Unbound Prometheus Program - Summer 2017

NONLINEAR WAVES: THEORY, APPLICATIONS, AND COMPUTATION

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SUMMARY/MOTIVATION:

The study of nonlinear systems has quietly and steadily revolutionized the realm of science over recent years. Nonlinear systems support emerging structures that have unique features and peculiar ways of interacting. Examples of such structures abound in nature and include: vortices (like tornadoes or eddies in water tanks), solitons (bits of information used in optical fiber communications, matter/plasma waves, water waves, and even tsunamis!), spirals (biological aggregates and chemical reactions), etc. This course is intended as an introduction to Nonlinear Waves and their applications. It is designed for senior undergraduates and graduate students in Applied Mathematics, Physics, Computational Science, Engineering, etc. Most of the concepts and examples will be supplemented with Matlab-based codes and visualizations. The course will include a unique hands-on computer component where students will actively learn to develop and use codes to study nonlinear wave dynamics. Applications to water waves, optical fiber transmission and matter waves will be covered.

MISSION OF THE COURSE:

The course is designed to introduce senior undergraduate and graduate students without previous background in the topic, to the exciting realm of Nonlinear Waves through a synergistic combination of theory, applications and computations.

SCHEDULE:

Monday to Friday lectures (4 hrs/day) for two weeks: Jul 24th – August 4th, 2017. There will also be some extra informal meetings to help you with the numerical methods.

CREDITS/UNITS:

Students will receive a transcript with the equivalent of 3 units (US) or 4.5 ECTS credits (Europe) from the (credited) Technological and Educational Institute (TEI) of Kavala, Greece. These units should be completely transferable to a US institution towards a 3 units upper division class for undergraduates or towards a 3 units introductory course for graduate students.

PREREQUISITES:

The course is structured having in mind a diverse student population and background in science and engineering. A complete sequence of calculus, some knowledge on differential equations and programming are the minimal requirements.

LAPTOP:

Since the course is designed to be hands on, students will be required to bring a laptop and obtain a copy of Matlab. Make sure to install/test Matlab it prior to the start of the course. Note: you should buy in advance a European plug converter. Most laptop chargers should accommodate the European 220 Volts, however a European plug converter will still be needed (you can get standard plug converters [$4 for one and $8 for a 3-pack] or universal plug converters [$15-$18] from Target or online).

LEARNING OUTCOMES:

At the end of the course students will gain the following knowledge:

- ⊕ contrast linear vs. nonlinear wave behavior
- ⊕ familiarity with the concept of solitary waves in nonlinear systems
- ⊕ reduce a nonlinear wave propagation equation using the traveling wave assumption
- ⊕ characterize wave types using phase-portrait analysis
- ⊕ familiarity with shallow wave water equations and their basic nonlinear solutions
- ⊕ familiarity with optical fiber and matter wave solitons
- ⊕ design Matlab-based codes to integrate and visualize wave propagation

TEXTBOOK/NOTES: (I will distribute PDF versions of the following material)

- ⊕ *Nonlinear waves, an introduction*. Manuscript in progress. Most of the core material for the course will be based on this introductory textbook on nonlinear waves that I am writing.
- ⊕ Supplemental reading material:
  - *Avoiding infrared catastrophes in trapped Bose-Einstein condensates.*
    P.G. Kevrekidis et al., Phys. Rev. A **70** (2004) 023602.
  - *Variational approach to nonlinear pulse propagation in optical fibers*.
    D. Anderson, Phys. Rev. A **27** (1983) 3135.
  - *Variational methods in nonlinear fiber optics and related fields.*
    B.A. Malomed, Progress in optics **43** (2002) 71-194.
  - Nonlinear Schrödinger equation and *N*-soliton interactions.
    V.S. Gerdijikov et al., Phys. Rev. E **55** (1997) 6039.
  - *Dark solitons in atomic Bose–Einstein condensates: from theory to experiments*,
    D.J. Frantzeskakis, J. Phys. A: Math. Theor. **43** (2010) 213001 .
  - *Perturbation-induced dynamics of dark solitons,*
    Yu.S. Kivshar & X. Yang, Phys. Rev. E **49** (1994) 1657.

TEACHING METHOD:

Each class will be designed to have two components (often these two components will be tightly intertwined): a theory portion where we will cover the basic (physical and mathematical) principles and a second portion where we will use and develop Matlab-based codes to visualize solutions and understand their behavior in a discovery-type environment. The course will be very much hands-on and constant participation from the class will be fostered and encouraged.

SPECIFIC TOPICS TO BE COVERED:

- ⊕ **Linear Waves:**
  Waves on a string + Linear wave equation + Superposition principle + initial conditions + Dissipation + Dispersion.
- ⊕ **Nonlinear Waves:**
  Nonlinearity + Method of characteristics + Wave breaking
- ⊕ **Shallow water waves: the Korteweg-de Vries (KdV) equation**
  Euler Eqs. + Boussinesq → KdV + scale/galilean invariance + KdV soliton + cnoidal waves in KdV + conservation laws
- ⊕ **Optical fiber and matter waves: The Nonlinear Schrödinger (NLS) equation**
  Envelope waves + Solitons in NLS + Focusing → bright solitons + Defocusing → dark solitons + Galilean boost for NLS + Modulational Instability for NLS + Variational Approximation
- ⊕ **Numerical Methods:**
  Steady states (Newton iterations) + stability + numerical integration for nonlinear PDEs.

ASSIGNMENTS:

There will be assignments every other lecture. Assignments will consist of a combination of analytical computations and numerical results. I will post a comprehensive set of basic Matlab codes that students will be able to download and modify for the assignments. When relevant, I will indicate which assignments/numerics/graphs will be submitted electronically by email.

FINAL PROJECT:

A final project will be assigned and it will be due 3 weeks after the completion of the course. Possible topics for the final project will be posted on our website. I encourage students to propose their own projects that might be related to something they have already been working on or something that they have always found interesting/appealing. Such proposal for final projects will have to be approved by me before the end of the course.

GRADING POLICY:

Grades will be assigned using the following weights:

1. Assignments: 50%
2. Final project: 30%
3. Attendance and in-class participation/interaction: 20%

with the following grading scale:

- ▹ A: 90-100
- ▹ B: 80-89
- ▹ C: 70-79
- ▹ D: 60-69
- ▹ F: 0-59