Strategic Allocation to Premiums in the Equity Market

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Abstract

Investors tend to focus on harvesting the risk premiums offered by traditional asset classes when making their strategic investment decisions. Some recent papers, however, argue that investors should also consider various other premiums for possible inclusion in the strategic asset allocation. Examples of such premiums that have been documented for the equity market are the size, value, momentum and low-volatility effects. In this paper we show that the theoretically optimal strategic allocation to these premiums is sizable, even when using highly conservative assumptions regarding their future expected magnitudes. We also discuss the pros and cons of two ways of obtaining the implied exposures in practice, specifically passively managed index funds versus actively managed quant funds.

1. Introduction

Investors tend to focus on harvesting the risk premiums offered by traditional asset classes when making their strategic investment decisions. An abundance of literature, however, shows that various other premiums may be captured in financial markets. Examples are the well-known value and momentum premiums, which have been found to be present within many asset classes (see, e.g., Asness, Moskowitz and Pedersen, 2009, for a good overview). In a recent paper, researchers from MSCI Barra argue that significant efficiency gains may be realized by expanding the traditional investment opportunity set based on asset classes with such premiums (Bender, Briand, Nielsen and Stefek, 2010). In the same spirit, a key recommendation of finance professors, asked by Norway's Government Pension Fund to evaluate its investment performance, is that the fund should seek explicit exposure to various non-standard premiums next to the risk premiums offered by established asset classes (Ang, Goetzmann and Schaefer, 2009).

Inspired by these studies, we examine if it indeed makes sense for investors to include non-traditional premiums in the strategic asset mix and how such an investment approach can be implemented in practice. We focus our analysis on the allocation to premiums directly related to the equity market. We find that the theoretically optimal allocation to various premiums in the equity market is sizable, even when using highly conservative assumptions regarding their future expected magnitudes. We next argue that it should be a strategic decision to allocate to these premiums, rather than a decision that can be postponed to some later stage of the investment process. Finally, we discuss the pros and cons of two ways of obtaining exposure to non-traditional premiums in practice, specifically passively managed index funds versus actively managed quant funds.

2. Premiums in the equity market

Investors typically include equities in their strategic asset allocation in order to earn the expected equity premium. Empirical research supports the existence of

such a premium by showing that, historically, equities have offered a higher average return than a risk-free investment in Treasury bills or Treasury bonds. However, empirical research has also shown that more return premiums may be captured in equity markets. Examples include the size premium (Banz, 1981), the value premium (Fama and French, 1992), the momentum premium (Jegadeesh and Titman, 1993) and the low-volatility (or low-beta) premium (Black, Jensen and Scholes, 1972[1]). In the literature the existence of these premiums is widely acknowledged, and the debate has moved from whether these premiums exist, to why they exist. One stream of literature argues that a systematic return premium should reflect a compensation for bearing a certain risk, while another stream of literature argues that these premiums are the result of structural mispricing, arising from systematic behavioral biases of investors. For the purposes of this paper it actually does not really matter which view one believes to be true. Once one acknowledges that, for whatever reasons, certain premiums have been and will continue to be present in the equity market, the question arises as to which investment implications follow from this.

In Exhibit 1 we quantify the historical equity, size, value, momentum and low-volatility premiums for the U.S. equity market over the period July 1963 to December 2009. The equity premium is based on the Market-Rf factor of Kenneth French[2], which represents the value-weighted return in excess of the risk-free return of the entire CRSP universe at each point in time. It is also used to calculate CAPM betas. The small stock portfolio is the 'med 40' size portfolio of Kenneth French, which consists of the stocks in the CRSP universe with a market capitalization between the 30th and 70th percentile of NYSE stocks. The value stock portfolio is the 'big-value' portfolio of Kenneth French, which consists of the 30% highest book-to-market stocks among the stocks in the CRSP universe with an above NYSE-median market capitalization. The momentum stock portfolio is the 'big-momentum' portfolio of Kenneth French, which consists of the 30% highest past 12-1 month return stocks among the stocks in the CRSP

[1] Or, more recently, Blitz and van Vliet (2007) and Baker, Bradley and Wurgler (2011).
[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

universe with an above NYSE-median market capitalization. The low-volatility stock portfolio is a ‘big-low-volatility’ portfolio constructed in the same spirit as the ‘big-value’ and ‘big-momentum’ portfolios of Kenneth French, consisting of the 30% lowest past-36 month total-volatility stocks among the stocks in the CRSP universe with an above NYSE-median market capitalization. All portfolios are constructed without look-ahead bias, i.e., based on information that was actually available at each point in time, and without survivorship bias. The small, value, momentum and low-volatility stock portfolios are considered on an equally-weighted basis as well as on a value-weighted basis. Please note that this should not be interpreted as an exhaustive list of premiums documented for the equity market, but that we simply focus on premiums that are widely known and for which data are readily available. By identifying additional premiums investors may expand their opportunity set and unlock additional performance potential.

We focus on long-only, big-cap portfolios in order to ensure that projected return levels are attainable in reality, even when applying the proposed allocations on a large scale. This is different from many other studies, which consider long/short premium portfolios (e.g. the related work of Bender, Briand, Nielsen and Stefek, 2010) and/or premium portfolios which are, to a large extent, driven by small- and micro-cap stocks (e.g. the popular Fama-French factors). By considering long-only portfolios we do not need to assume that many securities can be easily and cheaply shorted, and by focusing on stocks with an above NYSE-median market capitalization we exclude illiquid small- and micro-cap stocks in the definition of our value, momentum and low-volatility portfolios. On average, the resulting portfolios consist of the approximately 800 largest, most liquid U.S. stocks. By considering the ‘med 40’ size portfolio of Kenneth French, our small stock portfolio also represents a liquid small-cap portfolio, which excludes thousands of illiquid micro-cap stocks. As such, it should be no problem to invest, say, $1 billion in the equally-weighted premium portfolios and, say, $10

billion in the value-weighted premium portfolios. This makes our approach relevant for all but perhaps the very largest institutional investors.[3]

INSERT EXHIBIT 1 ABOUT HERE

Panel A of Exhibit 1 shows that the historical equity premium, i.e. the return of the market portfolio in excess of Treasury bills, amounts to 3.9% per annum. However, it also provides clear evidence of the existence of value, momentum and low-volatility premiums. The Sharpe ratios and CAPM alphas (Panel C) are particularly relevant in this regard, as these show the added value vis-à-vis the market portfolio after adjusting for differences in risk, using volatility and beta as risk measures respectively. Equally-weighted and value-weighted premium portfolios both exhibit attractive Sharpe ratios and CAPM alphas, with the highest levels being observed for equally-weighted portfolios. Only the evidence for a size premium is relatively weak.

Exhibit 1 also shows the cross-correlations between the raw excess returns of the various portfolios (Panel B) as well as the cross-correlations between the alphas of the various portfolios (Panel D). We find that all raw-return correlations are high (above 0.75), which can be explained by the fact that all portfolios have a sizable common exposure to the market factor. Nevertheless, we observe high tracking errors of 6-9% between the various stock portfolios and the market portfolio.[4] Moreover, the correlations between the alphas of the various premiums are typically low, varying between -0.45 and 0.64.[5] This indicates that the different premiums reflect distinct effects and that diversification benefits may be obtained by combining them into one portfolio.

[3] For the very largest institutional investors with equity portfolios in excess of, say, over $100 billion, any deviation from the capitalization-weighted index is likely to raise immediate liquidity concerns.

[4] We define the tracking error between a portfolio and an index as the (annualized) standard deviation of their return differences.

[5] The highest long-term correlation (0.64) is observed between the alphas of the equally-weighted value and low-volatility stock portfolios. Although the correlation between these two effects is indeed relatively high most of the time, it can also be low or even negative during specific periods, such as recessions (when value stocks tend to become high-risk). A more thorough analysis might therefore also take time-varying correlation properties into account.

3. Allocating to premiums

Before examining the desired strategic exposure to the various premiums, we address two important issues. First, the effect of transaction costs. Because the market portfolio is essentially a buy-and-hold portfolio, the impact of transaction costs on the equity premium is close to negligible. The size, low-volatility and value premiums are associated with modest amounts of turnover, because these stock characteristics tend to persist over time (for instance, a stock with low volatility in year *t* is usually still a stock with low volatility in year *t*+1).[6] This implies a limited impact of transaction costs. Most turnover is associated with the momentum premium, because it dynamically selects stocks based on their past twelve month return. So although at first glance momentum may appear to offer the largest premium, it is also the effect which will be eroded most by transaction costs in practical applications.

A second issue that needs to be addressed is whether the magnitude of premiums in the past is representative for the future. There is no consensus on the future expected equity premium, let alone on the expectations for the other premiums in the equity market. An understanding of the root causes behind the premiums may be relevant for developing a view on this. If an investor beliefs that a premium reflects a compensation for some sort of risk, the question is whether this risk will continue to be present and rewarded going forward. Alternatively, if an investor beliefs that a premium is caused by mispricing, the question is whether such mispricing is likely to persist going forward.

In order to address these concerns we propose to use conservative return expectations for strategic asset allocation purposes. As an example, we will use the following assumptions:

[6] To illustrate, funds designed to capture the small-cap premium (such as the Vanguard Small Cap Index fund and the Dimensional Fund Advisors Small Cap fund), funds designed to capture the value premium (such as the Vanguard Value Index fund and the Dimensional Fund Advisors Large-Cap Value fund), indices designed to capture the value premium (such as the FTSE/RAFI fundamental indices) and indices designed to capture the low-volatility premium (such as the MSCI Minimum Volatility indices) all report low turnover levels, in the range of 10-30% per annum.

- expected equity premium: 3% per annum
- expected alpha premium for small stock portfolio: 0% per annum
- expected alpha premium for value and momentum stock portfolios: 1% per annum
- same expected return for low-volatility stock portfolio as for market portfolio[7]
- volatilities, correlations and betas as implied by the historical data discussed above (Exhibit 1)
- all weights should be non-negative (no short-selling allowed) and sum to 100%
- weights of the size, value, momentum and low-volatility stock portfolios are constrained to a maximum of 40% (each)[8]

Based on these assumptions we conduct a simple portfolio optimization aimed at maximizing the Sharpe ratio. In addition, we consider a simple but arguably more robust 1/N portfolio, which invests 25% each in the market, value, momentum and low-volatility portfolios. The results for both equally-weighted and value-weighted premium portfolios are shown in Exhibit 2.

INSERT EXHIBIT 2 ABOUT HERE

Panel A of Exhibit 2 shows that the optimal allocations to the value, momentum and low-volatility portfolios are large, despite the use of highly conservative expected return assumptions for these portfolios. In fact, the mean/variance optimized portfolio is invested entirely in a combination of these premiums, and not at all in the market portfolio.[9] Panel B shows that the result is

[7] Given the lower beta of low-volatility stocks this implies an alpha of around 0.70-0.75% per annum.

[8] We impose this constraint in order to mitigate the tendency of mean/variance optimization to converge to a corner solution which represents an extreme portfolio. Imposing constraints has been shown to have a similar effect as more sophisticated robust optimization techniques, such as shrinkage of the covariance matrix, see, e.g., Jagannathan and Ravi (2003).

[9] This result suggests that the standard capitalization-weight market portfolio serves no value in strategic asset allocation if size, value, momentum and low-volatility stock portfolios are also in the opportunity set of investors. However, the alternative portfolios shown in Exhibit 2 still have a large implicit exposure to the market portfolio. Regression results (unreported) indicate that for each of the alternative portfolios over 90% of the return variability can be attributed to (implicit)

a 25-30% improvement of the expected Sharpe ratio. Interestingly, the simple 1/N portfolio is able to capture most of this improvement, with slightly more emphasis on risk reduction than on return enhancement. The historical performance of these portfolios is even more spectacular, with Sharpe ratio improvements in the range of 50-100% compared to the market portfolio. This can be explained by the fact that the (raw) historical premiums have been much larger than our conservative forward-looking assumptions. The results also confirm that significant diversification benefits can be obtained by combining different premiums into one portfolio, as the tracking error of the multi-premium portfolios are significantly lower (4-5% for the optimized portfolios and 3-4% for the 1/N portfolios) than the levels we observed before for the various premium portfolios in isolation (6-9%).

We conclude that investors can substantially improve the risk/return characteristics of their strategic asset allocation by considering not only the classic equity premium, but also other premiums present in the equity market. Moreover, even when one expects premiums to be smaller in the future than in the past, optimal allocations remain sizable.

4. Capturing these premiums should be a strategic investment decision

Having concluded that it is attractive to obtain exposure to premiums such as the value, momentum and low-volatility effects, we now address the question why this should be a strategic investment decision. Alternatively, investors could restrict their strategic asset allocation to traditional asset classes but still be able to benefit from these premiums, simply by selecting funds which, implicitly or explicitly, capitalize on these premiums. Specifically, one might argue that if value, momentum and low-volatility effects can be profitably exploited, the managers doing so will be able to boast successful track records and thereby make it to short-lists in manager selection processes in a natural manner. In fact, one of the key insights of the seminal Carhart (1997) study on mutual fund

exposure to the market portfolio. This can be explained by the long-only nature of our premium portfolios, as a result of which each of these portfolios provides combined exposure to a specific premium and the classic equity risk premium.

performance is that exposures to the size, value and momentum effects are the driving force behind persistence in the performance of actually managed funds.

One problem with the notion that attractive premium exposures will find their way in the portfolio through manager selection is that it requires a belief in added value of active management. Ironically, the findings of Carhart (1997) are often construed to imply that active management does not add value, and that low-cost passive management is a superior long-term investment approach. If passive investing is equated to passively following the market portfolio, this means missing out entirely on the various other premiums that are present in the equity market. The definition of passive investing used by Carhart (1997) is broader though, as the performance of actively managed funds can only be matched by including size, value and momentum portfolios as passive investment alternatives. In the spirit of Carhart (1997), therefore, passive investing does not mean merely following the market portfolio, but also entails investing in other known premiums in the equity market. This implies that passive investors should first determine optimal strategic exposures to not only the equity risk premium, but also to the other premiums in the equity market. The next step is to select passive vehicles which efficiently provide the desired exposures.

However, also investors who are open to the idea of investing in actively managed funds have good reasons to allocate strategically to the various premiums, rather than relying on premium exposures to find their way in the portfolio as a natural result of the manager selection process. One reason is a difference in investment horizons, and a second reason is to better control overall portfolio risk.[10] The first argument is based on the fact that, similar to the equity risk premium, the premiums associated with the value, momentum and low-volatility effects are strong in the long-run, but can be weak or even negative over shorter periods of time. For example, the alpha of the value stock portfolio in our dataset is negative in around 25% of all 3-year periods and 12% of all 5-year periods. The decision to gain exposure to such premiums should therefore be a

[10] For a formal analysis of how these (and other) factors can lead to inefficient outcomes in a two-stage investment process we refer to van Binsbergen, Brandt and Koijen (2008).

strategic one, based on long-term risk/return expectations. Actively managed funds, on the other hand, are typically selected based on much shorter (e.g. past 3- or 5-year) performance track-records. Similarly, they are often dismissed if performance in several consecutive years falls short of expectations. Given the short-term volatility in premium returns, as also illustrated by the high tracking errors of the various premium portfolios, prevailing institutional manager selection approaches tend to favor recent winning styles and are not well-suited for establishing balanced long-term exposures to a range of premiums with low cross-correlations.

A second argument for determining desired premium exposures at the strategic level is that overall portfolio risk can be better controlled in this way. To illustrate this point, consider the alternative situation in which the strategic asset allocation only implies a total allocation to equities, and in which exposures to other premiums in the equity market may only arise as a by-product of manager selection. Because the risk profile of the various premium portfolios can differ profoundly from that of the capitalization-weighted market portfolio, the ensuing risk profile of the overall portfolio may deviate significantly from the intended level determined in the strategic asset allocation process. For example, in Exhibit 1 we observed that the momentum portfolio is associated with higher beta and volatility than the market portfolio, while the low-volatility portfolio is associated with lower beta and volatility than the market portfolio. Overall portfolio risk, including correlation effects with other asset classes, can be most efficiently managed by determining premium exposures explicitly in the strategic asset allocation.

5. Capturing premiums in practice

Assuming that an investor has decided to obtain strategic exposure to premiums such as value, momentum and low-volatility, the next question is how investors can obtain the actual exposures in practice. We distinguish between two

competing approaches: passively managed index funds versus actively managed quant funds.[11]

Traditional index funds have been around since the nineteen seventies and are designed to mechanically follow an index which provides direct exposure to a certain premium. For example, a traditional value index fund mimics a certain value index, which aims to capture the value premium on top of the classic equity premium. More recently, a number of 'alternative' indices have been proposed. Although the philosophy behind these indices may appear to be different, it turns out that their added value is essentially the result of indirect exposures to classic premiums. For example, Asness (2006) and Blitz and Swinkels (2008) attribute the added value of the fundamental indexation approach, as introduced by Arnott, Hsu and Moore (2005), entirely to loadings on the value premium, while Scherer (2010) shows that minimum-variance indices simply capture the low-volatility premium. More generally, Chow, Hsu, Kalesnik and Little (2011) show that the superior performance of the seven most popular alternative equity indices can be largely explained by implicit loadings of these indices on classic premiums, in particular the value and size effects. This implies that, like traditional index funds, alternative index funds may be considered as instruments for capturing various premiums in the equity market.

The main advantages that are offered by passively managed index funds are low costs and simplicity. However, the index fund approach is also associated with several important drawbacks. First, for some premiums the required index funds have been simply unavailable to investors, e.g. for the momentum effect.[12] Second, a passively managed index fund is not necessarily the most efficient way to exploit a certain premium. To understand this, it is important to realize that size, value, momentum and low-volatility portfolios can be constructed in many different ways. For example, a value strategy can be

[11] Of course, traditional actively managed funds may also exhibit significant exposures to premiums *ex post*, but if they do not explicitly indicate *ex ante* that it is their actual objective to capture these premiums in a systematic manner, we would be hesitant to consider them as suitable instruments for gaining strategic exposure to premiums in the equity market.

[12] Only very recently the first passive momentum funds have been introduced. However, these are still small, unproven and unknown by most investors.

based on the book-to-market ratio, but also on the earnings-to-price ratio or the free cash-flow-to-price ratio. Buy and sell rules can also be defined in numerous ways. For example, should a high-momentum stock be replaced if it no longer ranks among the top 20% of the universe, or should one impose some other sell threshold? Third, it tends to be more efficient to exploit multiple premiums simultaneously instead of separately. For example, value and momentum exhibit a strong negative correlation, because value stocks typically score low on momentum, while high-momentum stocks typically have high valuations. By considering value and momentum characteristics simultaneously it is possible to create a more efficient portfolio which avoids contradictory positions and which maximizes diversification benefits. Fourth, sophisticated risk management is required when investing in multiple premiums, because the risk associated with the different premiums can vary significantly over time.

A solution to these issues associated with passively managed index funds is provided by actively managed quantitative investment funds, which are designed to exploit a combination of premiums in the most efficient and risk-controlled manner. The drawback of these more sophisticated actively managed quantitative funds is that they tend to charge higher costs than passively managed funds and that they tend to be less transparent to outsiders, because portfolio weights are based on proprietary modeling rather than some simple formula. In the end, therefore, the choice between passive versus active funds as preferred instruments for obtaining desired exposures to premiums in the equity market comes down to a cost-benefit assessment that each investor will have to make individually.

6. Summary

Investors tend to focus on the risk premiums offered by traditional asset classes when making their strategic investment decisions. Other premiums that are known to exist in financial markets are typically considered in a more ad hoc manner, e.g. as part of the manager selection process, or ignored altogether. In this paper we have argued that investors should explicitly consider all the

premiums that are offered by the equity market when making their strategic investment decisions. We have shown that value, momentum and low-volatility have historically delivered significant premiums, and that, even in case of conservative assumptions regarding their net profitability going forward, the optimal allocation to these premiums is sizable. Finally, we have discussed the pros and cons of different ways to obtain exposure to these premiums in practice: passively managed index funds versus actively managed quant funds.

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Exhibit 1: Historical premiums

This table shows returns characteristics of various U.S. equity portfolios over the sample period 1963:07 until 2009:12. The market portfolio (column 1) is based on the Market-Rf factor of Kenneth French, which represents the value-weighted return in excess of the risk-free return of the entire CRSP universe at each point in time. It is also used to calculate CAPM betas. The small stock portfolio is the 'med 40' size portfolio of Kenneth French, which consists of the stocks in the CRSP universe with a market capitalization between the 30th and 70th percentile of NYSE stocks. The value stock portfolio is the 'big-value' portfolio of Kenneth French, which consists of the 30% highest book-to-market stocks among the stocks in the CRSP universe with an above NYSE-median market capitalization. The momentum stock portfolio is the 'big-momentum' portfolio of Kenneth French, which consists of the 30% highest past 12-1 month return stocks among the stocks in the CRSP universe with an above NYSE-median market capitalization. The low-volatility stock portfolio is a 'big-low-volatility' portfolio constructed in the same spirit as the 'big-value' and 'big-momentum' portfolios of Kenneth French, consisting of the 30% lowest past-36 month total-volatility stocks among the stocks in the CRSP universe with an above NYSE-median market capitalization. All portfolios are constructed without look-ahead bias, i.e., based on information that was actually available at each point in time, and without survivorship bias. The small, value, momentum and low-volatility stock portfolios are considered on an equally-weighted basis (columns 2-5) as well as on a value-weighted basis (columns 6-9).

	value-weighted	equally-weighted premium portfolios				value-weighted premium portfolios			
	market portfolio	small stocks	value stocks	mom stocks	low-vol stocks	small stocks	value stocks	mom stocks	low-vol stocks
Panel A: raw return characteristics									
excess return (geometric, ann.)	3.9%	5.7%	8.3%	8.8%	5.9%	6.0%	6.5%	7.2%	4.3%
volatility excess returns (ann.)	15.6%	20.0%	17.4%	18.5%	13.3%	18.7%	16.2%	17.0%	13.1%
Sharpe ratio	0.25	0.29	0.48	0.48	0.44	0.32	0.40	0.42	0.33
ann. tracking error	0.0%	8.0%	8.6%	7.5%	7.3%	6.6%	8.2%	6.8%	6.4%
Panel B: correlation excess returns									
market portfolio	1	0.93	0.87	0.92	0.89	0.94	0.87	0.92	0.92
small stocks		1	0.89	0.89	0.83	1	0.84	0.86	0.80
value stocks			1	0.79	0.92		1	0.76	0.87
momentum stocks				1	0.79			1	0.83
low-volatility stocks					1				1
Panel C: CAPM-adjustment									
CAPM beta	1.00	1.19	0.97	1.09	0.76	1.13	0.90	1.00	0.77
CAPM alpha (ann.)	0.0%	1.1%	4.6%	4.6%	3.0%	1.7%	3.0%	3.3%	1.3%

Exhibit 1 (continued)

	value-weighted market portfolio	equally-weighted premium portfolios				value-weighted premium portfolios			
		small stocks	value stocks	mom stocks	low-vol stocks	small stocks	value stocks	mom stocks	low-vol stocks
Panel D: correlation CAPM alphas									
small stocks		1	0.44	0.23	0.08	1	0.11	0.01	-0.45
value stocks			1	-0.05	0.64		1	-0.16	0.39
momentum stocks				1	-0.13			1	-0.04
low-volatility stocks					1				1

Exhibit 2: Portfolio comparison

This table compares three different portfolios: first, a benchmark which represents the value-weighted market portfolio (column 1), second, a 1/N portfolio invested equally in the market portfolio, value stocks, momentum stocks and low-volatility stocks and third, a portfolio optimized for maximum Sharpe ratio under the following assumptions: (i) expected equity premium: 3% per annum, (ii) expected alpha premium for small stock portfolio: 0% per annum, (iii) expected alpha premium for value and momentum stock portfolios: 1% per annum, (iv) same expected return for low-volatility stock portfolio as for market portfolio, (v) volatilities, correlations and betas as implied by the historical data in Exhibit 1, (vi) all weights should be non-negative (no short-selling allowed) and sum to 100%, and (vii) weights of the size, value, momentum and low-volatility stock portfolios are constrained to a maximum of 40% (each). The table shows results for equally-weighted size, value, momentum and low-volatility stock portfolios (columns 2-3) as well as for value-weighted size, value, momentum and low-volatility stock portfolios (columns 4-5). Historical returns are based on the data as in Exhibit 1, ignoring transaction costs and management fees.

		equally-weighted premium portfolios		value-weighted premium portfolios	
	benchmark portfolio	simple 1/N portfolio	optimized portfolio	simple 1/N portfolio	optimized portfolio
Panel A: portfolio weights					
market portfolio	100.0%	25.0%	-	25.0%	-
small stocks	-	-	-	-	-
value stocks	-	25.0%	23.1%	25.0%	37.4%
momentum stocks	-	25.0%	40.0%	25.0%	40.0%
low-volatility stocks	-	25.0%	36.9%	25.0%	22.6%
Panel B: return characteristics					
volatility (ann.)	15.6%	15.3%	15.4%	14.6%	14.8%
expected excess return	3.0%	3.5%	3.7%	3.4%	3.7%
expected Sharpe ratio	0.19	0.23	0.24	0.23	0.25
historical excess return	3.9%	6.9%	7.8%	5.6%	6.4%
historical Sharpe ratio	0.25	0.45	0.51	0.38	0.43
tracking error (ann.)	0.0%	3.8%	4.7%	3.3%	4.4%