities to cultivate their own knowledge and understandings prior to being given the algorithm, they are able to make meaning of the mathematics they are meant to learn in a context that resonates in their own culturally relevant way.

Another way for us to have a culturally responsive classroom is to create an environment in which multiple strategies and methods for problem solving are honored. When students can display their thinking for others to see and critique, we honor the collaborative nature of learning that many cultures emphasize. For peer critique to function effectively, we must spend time teaching students how to provide feedback, so that it is palatable for their classmates to hear. Using a classroom structure that incorporates the strengths of a variety of cultures, such as cooperative learning and thinking, rather than just the strength of ones that value linear thought and independence, help all children learn.

Over the past generation, our charge as educators has changed from sifting students into groups based on levels of achievement to one that asks us to educate all students. This fundamental shift in philosophy has prompted many changes in our schools, such as heterogeneous grouping, support classes, and the Common Core Standards. As educators who are required to meet the needs of a more diverse group than ever before, we need to understand how our cultural backgrounds influence our classroom management, our instructional style, and our classroom expectations for how students work and learn. While there has always been the need for teachers to be flexible and mindful of student learning styles, we must now learn to go beyond this one dimension of differentiation and learn how to respond to the diverse cultural needs of our students as well.

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Nuri-Robins, K. N., Lindsey, D. B., Lindsey, R. B., & Terrell, R. D. (2005). Culturally Proficient Instruction: A Guide for People Who Teach (2nd ed.). Newbury Park, CA: Corwin Press.

FROM COLLEGE TO THE CLASSROOM

Best Practices for Flipping a Classroom

Spencer Bagley, University of Northern Colorado

As we are about to start another school year, we are thinking about different models of teaching that we can incorporate to enhance student active engagement in classrooms. One such "new" model of teaching is called the flipped, or inverted, classroom model. This model started in K–12 classrooms (Bergmann & Sams, 2008) and is now gaining momentum in mathematics classrooms at all levels, including college level courses. Many teachers are interested in the flipped classroom model for the many possible opportunities that it can provide. While there are many variations on this model, the basic idea is that some portion of the class content, which would normally be delivered to students during classroom time, is instead delivered before class, often through custom-created online videos. This frees up class time for students to participate in interesting, challenging, and active learning tasks. Researchers have demonstrated that students perform better when their classrooms are structured around active learning (Freeman et al., 2014), and the flipped class model allows teachers to give their students time to engage in active learning without sacrificing course content.

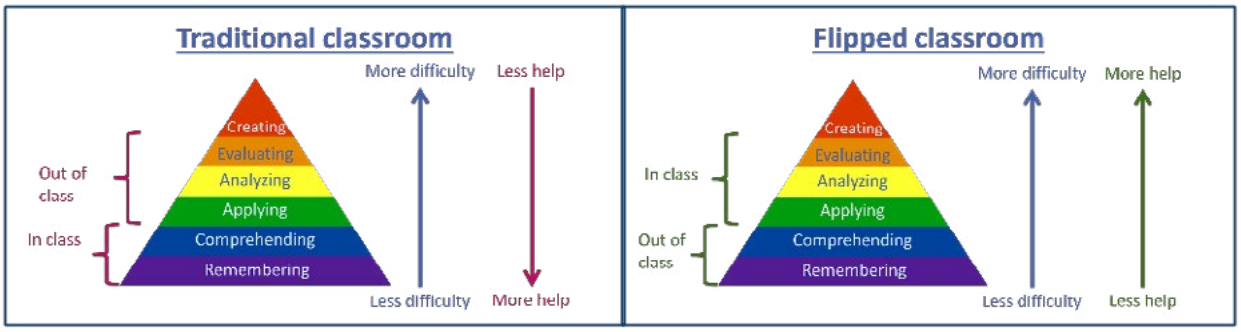

Figure 1: Traditional vs. flipped classroom. Adapted with permission from Robert Talbert.

One main benefit of the flipped class model is that it allows students to have more help when they need it (see Figure 1). In the traditional classroom, the less cognitively-demanding "remembering" and "comprehending" tasks occur inside the classroom, while the more cognitively-demanding "application" and "analysis" tasks occur outside class time, when the instructor is not immediately available. The flipped class model provides an opportunity to move higher-level tasks into class time, when the instructor can provide guidance directly.

Just as with any pedagogical strategy, there are certain best practices for the flipped class that should be followed to enable its success. In this article, I share some findings from my dissertation study in which I compared student success in four different college calculus classes, one of which was flipped. As part of this study, I read many reports of successful flipped classes from the research literature, and identified three commonalities in the way each one was implemented: pre-class activities were closely tailored to the class and included some accountability; in-class activities were "active learning" of some sort; and the instructors were involved in every step of the process. The flipped class in my study did not follow these best practices, and this absence negatively impacted both students' performance and their perceptions of the class.

Tailored pre-class activities

In successful flipped classrooms, instructors create the pre-class activities themselves or carefully

check to make sure the activities closely align with the day's learning objectives. Many college instructors start by recording the lectures they gave in the previous term, or by turning PowerPoint presentations into reading outlines or worksheets.

In the flipped class I studied, pre-class videos were chosen exclusively from external resources such as Khan Academy and HippoCampus. Students in this class reported that they felt like these videos were "not applicable to the work," and that they wished the videos had "more direct correlation to the class." Even when they watched the videos before class, they felt unprepared for the in-class work, which had adverse effects on their confidence: "I watched the videos and I understand it going in, I feel very confident, and then I get that paper [the in-class worksheet] and I'm like, well, I give up already."

External resources can still work well, as long as they are carefully chosen to align with course objectives. One student participant of the course observed that "at the beginning of the year, the videos corresponded well, because it was just a lot of simple stuff." The feeling of disconnect between pre-class videos and in-class activities was less noticeable for the early foundational material in the course.

Another best practice for pre-class activities is that students should be held accountable in some way for completing them. It's no surprise that students often don't do things they aren't accountable for. Accountability for pre-class activities can take many forms. Some instructors choose to quiz students on the assigned content at the beginning of each class session. Others assign a small amount of homework as part of pre-class activities, to be turned in and graded for completion at the beginning of class. Yet others, including Robert Talbert, simply maintain an expectation that students come prepared to class. These instructors move on as if everyone is prepared, even if some students are not. In the flipped class I studied, there was no specific accountability mechanism, so many students did not watch the videos before class, relying instead on previous calculus knowledge or on other students in the class.

Active learning in class

In successful flipped classrooms, instructors use the freed-up class time to engage students in well-structured active-learning activities. Some instructors combine clicker questions with peer discussion. Others involve students in demonstrations or experiments (one economics class, for instance, held an auction for a can of cola to help students understand the effect of demand on prices). Others use inquiry-based learning, providing a scaffold to help students work together in small groups to reinvent key ideas. The structure provided by the instructor helps students who may not otherwise have sophisticated time management skills (Talbert, 2014).

Lecture is not evil; it is a good tool for quickly and clearly communicating information. However, lectures in a flipped class should be brief (maybe seven minutes or less!), targeted at a specific topic, and not a review of pre-class activities. Again, one key benefit of the flipped model is having students work on more conceptually-demanding tasks with the instructor present; this benefit is erased if class time is spent only in lecture.

In the flipped class I studied, students spent the 100 minutes of daily class time working through a worksheet with about 20 calculus problems, either individually or in small groups of their own choosing. While students were actively engaged with this activity, class time was not well structured. Students complained that "an hour and forty minutes straight of doing word problems is kind of a lot," and that they frequently got distracted while trying to maintain focus on the worksheet. Providing more structure and creating more active-learning activities would have helped these students stay focused and engaged.

Instructor involvement

The flipped class model provides an opportunity for teachers to observe their students' thinking while students are actively engaged in tasks. Teachers can assess the progress of their students and can modify their immediate and future lesson plans. Such teacher involvement provides instructional strategies with the arrows in Figure 1 pointing in the right direction.

Some critics of the flipped class model argue that it is a "get-out-of-teaching-free" card. This is certainly not the case. In successful flipped classrooms, instructors are substantially involved in all aspects of the classroom. As discussed earlier, successful flipped-class instructors take an active role in creating or selecting pre-class activities; additionally, they also take an active role in monitoring, assessing, and supporting students' learning through in-class activities.

The flipped class I studied was part of an experiment to see if the flipped class model could be used as a more economical way to teach a large number of students. Therefore, the instructor did not come to class; instead, three graduate teaching assistants provided help and guidance to the students completing the daily worksheets. The instructor remained involved with the class by designing the daily worksheets and writing and grading the exams. The students in this class reported that they felt disconnected from the professor; they also reported feeling that their instructor's absence negatively impacted their learning.

The instructor's absence is a reasonable explanation for the lack of positive results from this flipped class; even though the arrows in Figure 1 pointed the right way, the instructor's absence weakened the second arrow. To achieve the full benefit of the flipped class model, the instructor must be present and actively involved; Talbert (2014) writes that "open lines of communication between the instructor and the students are critical to the success of the inverted classroom" (p. 365).

Best practices

The three best practices I suggested sound to many people like just plain good teaching; this is probably true. However, I see this as a good thing, because it means that the flipped class model doesn't require instructors to make drastic changes to their current teaching philosophy. Therefore, successful flipped classes are easily within reach.

In fact, these best practices open opportunities for teachers to strengthen their use of NCTM's eight mathematics teaching practices that are provided in the *Principles to Actions* (2014) document. In particular, active-learning exercises allow teachers to "implement tasks that promote reasoning and problem solving" (p.10) in a much more robust way than in lecture-based classrooms. Teachers who take an active role in in-class work are in an excellent position to "facilitate meaningful mathematical discourse" among their students, "pose purposeful questions… to assess and advance students' reasoning," and "support productive struggle in learning mathematics" (p.10). Additionally, teachers' greater involvement in the mathematical discourse and productive struggle of students gives them a greater ability to "elicit and use evidence of student thinking" (p. 10); teachers can be responsive to student thinking when creating or selecting pre-class activities to maximize their benefit.

With carefully selected pre-class activities, engaging in-class activities, and hands-on involvement from the instructor at every step, the flipped class model has the potential to improve both student learning and instructor understanding of where their students stand.

References

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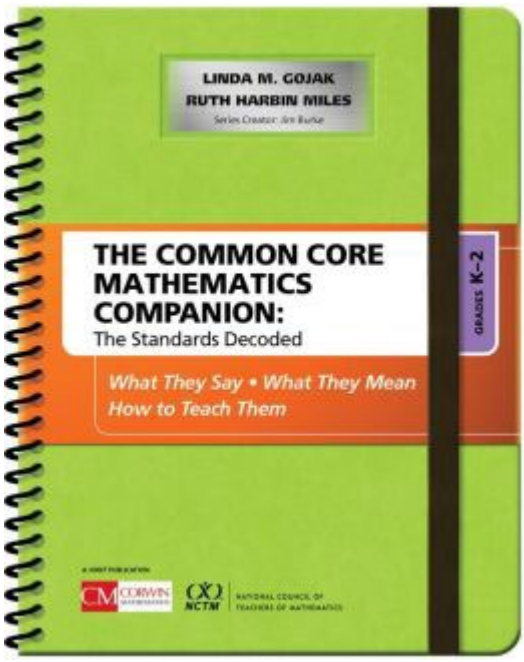
FOR YOUR BOOKSHELF

The Common Core Mathematics Companion: The Standards Decoded, K–2

Reviewed by Juli Lenzotti, CCTM Treasurer & Nicole Wimsatt, CCTM Regional Representative

In working with elementary teachers this past year to support their understanding and implementation of Common Core English Language Arts Standards, two resources were extremely helpful: *The Common Core Companion: The Standards Decoded, Grades K–2* by Sharon Taberski with Jim Burke and *The Common Core Companion: The Standards Decoded, Grades 3–5* by Leslie Blauman with Jim Burke. As they realized the wealth of information provided in those texts, math teachers were hopeful for similar resources. Finally! *The Common Core Mathematics Companion: The Standards Decoded, Grades K–2* is now available, with a version for Grades 3–5 on the way!



The authors, NCTM past-president Linda Gojak and mathematics coach Ruth Harbin Miles, explain: "The purpose of this book is to help teachers more deeply understand the mathematical meaning of each cluster and standard with the five domains of Grades K–2. We want this book to be your toolkit for teaching the mathematics standards, and we have left ample space for you to take notes, add ideas, and note other resources you have found to be helpful." (How to Use This Book, page xxviii *The Common Core Mathematics Companion: The Standards Decoded, Grades K–2*)

And, toolkit it is! Each domain—larger group of related standards—begins with a brief description of the big ideas (**Domain Overview**), a description that shows how mathematical ideas build across the grades. The **Suggested Materials** page lists helpful materials that might be used to introduce concepts within the domain. Materials labeled "Reproducible" indicate that there is a reproducible available in the Resources section in the back of the book. **Key Vocabulary** highlights grade-specific vocabulary, as well as the grade level at which terms are used. The mathematics for each standard is clearly mapped out with ideas for what teaching and learning might look like in kindergarten, and first and second grade classrooms. In addition to the Content Standards, connections are made to highlight relevant Mathematical Practice Standards for each cluster. **Addressing Student Misconceptions and Common Errors** is a valuable component with suggestions given for strategies and actions to address misconceptions and errors.

With each section of standards, the authors have created a **Sample PLANNING PAGE** for one individual standard which includes the purpose of the lesson, suggested mathematical practices that could be emphasized, possible questions to build student understanding, and instructions for adjusting the lesson to support struggling students and to extend the lesson for students who demonstrate understanding of the mathematics. A planning template similar to the Sample Planning Page is included.

The **Resources** section contains a wealth of tools. A wide range of situations and strategies for addition and subtraction are modeled through a variety of representations including words, models, and equations. Suggestions for scaffolding addition and subtraction are also helpful for supporting struggling learners. A detailed description of teacher and student actions regarding the eight Standards for Mathematical Practice and the eight Effective Teaching Practices (as identified in *Principles To Actions)* can be a useful tool to support the integration of these important practices into daily lessons. Finally, the **Where to Focus** page provides an overview of the major, supporting, and additional clusters, as well as required fluencies for each grade level.

The Common Core Mathematics Companion: The Standards Decoded, Grades K–2 is an easy-to-use tool for deciphering the standards, implementing effective teaching practices, and designing meaningful learning activities for students. Gojak and Miles encourage readers to collaborate with colleagues and to, "Keep in mind that implementation of the standards and practices is a process and may take time to do well." Digging into this work is certainly an investment that can transform instruction and allow students to deeply understand mathematics content. Take the next step in preparing YOUR students for college, career, and LIFE…read *The Common Core Mathematics Companion: The Standards Decoded, Grades K–2* (or *3–5*, available on Amazon in July).

NCTM CORNER

Resource Updates

Cathy Martin, CCTM Past President and NCTM Board Member

NCTM has added many new exciting resources to support teaching robust mathematics and making it accessible to all students. Be sure to explore these as you prepare for a new school year.

New resources for Effective Teaching Practices and Guiding Principles in *Principles to Actions: Ensuring Mathematical Success for All*

NCTM has produced the Principles to Actions Toolkit (http://www.nctm.org/PtAToolkit/) containing resources to support the analysis and development of each of the eight Effective Teaching Practices and one of the Guiding Principles. These resources provide videos and vignettes that illustrate the Effective Teaching Practices in action in the classroom along with the resources to facilitate professional learning around the Practice. Be sure to routinely check the website, as more resources will be added in the coming months.

NCTM-Hunt Institute Video Project: *Teaching and Learning Mathematics with the Common Core*

(http://www.nctm.org/Standards-and-Positions/Common-Core-State-Standards/Teaching-and-Learning-Mathematics-with-the-Common-Core/) NCTM and the Hunt Institute have produced a series of videos to enhance understanding of the mathematics that students need to succeed in college, life, and careers. Beginning in the primary grades, the videos address the importance of developing a solid foundation for algebra, as well as laying the groundwork for calculus and other postsecondary mathematics coursework. The series also covers the Standards for Mathematical Practice as elaborated in the Common Core State Standards for Mathematics and examines why developing conceptual understanding requires a different approach to teaching and learning. These videos could be used in many venues—with teachers, school leaders, community members, and parents.

New E-Book Series

(eBook) *Discovering Lessons for the Common Core State Standards in Grades K–5* (PDF). This guide

aligns high-quality, engaging activities with specific content and practice standards in the Common Core State Standards for Mathematics (CCSSM). The authors have selected peer-reviewed articles from NCTM's award-winning journal for elementary teachers, *Teaching Children Mathematics,* for this rich assortment of classroom activities aligned directly with CCSSM.

(eBook) *Discovering Lessons for the Common Core State Standards in Grades 6–8* (PDF). This guide provides ready-made alignment to high-quality, peer-reviewed, engaging activities reviewed by NCTM members that they can use with their students. Teachers will find this resource invaluable in locating activities that align to the CCSSM content standards and standards

for mathematical practice.

New NCTM Publications

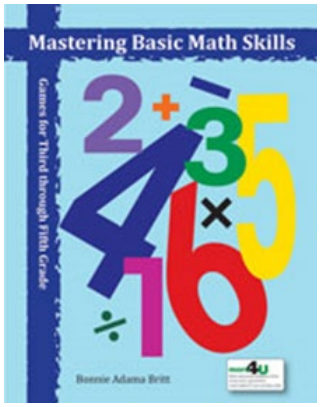
Putting Essential Understanding into Practice: Statistics, 9–12

This book focuses on the specialized pedagogical content knowledge that will support teachers teaching statistics effectively in grades 9–12. The authors demonstrate how to use this multifaceted knowledge to address the big ideas and essential understandings that high school students must develop for success with statistics—not only in their current work, but also in higher-level mathematics and real-world contexts.

Explore rich, research-based strategies and tasks that show how students are reasoning about and making sense of statistics. Use the opportunities that these and similar tasks provide to build on students' understanding while identifying and correcting misunderstandings that may be keeping students from taking the next steps in learning.

Mastering Basic Math Skills: Games for Third through Fifth Grade

Fractions, decimals, integers … children in third through fifth grade encounter new types of numbers and new concepts, and need to develop new skills to meet these challenges. This book provides a collection of games for students in grades three through five to address these challenges while also providing opportunities for students to have fun. Games are organized in each chapter from simplest to more advanced, and many have variations that change the rules of play or make it more challenging once the basic game is mastered. Math games keep children engaged while providing the practice they need to learn new concepts and math facts. All games are correlated to the Common Core State Standards.



FAST CONNECTIONS

Realistic Mathematics Conference

David Webb, University of Colorado, Boulder

It's an odd year, which means the Realistic Mathematics Education Conference is coming to the University of Colorado, Boulder, September 18–20. This biennial conference, hosted by the Freudenthal Institute US at the University of Colorado Boulder since 2009, regularly draws presenters and participants from around Colorado, the United States, and abroad. This year's conference (RME5) will focus on the various ways in which realistic contexts, emergent models, and representations support mathematical reasoning. Plenary and interactive sessions will explore curriculum and assessment design, teacher learning and practice, and studies of student learning.

Further information on the Freudenthal Institute US, as well as conference registration information, program updates, and session abstracts can be found at www.fius.org. We look forward to seeing you at RME5!