COLORADO MATHEMATICS TEACHER - WINTER 2016

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Thinking about Assessment? From *Principles to Actions*: Pose Purposeful Questions; and Elicit and Use Evidence of Student Thinking

CONTENTS & FEATURES

The CMT presents a variety of viewpoints. Unless otherwise noted, these views should not be interpreted as official positions of CCTM or CMT.

From The Editor's Desk

Sandie Gilliam, CCTM Editor

STEPPING BACK AND LOOKING FORWARD

For long-time *CMT* readers, you will remember when our journal was black and white, sent to a typesetter, and then mailed to you.

For long-time high school teachers, you will remember when "stand and deliver" was the norm and focusing on the numerical answer to a math problem was your sole concern.

Long-time teachers remember when state testing was an incidental part of our year, designed primarily for our own personal goals to gauge student learning and tweak our classroom instruction.

Reflecting on the 2015 CMT—now accessible to all in an on-line, colorfully designed format, where a click in the table of contents and in the articles themselves brings you to the desired pages and links—we presented articles on:

- Getting ready for PARCC.
- Standards for Mathematical Practice.
  - #7: Look for and make sense of structure.
  - #8: Look for and express regularity in repeated reasoning.
- Equity:
  - Reading strategies for ELLs.
  - Culturally responsive instruction.
  - Supporting the success of diverse, low-income learners.
- Book reviews:
  - *Principles to Actions: Ensuring mathematical success for all,*
  - *Beyond the Common Core: A handbook for mathematics in a PLC at work, and*
  - *The Common Core Mathematics Companion: The standards decoded K–2.*
- Strategies and resources for you to use with parents.
- Classroom, school, and district math activities and resources that have been used *in the field*.
- College transitions/courses given the pre-service teachers of mathematics that led to practicums in your classrooms.
- Resources from NCTM including the full article, *13 Rules that Expire.*
- *Principles to Actions (PtA)* teaching practice:
  - Establish goals to focus learning.

Look back and read something of particular interest to you!

So what's ahead in the CMT for 2016?

Notice the new logo at the top of page 1? CCTM has updated our logo, giving it a fresh Colorado look!

Since PARCC has given us our first exam and the results are in, where do we go, now? The focus of this issue is **assessment**. Reflect on the articles and determine your level of understanding assessment; then try out some of the suggestions. An additional focus on *PtA*, continues with information on the two additional **math teaching practices:** 1) Pose purposeful questions and 2) Elicit and use evidence of student thinking. How might these practices support classroom assessment?

An NCTM journal article,**12 Rules that Expire**, is included. This middle school math set of rules is a companion to the elementary version that can be found in the *CMT* Fall 2015 issue. No matter what level you teach, pre-k through college, glean insights into where your students have been and where they might go. In addition, other NCTM resources to consider for your professional library can be found within.

Perhaps your school has decided that with CCSS-M in place and mediocre PARCC results, it's time to reflect on your curriculum and adopt new resources. A **curriculum process** already designed with your use in mind–and presented to teachers in Denver Public Schools–is presented.

Did you miss the **CCTM Fall Conference**, or do you need to reflect on the ideas presented by the major speakers? Past and current articles, including one on P**romoting Access and Equity in Mathematics,** and links might help.

Honoring our own teachers of mathematics is important. Read what they've been doing for ideas you might try. Then think about applying for this award yourself in the spring!

Consider a 2016 New Year's resolution of writing an article for a professional magazine! Many of the authors whose articles you read in the *CMT* are uncomfortable with writing, but believe in the importance of sharing best practices with colleagues. Others are on a journey of improving their practice, creating an artifact to support Standard V: Teachers demonstrate leadership (*Colorado Professional Teaching Standards*), or pursuing National Board Certification or the Presidential Award for Excellence in Mathematics Teaching.

The newly formed *CMT* Editorial Board (see p. 53) will support you with ideas for revision, suggestions for pictures—and even help for grammar and structure!

The focus of the **next issue** will be: Support productive struggle in learning mathematics (from *PtA*.) This issue will also present ideas for your own summer professional growth, or to try out in summer school classes you might be teaching. We are looking for *in the field articles* on both what you have found instrumental in a "growth mindset," or an engaging summer school math class. Articles are due to me by **March 1, 2016**.

With your well-deserved winter break behind you, I hope your new semester brings you a fresh look at ideas presented in this magazine, and inspiration to try them out in your classroom.

President's Message

Joanie Funderburk, CCTM President

THIS ISSUE OF *Colorado Mathematics Teacher* focuses on assessment. Recently, there has been a lot of conversation about assessment at the state and national levels. Many of these conversations, particularly those external to educators, tend to focus on high stakes, summative state assessments. But as a classroom teacher, assessment refers to more than just these types of tests. We as teachers assess students every time we ask a question, have students share work on the board, review a homework assignment, or solicit student questions during a lesson.

Statewide summative assessments have an important place in the world of math teaching, but let's consider the more immediate and classroom-oriented practices that support learning. The eight Mathematics Teaching Practices in *Principles to Actions* serve as a guide to effective math teaching and learning, and in particular, *two* of the eight help teachers think about effective formative assessment: *Pose Purposeful Questions* and *Elicit and Use Evidence of Student Thinking.*

"Effective teaching of mathematics uses *purposeful questions* to assess and advance students' reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014, p. 35). We all ask questions all day long, in every aspect of our lives! However, when it comes to effective math teaching, questioning is quite complex, and when done well, can have a huge impact on students' understanding of important mathematical ideas. It is well worth the time teachers take to carefully craft questions and plan for their strategic use during a lesson. Good, purposeful questions can guide students to deeper levels of sense-making during math class, whereas poor ones can frustrate students and teachers alike.

In considering what is meant by "purposeful questions," teachers must attend to both the types of questions being asked, as well as the patterning of the questions. High quality questions will focus on revealing students' understanding of key mathematical ideas, and will lead to adjustments in the instruction as a result of student responses. The questions asked should gather information, probe thinking, make the mathematics visible, and encourage reasoning and justification.

Additionally, the patterning of questions is a component of effective questioning. When the questions force students into a singular path of thinking, or are disconnected from the student responses (a pattern called "funneling"), student understanding is less likely to be enhanced or developed by the questions. Instead, a question pattern that meets students in their current thinking and extends it toward important mathematical ideas (a "focusing" pattern) can be a powerful teaching tool during instruction.

Principles to Actions includes many sample questions and guidelines for teachers. In addition, my two favorite go-to sites for additional resources on this topic are: 1) Asking Effective Questions and 2) *Never Say Anything a Kid Can Say !* Here you can find sample questions to use with students, such as: What led you to that idea? and How does your idea connect to <another student>'s idea?"; and guidelines to consider while formulating your questions during lesson planning such as posing open-ended questions, using verbs that connect to higher levels on Bloom's Taxonomy, or attending to wait time.

Closely related to questioning is the teaching practice of *eliciting and using evidence of student thinking*. "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (NCTM, 2014, p. 53). Questioning, and other forms of formal and informal assessment, allow teachers to elicit evidence of student thinking, and use this evidence strategically in instruction.

It is important that students can find correct answers to math problems; however, correct answers are only one part of a deep understanding of mathematics. The Progressions Documents for the

Common Core Math Standards provide guidance for teachers in identifying indicators of student understanding. When the teacher has a learning trajectory in mind, as well as common pitfalls and misunderstandings that accompany the specific math concepts with which students will work, the required instructional adaptations become more fluid.

Using high-level tasks with strategically planned questions—at key points during the lesson—creates the opportunity for teachers to interpret and respond to students' learning during class. Effective teacher responses will guide students to a deeper conceptual understanding while supporting their advanced math reasoning. When these instructional methods are a regular part of the teacher's practice, students can become more adept at monitoring their own learning. When they can reflect on their mistakes or misconceptions and ask questions that add clarity and purpose, students are even more motivated and engaged in their math experiences.

Planning for effective questioning and determining how to elicit student thinking are great conversations for grade-level or PLC meetings. *Principles to Actions* and other books and articles from NCTM provide guidance as teachers engage in this work together. Teaching math is a craft, and when teachers collaborate to improve their classroom practice, students reap benefits in achievement, engagement, and enjoyment.

References

National Council of Teachers of Mathematics (2014). P*rinciples to Action: Ensuring Mathematical Success for All*. Reston VA: NCTM.

ASSESSMENT

Formative Assessments: You Have Options

Shelly Ray Parsons, Aims Community College

Formative assessment provides us with powerful opportunities to enhance and improve classroom experiences. Using what we know about students' learning through formative assessment can provide us with meaningful information about what they know, can do, and understand. As educators, we have the ability to empower our students every day in the classroom through active learning opportunities using formative assessment strategies.

Introducing Formative Assessment

Let's begin with a brief review of assessment in general. Assessment is frequently divided into three main categories: diagnostic, formative, and summative.

Diagnostic assessments typically occur prior to learning. They are used to determine a learner's prior knowledge and aptitude, and identify some characteristics such as strengths, weaknesses, knowledge gaps, and appropriate course level placement.

Formative assessments (FA) occur throughout the learning process and do not cause interruption to it. They serve as a tool to improve the quality of student learning and inform essential curriculum modifications. Formative assessments typically do not involve a grade or have a point value, are non-threatening in nature, and provide students with timely feedback. In addition, they have the power to provide feedback to the teacher about student understanding prior to any high stakes assessment.

Summative assessments are used to check the level of learning at the end of a lesson, unit, or course. These assessments are a formal measure of student learning and are relative to predetermined learning outcomes in the course. Summative assessments are often comprehensive in nature and carry a higher weight in the course. They are used to provide accountability at the end of a unit and, in mathematics courses, are frequently an exam.

For the purposes of this article, I will focus solely on formative assessment, highlighting ways of examining how students can improve the focus and depth of their conceptual understanding in our classrooms, thus improving student learning.

Formative Assessment for your classroom

Formative assessment can occur spontaneously, or it can be deliberately planned.

During a lesson a teacher may hear misconceptions that lead to a teachable moment. FA can also be intentionally designed to (1) elicit student thinking during the course of instruction in such a way that students uncover big ideas surrounding a concept, or (2) be embedded into the curriculum to solicit feedback at key points and highlight essential elements to provide deeper conceptual development. But whatever you do, focus your efforts on:

- Assessing prior knowledge and new learnings to plan instruction,
- Examining student work,
- Probing student thinking, and
- Providing appropriate feedback.

Prior Knowledge

Fundamental to any instruction is an awareness of what each student already knows and understands. The ultimate goal of any lesson is to deepen the students' knowledge and understanding of this concept—along with making connections to new material.

- Prior to starting your lesson for the day, consider the standard(s) you are addressing and determine what your students already know about the content in the lesson.

- Ask a variety of questions in order to understand the depth or absence of knowledge on the topic being taught. These questions can help determine how much students already know, what they understand, and what they are able to do with the material. The questions should be brief and take no more than 5 to 10 minutes depending on the length of your class period. They could be completed on a note card, at the board, or as part of a small group discussion.

Assessing Learning (so far)

Prior to students leaving class for the day, it is essential to assess learning, and this can be done without having to grade endless amounts of homework sets. Gallery Walks, Observation Protocols, and Round Robin activities all provide opportunities to do so.

Gallery Walks are a forum in which students rotate through a series of stations to demonstrate a level of mastery by posting a comment, a strategy, a question, or a solution. Critical thinking, written expression, and oral communication are all critical components. As a formative assessment, gallery walks may take an entire class period, can persist through a unit, or last only 10–15 minutes. Consider ways in which students share their work to help create a community of learners. To integrate a Gallery Walk into a lesson, first divide students into small groups and observe students as they discuss their work. Around the room, post a series of questions which require students to predict possible approaches to problems, analyze solution approaches, compare and contrast possible correct approaches to problems, and justify their thinking while making connections to previous mathematical experiences—all which require higher cognitive loads. All small groups should also address questions from other groups in the class. In doing so, each group can make connections between their group and the other groups' work.

Observation Protocols provide a format for evidence of student progress and inform instructional modifications or revisions based on this progress. Their focus depends on the intent of the particular activity. For example, students are working on a problem solving activity which requires they share their strategies, communicate their work both orally and symbolically, justify their answers both orally and in written form, and finally report their findings as a group to the class. A protocol can be designed to record each student's level of competence in each area, in addition to when a students get frustrated and gives up.

Round Robin Activities incorporate each student's work (turn) into a group solution. The key is that individual student work is still visible. This can be very beneficial as it makes the teacher aware of each student's struggles and misconceptions. These types of activities are designed to allow the teacher to assess each student as he or she works, intervene if necessary to address misconceptions or gaps in each student's understanding, and provide the student with immediate feedback.

Here is one example of how to use a Round Robin activity.

- Six problems related to a certain concept are written on the board (I have six whiteboards in my classroom). All students write down all six problems to work out again on their own, later.
- Six students are asked to come to the boards with one student per problem. Each student works the first step of the problem. The rest of the class can be assigned a certain problem and asked to work on it while these students are at the board, or simply watch what these students are doing.
- Students at the boards then rotate whichever direction the teacher indicates to a new problem. The students now work the second step to this problem.
- Students rotate in the same direction to a new problem and work the third step of this new problem.
- This continues until the problem is solved.
- Have students at the board discuss any errors, corrections, and misconceptions. Then ask students who are not at the board for additional comments or questions.
- Erase the board and write six new problems.

Have six students work these problems in the same fashion until all students in the class have been at the board.

- Some tips...
  - Try to use six different colors of markers so it is easy to tell which students are working.
  - If the previous student makes a mistake, do not erase. Circle the entire line of work, move below and do the correct work. This will be the work for that rotation.

Student thinking can also be formatively assessed with questioning and discourse strategies.

Probing Student Thinking

Take a moment and think about how you ask students questions during your classes. Do you engage all of the students, or are you having a conversation with a single student? Many times we call on only one student or pose a question, then wait for one or two (frequently the same students) to answer. These individual conversations allow other students in the class to disengage while one student and the teacher discuss the individual's response. Other approaches that can be used, and that you may already employ, requires active listening:

Focused Questions (specifically using Think-Pair-Share). First pose a question to the entire class. Have each student write down his or her thoughts, ideas, and/or work depending on the type of question posed. Be sure to allow enough time for students to compose an appropriate response. Then have students pair up to compare and share. This type of sharing is often called a Think-Pair-Share. Students can remain in pairs and share their thoughts with the class or broaden the dialogue by having students move into groups of four and then report out.

Hinge Questions are similar in nature, but depend on what students may be doing or saying while they are working in pairs, small groups, or individually. There are three forms of Hinge Questions to consider depending on your lesson: engaging, refocusing, or clarifying. As a lesson or activity unfolds, we often know where students are going to have difficulties. In preparing the lesson or activity, it will be helpful to have a series of relevant questions ready to ask students should they require assistance with engaging, refocusing, or clarifying with respect to the lesson or activity. Keep in mind the best questions require deep thinking, meaningful connections, and taking advantage of those teachable moments.

- *Engaging Questions* - When we begin any lesson or activity, we often have a series of basic questions to get students started. These questions direct students on the path needed to appropriately dissect a problem. Once we ask these types of questions and know that students understand the task at hand, we can then engage them further by asking questions that require examination of the problem in more detail, ask for individual contributions and group summaries, and produce results which support students' conceptual understanding—in addition to providing details about students' procedural approach. This questioning is the heart of formative assessment, assessing student understanding to properly adjust instruction.
- *Refocusing Questions* - How many times have you watched a student choose an approach to solving a problem that you know is incorrect? Often we want to "save" them the time and trouble so we intervene too soon. We must further our assessment of student understanding—even as they struggle to maintain lesson focus. When students drift off and discover they have chosen the wrong path, guide them

to a new path using refocusing questions. One question may not be enough, so be prepared with a series of questions that will refocus students that are off course—without just telling them what to do and how to do it. This formative assessment strategy will afford you knowledge of student understanding so that you can adjust instruction in the moment.

- *Clarifying Questions* - What teachers say and what students hear are not always the same thing. Clarifying questions can be used in many forms. The most basic version of a clarifying question is asking a student to repeat directions or to summarize an assignment. A more in-depth version of a clarifying question is to have students explain what they are thinking, either verbally or in a written format. Teachers can then experience a deeper look into student thinking about a concept.

Mathematical Discourse

Students of all ages are asked to communicate mathematics. To communicate what they know, students must also make connections. *Principles and Standards for School Mathematics*, published by NCTM in 2000, outlined the essential components of a high-quality school mathematics program, including Process Standards for Communication, Connections, and Reasoning and Proof. Probing student thinking via mathematical discourse makes use of these foundational elements.

Facilitating effective dialogue in the classroom can be a challenge. It falls on the teacher to ensure that classroom discussions focus on involving all students at a high level of cognitive demand with a focus on knowledge and reasoning about mathematical evidence, in order to achieve meaningful mathematical discourse. To achieve such a state, teachers of mathematics should pose questions that invite, engage, and challenge students' thinking while listening carefully to students' ideas and thoughts. Reflective listening statements and oral summaries will encourage more students to add to and redefine previous students' statements. To achieve deep and meaningful discourse, teachers must ask students to clarify and justify their thinking through multiple approaches and in both orally and written formats. Determining the depth of a discussion on a particular concept should be based on students' contributions during a discussion. Teachers must be attentive to include appropriate notation and structure as concepts are explored. Finally, it is essential to orchestrate successful discourse to determine when to provide information—rather than let students discover it on their own—when to correct or clarify misconceptions, and when to model or to lead—instead of letting students struggle to clarify something on their own.

Using assessment to plan instruction utilizes assessing prior knowledge, assessing students' work, and probing students' thinking. It also requires sharing feedback with students. Feedback can be given in a variety of forms, but its intent and approach should be examined prior to doing so.

Written Feedback

When returning student work, what do students look for on their papers? Are they looking for feedback or a grade? If the grade is good, students will typically keep that paper. If the grade is poor, what do student do with it? While they should keep it and determine where and why their errors were made, typically the paper with the poor grade is thrown away.

Consider this...Does every paper need a grade or can feedback be provided in such a manner as to promote a focus on the learning, rather than the grade? If written feedback is given in place of a grade, students will read the feedback and give more serious consideration to what is being said. To have a more positive impact on student learning, more informative and less judgmental feedback needs to be provided.

Of the feedback that can be given, we must consider the type that best addresses the outcomes we seek:

Motivational feedback helps learners feel that their work is recognized, allows learners to know they are making progress, and encourages and supports them.

Evaluative feedback measures student achievement through a grade or with a score, and summarizes achievement.

Descriptive feedback gives information about the work—not the student—in a neutral timely manner, is constructive in nature, and is based on accurate standards for performance.

Effective feedback moves a student's understanding forward, provides the necessary support for cognitive connections and internalization of approaches to learning, allows learners to move their reasoning to a higher level but at their own pace, places the focus on mastery and may allow student to revise or redo work, and encourages self-reflection and metacognition. In planning your instruction, also plan your approach to giving feedback. The type of feedback given may impact how students approach and engage with an activity or lesson. Do they feel supported and encouraged through the learning process? Do they understand productive struggle is a foundational part of deep and meaningful learning?

Conclusion

Incorporating formative assessment strategies in the classroom can provide educators with a significant amount of information to both improve and enhance student learning. However, it is crucial to be both purposeful and intentional in your choice of assessment. Be cautious about overwhelming yourself and your students with new approaches. It takes time to seamlessly integrate new assessment strategies into your lessons. A suggested approach to expanding one's formative assessment repertoire would be to choose one new formative assessment. Add it to your educational practice to inform and improve student learning. Perfect it, and then incorporate another.

ASSESSMENT

Formative Assessment: What We Don't Learn from "Just Answers"

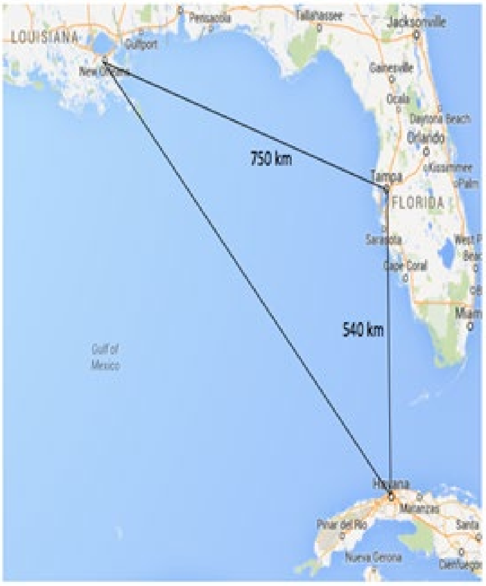
Shelbi K. Cole, Student Achievement Partners

High stakes summative assessments often use test questions that are proxies of the expectations outlined in standards as evidence of students' proficiency in mathematics. While these proxies can provide valuable information as part of a snapshot of student learning in the context of summative assessment, this value does not always neatly translate to classrooms in support of formative assessment. In 2006, the Council of Chief State School Officers (CCSSO) Formative Assessment for Students and Teachers State Collaborative on Assessment and Student Standards (FAST SCASS) defined formative assessment as "...a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievements of intended instructional outcomes." Defined this way, we should make a clear distinction between "testing" and the broader term, "assessment." We might even make the claim that there is no plural form of "formative assessment." The moment you hear the plural "formative assessments," it implies that the phrase is no longer defined by a process, but rather by more frequent testing. This article was written using the definition of formative assessment offered by the FAST SCASS, and will focus on a single mathematical modeling problem to highlight some important differences between formative assessment and other types of assessment.

Mathematical Modeling Problem and Student Responses

Consider the mathematical modeling problem shown in Figure 1. The problem was originally developed as part of an effort to illustrate the standards in the Smarter Balanced Assessment Consortium's[1] Item Specifications document, which now includes over one thousand examples of mathematics summative/interim assessment questions for grades 3–11. To determine whether to include the item as shown (i.e., with no explanation required), the problem was administered to eighty-five grades 4 and 5 students. The version administered to students required an explanation, but the version presented in Figure 1, intended for the summative assessment, did not require an explanation. The purpose of the small scale administration was to evaluate the information, or evidence of student learning, that might be lost by not asking students to write about the mathematics that they were using to provide a reasoned estimate in the problem. The alignment of the problem to the Common Core State Standards for Mathematics, also shown in Figure 1, illustrates that mathematical modeling problems often ask students to apply skills that they have developed over multiple years of learning. The ability to compare the lengths of three line segments is an expectation of the grade 1 standards, while the actual comparison given the specific measurements provided in the problem raises the problem to about grade 4. The problem was also given to grade 5 students to ensure that the 4th grade standards could be classified as securely held content for at least part of the sample.

1 Smarter Balanced Assessment Consortium (SBAC) is the counterpart to PARCC. For futher information, go to: http://www.smarterbalanced.org/smarter-balanced-assessments/



The distance between New Orleans and Tampa is about 750 kilometers. The distance between Tampa and Havana is about 540 kilometers. Estimate how far it is between New Orleans and Havana.

Alignment to CCSS-M:
MP 4. Model with mathematics.
MP 1. Make sense of problems and persevere in solving them.
MP 2. Reason abstractly and quantitatively.
1.MD.A.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.
4.MD.A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
4.NBT.A. Generalize place value understanding for multi-digit whole numbers.

Figure 1. Tampa to Havana problem and Common Core alignment.

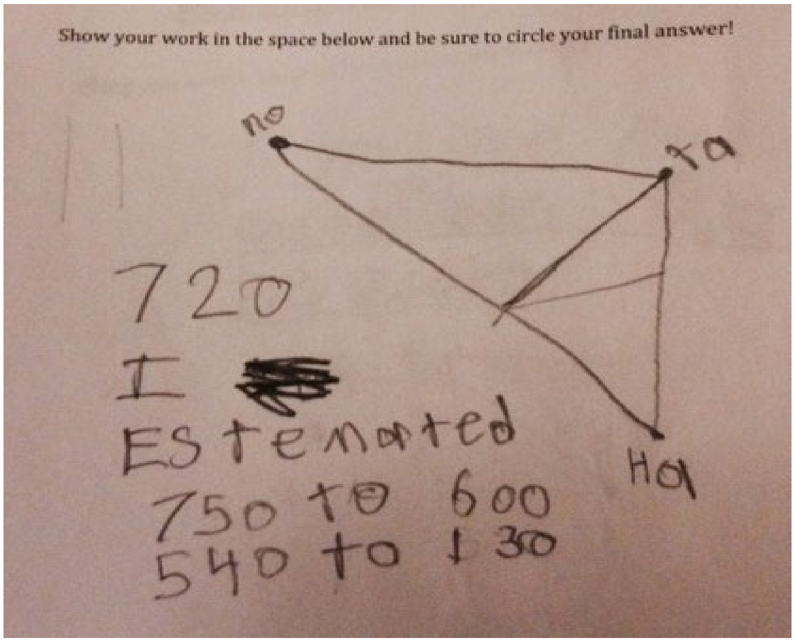
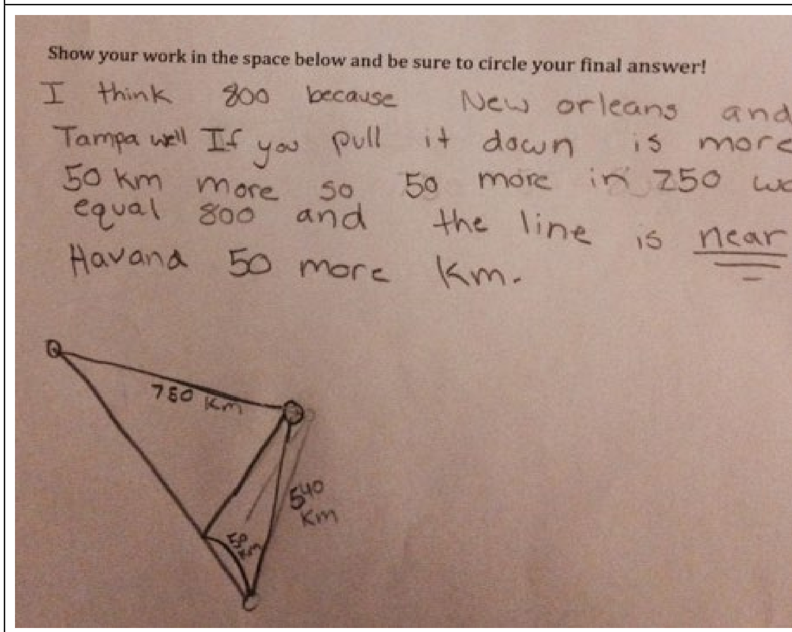
First, let's examine five student responses as they would appear absent of any explanation of their thinking: 720, 800, 870, 1000, 1290. Table 1 provides some potential inferences that a teacher might make given just these answers to the problem.

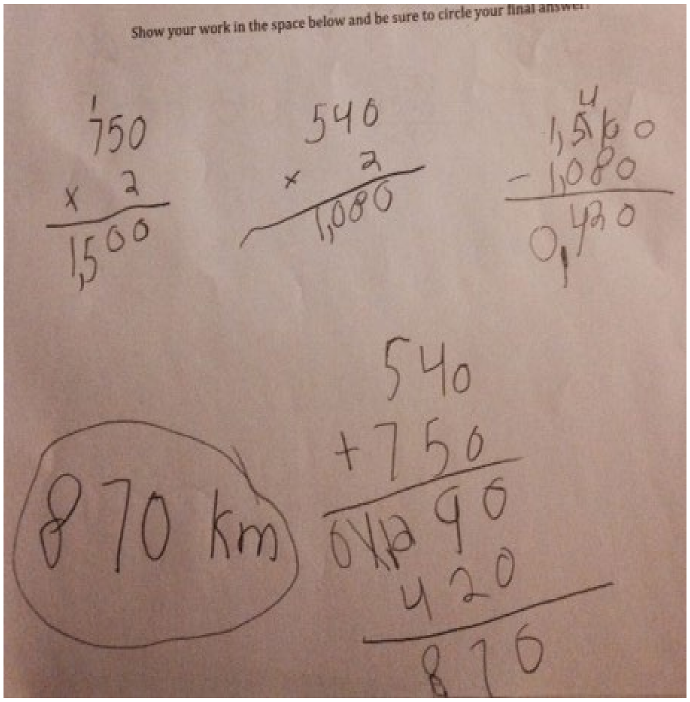
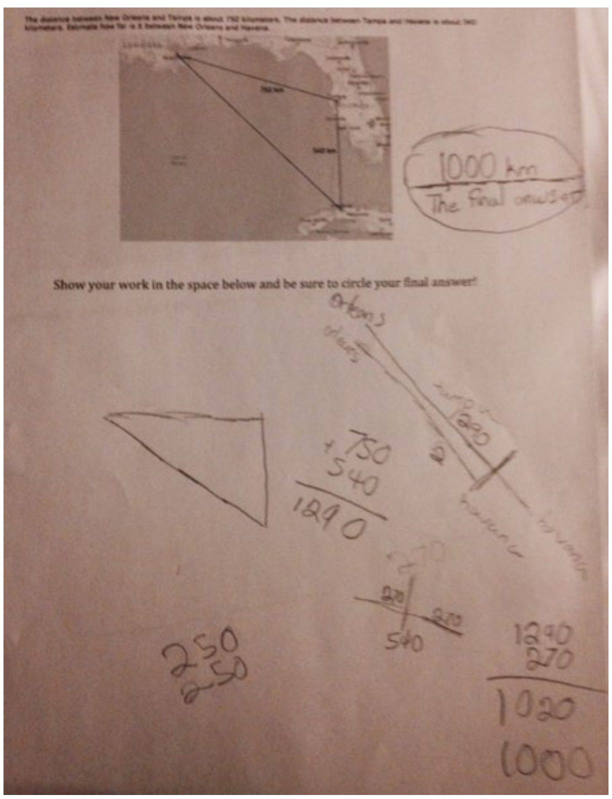
Table 1
Answers without Explanations and Potential Inferences

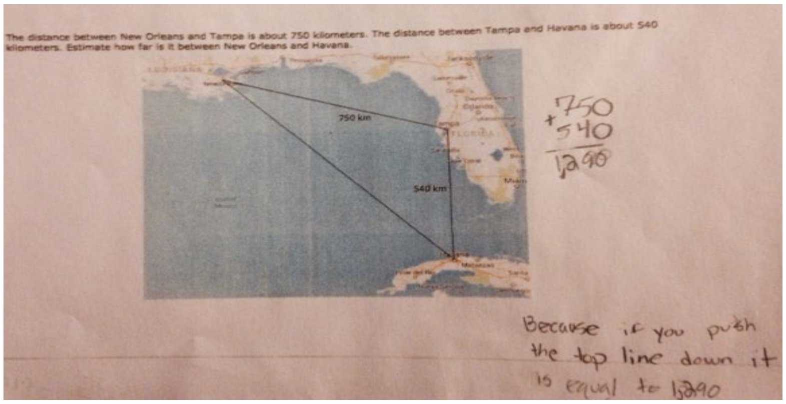
Answer	Potential Inferences Based on Answer
720	Student doesn't recognize that the unknown side length is longer than 750 km.
800	Student recognizes that the length is more than 750 km and less than 1290 km, but underestimates a bit.
870	Student is within a reasonable range for grades 4-5.
1000	Student is within a reasonable range for grades 4-5.
1290	Student seems to have just added the two numbers. He/she may have applied a methodology that works with other problems that "look" like this one, rather than making sense of the problem and given information in the context of the problem.

Now, let's examine the actual student work on which these responses are based. In each case, there is more that can be inferred about student understanding from the work students showed than could be gleaned from the answers alone.

Table 2
Student Work and Potential Teacher Inferences for Problem from Figure 1

Answer	Student Work, Teacher Inferences and Follow Up
 720	We see from the student's drawing that he dropped a perpendicular to partition the longest side. He then estimated the length of each segment of the partition, but provided estimates that were too low and then an incorrect sum of the segments. The student appears to have deployed a potentially successful (even sophisticated) strategy, but did not assess the reasonableness of his answer. A teacher may want to follow up with this student to get more information about the decisions he made while solving the problem.
 800	This student creates an isosceles triangle to replicate the 750 km length on the unknown side. She then provides an estimate of the remaining portion of the side length, which falls a bit short of being "close." This is exacerbated by the student's drawing not mirroring closely the original. This student appears to have a sophisticated understanding of the mathematics computations and strategies needed for this problem. The teacher may want to follow up to help the student find ways to get a more precise estimate of the 50 km.

 870	The mathematics seems to be an arbitrary collection of computations that lead to what could be considered a reasonable estimate of the unknown length. Does this student have a sense of what the "answer should be," but doesn't know how to articulate it or is something else going on here? The teacher could follow up with questions to find out more from the student about his thinking during each step.
 1000	This student sums 750 and 540, then appears to recognize that the length is less than that sum so divides the 540 in half, getting 270. He then subtracts the 270 from the "too big" 1290 and gets 1020. At the end, the student rounds this to 1000. After a mathematically sound process and reasonable estimate, the final step may represent some confusion between estimation and rounding.



1290

Although this student does what many others did (simply add the two numbers), she adds a note at the bottom of her paper that suggests that there is a bit more going on here. Her note, "Because if you push the top line down it is equal to 1,290" suggests that she may be visualizing "straightening out" the shorter two segments into a single segment. The teacher might offer physical objects, such as spaghetti, that the student could use to represent the 750 km and 540 km lengths to help her model pushing the "top line down."

From Individual to Group Information

While we can learn a lot about individual students from their mathematical explanations, we can also use item level classification analyses based on groups of students to make inferences about changes to instruction and/or curriculum. If we take the same problem as shown above and examine the group results classified based on students' approaches to the problem, we begin to get a picture of general trends in the kinds of strategies students are using and a sense of some of the more global misconceptions that students have about the mathematics.

Table 3
Classification of Student Strategies at the Group Level

Strategy	Student Response Classification	Number of Students (n=85)
1	Subtracted to find distance	5
2	Subtracted to find distance after rounding both numbers	2
3	Added to find the distance (including responses with minor computation errors)	33
4	Added to find distance and then rounded sum	9

5	Rounded both numbers to the nearest hundred and then added to find distance	11
6	Estimated distance at greater than 750 and less than 1290 using valid mathematical strategies	14
7	Used other not valid computational strategies (e.g., multiplied 750 and 540)	8
8	Used a potentially viable strategy, but estimated less than 750	2
9	Estimated distance at greater than 750 and less than 1290 using mathematical strategies that do not appear valid	1

At least two of the classified strategies stand out as potential curricular/instructional issues. First, 33 out of 85 students simply added the two numbers shown in the problem. This implies that a large number of students seem to dive into the problem with a familiar strategy without making sense of the problem. These students may need more work on Standards for Mathematical Practice MP1: Make sense of problems and persevere in solving them. Students may be accustomed to solving perimeter problems where all of the side lengths are given, and may be inclined to use the strategy: add up the numbers you see. Unfortunately, that strategy does not work here and this problem has both an unknown perimeter and an unknown side length, which takes it from a more general problem solving question into the mathematical modeling category. Students need opportunities to grapple with problems where reasonable estimates are required as part of the problem solving process. In early grades, students should be presented with questions like, "What else do I need to know to solve this problem?" In a growing arc of sophistication with mathematical modeling and as students progress through mathematics, they should encounter more and more problems in which making assumptions, weeding out extraneous information, and retrieving information from external resources are expectations of the problems they are solving.

Another notable trend looking at the table is the number of students who thought that rounding was an essential component of the problem solving process. Strategies 2, 4, and 5 indicate that at least 22 students believed that rounding was an important component of this problem. In this case, teachers may want to evaluate whether "rounding" and "estimation" are being used synonymously either during instruction or in the school's curriculum resources, or whether students are only being presented with problems that ask them to round, and not being asked to think more broadly about the verb "estimate." This problem highlights an important use of the word "estimate," where it is asking students to provide an estimate of a distance within a reasonable range of the true distance from Havana to New Orleans. A reasonable expectation based on the grades 1 and 4 content standards in conjunction with the Standards for Mathematical Practice would be that students could recognize that the distance is greater than 750 km and less than 1290 km.

Conclusion

Ultimately, content experts representing the states in the Smarter Balanced Assessment Consortium decided not to include the problem in the Grades 3-5 Item Specifications. Since the grades 3-5 computer adaptive test does not currently include problems that require explanations (although the performance task section does), the content experts felt that too much information was lost from this particular problem without the student explanations, including some students who would have gotten the problem correct without the required mathematical understandings and other students who would have gotten the problem incorrect who seemed to have sophisticated mathematical modeling strategies.

Although this problem did not make the cut as a proxy for student performance within the summative assessment, it is useful in highlighting the difference between students giving just answers and students providing mathematical explanations of their thinking. This problem and student responses illustrate the need to continue to require students to demonstrate their mathematical thinking in writing, even when summative assessments use other proxies for mathematical thinking. The formative value of the work that students do provides meaningful information about individual student understanding of the mathematics, as well as group level information that may highlight the need to revisit and modify curriculum and/or instruction.

We often treat the terms “testing” and “assessment” as synonymous. It is important to recognize that assessment is a much broader term that encompasses all of the activities that educators use to understand student learning and processes that allow students to understand their own learning. Establishing a purpose first and foremost for the things that we ask students to do, can help us ensure that what they produce will lead to useful information that drives teaching and learning.

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ASSESSMENT

Developing purposeful questions and analyzing student reasoning: Two tools

Frederick Peck, University of Montana & Jessica Alzen, Derek Briggs, and Raymond Johnson, University of Colorado

In this paper we introduce two tools to help teachers develop purposeful questions and collaboratively analyze student reasoning. We developed these tools during a two-year research-practice partnership between researchers from the Center for Assessment, Design, Research, and Evaluation (CADRE) at the University of Colorado, and elementary, middle, and high school math teachers in Colorado (due to logistical conflicts, the middle school teachers only participated for one year).

Together, we developed a framework for learning and assessment called the *Learning Progression Framework* (LPF). The framework has its roots in the National Research Council's (2001) report, *Knowing What Students Know*. This report introduced the concept of the "assessment triangle," consisting of three interconnected elements (represented as vertices) that should be the basis for any high quality student assessment: (1) the *cognition* vertex is a model of how knowledge develops, (2) the *observation* vertex is a method of collecting evidence about student cognition (e.g., tasks or other observable activities), and (3) the *interpretation* vertex is a method of making inferences about the observations with respect to the model of cognition.

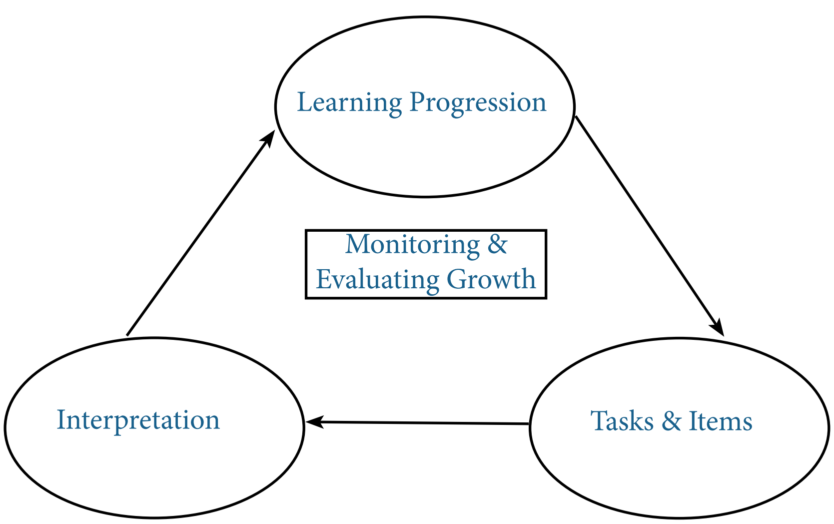


Figure 1. The learning progression framework (LPF).

In the LPF, we operationalized the assessment triangle using *learning progressions* (LPs; Anderson et al., 2012; Clements & Sarama, 2004; Daro, Mosher, & Corcoran, 2011) as our models of cognition, as shown in Figure 1. Learning progressions are "empirically supported hypotheses about the levels or waypoints of thinking, knowledge, and skill in using knowledge, that students are likely to go through as they learn mathematics" (Daro et al., 2011, p. 12). Learning progressions are often created by researchers in mathematics education, after years of careful study of how students learn a particular topic. In our collaboration, teachers combined these researcher-created progressions with the progressions inherent in the Common Core State Standards for Mathematics to create *conjectured learning progressions* in a single domain at each level: place value in elementary school, proportional reasoning in middle school, and algebraic manipulation in high school. As the arrows in Figure 1 make clear, in the LPF we do not consider these learning progressions to be fixed. Rather, they are conjectures about how learning happens, and as such they can be (and were) refined over time based on teachers' observations of student learning.

In this paper, our focus is on the other two pillars of the assessment triangle, so we will not discuss the process of creating or refining an LP further here. A complete discussion is available in the reports on the CADRE website, http://www.colorado.edu/cadre/learning-progressions-project.

For the observation and interpretation pillars, we developed and refined **two tools** during our collaboration: **A task and assessment analysis tool**, which is primarily focused on the observation pillar; and a protocol for collaborative, structured conversations of tasks and student reasoning—focused on both the observation and interpretation pillars—called **student focus sessions**. In this paper we describe these two tools, and discuss how teachers used them to create purposeful questions and engage in collaborative and purposeful analysis of student reasoning.

Task and Assessment Analysis Tool

The **task and assessment analysis tool** describes five *considerations* that emerged as being especially important for developing purposeful questions:

1. *Relevance to the learning progression:* the extent to which a given assessment task and its scoring rubric are likely to provide evidence relevant to the LP.
2. *Options for expressing understanding:* whether the task provides students with only one way to express their understanding (such as with a closed-ended problem like multiple choice or fill-in-the-blank, or tasks that ask for direct applications of routine procedures), or multiple ways to express their understanding (such as with open-ended problems that ask for multiple representations of a solution, or a task that asks for a mathematical procedure with a written justification).
3. *Cognitive demand required:* the extent to which tasks ask students to engage in high-level cognitive processes. There are four levels of cognitive demand (Stein, Grover, & Henningsen, 1996; Stein, Smith, Henningsen, & Silver, 2009):

- Level 1: Tasks that rely primarily on *memorization*.
- Level 2: Tasks that ask students to execute well-known *procedures without connections* to the underlying concepts.
- Level 3: Tasks that ask students to execute *procedures with connections* to underlying concepts.
- Level 4: Tasks that engage students in *doing mathematics*, which includes "framing problems, making conjectures, justifying, [and] explaining" (Stein et al., 1996, p. 464).

4. *Rubric quality:* including:

- Rubric reliability: Indicates whether there is a high probability that the task could be scored reliably by any teacher in the respective area and grade level.
- Rubric validity: Indicates that: (a) the rubric covers everything that students are asked to do (e.g., if the task asks students to "show work" the rubric gives guidance as to how to score the work), and (b) the rubric comprehensively covers the range of possible student responses. If there are multiple possible responses, the rubric gives guidance as to how to score likely or common responses.
- Rubric specificity: Indicates that all adjectives and general statements (e.g., "shows understanding" or "solves problem correctly") in the rubric are accompanied by specific descriptors related to the problem. For example, if the rubric says "solves problem correctly" the correct answer(s) for the problem is given in the rubric.

5. *Accessibility*, including:

- Fairness: Indicates whether the material is familiar to students from identifiable cultural, gender, linguistic, and other groups; is free of stereotypes; can be reasonably completed under the specified conditions; and if students will all have access to resources necessary for task completion (e.g. Internet, calculators, etc.).
- Clarity: Indicates whether the wording in the task and instructions are clear; grammatically correct; and free of wordiness, irrelevant information, unusual words, and ambiguous words.

Teachers used this tool to analyze existing tasks, identify weaknesses or gaps, and take action to make improvements. For example, in a session in the beginning of the second year, the elementary teachers used an early version of the tool to analyze an assessment provided by the district. At first, many questions on the assessment appeared be aligned to the place value LP, including two tasks that asked students to create an addition expression equal to a given teen number (e.g., ____ + _____ = 19). However, as the teachers analyzed these questions, they found that the tasks were not well aligned to their learning progression and hence would not support related inferences about student knowledge and understanding (consideration 1). As they discussed the task, they realized that the key aspect from a place value perspective was decomposing the teen number into tens and ones, and that the "blank plus blank" task may not give teachers evidence about a student's ability to decompose a teen number in this way. A teacher explained to her colleagues:

> Our [learning progression] is composing and decomposing a teen number, breaking it into ten plus how many ones, whereas these are just blank plus blank. Do you know what I mean?

In this way, the first consideration helped teachers scrutinize tasks for particular mathematical content, and helped teachers make purposeful selections given their content objectives. Ultimately, the teachers found that none of the items on the district assessment were aligned to the place-value LP, so they examined other resources and found tasks that were more targeted.

In high school, teachers had created a bank of assessment tasks during the first year. These tasks were largely procedural, asking students to engage in routine—if often difficult—algebraic manipulations to solve for the value of a variable given an algebraic equation. They provided students with little opportunity to express understanding in more than one way or to make connections to underlying concepts, including properties of equality, properties of operations, and the meaning of solutions to algebraic equations. In the second year, the teachers used the task and assessment analysis tool to improve these questions by providing students with multiple ways to express understanding and by asking students to link the procedures with underlying concepts. For example, the teachers discussed single-variable equations with infinite or no solutions (e.g., $2x + 4 = 8 + 2x$, which has no solutions). They suspected that students often execute a solution procedure correctly, without understanding what the result of the procedure (e.g., $4=8$) means. To assess whether students could link the procedure to the underlying concept, they asked students to solve the equation, $2x + 4 = 8 + 2x$, and then explain the meaning of the solution.

As the teachers created questions that asked students to make connections in writing, they were concerned that analyzing and scoring student responses would be "too subjective." They wanted to analyze student reasoning, but they did not currently have a structure that enabled them to do so collaboratively. To address this, we developed a protocol for collaborative analysis of student reasoning called **student focus sessions.**

Student Focus Sessions

Student focus sessions are conversations that are structured to enable collaborative analysis of student reasoning. They are designed to be conducted by groups of teachers. Below, we outline the main features of student focus sessions. A reference guide written for teachers that describes the process in detail, is available at: https://www.colorado.edu/education/node/1791/attachment.

Student focus sessions have three goals: (1) to learn more about how students are reasoning about tasks, (2) to design instructional moves and classroom activities that are responsive to student reasoning, and (3) to improve the reliability and validity of assessment tasks and rubrics. In a student focus session, teachers examine approximately five examples of student work on two tasks from a common assessment. Although there is no hard-and-fast rule about the quantity of student work, we found five students and two tasks was a sufficient amount of student work to represent a range of diverse responses, while being small enough to enable deep discussions about each student's reasoning.

Student focus sessions have two phases, each lasting about one hour. They can be held in a single two-hour session, or they can be broken into two one-hour sessions in order to fit into the one-hour meeting times that are common in many schools. Again, there is no hard-and-fast rule about the timing, but in our experience this timing worked well.

Phase I

The goal of Phase I is to improve the reliability of task scores by revising tasks and rubrics so as to minimize ambiguity in scoring rules. In this phase, all participants score the student work on common tasks. They then examine any instances where there is substantial disagreement in their scores. They discuss these disagreements, focusing closely on student reasoning, and arrive at a consensus score. They then discuss ways to modify the tasks and/or rubrics so that such scoring discrepancies can be minimized.

Teachers' discussions in this phase often centered on clarifying vague terms used in rubrics. For example, the high school teachers discussed the task and rubric shown in Figure 2.

Task:
Solve for b_1 : $\frac{(b_1+b_2)}{2}H = A$

Rubric:

Description	Score
Completely and correctly solves for b_1	2
Generally appropriate strategy, however b_1 may not be completely solved for or there may be algebraic mistakes.	1

Figure 2. A high-school task.

Notice that the description for score level 1 in the rubric includes the term *generally appropriate strategy*. The teachers discussed the need to clarify this term. In their conversations, the teachers used the term, "good algebra", as shown below:

Teacher A: What would you define as "good algebra?"

Teacher B: In a multiple step problem, multiple steps... I mean, I don't-

Teacher C: It's impossible to define.

Teacher B: Yeah.

Teacher A: Right, but like, what mistakes could they make to get a one?

Teacher D: I think the one I described, where they put it all over h (referencing an earlier part of the discussion).

Teacher E: So we just need to define it better in the rubric. And show what mistakes are okay. (crosstalk) It IS a common mistake that they divide the whole thing by h, not just the $2a$, but $2a$ minus b_2 over h. That's a reasonable mistake that they're gonna make. So I think we take out the words 'good algebra' and say these are the- this is what we're looking for.

Of particular interest here is the way that the teachers, in searching for consensus, do more than clarify an ambiguous term like "good algebra." In addition, they clarify for themselves what, exactly, they are looking for in the problem. This was a common occurrence in student focus sessions, and at the end of the project many teachers commented on how student focus sessions helped to make tasks more targeted. A high school teacher explained:

You really need to ask yourself, 'what are you trying to understand about their [students'] understanding?' Because you can change a task in the most- in such a small way, and suddenly you're addressing a totally different issue.

Phase II

Phase II has three goals: (1) to improve the validity of the tasks by strengthening the connection between the task and the learning progression, (2) to generate a deep understanding of each student's reasoning, and (3) to develop responsive classroom activities.

First, participants qualitatively analyze students and tasks with respect to the LP. They place students in order with respect to the LP based on a holistic analysis of each student's work, and they place tasks in order of difficulty with respect to the LP based on a holistic analysis of the student reasoning on each task. After coming to a consensus ordering of both students and tasks, they compare this ordering to the ordering inherent in the quantitative scores from Phase 1. If the orderings do not match, this likely indicates that there are important distinctions in student reasoning that are apparent to the teachers, but which are not being captured by the rubric. Participants discuss ways to improve the validity of the task and rubric by making sure that the rubric captures these distinctions.

Participants then focus on understanding each student's reasoning. For each student, they analyze the student's work on both tasks and use this analysis to create a narrative summary of the student. They then use this summary to devise instructional strategies that build on the reasoning and understandings that the student demonstrates in order to help her move along the learning progression. In this way, the instructional strategies gain nuance and go beyond simple decisions to "re-teach or move-on." As one veteran high school teacher explained at the end of the project:

> I started looking more directly at their [students'] work again. I mean I did that a long time ago, but what this has helped me do when I look directly at their work I don't teach a whole concept, I say 'okay this is where I notice a lot of kids are stumbling.' So 'you guys know a lot more than you give yourself credit for, so keep doing what you're doing, and that's where you've got to get a little more focused.'

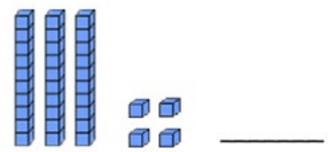
Similarly, an elementary teacher explained how student focus sessions helped to focus her instruction:

> [W]e did a student focus session around the last task, the second to last task that we had given students, kinda dealing with 10 frames and decomposing numbers, and I think that ... it helped us to see exactly what students were missing so to really look at, you know, what concepts they understand and what we need to hit back on.

As described by the teachers above, the conversations in student focus sessions help prompt teachers to focus on student *reasoning*, as opposed to simply determining whether an answer is correct or incorrect. In our experience, some of the best conversations happened when teachers had to confront two students who both had the correct answer or both had an incorrect answer, but whose reasoning was qualitatively different. For example, the elementary teachers discussed the task shown in Figure 3.

Task:

a) What number am I?



b) What number would I be if there were 7 more ? ______

Student responses:

Randy:
a) 34
b) 8

Salvador:
a) 34
b) 42 *(including drawing 8 cubes onto the figure in part a)*

Figure 3. An elementary school task.

As shown, two students, who we'll call Randy and Salvador, each wrote 34 for part (a). Randy wrote 8 for part (b), while Salvador drew 8 cubes onto the figure in part (a), and wrote 42 for part (b). Both of the students had an incorrect answer for part (b), and using the original rubric—which focused solely on whether the students' answers were correct—both students had the same score. However, the student focus session prompted teachers to look closer at each student's reasoning. Even as some teachers argued that the score on the tasks should be based entirely on correctness, they all agreed that Salvador showed more sophisticated understanding of place value (for example, Salvador correctly grouped 10 ones into one ten, and accurately adjusted digits in both the tens and ones places). Furthermore, even if the teachers disagreed about whether this distinction should be captured in the score, they all agreed that this sort of analysis of student reasoning was important for instructional purposes. During the discussion, one teacher captured the sentiment in the room:

> So I think that, what the student was thinking and us being able to look at these two students, as a teacher and have that direct my instruction, I'm able to say, okay, I know that Salvador has a better understanding of this than Randy. So when I group my students I'm going to group them differently and my instruction is going to look

> different for these two students. But as far as my data tracker goes, I guess I'm not sure how that is going to look when they're both wrong answers.

Over the course of the project these conversations started to have an effect on grading practices. One high school teacher explained the effect of student focus sessions on grading practices in the math department:

> I think we've all kinda gotten past the point of right and wrong answers, versus, observing, you know, what– not so much common mistakes, but different thinking kids have through the problem.

Similarly, an elementary teacher described how she struggled between scoring a task based on correctness vs. the sophistication of student reasoning. Ultimately, she scored the task based on the student's reasoning:

> I struggled with do I give this student two full points for their explanation or 0? I ended up giving him 2 because I think he explained using 10s and 1s. He just explained the wrong number. [...] I was like 'can he show the concept that I'm asking? That he understands the concept?'

Student focus sessions are powerful because they give teachers an opportunity to collaboratively engage in analysis of student reasoning. They also support *Principles to Actions* (National Council of Teachers of Mathematics, 2014) mathematical teaching practice: "Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning." These conversations lead to nuanced understandings of students and more responsive classroom instruction, and they seem to have an effect on teachers' grading practices. However, they require dedicated and repeated time throughout the year. Each session takes two hours, and the process should be completed multiple times over the year. We found that it was unrealistic to expect teachers to conduct these sessions unless they were provided with dedicated time and support to prepare for, conduct, and follow-up with the sessions. In some cases, this support may include having a math coach act as a facilitator for the session.

Conclusion

In this paper, we introduced two tools that can help teachers create more purposeful questions and collaboratively analyze student reasoning. The *task and assessment analysis tool* describes five considerations that help to make tasks more purposeful. *Student focus sessions* allow teams of teachers to have structured conversations about student reasoning, leading to improved assessment tasks, deeper understanding of students, and more-responsive classroom activities. Together, these tools can help teachers create assessments that are grounded in the assessment triangle, and create stronger links between learning and assessment in their classrooms.

Both tools are ready to be used by other teachers, and both are available on the CADRE website: http://www.colorado.edu/cadre/learning-progressions-project.

Acknowledgements

We gratefully acknowledge the teachers who dedicated hundreds of hours to our collaboration. Although they have to remain anonymous, their contributions to the project are manifest throughout this paper.

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IN THE FIELD

The Curriculum Review Process: A Powerful Learning Opportunity

Laura Neuberg, Curriculum Specialist, and Juli Lenzotti, Math Academic Manager, Denver Public Schools

Are You Able to Participate in Curriculum Adoption?
(Note from the Editorial Panel)

In this article, the authors describe the value and learnings that come from participating in a curriculum review and adoption process. However, we recognize that not all educators have this opportunity. Some districts support and seek out teacher contributions to this process, while others complete the process entirely at the central administration level. As an educator, I have been a part of both types of districts. I have been fortunate enough to have felt valued as I have participated in curriculum adoption in some districts, while I have also felt the frustration in my inability to give input or influence curriculum decisions in others. With that being said, this article presents beneficial information for all educators despite their ability to contribute to decisions. First of all, this article introduces many resources that would provide professional development opportunities for any teacher of mathematics. Second, all teachers, no matter the curriculum used or district worked for, make curricular decisions as they teach. This article provides a good framework for deciding what lessons/materials/activities/etc. are valuable and worth precious classroom time. The questions that the committee asks are questions that we should be asking on a daily basis when teaching. I hope that, as you read this article, you feel empowered to make better decisions regarding the curriculum you teach every day. - Tessa Ziser

"ADOPTION OF CURRICULUM MATERIALS is one of the most important decisions a teacher, school, or district can make." - Diane Briars

How do you bring a team of twenty-seven teachers and district support staff together to reach consensus in recommending math curriculum? As members of our district's math team, we began to realize the importance of Diane's statement at the outset of the curriculum adoption process for elementary mathematics. We saw this as an opportunity to build a common language and understanding not only of content and practice standards, but of effective mathematics teaching practices. In this article we describe the curriculum review process, the professional learning in which we engaged, and the outcome of our group's learning.

Session 1

As everyone gathered for the first work session, the diversity of the group was obvious: teachers representing grades K–5; representative teams from the English Language Acquisition (ELA) Instructional Practices, Student Services, Early Childhood Education (ECE), and Educational Technology; a teacher effectiveness coach; an administrative intern; and math team members. All were committed to the task of reviewing and recommending standards-aligned curriculum resources to best meet the needs of all students to achieve the Common Core State Standards–Mathematics (CCSS-M).

What Is Curriculum?

The meeting began by developing a common understanding of curriculum using "The Notion of Curriculum" (Niss, 2014). In this presentation, Niss identifies and describes six facets of curriculum within an educational setting: goals or standards, content, materials, forms of teaching, student activities, and assessment.

The Notion of Curriculum

"The curriculum can be seen as an amalgam of goals, content, instruction, assessment and materials." (Kilpatrick, 1994)

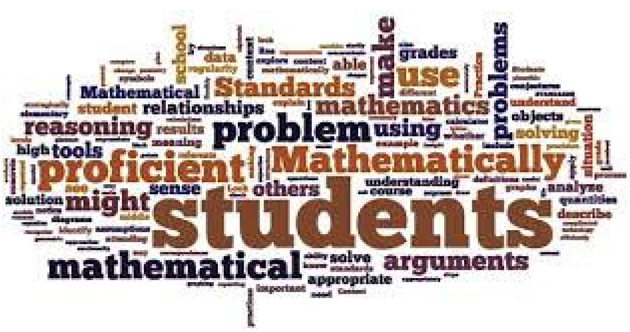
". . . we use the term curriculum broadly to include mathematics materials and textbooks, curriculum goals as intended by teachers, and the curriculum that is

enacted in the classroom." (Stein et al., 2007)

Six Facets of Curriculum within an Educational Setting:

- *Goals or Standards: overarching purpose, desirable learning outcomes, specific objectives/aims*
- *Content: topic areas, concepts, methods techniques, procedures*
- *Materials: instructional materials and resources, including textbooks and IT-systems*
- *Forms of Teaching: tasks, activities and modes of operation of the teacher*
- *Student Activities: activities of and tasks and assignments for students*
- *Assessment: goals, modes, forms and instruments of formative and summative assessment*

Committee members' conversations about this text focused on how their view of curriculum was expanded, and they identified implications for thinking about the materials to be reviewed.



How Is Quality Curriculum Identified?

The next stop in our grounding conversations about curriculum explored "Curriculum Materials Matter: Evaluating the Evaluation Process" (2014). In it, Diane Briars, National Council of Teachers of Mathematics (NCTM) President, discusses "Top Lessons Learned" during her tenure as mathematics director for the Pittsburgh Public Schools. Briars' seven key tenets about the effective evaluation of curriculum materials provided a bridge from the theory of "The Notion of Curriculum" to the practical aspects of reviewing curriculum. They included:

1. *Focus on the central evaluation question: What curriculum materials best support students' learning of the standards?*
2. *Remember that content analysis is much more than alignment.*
3. *Analyze the nature of the instructional tasks and activities—this is as important as analyzing content.*
4. *Focus initial reviews on student materials and teacher editions of the materials.*
5. *Consider equity, diversity, and access.*
6. *Recognize that all omissions or gaps are not the same.*
7. *Recognize that additional content is less problematic than gaps that are difficult to fill.*

Committee members read Diane's message and the specifics for each of the lessons learned, making note of new ideas for consideration throughout the curriculum review process. The second piece of the "What-is-curriculum?" puzzle grounded the group's conversations in the nuts and bolts of what to consider, and narrowed the focus to a discussion of the importance of quality tasks. Diane asserts that it is important to consider "how the materials support students' learning through opportunities to engage in tasks that promote reasoning and problem solving" (2014). Our group also examined several other thought-provoking statements highlighting the critical nature of choosing tasks wisely:

- "Student learning gains were greatest in classrooms in which instructional tasks consistently encouraged high-level student thinking and reasoning and less in classrooms in which instructional tasks were consistently procedural in nature." (Stein, Smith, Henningsen & Silver, 2009)
- Students' academic work in school is defined by the academic tasks that are embedded in the content they encounter on a daily basis. (Stein et al., 2009)
- Students develop their understanding of the nature of mathematics from the tasks they experience. (Stein et al., 2009)

- " . . . it is the level and kind of thinking in which students engage that determines what they will learn." (NCTM, 1991)
- Differences in the level and kind of thinking of tasks used by different teachers, schools, and districts, is a major source of inequity in students' opportunities to learn mathematics. (Briars, 2010).

Beginning the Review Process

With this foundation, the group was ready to begin its initial analysis of the curriculum using a rubric adapted from the Instructional Materials Evaluation Tool (IMET) developed by Student Achievement Partners, Achieve, and Council of the Great City Schools (CGCS) and used by states and school districts across the country (see Appendix A).

Grade-level teams explored the alignment of the curriculum to the depth and spirit of both the Common Core Content Standards and Standards for Mathematical Practice. Teams measured the content and practices of each curriculum using the following:

- Content: Quantity, pacing and placement of lessons and units
- New concepts developed on previous understandings, knowledge and skills
- Opportunities for students to connect to real-world situations and wrestle with challenging problems
- Conceptual understanding developed through questioning, multiple representations, written explanations and discussion
- Appropriate guidelines for procedural skills and fluency
    - Practices: Standards for Mathematical Practice embedded in lessons and are primary vehicle for learning
    - Standards for Mathematical Practice used to promote thinking that is rich, challenging, and useful ("habits of mind")
    - Assessments provided that demonstrate students' proficiency of Standards for Mathematical Practice

As committee members gathered data, teams created posters capturing positives and challenges (based on evidence from the curriculum—not on personal experience or preference), and presented their findings to the group. While many group members' conversations and noticings aligned with the day's learning, there were still areas where misalignment and misconceptions prevailed, specifically surrounding rigor. "Do we (the committee) have the same definition of rigor?" and "Is there a way to create a common definition of 'rigor' in order to view the materials with that shared understanding in mind?" and "Can we have a discussion with the entire group addressing rigor?" were a few of the questions that surfaced on the end-of-day feedback cards, highlighting further work needed to build and align our group's understanding of effective teaching practices. Participants left for the day with an assignment to read an excerpt from K–8 Publishers' Criteria for the Common Core State Standards for Mathematics (2012) before the next session.

Session 2

The Committee began Session 2 with a discussion of six poignant quotes from the reading assignment. We engaged in rich discussions highlighting topics from the reading: extensive work with grade-level problems, explicit attention to the specialized language of mathematics and careful consideration of each practice standard and individual standards that set an expectation of fluency. Particularly notable were conversations about the three aspects of rigor, clearly highlighting the importance of and need for creating a shared understanding of rigor for the group. So what is a rigorous task? Our group was about to find out!

Definition and Review of Rigor

Criteria for rigorous tasks were identified, and participants understood that a worthwhile, rigorous task could be described as being one that:

- Is open-ended,
- Does not have a solution path that is immediately obvious (or implied),
- Requires students to think and not just rely on memorized procedures,
- Requires students to connect mathematical skill, understanding, and reason, and

- Requires students to interpret and communicate results.

Next, committee members in grade bands analyzed grade-specific tasks from the curricula being reviewed for rigor, ranking items as being high, medium, or low rigor. Debate about the task rankings included evidence statements from the criteria and language about rigor from the K–8 Publishers' Criteria reading discussed earlier. With a more solid understanding of rigor, the group began the second round of curriculum review, continuing to use the Mathematics Grade-Level Instructional Materials Evaluation Tool (GIMET). Again, participants created posters capturing the positives and challenges, reaching consensus on a rating (yes, no, maybe) for each of the curricula.

A final review for Equity, Spanish Parity, Assessment, or Design and Usability found the committee exploring the metrics for these critical considerations, ready to share evidence that might change their grade-band's initial rating of the materials. (See Appendix A.)

At the end of the second session, committee members—based on the day's learning about rigorous tasks and publisher's criteria—articulated differences between the programs being reviewed, noting an accumulating body of evidence both for keeping and for eliminating curricula. To prepare for engaging in both a review for vertical alignment (Grades K–2 and 3–5) as well as a lesson-level view of the curricula during the next sessions, committee members' reading assignment was "13 Rules that Expire" (Karp, Bush, & Dougherty, 2014).

Session 3

A lively discussion around "13 Rules That Expire" ensued as Session 3 began! In the article, the authors outline common rules and vocabulary which teachers share and elementary school students tend to overgeneralize—tips and tricks that do not promote conceptual understanding, rules that "expire" later in students' mathematics careers, or vocabulary that is not precise (Karp, Bush & Dougherty, 2014). Committee members talked about rules they have encountered in their work as well as advantages and disadvantages for students in using such rules.

Next we watched the video "Did You Know? 2014" (Creative Thinking–University of Hawaii, Kapi'olani

Community College, 2014). This thought-provoking look at the exponential times in which we live, juxtaposed with "13 Rules That Expire" highlighted implications for both using mathematical rules in classroom work and reviewing curriculum materials. The Committee's text and video discussions highlighted the need for today's students to be savvy consumers of information and to be problem solvers with well-honed skills to use and apply both the mathematics content as well as the habits of mind (Standards for Mathematical Practice). Conversations addressed the fact that while teaching a tip or shortcut might make students' learning easier in the moment, it is not helpful to provide students with a collection of "explicit, yet arbitrary, rules that do not link to reasoned judgment but instead to learning without thought" (Karp et al., 2014). The committee vowed to be on the lookout for opportunities in the curricula being reviewed to build students' conceptual understanding instead of sacrificing understanding for procedural speed.

Definition and Review of Fluency

Before we could review the curricula for vertical alignment and coherence of models, fluency expectations for basic facts, and strategies for solving word problems, it was necessary that the committee have a common understanding of "fluency." Arthur Baroody's article "Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them" (2006) was an optimal vehicle to provide insight into two perspectives of fluency and support us with a common lens to analyze how various curricula addressed fluency. The committee was divided into two groups for this part of the review: Grades K–2 Addition and Subtraction, and Grades 3–5 Multiplication and Division. Members were paired and used a note catcher to record findings related to the guiding questions:

- What models for addition and subtraction (multiplication/division) are introduced and used?
- What strategies are used to support fluency development?
- How are students supported in solving word problems involving common addition and subtraction (multiplication/division) situations? (CCSS-M, pp. 88–89).

Following the review, the committee compared their findings across grade bands to capture the coherence of models, fluency, and strategies to solve word problems for each of the curricula. At this point in the analysis, the committee was able to recommend which curricula would continue in the review process, and which was eliminated from the review because it didn't meet the criteria described previously.

Since the upcoming session included vendor presentations, the committee was given the opportunity to write questions they wanted vendors to address in their presentations. Vendors were sent an agenda which included points to address and questions from the committee.

Session 4

The morning of Session 4 was designated for vendor presentations of the curriculum materials that the committee voted to continue to the next phase of the review process. Three vendors were slated for this session, and one vendor was scheduled to present on another date. Each vendor was given one hour to showcase their curriculum and answer the questions prepared by the committee. After each presentation, committee members broke into grade level bands (K–1, 2–3, 4–5) to debrief and capture pros, challenges, and additional questions for that particular set of materials. During the afternoon, committee members came together as a group to share the information collected during the grade band debrief sessions.

For one final learning opportunity, we read two articles—"Math Lies We Tell Students" (Graybeal, 2014) and "Rules or Understanding?"(Martinie, 2005)—emphasizing the importance of mathematical reasoning and making sense vs. using clues and viewing math as a set of rules. These text resources set the stage for Session 5's analysis of how conceptual understanding is built within each grade and over grade bands for each of the curricula.

Session 5

As we gathered for our final work day as a committee, we were filled with hope and anticipation, knowing there was more important work ahead of us: one more vendor presentation, a study of lessons focusing on conceptual understanding for each grade level in each curricula, a technology review, the final recommendation, and recommendations for professional development to support teachers and leaders with the adopted curriculum.

Again, we grounded our work for the day with professional learning focusing on beliefs about teaching and learning of mathematics. The book, *Principles to Actions*, provided a springboard into that learning. We started by analyzing "Unproductive" and "Productive Beliefs" (NCTM 2014, p. 11). Next, committee members explored two Mathematics Teaching Practices: supporting productive struggle in learning mathematics and building procedural fluency from conceptual understanding.

Final Review

This work laid the foundation for our next step: studying how each of the curricula engaged students in learning a grade-specific CCSS-M content standard. Guiding questions that focused our work included:

- What role does conceptual understanding play in the lesson? What models are used?
- How does procedural understanding follow from conceptual understanding rather than following from rules?
- How does the instructional guidance for teachers support productive beliefs about teaching and learning mathematics?
- How does the instructional guidance of the lesson support teachers to meet the expectations of the instructional indicators called out in the Framework for Effective Teaching (our district's educator effectiveness tool)?

Grade bands debriefed and created a poster to capture evidence that supported their findings. Poster findings were shared with the whole group, and the committee discussed important points and implications for each of the curricula. To our surprise, several lessons from one of the curricula specifically taught one of the "13 Rules That Expire"!

The committee explored and reviewed the technology supports for each of the curricula, capturing the pros and challenges of the technology resources for each of the curricula on a poster to share with the whole committee.

Recommendation of Curriculum

With analysis of the curricula complete, it was now time for the committee to come to a consensus on which curriculum would best meet the needs of our students and make a recommendation for adoption. Committee members were given a ballot in which they were to respond individually to the following prompts for each set of materials:

- I fully support this curricular resource, OR
- I support this curricular resource with some reservation, OR
- I do not support this curricular resource.

After each statement, members were asked to provide reasons for their responses based on evidence uncovered through our process. Ballots were tallied and the results indicated an overwhelming consensus that two curricula were viable options for adoption.

Our last task for the committee was to brainstorm recommendations for professional development structures and venues that would support teachers and school leaders with their learning around the two curricula options.

Reflections of the Process

In closing, we asked the committee members to reflect on the process that guided our curriculum review work. We were impressed with each person's dedication and willingness to follow the process and allow themselves to be open to new learning. Here are a few of their comments:

> "In order to provide a fully informed recommendation, each step of this process was meaningful and necessary. There were many layers to each set of curriculum resources that needed to be effectively uncovered. The struggle was productive and engaging."

> "I'm taking away a profound difference in the way I see teaching of mathematics. I leave feeling validated about my own beliefs yet determined to continue these conversations to help me grow. I thoroughly enjoyed this process. It helped me expand my thinking."

> "This opportunity helped me deepen my math understanding. It also gave me a new way to look at the curriculum or materials that I will potentially use in my classroom to make sure they are sound/strong materials that support teaching and learning."

> "I'm taking away a new lens in which to analyze materials. The depth needed to truly understand a curriculum is more than I knew. All of the professional readings were critical to this work and applicable to my instruction. I feel empowered to change my teaching for the better."

> "I loved it! Thinking about a curriculum critically, being open-minded to alternative perspectives. Learning about best practices in math—both teaching and student learning."

Conclusion

Diane Briars was right—curriculum materials do matter! What began as a daunting task—the review process for adoption of mathematics curriculum—quickly became an opportunity for professional

learning and deepening everyone's understanding of effective mathematics teaching and learning! Here is our Top Ten List of Lessons Learned:

#10: Use and adapt high-quality rubrics and resources, such as the IMET and GIMET; don't try to reinvent the wheel.

9: Look for opportunities to make connections

to educator effectiveness—whether it's through a formal evaluation tool or by looking specifically at effective teaching and learning of mathematics.

8: Throughout the process, continue to require that evidence is based on rubrics rather than feelings.

7: Ensure enough facilitators to lead small group discussions.

6: Be intentional to pair those with strong math backgrounds with other committee members throughout the review.

#5: "Experts" from other departments (English Language Acquisition, Student Services, Early Childhood Education, Education Technology, etc.) bring depth to the analysis, providing insights regarding how the curriculum might or might not support all students.

4: Use feedback—observations, "exit tickets"—from each session to strategically plan for the next session(s).

#3: Group and regroup committee members with various partners throughout the process, providing collaboration opportunities with different people to hear all voices.

#2: Grounding discussions in professional readings is key in building understanding for all committee members.

AND THE NUMBER ONE LESSON LEARNED FROM THE CURRICULUM PROCESS IS:

#1: Of utmost importance is looking at materials multiple times in multiple ways as reviewers deepen their understanding of quality materials.

Nonie Lesaux (Harvard Graduate School of Education) said it best, "We can't confuse curricular materials with good teaching, but we can support good teaching with high quality, comprehensive curricular materials." The curriculum review process in which we engaged not only identified high quality, comprehensive curricular materials, but also provided a powerful professional learning opportunity to support effective mathematics teaching.

Authors note: *Laura Neuberg and Juli Lenzotti are members of Denver Public Schools' Math Department, directed by Cathy Martin. Under Cathy's leadership, the curriculum adoption process—for middle school mathematics curriculum last year and elementary (K–5) curriculum this year—is clearly a professional development opportunity for everyone involved.*

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Appendix A: K–5 Math Materials Evaluation Tool

<table>
<tr><th colspan="3">Section II: Critical Criteria</th></tr>
<tr><td colspan="3">N (not found)--Materials do not support this criteria.
L (low)—Materials contain limited support for this criteria but support is not embedded or consistent within/across grades
M (moderate)—Materials contain support for this criteria but support is not embedded or consistent within/across grades.
H (high)—Curriculum materials contain embedded support for this element that is consistently present within/across grades.</td></tr>
<tr><td>Criteria</td><td>Metrics</td><td>Notes & Score</td></tr>
<tr><td>3) Equity</td><td>3a) Materials provide teachers with strategies for meeting the needs of a range of learners.
3b) Materials provide instructional support to help teachers sequence or scaffold lessons so that students build understanding from previous knowledge.
3c) Materials provide opportunities for teachers to use a variety of grouping strategies.
3d) Materials embed tasks with multiple-entry points that can be solved using a variety of solution strategies or representations.
3e) Materials suggest scaffolds for English language learners that will support their regular and active participation in learning mathematics.
3f) Materials provide opportunities for advanced students to investigate mathematics content at great depth.</td><td>Score: _____</td></tr>
<tr><td>4) Spanish Parity
Materials of comparable quality are available in Spanish.</td><td>4a) Materials are packaged and presented in Spanish in equal quality and format and meet the criteria for math content and practices.
4b) Teacher resources provide teaching scripts, prompting and reinforcing in Spanish, using Spanish academic language.
4c) Materials support math instruction and Spanish language development.
4d) Materials include explicit opportunities for the transfer of concepts and language to support the development of biliteracy. Spanish and English materials are aligned in order to allow for strategic language instruction.</td><td>Score: _____</td></tr>
<tr><td>5) Assessment</td><td>5a) Materials provide strategies for teachers to identify common student errors and misconceptions.
5b) Materials assess students at a variety of knowledge levels (e.g., memorization, understanding, reasoning, problem solving).
5c) Materials encourage students to monitor their own progress.
5d) Materials provide opportunities for ongoing review and practice with feedback related to learning concepts and skills.
5e) Materials provide support for a varied system of on-going formative and summative assessment.
5f) Assessment materials are available in Spanish.
5g) Materials provide multiple ways to show proficiency: i) multiple opportunities for written responses; ii) performance tasks and projects; and iii) multiple ways to represent understanding of concepts (including enrichment opportunities).</td><td>Score: _____</td></tr>
</table>

6) Design and Usability of Resources	6a) Materials include clear and sufficient guidance to support teaching and learning of the targeted standards. 6b) Materials are easy to use and cleanly laid out for students and teachers. 6c) Materials address instructional expectations and contain clear statements and explanation of purpose, goals, and expected outcomes. 6d) Materials can be reasonably completed within a regular school year and provide clear guidance to teachers about the amount of time each lesson/activity might reasonably take. 6e) Manipulative materials and disposable materials (e.g., dri-erase markers) are priced competitively.	Score: _____
7) Electronic/Online Resources	7a) Materials integrate technology such as interactive tools, virtual manipulatives/objects, and dynamic mathematics software in ways that engage students in the Mathematical Practices. 7b) Materials include opportunities to assess student mathematical understandings and knowledge of procedural skills using technology. 7c) Materials include teacher guidance for the mindful use of embedded technology to support and enhance student learning. 7d) Materials support differentiation for individual student needs, strengths, and interests. 7e) Resources are user-friendly and interactive and have an easy-to operate interface.	Score: _____

IN THE FIELD

12 Rules That Expire

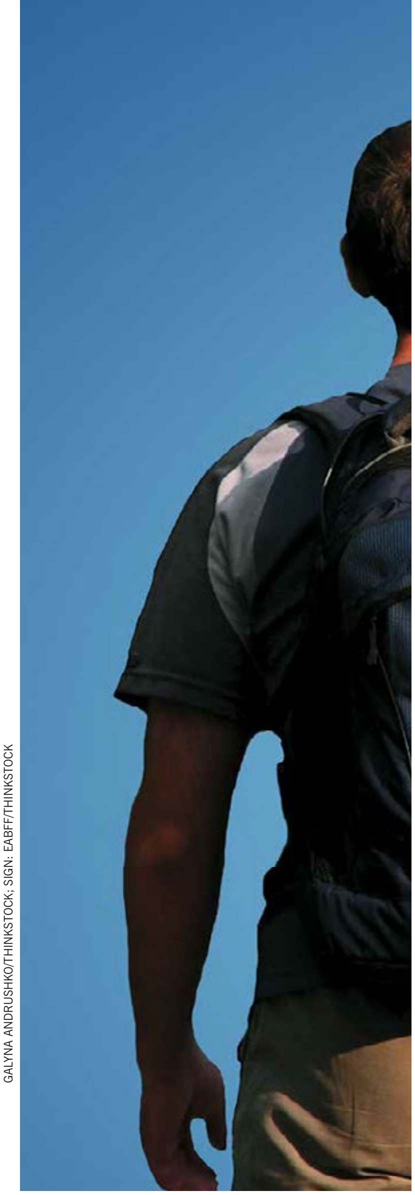
12 Math Rules That EXPIRE in the MIDDLE GRADES

Turn away from overgeneralizations and consider alternative terminology and notation to support student understanding.

Karen S. Karp, Sarah B. Bush, and Barbara J. Dougherty

Many rules taught in mathematics classrooms "expire" when students develop knowledge that is more sophisticated, such as using new number systems. For example, in elementary grades, students are sometimes taught that "addition makes bigger" or "subtraction makes smaller" when learning to compute with whole numbers, only to find that these rules expire when they begin computing with integers (Karp, Bush, and Dougherty 2014). However, middle-grades students, especially those who are struggling, often try to force-fit the rules that they remember from the elementary grades to new concepts or skills.

In this article, we present 12 persistent rules that expire. These are "rules"



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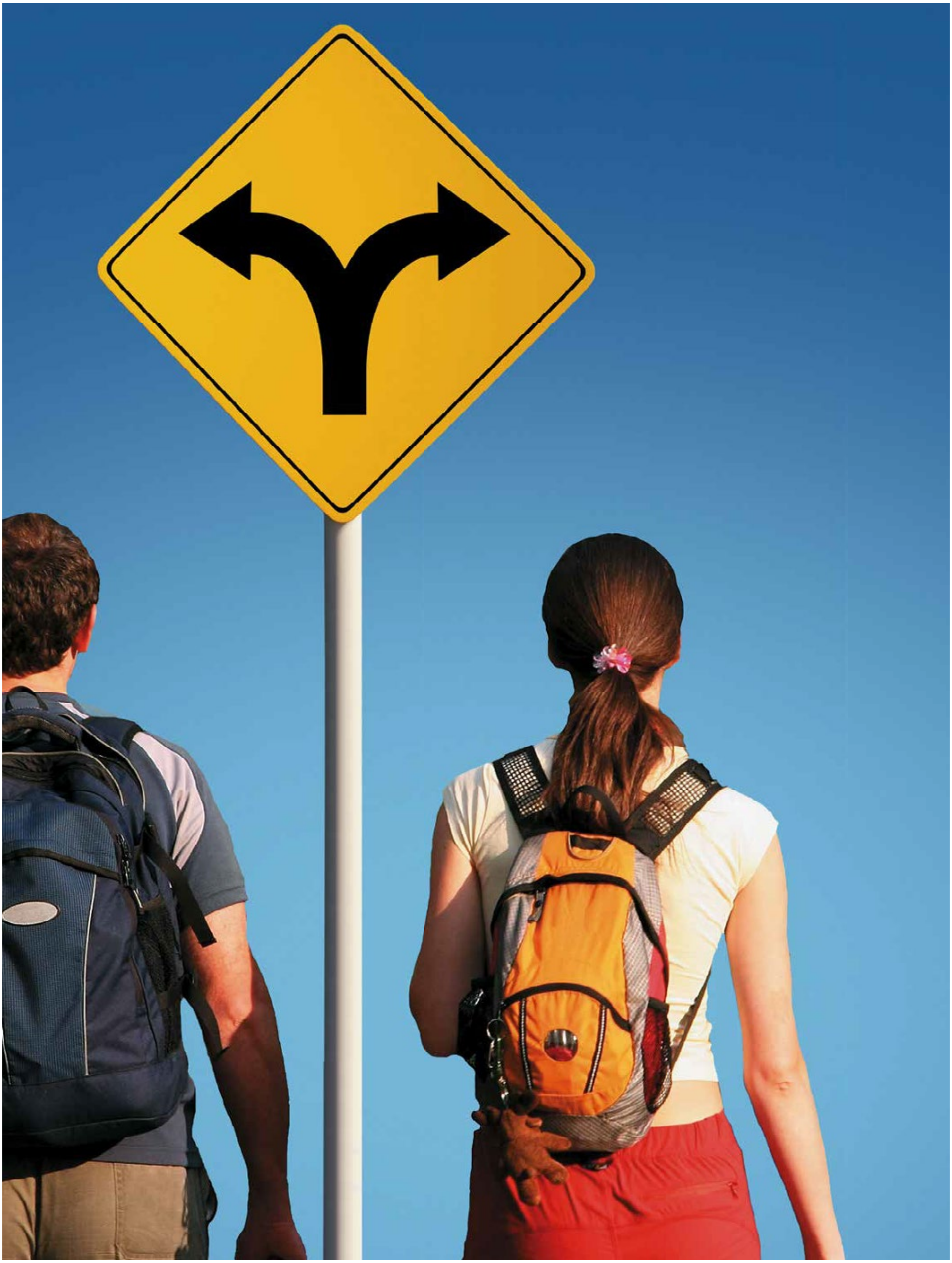


Table 1 Students' analyses of reasoning provide illuminating information.

Equation Chosen as Easy	Student Reasoning	Percentage of Students Choosing the Equation (n = 50)
$-x + (5) = 8$	• "I chose this because I can rewrite the equation as $5 - x = 8$. That's easier to solve." • "It looks normal like stuff I did in first grade."	36
$-6 = 3(x + 1)$	• "I don't like this one 'cause you have to do parentheses." • "My teacher last year said that you have to flip this one before you can do anything because the letter has to be on the left. I don't like doing that."	18
$3x + 8 = -10$	• "I picked this one because the letter is on the left and it's supposed to be."	44
$-8 = 2x + 20$	• "You just have to turn this one around and then it's easy. You gotta make sure the letter is on the left. I don't know why math teachers put letters on the right."	2

that we have found prevalent in our many years of working with students, from mathematics education literature, or in some cases, rules that we ourselves have taught and later regretted. In each case, we offer mathematically correct and more helpful alternatives.

The Common Core's Standards for Mathematical Practice (SMP) encourage precision, including the appropriate use of mathematics vocabulary and notation and the reasoned application of "rules" (CCSSI 2010). This leads to instruction focused on sense making and reasoning—the very experience described in NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (2014).

GETTING STARTED

Imagine this scenario in your mathematics classroom, in which you present the following set of one-variable equations:

1. $-x + (5) = 8$
2. $-6 = 3(x + 1)$
3. $3x + 8 = -10$
4. $-8 = 2x + 20$

Which of these equations would your students choose to solve first or find the easiest? In our work, we found that students were comfortable solving equations 1 and 3 because they "looked normal" with the "operation" first (on the left side), followed by the "answer" (on the right side). Students often hesitated at equations that were similar to 2 and 4 because the perceived "operation" and "answer" were arranged in a seemingly reversed order (see **table 1**). This inflexibility can be linked to the idea that middle school students often have yet to interpret and understand the equal sign as a symbol indicating a relationship between two quantities (Mann 2004). Additionally, students may think that the solution to an equation always goes on the right side of the equal sign. These overgeneralizations are not helpful and can have a negative impact on students' conceptual understanding. We suggest that these students are experiencing rules that expire (Karp, Bush, and Dougherty 2014).

We highlight rules sometimes used with middle school students that seem to hold true at the moment, given the content the student is learning at that time. However, students will later find that these rules expire. Sometimes taught as shortcuts with content that students learned in the previous grades, these rules expire when students use them inappropriately with more advanced problems and find that they are incorrect. Such experiences can be frustrating and can promote the belief that mathematics is a mysterious set of tricks and tips to memorize rather than concepts that relate to one another. For each rule that expires, we do the following (similar to Karp, Bush, and Dougherty 2014):

1. State the rule.
2. Discuss how students overgeneralize it.
3. Provide counterexamples.
4. State the expiration date or the point when the rule begins to fall apart.

12 RULES THAT EXPIRE

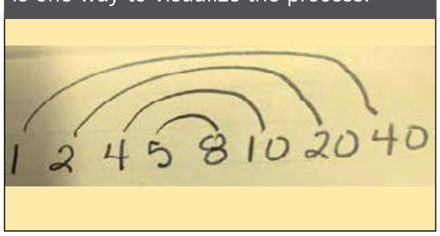
1. *KFC: Keep-Flip-Change.*
When learning to divide fractions, students are sometimes taught to KFC (Keep-Flip-Change) or told "Yours is not to reason why, just invert and multiply." Although both versions align with the standard algorithm, students might overgeneralize

> ***"My teacher last year said that you have to flip this one before you can do anything because the letter has to be on the left. I don't like doing that."***

this rule to other operations with fractions. Additionally, these mnemonics and sayings do not promote conceptual understanding, making it challenging for students to apply them in a problem-solving context. Instead, division of fractions can be linked to whole-number division by asking how many groups of the divisor make up the dividend. Although students will eventually use the algorithm, they should gain a conceptual understanding of dividing fractions through the use of physical models (Cramer et al. 2010) or other methods, such as the common denominator strategy.
Expiration date: Grade 6 (6.NS.1)

2. *Use the factor rainbow to factor.*
Sometimes students are taught to create a "factor rainbow" to list all factors of a number. For example, if finding the factors of 20, students would write 1 and 20, then 2 and 10, and 4 and 5 (see **fig. 1**). The rule is taught so that once you identify factors that are consecutive numbers (e.g., 4 and 5), you have identified all factors. However, factors can be missed with this approach. Some numbers do not have consecutive factor pairs, as in when finding the factors of 40: 5 and 8 are the closest factors. Moreover, this rule expires when factoring a square number (its factors are not all paired) or when working with non-whole-number factors, such as in algebra when students factor out fractional parts as well as variables.
Expiration date: Grade 6 (6.NS.4)

Fig. 1 Drawing a factor rainbow for 40 is one way to visualize the process.



3. *The absolute value is just the number.*
Students are sometimes told that the absolute value of a number is that number, with a positive sign. For example, $|-4| = 4$ because you drop the negative sign. Confusion sets in when students are presented with $-|-4|$ because they are unsure what this represents. How can absolute value be negative? Without making sense of the meaning of absolute value (that is, its distance from zero on a number line), students may not interpret it correctly within particular contexts.
Expiration date: Grade 6 (6.NS.7)

4. *Multiplication is repeated addition.*
Considering multiplication as only repeated addition can result in students thinking that the expression 3^3 is equivalent to 3 + 3+ 3. This thinking leads to overgeneralizations because students come to believe that 3 raised to the third power means that 3 is used as an addend 3 times. Writing such expressions in correct expanded form can help with this misunderstanding.
Expiration date: Grade 6 (6.EE.1)

5. *PEMDAS: Please Excuse My Dear Aunt Sally.*
This mnemonic phrase is sometimes taught when students solve numerical expressions involving multiple operations. At least three overgeneralizations commonly occur with this rule:

- Students incorrectly believe that they should always do multiplication before division, and addition before subtraction, because of their order in the mnemonic (Linchevski and Livneh 1999), instead of performing them in the order in which they appear in the expression.
- Students perceive that the order of PEMDAS is rigid. For example, in the expression

  $$30 - 4(3 + 8) + 9 \div 3,$$

  there are options as to where to begin. Students actually have a choice and may first simplify the 3 + 8 in the parentheses, distribute the 4 to the 3 and to the 8, or perform 9 ÷ 3 before doing any other computation—all without affecting an accurate outcome.
- The P in PEMDAS suggests that parentheses are first, but this should also represent other grouping symbols, including brackets, braces, square root symbols, and the horizontal fraction bar. We suggest making sense of a problem. However, if using a hierarchical model, consider this order: (a) Grouping symbols or exponents; (b) multiplication or division; and (c) addition or subtraction.
  Expiration date: Grade 6 (6.EE.2)

6. *A solution to an equation must be in the form x = □.*
Students are often taught that the variable and/or operation comes first, followed by the answer (e.g., the constant) in an algebraic equation

(Dougherty and Foegen 2011). However, this rule has no mathematical necessity because the equal sign indicates that two quantities are equivalent. Therefore, variables, operations, and constants can be located on either or both sides of the equal sign. Instead of overgeneralizing that an equation should "look" a certain way, we as teachers should promote flexibility in students' thinking. When the teacher uses a specified set of steps and the placement of the solution in that format, students lose sight of the conceptual aspects of equations and instead focus on implementing algorithmic steps.
Expiration date: Grade 6 (6.EE.4)

7. *The "Butterfly Method" for comparing fractions.*
Students are frequently taught the "Butterfly Method," which refers to cross multiplying two fractions to determine which fraction is greater. For example, in

$$\text{Which is greater, } \frac{3}{4} \text{ or } \frac{7}{8}?$$

students may draw a loop around the 3 and 8 and around the 4 and 7 (which looks like a butterfly) and multiply $3 \times 8 = 24$ and $4 \times 7 = 28$ to determine that 24 is less than 28, which tells them that

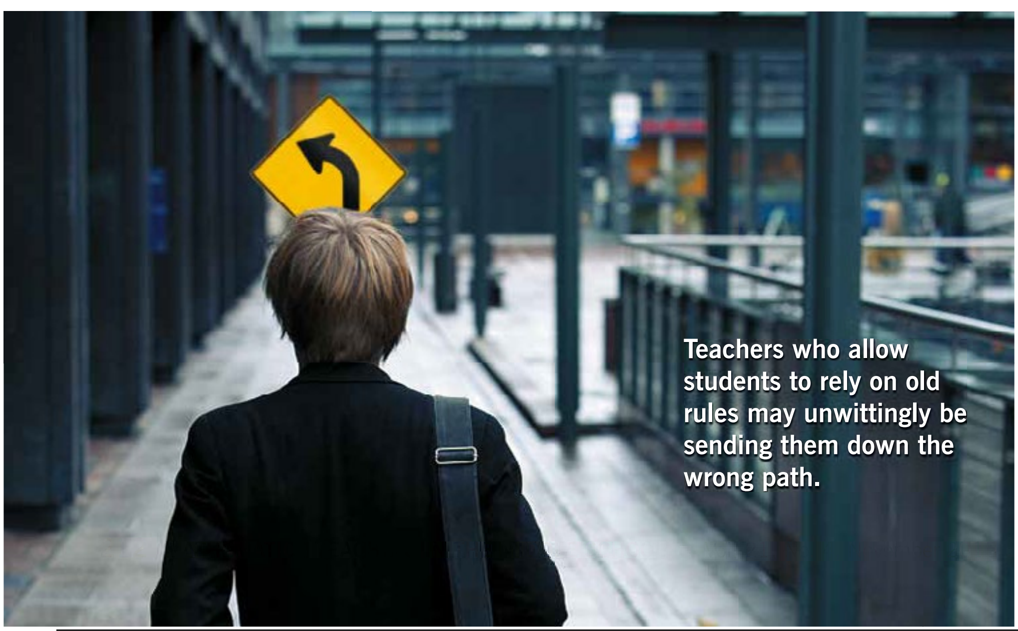
$$\frac{3}{4} \text{ is less than } \frac{7}{8}.$$

However, this rule is problematic for several reasons. First, it does not foster conceptual understanding of the numerical value of fractions because it removes the need to understand the relationship between the two fractions or consider the quantities they represent. Second, students begin to overgeneralize and incorrectly apply this rule to other situations whenever they see two fractions, such as when they add, subtract, multiply, or divide fractions.
Expiration date: Grade 7 (7.RP.2)

8. *The most you can have is 100 percent of something.*
Students are sometimes taught that because 100 percent is equivalent to 1 whole, that is the most they can have. However, increases and decreases can be of any size, including more than 100 percent. This rule expires as students work with ratios and proportional relationships involving markups, discounts, commissions, and so on. *Expiration date:* Grade 7 (7.RP.3)

9. *Two negatives make a positive.*
This rule may be taught when students learn about multiplication and division of integers and is used to help students quickly determine the sign of the product or quotient. However, this rule does not always hold true for addition and subtraction of integers, such as in $-5 + (-3) = -8$. Additionally, this rule does not foster the understanding of why the product or quotient of two or more integers is negative or positive. Instead of focusing on the rule, consider using patterns of products to develop generalizations about the relationship between factors and products.
Expiration date: Grade 7 (7.NS.2)

10. *Use keywords to solve word problems.*
A keyword approach is frequently introduced in the elementary grades and extends throughout a student's school career as a way to simplify the process of solving word problems. However, using keywords encourages students to overgeneralize by stripping numbers from the problem and using them to perform a computation outside the problem context (Clement and Bernhard 2005). This removes the act of making sense of the actual problem from the process of solving word problems. Many keywords are common English words that can be used in many different ways. Often a list of words and corresponding operations are given so that word problems can be translated into a symbolic, computational form. For example, students are told that if they see the word *of* in a problem, they should multiply all the numbers given in the problem. Likewise, although the keyword *quantity* sometimes signifies the need for the distributive property, at other times it does not. Keywords are especially troublesome in the middle grades as students explore multistep word problems and must decide which keywords work with which component of the problem. Although keywords can be informative, using them in conjunction with all other words in the problem is critical to grasping the full meaning.
Expiration date: Grade 7 (7.NS.3)



Teachers who allow students to rely on old rules may unwittingly be sending them down the wrong path.

LUMINIS/THINKSTOCK; SIGN: JOJO064/THINKSTOCK

Another student analyzed $-8 = 2x + 20$: "You just have to turn this one around and then it's easy. You gotta make sure the letter is on the left. I don't know why math teachers put letters on the right."

11. *A variable represents a specific unknown.*
When students work with one-variable equations, the solutions to the equations are almost always one specific value (e.g., $x = 5$ or $x = -3$). However, students overgeneralize this as being true for all situations involving variables, yet this rule quickly expires as variables take on other meanings, such as varying quantities or parameters (e.g., $y = mx + b$), labels ($A = bh$), or generalized unknowns. Additionally, students may not accept equations that represent identities (such that the variable can take on *any* value) or equations that have no solution (such as $3x + 4 = 3x - 4$). This rule expires when students begin to work with linear functions.
Expiration date: Grade 8 (8.EE.7)

12. *FOIL: First, Outer, Inner, Last.*
When learning to multiply two binomial expressions, students might be taught to FOIL, that is, to multiply the first term in the first binomial by the first term in the second, then multiply the outer terms of each binomial, then the inner terms of each binomial, and then the second (last) terms of each binomial. Although this rule works for binomials, it soon expires as students begin multiplying other polynomials, such as a binomial and a trinomial, or two trinomials. Instead, have students explore how they are really using the distributive property multiple times, to multiply each term in one polynomial by each term in the other polynomial.
Expiration date: High school (A.APR.1)

EXPIRED LANGUAGE AND NOTATION

We must also consider the mathematical language and notation that we use and that we allow our students to use. The ways in which we communicate about mathematics may bring with them connotations that result in students' misconceptions or misuses, many of which relate to the previously discussed Rules That Expire. Using terminology and notation that are accurate and precise (SMP 6) develops student understanding that withstands the growing complexity of the secondary grades. **Table 2** includes commonly used expired language and notation, gathered from our years of experience in the classroom, paired with alternatives that are more appropriate.

"BUT MS. JONES SAID SO"

Coherence is one of the major emphases in CCSSM. By having a series of rigorous standards at each grade, with less overlap and structured alignment, students can progress more purposefully through the content. By building a schoolwide plan for the consistent and precise presentation of rules, terminology, and notation used by all teachers, students will never get caught in the "But Ms. Jones said so" mode of finding something in their past instruction that is no longer accurate. Through such intentional consistency, students are able to focus on the new ideas presented as the language and tenets continue to be the foundation for lessons. Because the middle-grades years are pivotal in cementing the ideas from elementary school and building the concepts needed for high school, this explicit, systemic consistency is critical. As we avoid these 12 Rules That Expire, we instead find ways to present a seamless and logical world of mathematical ideas.

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Are There Other Rules That Expire?

We invite *MTMS* readers to submit additional instances of "rules that expire" or "expired language" that this article does not address. Join us as we continue this conversation on *MTMS*'s blog at **www.nctm.org/12rules**, or send your suggestions and thoughts to mtms@nctm.org. We look forward to your input.

Table 2 These alternatives can be used in place of expired language and/or notation.

What Is Stated and/or Notated	Alternative Appropriate Statements or Notations
Using the notation $8 + 4 = 12 + 5 = 17 + 3 = 20$ to symbolize a series of addition problems	Stringing together a series of additions (or other computations) cannot be connected with equal signs, as the components are unequal. Instead, use individual equations, each using the answer of the previous problem as the starting addend. Equal signs must connect equal quantities.
Using a diagonal bar in fraction notation	This notation becomes problematic with polynomials and for learners who often read the handwritten diagonal as a 1 (e.g., 3/4 is read as 314). Use a horizontal bar instead. For 1/2, write $\frac{1}{2}$.
Getting rid of the fraction or decimal	Students create an equivalent equation by multiplying or dividing and are not doing away with the fraction or decimal point at all. For example, $\frac{1}{2}x + 4 = \frac{1}{4}$ becomes $2x + 16 = 1$ by multiplying each term by 4.
Using *rounding* to mean the same as *estimating* Using *guess* to mean the same as *estimate*	An *estimate* is an educated approximation of a calculation of an amount of a given quantity. It is not a random guess. Rounding is one strategy to produce a computational estimate, but it is not synonymous with an estimate.
Using *point* to read a decimal, such as “three point four” for 3.4	Instead, read a decimal as a fraction: 3.4 is “three and four-tenths.” This will make converting decimals into fractions an easier task. Use the word *point* only when describing how a decimal is written or in a geometric context.
Reducing fractions	Using the term *reducing* may cause students to think the fraction value is getting smaller. Instead, use the term *simplifying fractions*, or instruct students to *write the fraction in simplest form or lowest term*s.
Plugging in a value for a variable	*Plugging in* is not a mathematical term. Instead, students should *substitute* a value.
Saying that fractions have a *top and bottom number*	A fraction is one number, one value. The *numerator* and *denominator* should be used to describe where different digits of a fraction are located. The words *top* and *bottom* have no mathematical meaning and may incorrectly imply that a fraction consists of more than one number.
Using the first letter of the word to describe the variable	For example, if you use the variable c to represent the number of cars in a problem, when students see $4c$ in the equation, they think it means 4 cars (using c as a label) rather than 4 times the number of cars. When you select a variable, avoid the first letter of the word and use instead an arbitrary letter to represent the *number* of cars.
Moving the decimal point when dividing decimals	The decimal point does not actually move. Rather, the digits are shifted when an alternative equation is made by changing the divisor and the dividend by multiplying (or dividing) both by a power of 10.

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Any thoughts on this article? Send an email to **mtms@nctm.org**.—*Ed.*

Karen S. Karp, kkarp1@jhu.edu, is a visiting professor at Johns Hopkins University in Baltimore, Maryland. She is professor emeritus at the University of Louisville in Kentucky, a past member of the NCTM Board of Directors, and a former president of the Association of Mathematics Teacher Educators. Her current scholarship focuses on teaching interventions for students in the elementary and middle grades who are struggling to learn mathematics. **Sarah B. Bush**, sbush@bellarmine.edu, an associate professor of mathematics education at Bellarmine University in Louisville, Kentucky, is a former middle-grades math teacher who is interested in relevant and engaging middle-grades math activities. **Barbara J. Dougherty**, barbdougherty32@icloud.com, a research professor for mathematics education at the University of Missouri–Columbia, is a past member of the NCTM Board of Directors and the editor for the Putting Essential Understandings into Practice series. She is a co-author of conceptual assessments for progress monitoring in algebra and curriculum modules for middle school interventions for students who struggle.

NCTM CORNER

Resource Updates

Catherine Martin, CCTM Past President and NCTM Board Member

New NCTM resources for your professional learning include interactive institutes and conferences along with new print resources. If you are not currently a member of NCTM and would like to join, you may use the code, BCM0616, to receive a discount on your membership.

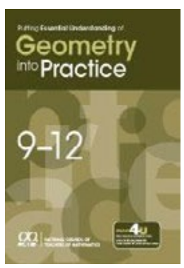
Professional Learning Opportunities

Interactive Institute: Effective Teaching with Principles to Actions: **Implementing College- and Career- Readiness Standards**; February 5–6, 2016 in Dallas, TX

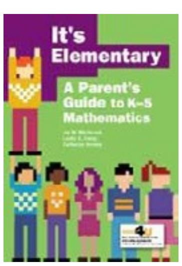
Annual conference and exposition: **Building a Bridge to Student Success;** April 13–16, 2016 in San Francisco, CA

New Books from NCTM

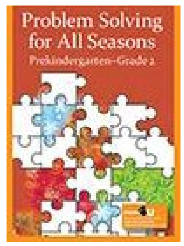
Putting Essential Understanding into Practice: Geometry 9-12. This book builds on *Developing Essential Understanding: Geometry, Grades 9–12*. The authors explore best practices for teaching the essential ideas of high school geometry. Classroom vignettes and samples of student work are included to connect directly to classroom practice.



It's Elementary: A Parent's Guide to K–5 Mathematics (Whitenack, Cavey, and Henney). This book is designed to support parents in understanding the fundamental concepts that their children need to understand to be successful mathematics learners. The authors show how children progress in their mathematical thinking and provide advice and guidance, from helping with homework to choosing math games to talking to children about math beyond homework and school.



Problem Solving for All Seasons: Prekindergarten–Grade 2 (Markworth, McCool, and Kosiak). The authors provide 32 mathematical tasks around holidays and seasonal activities to engage young learners. Tasks are tied to the Common Core State Standards or other content standards and designed to support students' participation in the Common Core Standards for Mathematical Practice.



Coming Soon

NCTM will publish a series of three books around the teaching practices in *Principles to Actions: Ensuring Mathematical Success for All*. These books, along with a research companion for *Principles to Actions*, will be released in April 2016.

Activities with Rigor and Coherence (ARCs)

These are a series of lessons that demonstrate the vision of *Principles to Actions*. In each ARC, a sequence of two to four lessons will address a mathematical topic. These will scaffold effective teaching and support enactment of the eight Mathematics Teaching Practices in *Principles to Actions* as well as the instructional guidance set forth in *5 Practices for Orchestrating Productive Mathematics Discussions*. ARCs will integrate the wide array of NCTM resources (e.g., *Student Math Notes*, Illuminations). Users might also be directed to a related professional development offering or journal article.

CONFERENCE AND PROFESSIONAL DEVELOPMENT

Recap of the CCTM Annual Conference

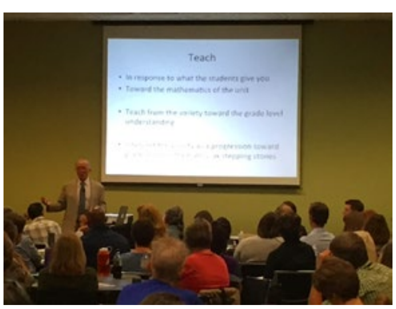
Leigh Ann Kudloff & Cassie Gannett, Conference and Program Chairs

The 2015 CCTM Annual Conference, *Putting the Pieces Together for Mathematical Success*, on September 24–25 was a celebration of mathematics education in our state. Over 650 participants attended this year's conference, enjoyed powerful pre-sessions and varied conference sessions, and recognized outstanding math teachers and leaders:

Matt Larson, President Elect of NCTM, led over 150 school and district leaders through an outline of high-leverage actions for principals, including the most effective ways to prepare students for next generation assessments, from the leader's guide of *Mathematics in a PLC at Work*. (See Matt's follow-up article, "Promoting Access and Equity in Mathematics" on p. 45.)

Matt with a team from Mesa County Valley School District 51

The teacher pre-session explored the progressions of math concepts in the Common Core with **Phil Daro,** from the Common Core writing team, who emphasized that the many ways that students think about a concept are used as stepping stones to bring students up to grade level understanding.



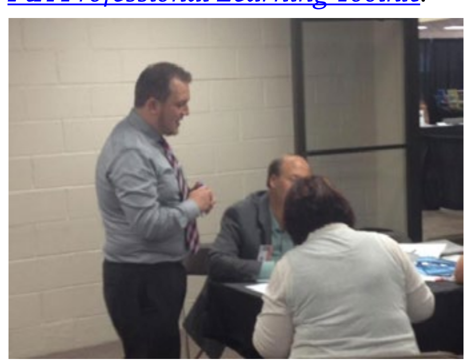
Thursday wrapped up with over 100 people attending the annual CCTM Awards Reception. Arlene Mitchell was presented with the Forest Fisch Award, and teachers from all regions of Colorado were recognized. (Information on each of the awardees can be found on pp. 48-52.)

On Friday, **Steve Leinwand**, our keynote speaker, shared his learnings from visiting over 1000 classrooms while challenging the audience's thinking about current practices. (Revisit Steve's [conference presentation slides]; and his article, ["Helping Parents Understand What We're Doing"] from the Fall 2015 issue of the *CMT*.)

Joanie Funderburk with Steve Leinwand

A special thank you to the over 100 presenters from around our state. We had professionals from pre-school through college sharing experiences and strategies. The strands that were offered included: STEM, Special Populations, Discourse, Effective Teaching Practices, Lessons and Activities, and General. For the "Effective Teaching Practices" strand, CCTM board members were able to continue our goal this year of focusing articles in the *CMT*on NCTM's *Principles to Action: (PtA),* by presenting sessions on the, *PtA Professional Learning Toolkit*.

Due to so many wonderful applicants for speakers, we were able to offer a wide



variety at every time slot. The sessions included four one-hour time slots, half-hour "Blast" sessions in the middle of the day, and an IGNITE session at the conclusion of the conference. Blast sessions allowed vendors to share specifics about their products in a small group format, and allowed first-time presenters a more formal opportunity to share ideas. Many thanks to all the exhibitors that provided a plethora of door prizes. The IGNITE session at the end of the day was a huge success with the largest turnout on memory staying until the very end to hear from Joanie Funderburk, Cathy Martin, Mary Pittman, and David Woodward.

NCSM Exhibitor *Excited door prize winner!*

We hope you'll join us again next year on **September 22–23, 2016.**

Conference Reflection

This last fall, I attended the CCTM conference for the first time. Although the entire conference provided new and exciting learning opportunities, here are some of the highlights for me:

- *Key-note speakers - Both Phil Daro and Steve Leinwand were engaging and provided new insights and encouragements in teaching mathematics at any level. In Phil Daro's Thursday evening address, I learned, in more depth, about the creation and intent of the Common Core State Standards. On Friday, Steve Leinwand encouraged the entire audience in some simple strategies to make mathematics instruction more concept-based and student-focused.*
- *Networking - The conference allowed me to talk with and learn from other educators in the field, both in my own district and from across the state. This was a unique opportunity that provided me with lots of new ideas and resources for my classroom.*

I look forward to another time of learning and encouragement with the mathematics teaching community next fall. I hope other educators, at any grade or experience level, have the opportunity to attend in the future.
-Tessa Ziser

EQUITY

Promoting Access and Equity in Mathematics

Matthew R. Larson, NCTM President-Elect

In *Principles to Actions: Ensuring Mathematical Success for All*, the National Council of Teachers of Mathematics [NCTM] argues in the Access and Equity Principle that

> An excellent mathematics program requires that all students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential (NCTM, 2014, p. 59).

The Access and Equity Principle is critical to student success in mathematics. After all, unless students have *access* to a powerful curriculum, *access* to appropriate tools and technology, *access* to meaningfully utilized assessments, and *access* to teachers who employ research-informed instructional strategies, it doesn't matter how outstanding these other programmatic elements are.

The purpose of this article is to describe some of the research-informed supports schools are putting in place to promote student access to meaningful mathematics and improve student learning.

Eliminating Low-Slow Groups

Schools that promote access and equity are eliminating their low or slow instructional groups. Although tracking is often viewed as a secondary concern, the reality is that "tracks" in mathematics are often established as early as the primary grades when students who struggle in kindergarten are placed in a low-slow mathematics group in first grade. Once placed in these low instructional groups, it is very difficult for students to move to an on-grade level group (Flores, 2008).

In many elementary schools, and even into the middle grades, math instruction is frequently organized into high, medium, and low groups, with corresponding expectations for students (Baifora & Ansalone, 2008). Students in high groups typically have access to mathematical ideas, concepts, and problem solving, whereas students in low-groups tend to repeat the same basic computational skills (Boaler, Wiliam, & Brown, 2000; Tate & Rousseau, 2002).

The research indicates that when students believed to be less capable of learning are given access to grade level curriculum and appropriate support, they are capable of being successful (Griffin, Case, & Siegler, 1994; Knapp, Shields, & Turnbull, 1995; Silver & Stein, 1996; Usiskin, 2007). Schools that maintain low instructional groups tend to see the achievement gap between students placed in low- and high-instructional groups widen over the years of schooling (Flores, 2008; Tate & Rousseau, 2002).

While the elimination of low-instructional groups is a critical first step to provide all students access to quality curriculum and effective instruction, it will not be successful without the simultaneous implementation of common assessments and differential instructional time and supports.

Common Formative Assessments

Student access to quality grade-level curriculum is made possible by the implementation of common assessments linked to additional targeted-instruction. Implementing common assessments means, for example, that every student in grade 2 takes the same assessment. These common assessments not only establish a common high expectation for student learning, but the key is to use the results formatively.

Additional Instructional Time and Support

One of the most effective interventions in mathematics is an approach to instruction that implements common formative assessments and then uses the results of the formative assessments to form smaller groups of students who receive *additional* instruction in the skills and concepts with which they are struggling (Baker, Gersten, & Lee, 2002). The critical point is that the targeted supplemental instruction takes place *in addition to* whole-class instruction instead of *in place of it*. Students are not removed from grade-level instruction to receive targeted support. In too many cases, traditional interventions have failed because they are not done *in addition to* whole-class instruction but *instead of it* (Larson et al., 2012).

The research is clear that highly effective schools allow instructional time and support to vary in order to meet student needs (DuFour et al., 2004; Lezotte, 1991). One way to accomplish this at the elementary level is to provide 60 minutes of daily whole class math instruction and compact other parts of the day to create a daily thirty-minute block of time designated for additional mathematics instruction. Teachers meet regularly in grade-level collaborative teams to identify student needs based on formative assessment results and students are re-grouped for targeted support during the intervention time.

At the middle level, one successful approach to provide additional time and instruction is "fluid" math intervention. With "fluid intervention," collaborative grade level or subject teams analyze common assessment results (administered every three weeks) and regroup students into three distinct one-week interventions for targeted support in addition to their regular math course. Some students might receive one-week of targeted support, others two, three, or none during each assessment cycle.

This approach is based on research that supports enrolling middle level students in on grade level courses with additional "workshop" support (Burris, Heubert, & Levin, 2006) driven by the results of frequent assessment (Baker, Gersten, & Lee, 2002; Buffum, Mattos, & Weber, 2012; NMAP, 2008). The key factor is that the targeted support is fluid, students move in and out as need arises, and support is linked directly to the grade level curriculum.

At the high school level, double-period versions of the core courses of algebra, geometry, and algebra 2 can be offered. The double-period approach permits students to receive the support they need to be successful, but does not put students behind in the mathematics curriculum, allowing them access to upper level mathematics courses before they graduate.

As early as 2000, the National Council of Teachers of Mathematics proposed that "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (p. 12). Schools that promote access and equity in mathematics make accommodations in time and instructional supports to ensure that all students have access and the support necessary to be successful in challenging grade level mathematics and upper level mathematics courses in high school.

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AWARDS

Arlene Mitchell Honored with CCTM Distinguished Service Award

John Sutton and Clare Heidema, RMC Research Corporation

RMC Research Corporation Senior Research Associate, Arlene Mitchell, was honored with the prestigious Forest N. Fisch Award from the Colorado Council of Teachers of Mathematics (CCTM). Arlene was presented the award for outstanding service in mathematics education at the Council's annual awards ceremony Thursday evening, September 24, 2015 at the Denver Mart, with more than 200 of her colleagues and fellow mathematicians and mathematics educators in attendance.

Arlene began her professional career teaching in Florida and Texas, before coming to Colorado where she taught in Greeley for over 20 years. After moving from the classroom to the research and technical assistance environment in the mid-90's, Arlene was an integral member of the High Plains Consortium for Mathematics and Science at the Mid-continent Regional Educational Laboratory (McREL) from 1997 until 2003. In 2003, Arlene joined the staff of RMC Research Corporation, where she has been a leader on a number of research, development, and evaluation projects. In addition, she has provided technical assistance and guidance to educators across multiple states through the Texas Comprehensive Center and the Central Regional Education Laboratory at Marzano Research. Arlene has been very active in the CCTM, serving as President (2005–2007) and as editor of the *Colorado Mathematics Teacher*. She also has been active in the National Council of Teachers of Mathematics (NCTM) serving as co-chair for NCTM's Regional Conference in 2010 and as a program member for the NCTM National Conference (2012) in Philadelphia, as well as chair and member of the NCTM Professional Development Services Committee (2006–2009), and a NCTM Algebra Institute session developer for Middle School (2010–2015).

Because of her depth of knowledge, passion, and advocacy for high quality mathematics teaching and learning, Arlene has helped teachers and students deepen their understanding of mathematics content and connections, and supported others' growth into leadership roles. Tom Peters in his book *In Search of Excellence* said, "Leaders don't create followers, they create more leaders." This has been Arlene's role from the start: for her students; for her colleagues and associates; for teachers, specialists, and coaches; and for other mathematics leaders. Arlene is so deserving of CCTM's Forest N. Fisch Award – she has been and continues to be a professional who leads by example and instills in all a love for mathematics teaching and learning.

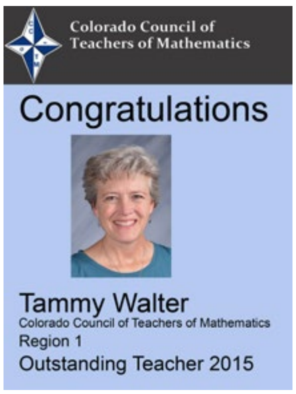
Sadly, our friend and colleague, Arlene Mitchell, quietly passed away on Saturday, December 12, 2015 at her home. Arlene was a strong contributor to and dynamo in the mathematics community, well respected for her knowledge and practice.

AWARDS

CCTM Teaching and Leadership Awards: Celebrating Colorado Classrooms

Rachel Risley, CCTM Awards Chair

The Colorado Council of Teachers of Mathematics (CCTM) teaching and leadership awards provide our community an opportunity to celebrate the accomplishments of our colleagues in the mathematics teaching and learning community. This year CCTM honored twelve teachers and two leaders at our annual conference. Please meet our 2015 awardees and join the CCTM board in hearty congratulations to all!



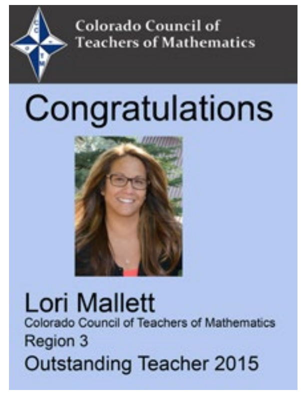
Tammy Walter has been teaching for 11 years and currently teaches 1st and 2nd grade at Carl Sandburg Elementary School in Littleton. Tammy's students discover that math is intertwined with everyday life. They investigate how much things cost, if they have enough money, if they have a fair share, and the difference between strategy and luck when playing a game. She believes that to someday become full participants in society, her students will need a rich understanding of math.

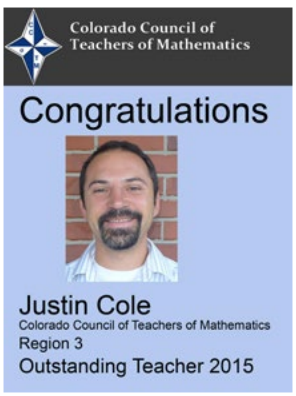
Jennifer Engbretson has been teaching for seven years and currently teaches at North High School, in Denver. A student once said that "Ms. Engbretson's room is so bright that it tricks us into liking math!" Not only is Jennifer's room bright, but her attitude and energy toward her students has been described as electrifying! Jennifer creates a classroom culture that values mistakes and wrong answers. Each day she reminds students that we learn the most when we make mistakes.

Bethany Kearl has been teaching for eight years and currently teaches at Louisville Middle School in Boulder Valley. There are few things that Bethany loves more than listening to students defend the thinking behind their strategies and solutions. Bethany's goal is to shift the emphasis from what she is doing and saying to what students are doing and saying—so that students become confident in their ongoing ability to be mathematicians.

Anita Chakraborty- Spotts has been teaching for 16 years and is currently teaching at Peak to Peak Charter School in Boulder Valley School District. Anita has found technology to be a useful teaching aid. For example, this year students investigated businesses from three different countries. The students created linear inequalities using Google Docs and iPads to show points for the companies. Anita has found that using technology and a 360 degree classroom has greatly improved her students' understanding of mathematics.

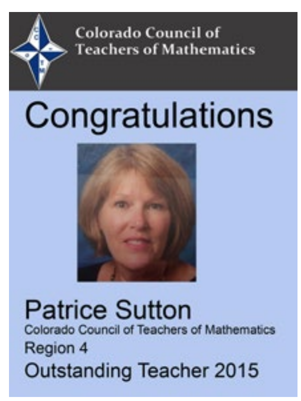
Lori Mallett has been teaching for 22 years and currently teaches at Liberty Middle School in Cherry Creek. Lori empowers her students to be problem solvers. Her classroom is a place where students talk with each other about their ideas and have opportunities to work on math tasks that are complex and involve more than one method of solution. Most importantly, Lori believes everyone *is a mathematician at heart* AND, *can learn to* do mathematics well!

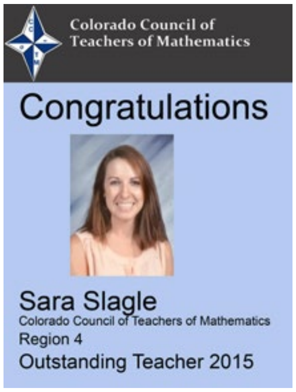




Justin Cole has been teaching for 10 years (most recently 7th grade at Thunder Ridge Middle School) and is now a STAR Mentor for Cherry Creek Schools. Justin's classroom is a place where students discover mathematics and where teaching is about providing opportunities for students to learn. Justin found that as a teacher, he had to pull back and let his students make sense of problems on their own. His students not only have the confidence and perseverance to find answers, but they also understand the meaning behind their solutions.

Patrice Sutton has been teaching for 13 years and currently teaches 3rd grade at O'Dea Core Knowledge Elementary School in Poudre School District. Patrice's beliefs and teaching style reflect a growth mindset philosophy. Her class is mixed ability grouped because she knows that this encourages collaboration, conversations, observations, thinking, and explorations when all students work together. To Patrice, the best sound is hearing the excitement when her students learn something new from their peers or through self-discovery. Patrice's students have learned how to teach each other, take risks, share their thinking, and, perhaps her absolute favorite, use meaningful, precise mathematical discourse.



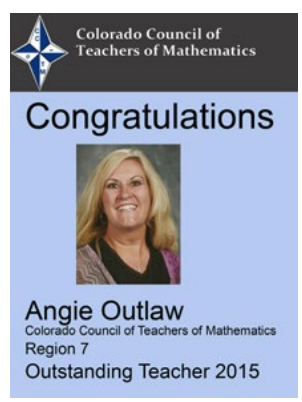


Sara Slagle has been teaching for six years and currently teaches at Fort Collins High School. Sara has the ability to relate to all of her students. Her students appreciate her enthusiasm and positive sense of humor, and they appreciate that she acknowledges their hard work with handwritten notes. Sara is passionate about integrating technology into her classroom. She feels rather than fight against student's craving for technology, we should be embracing it. Sara builds the use of clickers, graphing calculators, interactive websites, and online graphing tools into her lessons.

Jessy McKinley has been teaching for 13 years and currently teaches 2nd grade at Pomona Elementary in Grand Junction. Jessy loves seeing the light go on when kids really, truly understand a new concept. She loves hearing students explain their thinking to their peers and then struggle to understand how someone else solved the same problem differently. Jessy sees the teacher, student, and parents as part of a team. She believes that every child can succeed as a mathematician and makes sure her students believe it too.

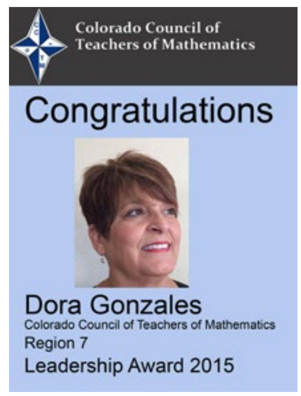
Carla Haas has been teaching for 22 years and currently teaches at Central High School in Grand Junction. Excitement spills out of Carla's classroom, which is productively noisy on a daily basis. Her students are constantly estimating *authentic* problems before they solve them and sometimes instead of finding an exact answer. For example, when students are working on trig applications, they think, "About how far is that?" Students are constantly asked to think of the mathematics in terms of their own world—because that is where they will need it most!

Angie Outlaw has been teaching for 12 years and currently teaches 4th grade at Pikes Peak Elementary School in Colorado Springs. Angie has said that today's students need to move beyond rote and repetition and solve actual real world problems. Everyday, Angie's students use accountable talk to explain how they have seen math used in the last 24 hours. This leads to fun discussions, especially when students talk about sports. Angie's students will definitely be prepared for whatever their future holds.



Veronica Layman has been teaching for nine years and most recently taught 6th grade at Fox Meadow Middle School in Colorado Springs. Veronica always teaches her students conceptually. For example, when teaching division of fractions, rather than teach her students the easy to show but hard to remember rule of "copy, dot, flip", her students use manipulatives to explore the division process and develop the concept and rule for themselves. That way, her students understand the "why" behind the concept, and when confused about the procedure, can relate it back to the concrete, instead of the abstract.

Dora Gonzales has been a teacher and mathematics leader for 30 years and is currently working for the Pikes Peak BOCES as a field coach and instructor for alternative licensure. She is a past president of CCTM, former math specialist in Colorado Springs District 11, and math consultant for Pueblo County Schools. Dora continues sharing her knowledge and mentoring others, as she has done throughout her years of service. She exemplifies leadership in the mathematics community throughout the state and is considered a great mentor by many teachers and leaders in Colorado.



Liz Zitterkopf has been a teacher and a mathematics leader for more than 20 years. She is currently a coordinator of mathematics coaches in Mesa Valley School District 51. Liz models life-long learning and a growth mindset and fosters these in others. Through coaching and professional learning, Liz has increased the equity and access to quality math instruction for all students.

Look for the 2016 CCTM Teaching and Leadership Awards nomination forms on our website, cctmath.org, and nominate a deserving candidate in your region.

MEET THE BOARD

CMT Editorial Staff

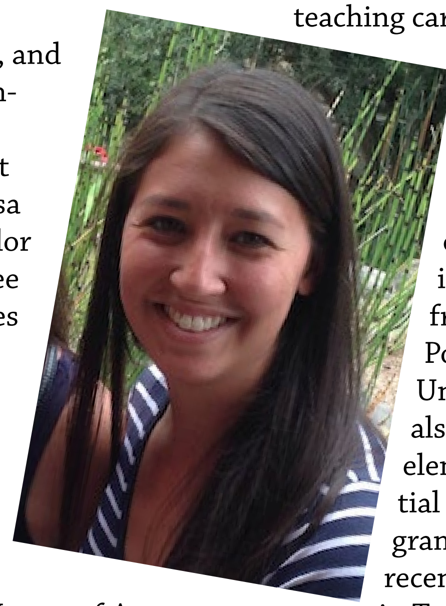
Sandie Gilliam, Editor

EVER WONDER WHO PUTS TOGETHER the *Colorado Mathematics Teacher*?

Sandie Gilliam is the Editor and a board member of CCTM. She selects the focus of each issue, solicits articles, and is part of the panel that edits the submitted articles and puts them all together for your exploration. Sandie is a 32-year veteran teacher in high school math classrooms who now teaches preservice teachers of mathematics at Colorado College. Gilliam was recently elected as the Western 1 Regional Director for the National Council of Supervisors of Mathematics. She is a National Board Certified Teacher in Adolescent and Young Adulthood Mathematics, Carnegie Scholar, and a recipient of the Presidential Award for Excellence in Mathematics Teaching. Sandie received both her BA in mathematics and her MA in educational leadership from San Jose State University.

Charlee Passig Archuleta is an editorial panel member for the *CMT*. She teaches at Rudy Elementary School in Colorado Springs; the 2015-2016 school year began her 25th year of teaching. She has taught regular education in grades Kindergarten, 4, and 5 as well as grades 3–5 special education, middle levels at a youth psychiatric facility, and was a behavior consultant. Charlee holds a BA in Elementary Education from Adams State College (now University), a MA in Special Education from University of Colorado, Colorado Springs, and a MA in Integrated Natural Sciences from Colorado College. She received the CCTM award, 2014 Outstanding Elementary Mathematics Teacher for Region 7. When Charlee is not busy with school and other organizational work, she enjoys reading, biking, cooking, and spending time with her family!

Tessa Zisser is an editorial panel member for the *CMT*, and a 2nd grade teacher at Academy International Elementary School in Colorado Springs. She began her teaching career in California, and has been teaching primary grades for the last seven years. Tessa received her Bachelor of Science degree in Liberal Studies from California Polytechnic State University, where she also completed the elementary credential program. She has also recently received a Master of Arts in Teaching degree from Colorado College. Tessa loves her husband, being outside, teaching, and, of course, math.



Larry Gilliam (Sandie's husband) is the *CMT*'s journal designer. He takes the edited articles, including the photos, drawings, and equations, and with Adobe InDesign creates the *CMT* copy you see. Larry, a retired aerospace vice president, spends his time volunteering for El Paso County Search and Rescue, Civil Air Patrol, and The Salvation Army, as well as the *CMT*. Sandie and Larry have three grown children and two grandchildren, and love to hike and travel around the world. They count themselves fortunate to have been to all seven continents!

FAST CONNECTIONS

Math on the "Planes"

CCTM is a proud supporter of the Colorado Council for Learning Disabilities' annual conference, Math on the "Planes," to be held February 26–27 in Centennial, CO:

Creating A Classroom Environment that Supports Students Who Struggle with Learning Mathematics

Dr. Eden M. Badertscher works to close opportunity gaps in mathematics education and ensure our system of mathematics education is equitable for all students. She brings extensive expertise in teacher education, mathematics curriculum and professional development. Dr. Badertscher provides experiences and tools that enable teachers and school administrators to effectively implement the Common Core Standards for Mathematical Practice.

Participants in this workshop will develop a knowledge base of how inequitable mathematical experiences and counterproductive identities develop and learn new ways to foster productive self-concepts. Participants will expand their knowledge of strategies that create classroom environments that empower students who struggle with learning mathematics. This nurturing classroom environment is designed to facilitate students' progress within the standards.

For more information, please visit the CCLD website: www.cocld.org