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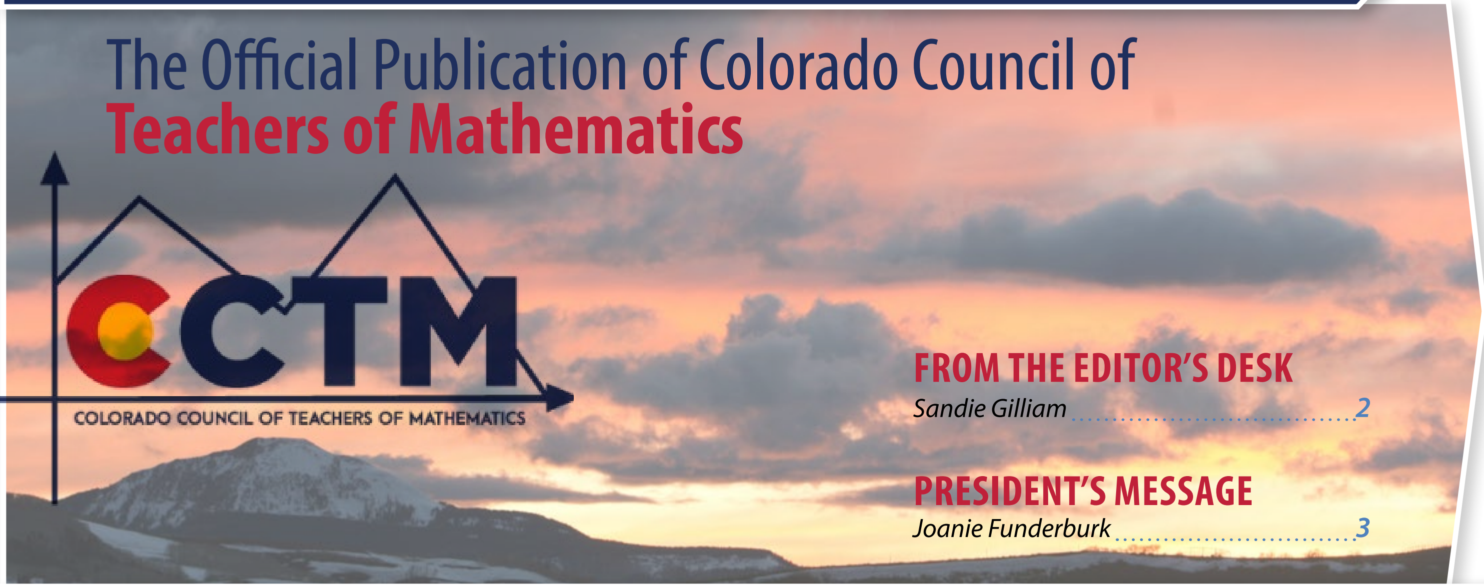

Reasoning and Problem Solving II

CONTENTS & FEATURES

The CMT presents a variety of viewpoints. Unless otherwise noted, these views should not be interpreted as official positions of CCTM or CMT.

From the Editor's Desk

Sandie Gilliam, Editor

ARE YOU AN EDUCATIONAL LEADER? Many consider this synonymous with school administration. But is it? Are leaders the people in schools with status and power? Others consider that certificated district office personnel out of the classroom—which include project director, learning coordinator, assessment facilitator, and the like—are educational leaders, as well. *Can classroom teachers fit this job description*, or only department chairs and grade-level representatives in a school?

What exactly is an educational leader? What role(s) do they take on? I read somewhere that "the great leader is a servant first." Serving others is important, but the most important thing is to serve the values and ideas that help shape the school as a community and provide the conditions where students and teachers can promote and sustain learning for themselves. *Does that mean that everyone is or should be an educational leader?* Like with any endeavor, everyone won't be interested in it, or cut out to be one in schools. We are all leaders at different times in different places—cooking in the kitchen, helping in church activities, pursuing the values and activities on boards such as the Salvation Army, to name a few. But some like using their classroom to give voice to new ideas, respond to standards, and reach attainable goals for students and teachers. They like discussing issues with colleagues and helping others figure out how to make each math lesson better. They use their knowledge to inspire and reach out to students, parents, and the community.

With this different meaning of instructional leader and roles, do you now consider yourself to be one? *Where in Colorado can one learn about, get on board with, and actively practice this role?* Are you familiar with Colorado Mathematics Leaders (CML)?

"The purpose of CML shall be:

- to build collaboration and share expertise among Colorado mathematics leaders;
- to provide ongoing experiences, information, and support to enable mathematics educators to assume leadership roles which lead to positive mathematics experiences for students in Colorado, and
- to establish and maintain a communication network among mathematics leaders and other related educational leaders and entities, including the Colorado Department of Education."

So, whether you are an instructional leader in mathematics inside or outside of the role as a classroom teacher, consider being a part of CML. You can help other teachers better accomplish their classroom responsibilities, and students become curious about and engaged in mathematics.

Contact Raymond Johnson, Johnson_R@cde.state.co.us, to get on the CML listserv, attend meetings in person or online, and become the leader you were meant to be!

President's Message

Joanie Funderburk, CCTM President

The Colorado Council of Teachers of Mathematics (CCTM) was founded in 1950, but teaching math then certainly looked different! Math teachers weren't talking about growth mindsets, STEM, or college-readiness. Yet, 67 years later, here we are! Our students are technology "natives," growing up in a world of cell phones, the Internet, and Google. As math educators, we have to work harder to keep up!

The CCTM Board is dedicated to ensuring that we are meeting the needs of our hard-working Colorado math teachers. We want to be sure that we are creating opportunities for educators to collaborate across the state, to come together to learn, to help one another analyze the strengths and concerns in our schools, and to provide the best possible classroom experiences in math for the students of Colorado. Yet, with all of this work, we need your help! We want to be sure that you are well-represented, and that your voice is heard to help guide our work in supporting you.

The CCTM Board is comprised of a representative from each region. Your representative's name and email address can be found below, as well as on our website, www.cctmath.org. Please make contact to let them know what you need and want from CCTM. We appreciate your feedback and will work to make your suggestions come true, so that we can better help with what you do every day with students. Reaching out to me directly is also an option! I'd love to hear what you are doing, what is going well, and what you need in order to do your work better.

Our annual conference is another avenue by which we gather your input! Based on feedback from our 2016 annual conference, the Board is revising our 2017 fall conference. The updated conference is filled with more focus, more opportunities to engage with other members and presenters, and some new structures for making the most of your professional learning time. The 2017 conference theme is "Captiv8: Captivating and Engaging Students' Mathematical Minds," and we are excited to be moving the date to September 14–15 and the location to the beautiful University of Denver campus for a more academic and flexible meeting space. Keep an eye out for our call for proposals (coming in May) and conference registration, opening in late July. We would love to have you volunteer for many of the committees and jobs that help make the conference spectacular. It's a great way to get involved at a commitment level you can be comfortable with.

As we move into the last month of the 2016–17 school year, we hope you'll tell us how to make your CCTM membership most beneficial. Send us an email today!

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IN THE CLASSROOM

The Redesigned SAT Calls for a Conceptual Approach

Jeff McCalla, St. Mary's Episcopal School, Memphis, Tennessee

I TEND TO OVERESTIMATE the math abilities of my students. But rarely am I this wrong. I posed this SAT-like question:

$f(x) = (x + 6)(x - 4)$

*Which of the following is an equivalent form of the function f where the vertex appears as a constants or coefficients?**

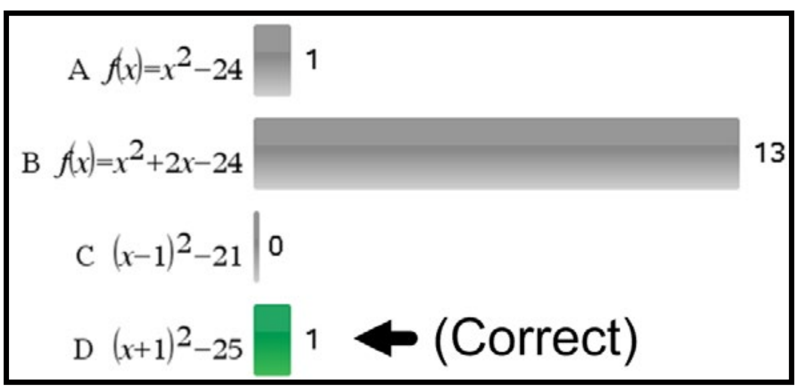
A) $f(x) = x^2 - 24$

B) $f(x) = x^2 + 2x - 24$

C) $f(x) = (x + 1)^2 - 21$

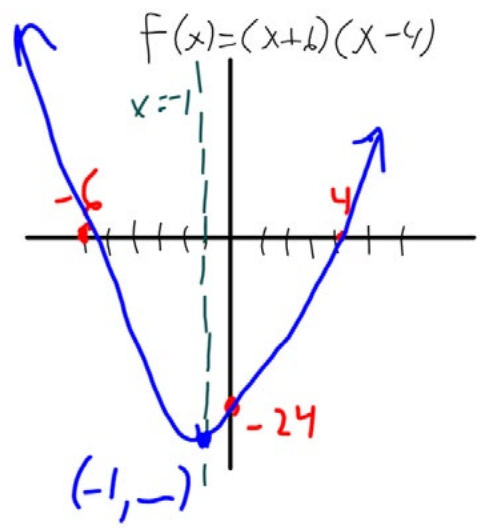
D) $f(x) = (x + 1)^2 - 25$

We had just finished a chapter on transformations of quadratics. I don't want to brag, but, my students were very good at transformations. Being good at procedures does not set up students for success on the SAT. Here is what happened:



Only one student (in all of my Honors Algebra II classes) got it correct? How could this be? Why did they get it wrong? I had so many questions. But right then, I knew that I needed to change the way I taught and assessed my students. I was not doing a good job of preparing my students for questions that were more conceptual than procedural.

I asked the student who got the question right how she solved it. It wasn't the way I expected. She didn't complete the square, instead, she sketched a graph.



She found the intercepts of the parabola and reasoned that the middle of the parabola (axis of symmetry) would be the average of the x-intercepts (-1) . She didn't take the time to find the minimum value of the function, but she knew from the sketch that it had to be smaller than -24 . What an elegant way to to solve this problem!

I am still a work in progress. But here is a summary of things I am learning.

Encourage students to think graphically.

As Jo Boaler says, all students understand math in a deeper way when they can see the graph. The textbook I use (like most textbooks) starts with factoring before it gets to solving a quadratic graphically. This may sound sacrilegious, but I have switched the order. From day one of factoring, I want students to make connections with the graph. The tougher the problem, the more it helps to sketch the graph. From my experience tutoring students, it is often the last thing they would think of doing.

Attend to precision with mathematics vocabulary.

Think about all of the math vocabulary in that opening question: equivalent, minimum value, constant, and coefficient. If students don't understand the language of math, they are going to struggle on many of the wordy SAT questions. Of course, it starts with the teacher, but I am more and more convinced that students don't really understand the math language until they are able to verbally explain a concept to other students. We have to get our students conversing about math to each other! Card sorts are my favorite way to do this. Pair students up and have them ask and answer questions to each other.

Solve problems elegantly.

Mathematics can be too compartmentalized, too prescribed. In Algebra II, I teach my students how to solve quadratics many different ways (factoring, quadratic formula, completing the square, graphing, etc). So, on a test, I'll ask them to solve a quadratic using a certain method. I'm not saying that is a bad way to do things, it assures that they know how to do each procedure. But, what kind of question could I add to my test that would force students to determine which method would be best? Maybe something like this: What method would you use to solve this quadratic? Explain why you chose the method you did. $x^2 + 12x = 64$. I want students to use the structure of a problem to help them decide on the method they would use to solve it.

Bottom line.

If your assessments contain only procedural questions, how can students be expected to perform well on a conceptual test? From my experience, that is not a good plan. I challenge you to be intentional about including conceptual questions on every assessment that students take.

*https://collegereadiness.collegeboard.org/pdf/sat-practice-test-4.pdf

Jeff McCalla teaches AP Statistics and Algebra II at St. Mary's Episcopal School in Memphis, Tennessee. In 2009, Jeff won the Presidential Award for Excellence in Science and Mathematics Teaching. Jeff is an instructor for T³™ (Teachers Teaching with Technology) and has written two books for Wiley Publishing, TI-Nspire for Dummies and TI-84 for Dummies.

REASONING AND PROBLEM SOLVING

Adapting a Number Sense Task to Learn More About K-5 Student Reasoning

Jody Guarino (CA), Chepina Rumsey (IA), Jennie Beltramini (WA), Angie Miller (WA), Kristin Gray (DE), Shelbi Cole (FL)

The fifth grade teachers are in charge of planning the annual Davis Elementary Fun Run. The teachers decide that each adult should run $6/4$ as far as each student in grade 5 and each student in grade 1 should run $3/4$ as far as each student in grade 5. Who has to run the longest distance? Who has to run the shortest distance? Explain your reasoning.

As the fifth grade students engaged with this task, many were struggling and we wondered why. Why couldn't they access the problem? What part of the mathematics was causing them confusion? What type of prior tasks or activities would have helped develop the reasoning necessary to be successful? We thought about the task and considered the knowledge students might access to solve the problem. The following ideas came to mind:

- Do students understand the magnitude of the values, knowing that $6/4$ is greater than $3/4$?
- Do students use their knowledge of benchmarks, knowing that $6/4$ is greater than 1 and $3/4$ is less than 1?
- Do they consider the relationship between the two fractions and notice $6/4$ is the double of $3/4$?
- Do they see the task as a comparison task, perhaps building on comparing fraction skills developed in CCSS in grades 3 and 4?
- Do they tap into their thinking around measurement?
- Do they think of math from an arithmetic stance, relying on computation to solve, or use an algebraic lens, focusing on the relationship of the values?

When we consider the Davis Fun Run problem, the primary alignment is to standard 5.NF.B.5 (See Table 1). However, it also uses language of multiplicative comparison from 4.OA.A.1 and requires a strong understanding of 3.NF.A which establishes that fractions are in fact numbers.

Table 1

Primary CCSSM Alignment to the "Davis Fun Run" Task

Grade and Standard	
5.NF.B.5	Interpret multiplication as scaling (resizing), by explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n\times a)/(n\times b)$ to the effect of multiplying a/b by 1.
4.OA.A. 1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
3.NF.A	Develop understanding of fractions as numbers.

Note: This is not an exhaustive list of CCSSM expectations, but rather a focused look at the ideas related to the task.

We wondered why students did not reason by saying, "Since $\frac{6}{4} > \frac{3}{4}$, and multiplying a number by $\frac{6}{4}$ produces a larger product than multiplying the same number by $\frac{3}{4}$, it is clear that the adults ran farther." In thinking about what students would need to understand in order to reason in that way, we considered another related task (*Figure* 1) from the Partnership for Assessment of Readiness for College and Career (PARCC).

Order the following expressions from least value to greatest value.

Drag and drop the expressions into the correct order

$19 \times \frac{3}{3}$	$19 \times \frac{1}{2}$	$19 \times \frac{3}{2}$	$19 \times \frac{2}{3}$

LEAST GREATEST

Figure 1. PARCC task.

Retrieved 7/13/2016 from: https://prc.parcconline.org/system/files/5th%20grade%20Math%20-%20EOY%20-%20Item%20Set_April%202016.pdf

A group of teachers were analyzing this fifth grade item during a professional development session on fractions when one declared, "This item is silly. All the student needs to do is order these by the size of the fraction." But that is precisely the understanding standard 5.NF.B.5 is targeting—that the size of the product is determined by the size of the fraction, and likely one of the key reasons why students struggle with the Davis Elementary Fun Run problem. Unlike previous state standards that often introduced fractions before students had a strong grasp of whole numbers, the CCSS require a strong foundation in whole number in K–2 before fractions are introduced, and then when fractions are introduced, the headline in Grade 3 is: "Develop understanding of fractions as numbers." While some may argue that students inherently understand fractions better when they are presented as partitioned shapes or food items, what is missed is the understanding that 3/2 is a number between 1 and 2 and more generally that fractions are an extension of the number system that students have been learning since kindergarten or earlier.

Wanting to further investigate some ideas that students would draw upon in solving the tasks described above, we decided to focus on student understanding of comparison and number sense.

The Mathematics: Number Sense and Comparison

Number sense is a foundation for mathematics (Shumway, 2011); therefore it is important for teachers to spend time developing students' understanding of quantitative relationships. Shumway writes about the interconnected web of components involved in number sense, and one of the specific understandings she discusses involves comparison:

> Students have "ability to make comparisons among quantities. For example, they know that 300 is 400 away from 700 by using a mental number line ... Students with strong number sense make comparisons using their sense of quantities, using landmarks such as 10, 50, and 100, and using a mental number line (understanding where numbers fall on a number line)." (p. 9)

Ordering numbers and making comparisons helps students to develop an understanding of quantitative relationships and builds a foundation that students will use in later grades. The idea of comparison is also emphasized by the authors of the CCSS across grade levels; they write, for example, that first grade students will have opportunities to "compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. ... Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes" (CCSS, 2010, Grade 1 Introduction, para 3, http://www.corestandards.org/Math/Content/1/introduction/).

Designing the Task

Curious how number sense and reasoning developed over time, we decided to begin by looking deeper into the understandings of students in primary grades, specifically starting with students in kindergarten and first grade. We looked for a task that could help us gain insight into student thinking. Our goal was to find a task with multiple entry points that could be easily adapted for students at varying grade levels and could afford us the opportunity to understand how students reason across grade levels. A first-grade task from Illustrative Mathematics was selected (See Figure 2). (http://

s3.amazonaws.com/illustrativemathematics/attachments/000/008/467/original/public_task_6.pdf?1462386961).

1.NBT Ordering Numbers

Malik is given a list of numbers:

1 5 10 50 100

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right):

49, 7, 22, 98, and 3

Where in the list should he put each of these numbers?

Figure 2. Original task from Illustrative Mathematics

Using this task as a framework, we considered the list of numbers Malik is asked to reason about and wondered how these could be modified for different grade levels. Knowing that the numbers students are given within the task matter (Land, Sweeney, Johnson & Franke, 2015), we drew upon our experience working with elementary students and designed number sets that would get at important ideas and common misconceptions in the understanding of numbers and the number system and allow opportunities for student reasoning. Table 2 shows numbers we considered, as well as the rationale in modifying the task for kindergarten and first grade students. Considerations included affordances of particular numbers and the understandings that could be elicited including number sequence, magnitude, place value, and use of benchmarks. We also drew upon common misconceptions such as teen numbers, zero, and numbers that students work with less frequently or may not have conceptual understanding of. A goal was to identify numbers that would create opportunities for discussion.

As we thought about how we would use and modify the task, we thought about the way in which the task was formatted and considered the spacing of the numbers. Would this spacing elicit specific understandings and misconceptions? Would creating a scaled list provide additional insight into student understanding? Are there affordances to using the number positioning of the original task? We decided to provide students with a scaled set of numbers to gain insight into student considerations when placing numbers in the sequence, as well as their understanding of the magnitude of numbers.

Table 2

Task Adaptations and Explanations for Grades K–1

Grade Level	Task Design	Number Choice Rationale
Kindergarten students, in Common Core State Standards, work with numbers 1–20. Students often struggle with teen numbers especially numbers like 11, 12 and 13 where the number names aren't clearly connected to the number system in the way that students recognize, such as 4 in fourteen.	Malik is given a list of numbers. 1 5 10 20 (numbers to scale) Where in the list should he put each of these numbers? 2, 4, 15, 0, 11, 19	2 or 4: Do students place between 1 and 5 or think about the magnitude and relationship to anchor numbers 1 or 5? 15: Is 15 placed directly between 10 and 20 or just somewhere between the two numbers? 19: Will students recognize this as 1 less than 20? 11: A number that students are confused by. 0: What do students understand about 0?
First Grade students, in Common Core State Standards, work with numbers 1–120. They develop an understanding of the base ten system and operate on numbers to 100.	Malik is given a list of numbers. 1 5 10 50 100 (numbers to scale) Where in the list should he put each of these numbers? 49, 7, 22, 0, 98, 3	Adapt the original task by adding the number below. 0: What do students understand about 0?

Implementing the Task and Reviewing Student Work

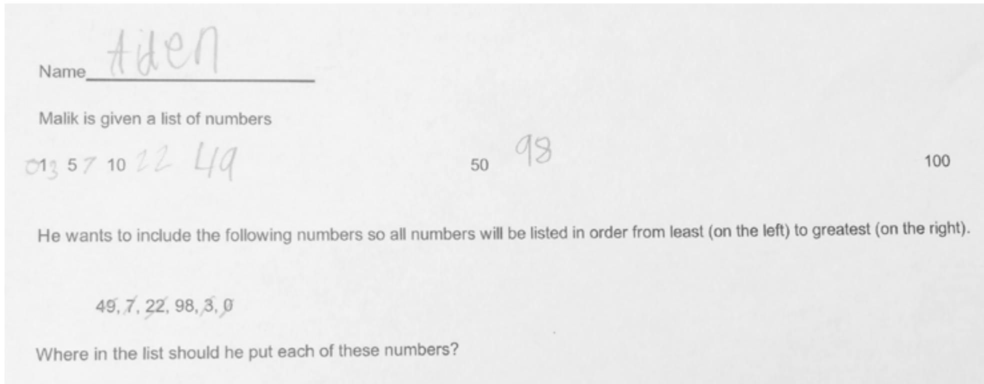
Wanting to learn more about how students reason around the numbers in the task, it was given to kindergarten and first grade students. We view kindergarten and first grade student understanding as a starting point to think about how early number sense might inform later number sense and develop over time. Our hope was to gain insight into understandings they built upon. We wondered:

- What understandings will students build upon?
- How will students think about the values of the number they are placing? Will they consider a number as being greater or less than numbers that are already given in the task?
- Will students think about placement of a number from a given benchmark or start at 1 and use some kind of counting to strategy to determine where to place a number?
- How will students articulate their thinking?

Before giving students the independent task to her first grade students, Mrs. Miller modeled a similar task on the board using numbers within 20. The numbers that needed to be added to the sequence were 0, 2, 9, 15, and 19. Mrs. Miller called on a student to share where 19 would go and to explain his thinking. He said to place the 19 in front of the 20 because 19 is right before 20 when counting. She called on another student to explain where to place 9. The student said, "between the 7 and 11." When Mrs. Miller questioned her about where 'exactly' to put it, the student said, "Exactly between the 7 and 11 because right after 7 is 8 and you have to leave room for 8. Also, right before 11 is 10 and you have to leave room for 10, so 9 would go in the exact middle." At this point students were given the task to complete independently.

A Kindergarten teacher, Mrs. Cardinale, also modeled a similar task on the board before giving the task independently to her students.

When reviewing the student work, we noticed that some students understood the concept of comparison, putting the numbers in the correct order however they didn't understand the relative value of the numbers as indicated by where they placed them in the sequence. One first grade student correctly placed 22 after 10, and then 49 after 22, but the numbers were written close together leaving a large space between 49 and 50, see Figure 3.


Name Aiden

Malik is given a list of numbers

0 1 3 5 7 10 22 49 50 98 100

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right).

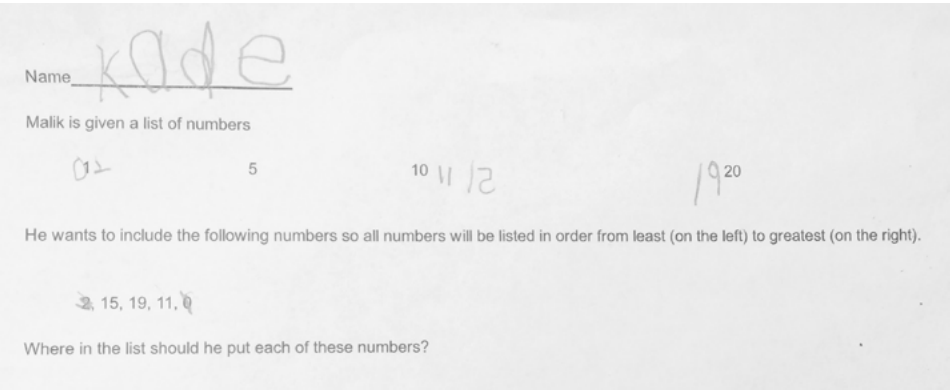
49, 7, 22, 98, 3, 0

Where in the list should he put each of these numbers?

Figure 3. First grade student's work: Aiden.

We find it interesting that these few students did not place the numbers in a correct location considering the magnitude. However, they do appear to understand the concept of greater and less. They did put the numbers in the correct order, but didn't connect to the number line, which would have helped them consider the magnitude. We wonder how teachers can help students connect to a mental number line when comparing and ordering numbers. We also wonder how we can facilitate students' reasoning of magnitude when ordering and comparing numbers.

We noticed that several kindergarten students were successful with placing numbers correctly in the sequence, that were adjacent to numbers on the list. For instance, it appears Kade understood 11 as 1 more than 10 and 19 as 1 less than 20, because he put them right next to the adjacent numbers, see Figure 4. However, Kade placed 15 in the correct order, after 11, but not in the correct place based on magnitude. He put 15 right next to 11, instead of in-between 10 and 20.



However, if the following student must place the card with a 2 on it, he or she must adjust the position of the 1 in relation to their placement of 2. While there are many ways to do this, one possible arrangement is given below.

Figure 4. Kindergarten student's work: Kade.

We think it could be interesting to consider ways we might modify the task to better understand students' reasoning. In this case, the number 15 was the only non-adjacent number. If we gave the task again, we would add another non-benchmark number that's not adjacent to given numbers, such as the number 8. This would tell us more about kindergarten students' ability to reason about the magnitude of numbers when comparing and ordering.

The Sequel...Developing Student Thinking

While tasks such these are invaluable ways to look deeper into individual student thinking, there are also many classroom activities that build this same reasoning, in particular a yarn number line and guess my number. A yarn number line, or clothesline, is exactly that—a piece of yarn hung across a classroom resembling a number line. Prior to starting, the students are not told what numbers will be placed on the line. Each student, or pair of students, is given a folded index card with a number on it. The students are called on randomly to place their card on the number line and are allowed to adjust the previously placed cards if needed. For example, a 5th grade student could place 1 at the end of the yarn line, believing the line only goes to 1 as pictured below:

This activity encourages students to think about not only the magnitude of numbers and distance between them but also their location in relation to one another. In an informal way, students reason about such things as midpoints and fractions of numbers. It is also an extremely flexible activity that can be adapted simply by changing the numbers on the cards and the distance between the starting and ending number.

Guess my number is another activity that engages students in thinking about number relationships, but in a more abstract way. Students play this game in groups of 2 with one player attempting to guess the number of their partner. These numbers can be chosen by the student or the teacher if they want to be more explicit in the numbers students are using. The partner who is guessing is allowed to ask only yes or no questions of the player whose number they are trying to guess. The conversation may go something like this in a 2nd grade classroom:

Student 1: Is your number less than 100?

Student 2: Yes

Student 1: Is your number between 30 and 50?

Student 2: No

Student 1: Do I say your number when I count by 5s?

Student 2: Yes

Student 1: Is it less than 60?

Student 2: Yes

Student 1: Is your number 55?

This activity is also very flexible in that it can be adapted by number choices and, for students who may struggle to visualize the numbers, by tools. An unmarked number line or 100 chart could be used to help students who need them.

Not only should students engage in these activities across all grade levels, but teachers should as well. Using an activity such as this in a PLC provides teachers with an opportunity to look closely at the standards at each grade level and think about the progression of these ideas in order to create a coherent learning experience for all students. We can imagine a group of teachers at various grade levels taking a task, adapting it for their grade level, and connecting the ideas of each adaptation to the [learning progressions] document. Table 3 includes some ways we thought about doing just that, following the line of thinking used to adapt the task for kindergarten and first grade students.

Table 3

Adapting a task to the grade level

Grade Level	Task Design	Number Choice Rationale
Second Grade students, in Common Core State Standards, work with numbers 1–1000. They extend their understanding of the base ten system and operate on numbers to 1000.	Malik is given a list of numbers. 1 250 1000 (numbers to scale) Where in the list should he put each of these numbers? 500, 0, 100, 300, 987, 5, 243, 50	500: How will students think of 500? Do they consider it directly between 1 and 1000? 0: What do students understand about 0? 100: How will students reason about the “middle-ness” between 1–250? Do they consider the relationship to 125? 300: How will students determine 50 more than 250? 987: How will students think about the value of this number? Will they consider the relation to 1000? 5: How close will students place to 1? 243: What reasoning will students use to place? Will they consider the distance to 250? 50: How will students think about 50 in relation to 250?

Third Grade students, in Common Core State Standards, work with numbers 1–1000 and are introduced to fractions as numbers. A common misconception is that fractions are between 0 and 1. Fractions equivalent to 1 ($\frac{3}{3}$) or other whole numbers ($\frac{4}{1}$) may also be confusing.	Malik is given a list of numbers. 0 1 2 (numbers to scale) Where in the list should he put each of these numbers? $\frac{1}{4}$ $\frac{3}{3}$ $\frac{4}{1}$ $\frac{7}{6}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{1}{2}$	$\frac{1}{4}$: How do students approach this, do they eyeball, make four hash marks, break into four equal parts and then label first? $\frac{3}{3}$: Will students recognize as equivalent to 1, understand relationship of numerator and denominator? $\frac{4}{1}$: Will students understand as 4? Where will they place it distance-wise from 2? $\frac{7}{6}$: How will students think about a unit over a whole with different denominator than others so far? $\frac{3}{4}$: How will students think about $\frac{3}{4}$? Will they use a benchmark number such as $\frac{1}{2}$ or 1 to determine where to place it? $\frac{1}{3}$: How will students think about this? Will they consider the relationship to $\frac{1}{4}$ and see it as more or less? Do they use whole number thinking, a common misconception and think a bigger denominator is a bigger number? $\frac{1}{2}$: How will students think about $\frac{1}{2}$? Will they see it as a number or place it in the halfway point of all numbers (half of something)? OR Intentionally not include $\frac{1}{2}$. Will anyone add $\frac{1}{2}$ to think about where other numbers might go? What are the affordances and constraints of including one or more fractions within the list of numbers provided to students?

Fourth Grade students, in Common Core State Standards, work with numbers to 1,000,000 including fractions and decimals with denominators 10 and 100.	Malik is given a list of numbers. 0 1 2 (numbers to scale) Where in the list should he put each of these numbers? $3/4$ $4/4$ $4/3$ $2/5$ $5/8$ $1\,7/8$ $1/100$ $12/8$	$3/4$: Will students use to help them place the fraction? Will they place it equidistant from $1/2$ and 1? $4/4$: Will students know the fraction is equal to 1? $4/3$: Will students know this fraction is greater than 1, and how will they reason about the unit fraction over the whole? $2/5$: Will students reason this fraction is just shy of the benchmark fraction $1/2$? $5/8$: Will students reason about this fraction in relation to the benchmark $1/2$ using its equivalent fraction $4/8$? $1\,7/8$: Will students know this fraction is $1/8$ from 2? How will they reason about the size of $1/8$? $1/100$: How will students reason about really small fractions? $12/8$: Will students know this fraction is equivalent to the benchmark of $1\,1/2$?
Fifth Grade students, in Common Core State Standards, work with numbers to 1,000,000 including fractions and decimals to thousandths.	Malik is given a list of numbers. 0 $1/3$ 1 2 3 (numbers to scale) Where in the list should he put each of these numbers? .333, 0.3, .005, $1/100$, 1.6, $1\,6/9$, $2\,7/8$, 2.8	.333: How will students think about this? Will they think about equivalence to $1/3$? 0.3: How will students think about this in relation to $1/3$ or .333? How will they determine if it's more or less and how much more or less? .005: How will students think about this in relation to 0? $1/100$: Will students reason that this is almost 0? Will they consider the magnitude and relationship to .005? 1.6: Will students use benchmarks to consider where to place this? $1\,6/9$: How will students think about the relationship of $1\,6/9$ and 1.6? $2\,7/8$: Will students think about this as almost 3? 2.8: How will students think about .8 in relation to $7/8$?

It is amazing we can learn so much about how students engage in a 5th grade task by looking back at how they think about numbers and their values in first grade. While we had the opportunity to look deeper into how students think through a task, there is still so much more to be understood. We continue to be intrigued by student thinking and curious…

What would we learn by interviewing students? Would we learn things about their thinking that weren't visible through the paper-pencil task?

We saw things we didn't anticipate in student work. What would we learn about the thinking of students at other grade levels?

Even though the task was originally designed to build student thinking, the Davis Fun Run task has immensely contributed to our thinking as educators!

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PROFESSIONAL DEVELOPMENT

Colorado Core Advocates–A teacher-led, teacher-driven network

Joanie Funderburk, CCTM President

Student Achievement Partners (SAP) supports a national network of educators called Core Advocates. Much like the other work and resources you can find on SAP's website, www.achievethecore.org, the work of Core Advocates is grounded in the math shifts of *focus*, *coherence* and *rigor*. Core Advocates work toward *instructional advocacy*: educators owning, supporting, and promoting the resources, tools, and practices that create learning environments in which students develop college- and career- readiness.

In February, nearly 40 Colorado math educators gathered in Denver to work on a Core Advocates' campaign focused on the three aspects of *rigor*: conceptual understanding, procedural skill and fluency, and application (with a focus on increasing students' conceptual understanding of important mathematical ideas). Read below to hear what some of them had to say about their weekend and the launch of this work statewide.

Liz Zitterkopf and Felicia Castro, District 51 Schools:

> "Colorado Core Advocates convened for a weekend of learning together at The Brown Palace Hotel in Denver February 11th and 12th. Our work kicked off with a deep dive into the shifts of *focus, coherence and rigor.* We worked in K–12 teams to identify which strands of rigor [were] called for in the Standards. One of our biggest take-aways was the reminder that each strand is equally important and required for a balanced approach to mathematics learning. We were reminded of the importance of balance among the three strands of rigor and selecting the tasks that match the strand of rigor and how they work together to develop a comprehensive understanding of mathematics. We are still wrestling with Jason Zimba's thinking around defining *application standards* as only those including the words "real-world" contexts. The idea being that when we think about standards that require the application of previous math understandings to new mathematical learning, it highlights coherence and progression across the grades. Therefore application of mathematical understanding to new mathematical context is not necessarily going to be defined as the application strand of rigor.
>
> We categorized mathematical tasks based on the shift that the task addressed, again, reminding us to consider the need for balance of *focus, coherence, and rigor* in our classroom activities. We also came to see that standards and tasks do not always fit nicely into a single strand of rigor. Mathematics learning is clearly fluid and not linear. This presents many challenges for our school communities. We really need to think of these more as spotlights rather than buckets. Strategically selected tasks are important.
>
> As our district is moving to become a performance based system, we are creating transparency around expectations. Teams of teachers are working alongside content facilitators to create standards based rubrics for all content areas. The mathematics teams are not only considering strands of rigor, but are challenged to provide examples of authentic rather than contrived application. We recognize that our ultimate goal is always college and career readiness. In designing aligned assessments we now know that "real world" context is necessary, although not sufficient to reach our goals.
>
> We were introduced to the Instructional Coaching Guide as a tool to give clear, intentional feedback for teachers. This is something we would like to spend more time using within coaching conversations and standards based lesson planning.
>
> We have already put some of our learning around conceptual understanding into practice in conversations with colleagues and using the Building Conceptual Understandings in Mathematics video to set the stage with parents for a family math night."

Melissa Higgins, Aurora Public Schools:

"For me, the most powerful part of our weekend time together was the opportunity to collaborate with other math teachers across the state that have the same philosophy about Common Core and teaching math. It was refreshing to hear other people have the same values and beliefs about what is truly important in teaching math to students.

Beyond being surrounded by like-minded people, it was valuable to be able to notice and name strategies that could be implemented into our classrooms in order to support all three aspects of rigor, while identifying why each is so important in developing well-rounded math students was very powerful.

I felt more confident in my knowledge and practice by collaborating with like-minded peers. This particular piece of the weekend was huge for me because I feel more confident in my understanding of the three aspects of rigor now, so I am prepared to discuss with confidence why they all need to be incorporated in all math classrooms. I personally use all three in my classroom, but didn't have the background knowledge before this weekend to be able to defend my perspective with someone who believes in only using procedural fluency. After attending this weekend, I feel that I have the necessary background to explain to any teacher, parent, or student why all three make for a well-rounded and successful math student."

Beth Sasse and John Ryan, Weld County Schools:

"In attending the Core Advocates Convening, we found the whole weekend informative and applicable to our classrooms. One of the most powerful aspects of the weekend for us was getting to work with teachers across all grades! Typically, we only get to experience professional development with other high school teachers, but at this conference we were sitting at a table with two elementary school teachers. Being able to look at these important ideas with teachers from various grade levels really opened our eyes to how the standards work as a whole. I am eager to be a part of more mixed-grade professional development opportunities!

Upon returning to school from the Core Advocates Convening, we had the opportunity to share our learning with our colleagues. We presented to the other teachers in our department, instructional coaches, principal, assistant principal, superintendent, and assistant superintendent. We were able to share about the shifts in the common core, but focused our discussion on rigor. After presenting rigor as the "three-legged stool" relying equally on conceptual understanding, application, and procedural skill and fluency, we then finished our meeting by talking in-depth about conceptual understanding and the Colorado Initiatives.

Going forward, we are looking at creating a three-year plan for our mathematics department. We will look at how to shift our teaching to include more conceptual understanding in our classrooms. We are excited to work together to find ways to enrich the mathematics in our classrooms!"

CDE CORNER

Connecting with a Broader Community: The Best of the Math Ed Web

Raymond Johnson, CDE Mathematics Content Specialist

Just as we want our students to feel they are part of a classroom and school community working towards common goals, I am a strong believer that teachers of mathematics should feel that they are part of a community, too. I hope every teacher feels that they are a key part of their own school community, but beyond those schools, we should all reach out to connect with the broader mathematics education community. By being a member of CCTM and reading articles in the *CMT Journal*, you are an important part of Colorado's community of math educators. I encourage you to write for the *CMT Journal* as well as read, and find other ways to participate in our Colorado community.

There are other communities of math educators beyond professional organizations like CCTM and NCTM. Many teachers seek out like-minded educators online. Here in Colorado, the COmath listserv has connected math teachers across the state since the 1990s. More recently, and broadly, social media and blogs have helped teachers of mathematics find each other. Currently, the most active people in these communities use Twitter and blogs to connect and share ideas. I have been part of this community since 2009, and since the start of 2016 I have been tracking the links teachers share most on Twitter each day. There's a lot of great stuff out there! Here are five of the best things shared in the first three months of 2017.

January 7: Michael Pershan (@mpershan), a math teacher in New York City, wrote a long and worthwhile post focused on helping a student named Rachel who struggles with her math facts. The post is called, "Missing Factors: On Learning What You Don't Know." Michael takes his teaching very seriously, but as we all know, sometimes a teachers' best ideas and efforts are met with little student progress. If it's any indication of Michael's struggles, Marilyn Burns left a comment to this post and used the phrase, "I've had students bring me to my pedagogical knees."

January 31: Even though the basic ideas behind Depth of Knowledge are pretty straightforward, it can be difficult to assign a single level to a particular task, especially if you don't have examples to work with. Robert Kaplinsky (@robertkaplinsky) helps by giving us some examples in his "Depth of Knowledge Matrix." Robert has actually created two matrices: one specifically about secondary math DOK, and an earlier one focused on elementary math DOK. Even if you don't agree with Robert's examples 100 percent of the time, these topic-by-topic, level-by-level matrices are useful for sharpening your DOK thinking.

February 22: Do you want to see what another teacher experienced during a thought-provoking day of math professional development? Gregory Taylor (@mathtans), a math teacher from British Columbia, Canada, had the privilege of welcoming Peter Liljedahl to his school to deliver a keynote address on a PD day. Peter Liljedahl is a professor of mathematics education at Simon Frasier university, and if you've heard the term "vertical non-permanent surfaces," that research comes from Peter's work and Gregory explains those and other ideas in this post.

March 18: Sometimes the math education community on Twitter shares the latest and greatest, and other times they share an older post that continues to resonate among the group. On March 18th, teachers were sharing a 2016 post by Mark Chubb (@MarkChubb3), an instructional coach in Ontario, Canada. The post is titled, "So You Want Your Students to Have a Growth Mindset?" and it's great for questioning your assumptions about what it means to have a growth mindset. Carol Dweck, the researcher behind the idea of mindsets, says we've missed the point, so I think this is a topic teachers should be looking at more closely.

March 26: Graham Fletcher (@gfletchy), a district math specialist from Atlanta, is well known for his 3-Act Tasks and now his *Making Sense Series* of videos. The latest is "The Progression of Early Number and Counting," and it is worth watching even if you aren't a K–2 teacher who works with these ideas every day.

These blog posts, videos, and other resources are just a sample of the great things that get created and shared every day in the online community of mathematics educators. If Twitter isn't for you, that's okay – you can follow blogs using a tool called an RSS reader and there are plenty of other teachers of mathematics that use other social media services, Internet forums, and listservs. Of course, I want you to feel part of your local and state communities as well. As with any community, you get more from it when you give back to it. If you have an idea, story, lesson plan, question, or a struggle on your mind, share it!

IN THE CLASSROOM

Letting Go

Kevin Junod, Centennial Elementary, Harrison School District 2

One of the most challenging tasks as a teacher has been changing perspectives and methods to be more problem-solving based. During my first year of teaching, my teammates and I taught our students computations. We thought having our students simply solve a calculation correctly was true understanding of mathematics. Teaching just computations to rote memory was easy to teach and felt rewarding. I felt like I was an amazing teacher because almost all of my students were "successful." I failed my students when they solved story problems or any problem that did not have an obvious equation to solve. My students were unsuccessful at these problems because the problems were too complex for them to identify the process to the solution. I realized I had taught my students to be like computers. If they saw a problem a certain way, they would be successful. Like a computer, when one thing differed from the known and expected, my students couldn't successfully solve the problem.

As my district began to rewrite our curriculum maps to mirror the expectations of Common Core State Standards, I began to research how my teaching style would have to change. I found my approach that first year was the complete opposite of what I would need to become. I realized I would have to transition to this new style of teaching. I did not then know the challenge I was facing

I began by planning my lessons differently. I intentionally placed more complex concepts throughout the lesson to help challenge my students. As anticipated, my students were stumped and stared at me like I gave them a problem in a foreign language. I then realized I was going to have to teach my students problem solving strategies to help identify the process to the solution. I began modeling how I would make sense out of complex problem. I was surprised that with this questioning and brainstorming as a group, it took over 10 minutes for just one problem. Deciding this was too long for a single problem, we would usually skip the next challenging problem and just accomplish the modeling problem. I kept trying to quicken the pacing of the modeling to squeeze in time for students to apply, but I never accomplished this. Thinking the most important component of a lesson was the practicing of computation problems still, I decided there was not enough time in the lesson for modeling and practicing of difficult problems.

Learning to let go of both the ideology that students need repeated practice of problems and the control of my classroom was the biggest challenge in creating a classroom that focused on problem solving skills. I had to learn to let my students fail on problems repeatedly to the point of frustration to allow them an opportunity to improve on their problem-solving skills. Just allowing them to struggle was not enough; I had to anticipate their struggles and create guiding questions that would support them in solving the problem. The transition from helping students with complex problems to letting them fail repeatedly was crucial and troublesome. Teachers want to be there to support their students and help them fix their mistakes, but by doing this I never allowed my students to learn how to catch and fix their mistakes on their own. They became dependent on me to be there to guide them through strenuous problems and tasks in my classroom. As I became better at this, I noticed that my students were gaining a better and deeper understanding of the material. They were also gaining a strong sense of accomplishment because they were able to do this without my help. Some of my students still depend upon me for assistance with complex problems, but for the majority of my students I have become more of a facilitator. This switch has made teaching more rewarding than having my students receive a problem, solve the problem, and then rinse and repeat.

NCTM CORNER

NCTM Board Report, February 2017

Catherine Martin, CCTM Past-President

The NCTM Board of Directors met in Reston, VA, at NCTM headquarters in February 2017 and took the following actions:

- The Board approved the FY2018 budget with a focus on aligning resources and priorities.
- The Board approved in concept a new membership model focused on building community and member value.
- The Board and staff continued its own professional development surrounding issues of access, equity, and empowerment. The discussion centered on recent response commentaries published in *The Journal of Urban Mathematics Education*.
- The Board approved the formation of a joint NCTM-NCSM task force on conferences.
- The Board approved a position statement on *Mathematics in Early Childhood*.
- The Board elected to merge the current Publishing Committee and Educational Materials Committee into a new single committee, which will be known as the Publishing Committee.
- The Board approved the 2017 Legislative Platform.
- The Board approved taking action to ensure the Research Conference and Annual Meeting overlap by a full day.
- The Board approved the implementation of a variety of community and student engagement activities for the DC Annual Meeting.

As you plan your summer, consider attending NCTM Interactive Institutes. These institutes offer two and a half days of face-to-face, in-depth professional development provided by experts in mathematics education for pre-K–grade 12 teachers and school leaders. They will include:

- Instruction aligned to **college- and career-ready standards**
- Effective **teaching strategies** through the *Principles to Actions* publication
- **Practical classroom strategies** to promote student success

Two institutes will be held this summer in Baltimore:

- Supporting Students' Productive Struggle: July 20–22, 2017 • Baltimore, MD
- Facilitating Meaningful Mathematical Discourse: July 17–19, 2017 • Baltimore, MD

And, be sure to check out NCTM resources at www.nctm.org/leaders to find **Tools for Classroom Instruction**

- Activities with Rigor and Coherence
- Classroom Resources Collaboration Center
- The Math Forum at NCTM: Problems of the Week, Ask Dr. Math , Teacher2Teacher, and Math Tools
- Research briefs, teaching tips, blog posts, and more!