

Free Choice for Simplification

Malte Willer

University of Chicago
willer@uchicago.edu

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Abstract

The fact that counterfactuals in general license simplification of disjunctive antecedents is a familiar problem for the traditional Lewis-Stalnaker variably strict analysis of counterfactuals. This paper demonstrates that the data are well explained by a dynamic strict analysis of counterfactuals that uses ideas from the inquisitive semantic tradition to provide a satisfying semantic explanation of the modal free choice effect. The analysis is general enough to predict other, frequently ignored simplification patterns, such as the one instantiated by *might*-counterfactuals with disjunctive consequents, and makes progress in developing a uniform perspective on the distinction between informative, inquisitive, and attentive content.

1 The Plot

On a textbook variably strict analysis a *would*-counterfactual $\lceil \phi \Box \rightarrow \psi \rceil$ is true at some possible world w iff $f_c(w, \llbracket \phi \rrbracket) \subseteq \llbracket \psi \rrbracket$, where $f_c(w, \llbracket \phi \rrbracket)$ denotes the ϕ -worlds closest to w given some contextually provided similarity relation between worlds (see Lewis 1973 and Stalnaker 1968 for seminal discussion). The fact that counterfactuals in general license SIMPLIFICATION OF DISJUNCTIVE ANTECEDENTS (SDA) is a familiar problem for this analysis (Fine 1975, Nute 1975):

$$\text{SDA } (\phi \vee \psi) \Box \rightarrow \chi \models \phi \Box \rightarrow \chi, \psi \Box \rightarrow \chi$$

Given classical disjunction, SDA inferences are unexpected in a variably strict setting since $f_c(w, \llbracket \phi \vee \psi \rrbracket) \neq f_c(w, \llbracket \phi \rrbracket) \cup f_c(w, \llbracket \psi \rrbracket)$ unless the closest ϕ -worlds are just as close to w as are the closest ψ -worlds, leaving it unexplained why (1) entails that the party would have been fun if Alice had come *and* that the party would have been fun if Bert had come:

- (1) If Alice or Bert had come to the party, it would have been fun.
 - a. \rightsquigarrow If Alice had come to the party, it would have been fun.
 - b. \rightsquigarrow If Bert had come to the party, it would have been fun.

The first observation of this paper—discussed in the remainder of this section—is that the trouble with simplification goes beyond SDA in interesting ways. The second observation is that the data are well explained by a dynamic strict analysis of counterfactuals that uses ideas from the inquisitive semantic tradition.

Semantic attempts to deal with the SDA trouble in a variably strict setting exist but do not address the full range of simplification data. One prominent idea is to go with a Hamblin-style analysis of disjunction and then let *if*-clauses be universal quantifiers so that $\lceil \phi \Box \rightarrow \psi \rceil$ is now true at some possible world w iff $f_c(w, p) \subseteq \llbracket \psi \rrbracket$ for all propositions p in the set of alternatives denoted by the antecedent ϕ —a singleton in case of non-disjunctive antecedents; a

set containing all the atomic propositional disjuncts if the antecedent is a disjunct (see Alonso-Ovalle 2009). Another prominent idea is to adopt an existential analysis of disjunction—the antecedent of (1), for instance, would be of the form ‘ $\exists x. Cx \wedge (x = \text{Alice} \vee x = \text{Bert})$ ’ with ‘C’ denoting the property of coming to the party—and a treatment of indices of evaluation as world-assignment pairs. If we now say that two such pairs are unconnected (and hence none of them more similar to the index of evaluation than the other) if their assignments differ, we predict that the counterfactual selection function includes indices at which Alice comes to the party as well as indices at which Bert comes to the party (see van Rooij 2006).

Both approaches explain why (1) simplifies but are hand-tailored to handle simplifications of *disjunctive antecedents*: what does the explanatory lifting in each approach is the special interaction between a non-classical analysis of disjunction with whatever is involved in interpreting *if*-clauses. And this cannot be the whole story since a counterfactual like (2) with a *negated conjunction* as antecedent also simplifies (see Nute 1980) and since a *might*-counterfactual such as (3) allows for simplification of its disjunctive *consequent*:

- (2) If Nixon and Agnew had not both resigned, Ford would never have become president.
 - a. \rightsquigarrow If Nixon had not resigned, Ford would never have become president.
 - b. \rightsquigarrow If Agnew had not resigned, Ford would never have become president.
- (3) If Mary had not gone to Pisa, she might have gone to Lisbon or Rome.
 - a. \rightsquigarrow If Mary had not gone to Pisa, she might have gone to Lisbon.
 - b. \rightsquigarrow If Mary had not gone to Pisa, she might have gone to Rome.

It will not do to just stipulate that (2) is evaluated by checking its consequent against the union of the closest worlds in which Nixon and Agnew do not resign, respectively: this fact calls for an explanation in terms of negation and conjunction just as much as SDA called for an explanation in terms of disjunction. Explaining the simplification pattern exhibited by *might*-counterfactuals such as (3) is also challenging in a variably strict setting under the reasonable assumption that ‘ $\diamond\rightarrow$ ’ and ‘ $\square\rightarrow$ ’ are duals ($\phi \diamond\rightarrow \psi =_{\text{def}} \neg(\phi \square\rightarrow \neg\psi)$). For observe that given some context c , the truth of ‘ $\neg(\phi \square\rightarrow \neg(\psi \vee \chi))$ ’ at some possible world w only requires that $f_c(w, \llbracket\phi\rrbracket) \cap \llbracket\psi\rrbracket \neq \emptyset$ or $f_c(w, \llbracket\phi\rrbracket) \cap \llbracket\chi\rrbracket \neq \emptyset$, not both. In any case, a comprehensive story about how and why counterfactuals simplify cannot be dependent on the interpretation of *if*-clauses (or on considerations about the similarity relation) alone since simplification is also a feature of disjunctive counterfactual consequents.

The Lewis-Stalnaker analysis for counterfactuals has a problem with simplification that goes beyond the familiar observations about SDA. One may, of course, hope that all these problems will eventually go away once we couple such an analysis with a sufficiently powerful pragmatic supplement. I leave a critical discussion of this strategy to another day, not least because existing pragmatic explanations of why counterfactuals should simplify in a variably strict analysis (Franke 2011, Klinedinst 2009, and van Rooij 2010) are no exception to the earlier observed trend of focussing on SDA at the expense of other simplification patterns, and it is simply not obvious how their stories can be generalized so that they address, for instance, the data surrounding *might*-counterfactuals with disjunctive consequents. Instead, I pick up the more interesting task of demonstrating that the simplification data already receive a straightforward semantic explanation in a suitably elaborated strict analysis of counterfactuals (von Fintel 2001, Gillies 2007, Willer 2013b) that uses insights from the inquisitive semantic tradition.

The proposal to be developed here is that a *would*-counterfactual ‘ $\phi \square\rightarrow \psi$ ’ is a strict material conditional ‘ $\square(\phi \supset \psi)$ ’ over a contextually determined but dynamically evolving domain of quantification presupposing that its antecedent ϕ is a possibility in that domain.

Assuming duality and that presuppositions project out of negation, a *might*-counterfactual ' $\phi \diamond \rightarrow \psi$ ' amounts to an assertion of ' $\diamond(\phi \wedge \psi)$ ' under the presupposition that ϕ is possible. Once we predict that ' $\diamond(\phi \vee \psi)$ ' and ' $\diamond\neg(\neg\phi \wedge \neg\psi)$ ' entail ' $\diamond\phi$ ' *as well as* ' $\diamond\psi$ ' via a semantic free choice effect, we predict that (1)–(3) simplify in the way they do.

On the view developed here a semantic explanation of why counterfactuals with disjunctive antecedents simplify flows from a semantic account of the free choice reading of disjunctions under existential modals, which is exhibited by (4) and (5):

- (4) You may take an apple or a pear.
 - a. \rightsquigarrow You may take an apple.
 - b. \rightsquigarrow You may take a pear.
- (5) Mary might be in Chicago or in New York.
 - a. \rightsquigarrow Mary might be in Chicago.
 - b. \rightsquigarrow Mary might be in New York.

Free choice readings are not generated by the standard semantic analysis of modals and disjunction.¹ The problem gets additional bite due to the fact that embedding a disjunctive possibility under negation reverts disjunction to its classical behavior:

- (6) You may not take an apple or a pear.
 - a. \rightsquigarrow You may not take an apple.
 - b. \rightsquigarrow You may not take a pear.
- (7) Mary cannot be in Chicago or in New York.
 - a. \rightsquigarrow Mary cannot be in Chicago.
 - b. \rightsquigarrow Mary cannot be in New York.

Observations along these lines suggest that free choice is best understood as a pragmatic phenomenon (and likewise simplification, assuming that the phenomena should receive a parallel treatment). Part of the exercise pursued here is to demonstrate that this conclusion is not irresistible: a semantic treatment of the free choice effect can accommodate the observations about (6) and (7) if combined with a suitably sophisticated analysis of negation. Such an analysis will also be in a position to bring the free choice effect to bear on an explanation of the fact that simplification is not limited to *would*-counterfactuals with disjunctive antecedents.

My plan is as follows. §2 offers a semantic analysis that delivers the free choice effect without negative side effects, combining insights from the dynamic and the inquisitive tradition in a nonclassical semantic outlook on modals and propositional connectives. §3 builds an analysis of counterfactuals on top of the framework developed in the previous section. §4 addresses some remaining issues, including the observation that counterfactuals with disjunctive antecedents do not always seem to simplify. §5 summarizes the key findings and briefly explores additional applications of the tools and techniques developed here.

¹Kamp's (1973, 1978) discussion of the phenomenon is seminal, though the label 'free choice' goes back at least to von Wright (1968). Semantic answers to the problem generally revise the semantics of disjunction or of the modals involved and include, among others, Aher 2012, Aloni 2007, Barker 2010, Geurts 2005, Simons 2005, and Zimmermann 2000. Pragmatic solutions, mostly Gricean in spirit, include Alonso-Ovalle 2006, Fox 2007, Franke 2011, Kratzer and Shimoyama 2002, and Schulz 2005. The observation that there is a striking parallel between the problem of simplification and the paradox of free choice is voiced in Franke 2011, Klindinst 2007, and van Rooij 2006.

2 Basic Framework

The target language \mathcal{L} contains a set of sentential atoms $\mathcal{A} = \{p, q, \dots\}$ and is closed under negation (\neg), conjunction (\wedge), disjunction (\vee), the modal possibility operator (\diamond), and the *would*-counterfactual ($\square\rightarrow$). Other connectives are defined in the usual manner. In this section I state the semantics for the modals and the propositional connectives. The subsequent section addresses the counterfactual connective.

2.1 Semantics: First Steps

The analysis of modals is in the spirit of Veltman's (1996) approach but here we do not treat input contexts as sets of possible worlds but as sets of consistent propositions (which I label *alternatives*).

Definition: Possible Worlds, Propositions. w is a *possible world* iff $w: \mathcal{A} \rightarrow \{0, 1\}$. W is the set of such w 's, $\mathcal{P}(W)$ is the powerset of W . The function $\llbracket \cdot \rrbracket$ assigns to nonmodal formulas of \mathcal{L} a *proposition* in the familiar fashion. \perp is the contradictory proposition (the empty set of possible worlds) while $\underline{\perp}$ is any consistent proposition.

Definition: States, Alternatives. A *state* $s \subseteq \mathcal{P}(W) \setminus \perp$ is any set of consistent propositions (*alternatives*). S is just the set of all such states. The *information* carried by a state s is the set of possible worlds compatible with it so that $\mathbf{info}(s) = \{\bigcup \sigma: \sigma \in s\}$. We refer to \emptyset as the *absurd* state and speak of $s_0 = \mathcal{P}(W) \setminus \perp$ as the *initial* state.

States thus have informational content in the sense that they rule out certain ways the world could be. In addition, they encode this information as a set of alternatives (which do not have to be mutually exclusive).

States are updated by updating each of their alternatives. Updates on an alternative σ are sensitive to the state s containing it since modals perform tests on the state's informational content. Furthermore, I will think of update rules as *relations* between alternatives to capture the inquisitive effect of disjunction and distinguish between a *positive* acceptance inducing update relation $[\cdot]_s^+$ and a *negative* rejection inducing update relation $[\cdot]_s^-$ to allow for inquisitive negation (inspired by Aher 2012 and Groenendijk and Roelofsen 2015). So for instance we shall say:

$$(\mathcal{A}) \quad \begin{array}{l} \sigma[p]_s^+ \tau \text{ iff } \tau = \sigma \cap \llbracket p \rrbracket \\ \sigma[p]_s^- \tau \text{ iff } \tau = \sigma \setminus \llbracket p \rrbracket \end{array} \quad (\neg) \quad \begin{array}{l} \sigma[\neg\phi]_s^+ \tau \text{ iff } \sigma[\phi]_s^- \tau \\ \sigma[\neg\phi]_s^- \tau \text{ iff } \sigma[\phi]_s^+ \tau \end{array}$$

A positive update with a sentential atom p eliminates from an alternative all possible worlds at which p is false while a negative update with p eliminates all possible worlds at which p is true. A positive update with $\neg\phi$ is just a negative update with ϕ and a negative update with $\neg\phi$ is just a positive update with ϕ .

The basic idea about possibility modals is that they test whether their prejacent relates the information carried by a state to a contradiction \perp or to a consistent proposition $\underline{\perp}$:

$$(\diamond) \quad \begin{array}{l} \sigma[\diamond\phi]_s^+ \tau \text{ iff } \tau = \{w \in \sigma: \langle \mathbf{info}(s), \perp \rangle \notin [\phi]_s^+\} \\ \sigma[\diamond\phi]_s^- \tau \text{ iff } \tau = \{w \in \sigma: \langle \mathbf{info}(s), \underline{\perp} \rangle \notin [\phi]_s^+\} \end{array}$$

For an alternative in a state s to pass the test imposed by a *positive* update with $\neg\phi$, the information carried by s must not be related to the *inconsistent* proposition via a positive

update with ϕ . For an alternative in a state s to pass the test imposed by a *negative* update with $\lceil \diamond\phi \rceil$ —that is, a positive update with $\lceil \Box\neg\phi \rceil$ —the information carried by s must not be related to a *consistent* proposition via a positive update with ϕ .

So far we only have a rewrite of classical Update Semantics but the present setup allows us to combine a test semantics for modals with an inquisitive analysis of disjunction and negated conjunction. Start by coupling each update rule for alternatives with a corresponding update procedure for states:

Definition: Updates on States. Define two update operations $\uparrow, \downarrow: (\mathcal{L} \times S) \mapsto S$:

1. $s \uparrow \phi = \{\tau \neq \perp : \exists \sigma \in s. \sigma[\phi]_s^+ \tau\}$
2. $s \downarrow \phi = \{\tau \neq \perp : \exists \sigma \in s. \sigma[\phi]_s^- \tau\}$

A positive/negative update of some state s with ϕ delivers all the alternatives that are positively/negatively related to some element of s via ϕ .

The proposal for disjunction is then the following one:

$$(\vee) \quad \begin{array}{l} \sigma[\phi \vee \psi]_s^+ \tau \text{ iff } \sigma[\phi]_s^+ \tau \text{ or } \sigma[\psi]_{s \downarrow \phi}^+ \tau \\ \sigma[\phi \vee \psi]_s^- \tau \text{ iff } \exists \nu: \sigma[\phi]_s^- \nu \text{ and } \nu[\psi]_{s \downarrow \phi}^- \tau \end{array}$$

This analysis captures two important intuitions about disjunctions: first, in addition to ruling out certain possibilities they raise each of their disjuncts as an issue in discourse. We capture this by letting a disjunction relate an input alternative to two potentially distinct alternatives: the result of updating with the first and the result of updating with the second disjunct. Moreover, in a sentence such as ‘Mary is in Chicago or she must be in New York’ the modal in the second disjunct naturally receives a modally subordinated interpretation: it is interpreted under the supposition that Mary is not in Chicago. We achieve this result by saying that whenever a disjunction is processed in light of some state s , its second disjunct is processed in light of a negative update of s with the first disjunct.

Given some state s , a positive update with a conjunction $\lceil \phi \wedge \psi \rceil$ proceeds via a positive update with ϕ light of s and then via a positive update with ψ in light of $s \uparrow \phi$:

$$(\wedge) \quad \begin{array}{l} \sigma[\phi \wedge \psi]_s^+ \tau \text{ iff } \exists \nu: \sigma[\phi]_s^+ \nu \text{ and } \nu[\psi]_{s \uparrow \phi}^+ \tau \\ \sigma[\phi \wedge \psi]_s^- \tau \text{ iff } \sigma[\phi]_s^- \tau \text{ or } \sigma[\psi]_{s \uparrow \phi}^- \tau \end{array}$$

The rules for negative updates with disjunctions and conjunctions enforce the validity of De Morgan’s Laws.

The notions of support, entailment, and consistency in the familiar dynamic fashion:

Definition: Support, Entailment, Consistency. Take any $s \in S$ and formulas of \mathcal{L} :

1. s supports ϕ , $s \Vdash \phi$, iff $s \uparrow \phi = s$
2. ϕ_1, \dots, ϕ_n entails ψ , $\phi_1, \dots, \phi_n \Vdash \psi$, iff for all $s \in S$, $s \uparrow \phi_1 \dots \uparrow \phi_n \Vdash \psi$
3. ϕ_1, \dots, ϕ_n is *consistent* iff for some $s \in S$: $s \uparrow \phi_1 \dots \uparrow \phi_n \neq \emptyset$

A state supports ϕ just in case a positive update of s with ϕ idles. Entailment is just guaranteed preservation of support and the consistency of a sequence requires that a positive update with it sometimes results in a non-absurd state. It would, of course, be possible to define the notions of entailment and consistency on the basis of \downarrow but I set an exploration of this interesting avenue aside for now. Instead, let me highlight a few crucial predictions that the framework developed so far makes.

2.2 Output

Disjunctions embedded under a possibility operator exhibit the free choice effect:

Fact 1. $\diamond(p \vee q) \models \diamond p \wedge \diamond q$

The underlying observation here is that $s \uparrow \diamond(p \vee q) \neq \emptyset$ only if $\langle \mathbf{info}(s), \underline{\perp} \rangle \notin [p \vee q]_s^+$. But suppose that $\mathbf{info}(s)$ fails to contain both p - and q -worlds: then $[p]_s^+$ or $[q]_s^+$ *does* relate $\mathbf{info}(s)$ to $\underline{\perp}$, hence $\mathbf{info}(s)[p \vee q]_s^+ \underline{\perp}$ and thus $\langle \mathbf{info}(s), \underline{\perp} \rangle \in [p \vee q]_s^+$ after all. So if $s \uparrow \diamond(p \vee q) \neq \emptyset$ then $s \uparrow \diamond p = s$ and $s \uparrow \diamond q = s$.

Note that $\diamond(p \vee q) \not\models \diamond(p \wedge q)$ since passing the test conditions under consideration does not require the presence of a $p \wedge q$ -world in $\mathbf{info}(s)$. Furthermore, it is easy to see that the free choice effect also arises if \diamond scopes over a negated conjunction since for all choices of $s \in S$ we have $[\neg(\neg\phi \wedge \neg\psi)]_s^+ = [\phi \vee \psi]_s^+$ by design.

We also account for the earlier stated observation that embedding a disjunctive possibility under negation reverts disjunction to its classical behavior:

Fact 2. $\neg\diamond(p \vee q) \models \neg\diamond p \wedge \neg\diamond q$

Observe that $s \uparrow \neg\diamond(p \vee q) \neq \emptyset$ only if $\langle \mathbf{info}(s), \underline{\perp} \rangle \notin [p \vee q]_s^+$. But suppose that $\mathbf{info}(s)$ contains a p - or a q -world: then $[p]_s^+$ or $[q]_s^+$ *does* relate $\mathbf{info}(s)$ to $\underline{\perp}$, hence $\mathbf{info}(s)[p \vee q]_s^+ \underline{\perp}$ and thus $\langle \mathbf{info}(s), \underline{\perp} \rangle \in [p \vee q]_s^+$ after all. So if $s \uparrow \neg\diamond(p \vee q) \neq \emptyset$ then $s \uparrow \neg\diamond p = s$ and $s \uparrow \neg\diamond q = s$.

We may also observe that the framework developed here preserves key insights from the dynamic analysis of modals, including the internal dynamics of conjunction:

Fact 3. $\neg p \wedge \diamond p$ is inconsistent

Here it pays off that updates are defined relative to a shifty state parameter s . Clearly $\mathbf{info}(s \uparrow \neg p)$ does not contain any p -worlds and so any update with with ‘ $\diamond p$ ’ in light of $s \uparrow \neg p$ is guaranteed to result in the absurd state.

Finally, let me just state some observations about the material conditional and the necessity operator that are of relevance for the upcoming discussion:

Fact 4. For all $s \in S$: $[\Box(\phi \supset \psi)]_s^+ = [\neg\diamond(\phi \wedge \neg\psi)]_s^+$ and $[\Box(\phi \supset \psi)]_s^- = [\diamond(\phi \wedge \neg\psi)]_s^+$

These identities follow immediately if we treat ‘ \diamond ’ and ‘ \Box ’ as duals and adopt the standard analysis of the material conditional in terms of conjunction and negation.

In sum, the proposal developed so far combines a test semantics for modals with an inquisitive approach to disjunction and negated conjunction in a way that captures the scope as well as the limits of the free choice effect. Let me now turn to counterfactuals.

3 Counterfactuals

I will first present an analysis of counterfactuals that explains the basic simplification patterns (§3.1). The semantics does not give us everything one might hope for but—as I will show in §3.2—the most immediate shortcomings are avoided by adding just a few bells and whistles to the basic story.

3.1 Simplification

A *would*-counterfactual is a strict material conditional presupposing that its antecedent is possible. Following standard protocol I treat *might*- and *would*-counterfactuals as duals and presuppositions as definedness conditions on updating (Heim 1982, Beaver 2001):

$$(\Box \rightarrow) \quad \begin{array}{l} \sigma[\phi \Box \rightarrow \psi]_s^+ \tau \text{ iff } \sigma[\Diamond \phi]_s^+ \sigma \text{ and } \sigma[\Box(\phi \supset \psi)]_s^+ \tau \\ \sigma[\phi \Box \rightarrow \psi]_s^- \tau \text{ iff } \sigma[\Diamond \phi]_s^+ \sigma \text{ and } \sigma[\Box(\phi \supset \psi)]_s^- \tau \end{array}$$

Given some state s , a positive or negative update with $\lceil \phi \Box \rightarrow \psi \rceil$ fails to relate an input alternative σ to any output in case the information carried by s is incompatible with ϕ (the presupposition thus projects out of negation). Assuming that the presupposition is satisfied, a positive update with $\lceil \phi \Box \rightarrow \psi \rceil$ then tests whether s supports $\lceil \phi \supset \psi \rceil$ while a negative update effectively asks whether $\lceil \phi \wedge \neg \psi \rceil$ is compatible with s .²

For convenience, let me state explicitly the update rules for *might*-counterfactuals:

$$(\Diamond \rightarrow) \quad \begin{array}{l} \sigma[\phi \Diamond \rightarrow \psi]_s^+ \tau \text{ iff } \sigma[\Diamond \phi]_s^+ \sigma \text{ and } \sigma[\Diamond(\phi \wedge \psi)]_s^+ \tau \\ \sigma[\phi \Diamond \rightarrow \psi]_s^- \tau \text{ iff } \sigma[\Diamond \phi]_s^+ \sigma \text{ and } \sigma[\Diamond(\phi \wedge \psi)]_s^- \tau \end{array}$$

These update rules are an immediate consequence of treating ‘ $\Box \rightarrow$ ’ and ‘ $\Diamond \rightarrow$ ’ as duals.

It is of course uncontroversial that this analysis predicts that counterfactuals simplify if their antecedents involve a disjunction or a negated conjunction:

Fact 5. $(p \vee q) \Box \rightarrow r \models p \Box \rightarrow r, q \Box \rightarrow r$ and $\neg(p \wedge q) \Box \rightarrow r \models \neg p \Box \rightarrow r, \neg q \Box \rightarrow r$

This is an immediate consequence of analyzing *would*-counterfactuals as strict material conditionals. The claim that counterfactuals presuppose the possibility of their antecedents, however, immediately predicts that $s \uparrow (p \vee q) \Box \rightarrow r = \emptyset$ unless $\mathbf{info}(s)$ includes p - as well as q -worlds due to the free choice effect. I will come back to this fact momentarily, but we can already at this stage observe the following fact about *might*-counterfactuals:

Fact 6. $p \Diamond \rightarrow (q \vee r) \models p \Diamond \rightarrow q, p \Diamond \rightarrow r$

Clearly a state s supports ‘ $p \Diamond \rightarrow (q \vee r)$ ’ just in case it supports ‘ $\Diamond p$ ’ and ‘ $\Diamond(q \vee r)$ ’ and hence—due to the free choice effect—both ‘ $\Diamond q$ ’ and ‘ $\Diamond r$ ’. So we predict that *might*-counterfactuals such as (3) simplify in the way the do. In contrast *would*-counterfactuals with disjunctive consequents rightly fail to simplify: a nonempty state supporting ‘ $p \wedge (q \wedge \neg r)$ ’, for instance, supports ‘ $p \Box \rightarrow (q \vee r)$ ’ without supporting ‘ $(p \Box \rightarrow r)$ ’.

This is all good news but it is easy to spot a shortcoming. Fine and Warmbröd worry that SDA entails the validity of ANTECEDENT STRENGTHENING (AS) assuming substitution of logical equivalents in conditional antecedents (Fine 1975, Warmbröd 1981). The simple argument here is that ϕ is equivalent with $\lceil (\phi \wedge \psi) \vee (\phi \wedge \neg \psi) \rceil$, hence $\lceil \phi \Box \rightarrow \chi \rceil$ entails $\lceil ((\phi \wedge \psi) \vee (\phi \wedge \neg \psi)) \Box \rightarrow \chi \rceil$ and thus, given SDA, must also entail $\lceil (\phi \wedge \psi) \Box \rightarrow \chi \rceil$. This

²Bringing presuppositions into the picture also raises the question of how they project and a proper answer requires minor modifications to some of our original update rules. For instance, in order to predict that presuppositions project out of the possibility operator one would need to say that $\sigma[\Diamond \phi]_s^+ \tau$ holds just in case $\tau = \{w \in \sigma : \langle \mathbf{info}(s), \perp \rangle \notin [\phi]_s^+\}$ and, moreover, $\exists \nu. \sigma[\phi]_s^+ \nu$. Likewise for the negative entry: $\sigma[\Diamond \phi]_s^- \tau$ holds just in case $\tau = \{w \in \sigma : \langle \mathbf{info}(s), \perp \rangle \notin [\phi]_s^+\}$ and, moreover, $\exists \nu. \sigma[\phi]_s^- \nu$. I set these additional complexities, which would also affect the update rules to disjunction, aside to streamline the notation and since getting all the facts about presupposition projection right goes beyond the scope of this investigation.

argument does not go through in the present framework. While ϕ and $\lceil(\phi \wedge \psi) \vee (\phi \wedge \neg\psi)\rceil$ entail each other, substitution requires a stronger notion of equivalence like the following one:

$$\phi, \psi \in \mathcal{L} \text{ are } \textit{equivalent} \text{ iff for all } s \in S, s \uparrow \phi = s \uparrow \psi.$$

The basic intuition here is that ‘ p ’ and ‘ $(p \wedge q) \vee (p \wedge \neg p)$ ’ differ in their inquisitive update potential: the latter, but not the former, raises the question whether or not q is the case. Since the semantics of the possibility modal and thus the semantics of counterfactuals are sensitive to this inquisitive dimension, it is not innocent to replace ‘ p ’ with ‘ $(p \wedge q) \vee (p \wedge \neg p)$ ’ in the antecedent of a counterfactual.

In fact, it is easy to verify that AS must fail since a state may support the possibility presupposition carried by $\lceil\phi \Box\rightarrow \chi\rceil$ without supporting the one carried by $\lceil(\phi \wedge \psi) \Box\rightarrow \chi\rceil$. However, the framework developed so far fails to leave room for the consistency of Sobel sequences such as (8):

- (8) If Mary had come to the party, it would have been fun. But if Bert had come too, it would not have been fun.

No consistent state compatible with Alice and Bert coming to the party can support the asserted contents of both counterfactuals, and every state that is incompatible with that possibility inevitably fails to satisfy the presuppositions carried by the first or the second member of the sequence. In any case updating with the sequence in (8) is guaranteed to result in the absurd state and thus counts as inconsistent, which is not a good result. The positive news is that it does not take much to modify the basic story so that it avoids the problem. Presuppositions in general and possibility presuppositions in particular are normally *accommodated* as discourse proceeds, allowing counterfactual domains of quantification to evolve dynamically, and this is what underlies the consistency of Sobel sequences. Let me explain.

3.2 Hyperstates

The twist to the basic story is the idea that modals and counterfactuals are quantifiers over a *minimal* and *dynamically evolving* domain of quantification. To make sense of this idea we let context determine a slightly more complex background for processing counterfactuals: instead of thinking of a context as providing a single state, we will think of it as providing a *set* of (nonempty) states. Intuitively, the information carried by each state can be understood as a domain of quantification, and counterfactuals then pertain to whatever states come with the strongest informational content.

Definition: Hyperstates. A *hyperstate* $\pi \subseteq \mathcal{P}(S) \setminus \emptyset$ is any set of nonempty states. We say that $s' \leq_{\pi} s$, s' is *at least as strong as* s in π , iff $s, s' \in \pi$ and $\mathbf{info}(s') \subseteq \mathbf{info}(s)$. Π is the set of all hyperstates. We refer to \emptyset as the *absurd* hyperstate and treat $\pi_0 = \mathcal{P}(S) \setminus \emptyset$ as the *initial* hyperstate.

Thinking of contexts as hyperstates requires some modifications to our update system. Fortunately, our update procedures for alternatives can stay the same. But states are now updated in light of a hyperstate and updates with modals now pertain to those elements of a hyperstate whose informational content is strongest. We achieve this by slightly modifying the update functions for states in the following manner:

Definition: Hyper-updates on States Define update functions $\uparrow_\pi, \downarrow_\pi: \mathcal{L} \mapsto (S \mapsto S)$ as follows:

1. $s \uparrow_\pi \phi = \{\tau \neq \perp : \exists \sigma \in s \exists s' \leq_\pi s. \sigma[\phi]_{s'}^+ \tau\}$
2. $s \downarrow_\pi \phi = \{\tau \neq \perp : \exists \sigma \in s \exists s' \leq_\pi s. \sigma[\phi]_{s'}^- \tau\}$

To see why these modifications matter, consider how a modal now interacts with some $s \in \pi$: an update of s with ' $\diamond p$ ' now tests whether there is some $s' \leq_\pi s$ whose informational content $\mathbf{info}(s')$ includes a p -world. Clearly, this is so just in case $\mathbf{info}(s)$ includes a p -world as well, and so possibility modals work exactly as before. But the twist does matter when it comes to an update with ' $\square p$ ': this now tests whether there is some $s' \leq_\pi s$ whose informational content $\mathbf{info}(s')$ exclusively consists of p -worlds, and that may be so even if $\mathbf{info}(s)$ itself includes a possible world at which p is false (though of course no state stronger than s' may contain a $\neg p$ -world). So in this sense ' \square ' becomes a strict quantifier over the informational content of the strongest members of a hyperstate. Updating with nonmodal formulas of \mathcal{L} stays the same.

We now define what it takes for a context understood as selecting a hyperstate to accept and admit ϕ and define updates on hyperstates on that basis:

Definition: Acceptance, Admission, Updates on Hyperstates. Consider arbitrary $\pi \in \Pi$ and $\phi \in \mathcal{L}$:

1. π *accepts* ϕ , $\pi \Vdash \phi$, iff for all $s' \in \pi$ there exists some $s \leq_\pi s'$: $s \uparrow_\pi \phi = s$
2. π *admits* ϕ , $\pi \triangleright \phi$, iff $\pi \not\vdash \neg \phi$
3. $\pi + \phi = \{s \uparrow_\pi \phi : s \in \pi \ \& \ \pi \triangleright \phi\} \setminus \emptyset$

Acceptance of ϕ amounts to support by the strongest states in a hyperstate. An update with ϕ is admitted as long as its negation is not accepted. And finally, a hyperstate is updated with ϕ by updating each of its elements with ϕ and collecting the nonempty results, provided that an update with ϕ is admissible.

Consistency is once again understood in the familiar dynamic fashion. And while there are several ways to define the notion of entailment, I go here for the following option: the conclusion is supported in any hyperstate that has been updated with the premises, and conditional on the conclusion's presuppositions (if any) being accommodated. As a preparation, say that $s \circ_\pi \phi = \{\sigma \in s : \exists \tau \in S \exists s' \leq_\pi s. \sigma[\phi]_{s'}^+ \tau\}$ and that $\pi \circ \phi = \{s : \exists s' \in \pi. s' \circ_\pi \phi = s\}$.

Definition: Entailment and Consistency (Hyperstates). Take any $\pi \in \Pi$ and formulas of \mathcal{L} :

1. ϕ_1, \dots, ϕ_n *entails* ψ , $\phi_1, \dots, \phi_n \vDash \psi$, iff for all $\pi \in \Pi$, $\pi + \phi_1 \dots + \phi_n \circ \psi \Vdash \psi$
2. ϕ_1, \dots, ϕ_n is *consistent* iff for some $\pi \in \Pi$: $\pi + \phi_1 \dots + \phi_n \neq \emptyset$

This setup preserves everything said in §3.1 but in addition allows for a Sobel sequence like ' $p \square \rightarrow r$ ' followed by ' $(p \wedge q) \square \rightarrow \neg r$ ' to be consistent. To see why, assume that $w_1 \in \llbracket p \wedge \neg q \wedge r \rrbracket$ and that $w_2 \in \llbracket p \wedge q \wedge \neg r \rrbracket$, and let $\pi = \{s, s'\}$ be such that $\mathbf{info}(s) = \{w_1\}$ while $\mathbf{info}(s') = \{w_1, w_2\}$. Then clearly both s and s' satisfy the presupposition carried by the first counterfactual in the sequence and since $s \leq_\pi s'$ and $s \uparrow_\pi p \square \rightarrow r = s$, we have $\pi + p \square \rightarrow r = \pi$. Notice furthermore that $\pi \not\vdash \neg((p \wedge q) \square \rightarrow r)$ since we have $s \uparrow_\pi \neg((p \wedge q) \square \rightarrow r) \neq s$: here the underlying observation is that s is the strongest state in π but fails support the counterfactual's possibility presupposition ' $\diamond(p \wedge q)$ '. So π admits an update with the second member of the Sobel sequence, resulting in a consistent hyperstate $\pi' = \{s'\}$, as desired. More complex Sobel sequences can be consistently processed in more complex hyperstates. I conclude that the validity of SDA is compatible with the fact that counterfactuals resist AS so that Sobel sequences are consistent.

4 Loose Ends

It is a familiar observation that disjunctive possibilities do not inevitably give rise to a free choice effect—an ignorance reading is available as well and can be explicitly enforced, as in the following example involving deontic permission:

- (9) You may have an apple or a pear, but I do not know which.

Following standard protocol I associate the ignorance reading with the disjunction taking wide scope at the level of logical form, and it is easy to verify that $\diamond\phi \vee \diamond\psi \neq \diamond(\phi \wedge \psi)$. The question of which syntactic principles are at play in generating the relevant scope possibilities (or whether syntactic principles alone are sufficient in the first place) is too complex to be efficiently addressed here. I refer the reader to Larson 1985 and Simons 2005 for key considerations but also point out that the need for such a story exists independently since, for instance, disjunctions under intensional verbs also give rise to scope ambiguities, as in ‘Mary is looking for a cook or a mate’ (Rooth and Partee 1983). Related facts are also at play when it comes to the observation that not all counterfactuals with negated conjunctions seem to simplify:

- (10) If John had not had that terrible accident last week and died, he would have been here today.

Here the intuition is that (10) does not license the inference of ‘If John had not died last week, he would have been here today’ since he still might have had that accident. What underlies this observation, I suggest, is the familiar fact that negated conjunctions sometimes give rise to a ‘neither’ rather than a ‘not both’ reading (Szabolsci and Haddican 2004). Not surprisingly, counterfactuals of the form $\ulcorner \neg\phi \wedge \neg\psi \urcorner \Box \rightarrow \chi$ do not license the inference of $\ulcorner \neg\phi \urcorner \Box \rightarrow \chi \wedge \ulcorner \neg\psi \urcorner \Box \rightarrow \chi$.

Certain apparent counterinstances to free choice and simplification inferences can thus be dispelled by independently motivated assumptions about logical form. Such considerations, however, do not apply to McKay and van Inwagen’s (1977) well-known case against simplification of disjunctive antecedents:

- (11) If Spain had fought for the Axis or the Allies, she would have fought for the Axis.
a. \rightsquigarrow If Spain had fought for the Axis, she would have fought for the Axis.
b. ??? If Spain had fought for the Allies, she would have fought for the Axis.

(11a) is of course trivial but (11b) is objectionable, contrary to what SDA predicts.

Part of the picture is that the problematic counterfactual implies that Spain would never have fought for Allies. In fact, explicitly acknowledging the possibility of Spain joining the Allies renders (11) unacceptable (see also Starr 2014):

- (12) Spain might have fought for the Allies. ???But if Spain had fought for the Axis or the Allies, she would have fought for the Axis.

Given this implication it is not surprising that a context that has been strengthened with (11) fails to support ‘If Spain had fought for the Allies, she would have fought for the Axis’ since it fails to satisfy the counterfactual’s possibility presupposition.

The formal response to McKay and van Inwagen’s case then is that we need to distinguish between a rule of inference being valid and its being applicable in a given context. The inference of $\ulcorner p \urcorner \Box \rightarrow q$ from $\ulcorner (p \Box \rightarrow q) \wedge \neg(p \Box \rightarrow q) \urcorner$ is valid, for instance, but has no purchase in actual discourse and reasoning since updating any hyperstate with its premise results in a hyperstate for which the conclusion fails to be defined. The suggestion then is that SDA is formally valid—and applicable in most of its instances—but fails to have a purchase in McKay and van

Inwagen’s scenario: in ordinary discourse situations anyway, updating the context with (11) results in a state in which (11a) fails to be defined.

One may think that the previous observation immediately undermines the proposal that counterfactuals with disjunctive antecedents presuppose the possibility of each disjunct. But this is not so: in ordinary circumstances anyway, the indicative conditional ‘If John wins the competition, then I am the Flying Dutchman’ does not carry the presupposition that John possibly wins the competition but there is no serious doubt that indicative conditionals in general presuppose that their antecedent is a possibility in the common ground (Stalnaker 1975). What is needed is a story about how the possibility presupposition carried by a conditional may at times be cancelled.

Here is a sketch of how such a story might go. It is a well-worn story that presuppositions sometimes conflict with implicatures and that in such cases the presupposition may be cancelled (Gazdar 1979, Stalnaker 1974, van der Sandt 1992). One way to make sense of this is to say that presuppositions impose a preference on the input context that is defeasible in the sense that it has to be balanced with other constraints such as the ones flowing from conversational implicatures. So in the Flying Dutchman conditional, the possibility presupposition carried by the antecedent imposes a possibility preference on the input context that conflicts with the implicature-based constraint on the context not to allow for that very same possibility. Assuming that the implicature based constraint is given priority—which is often but perhaps not always the case (see Beaver 2010 for detailed discussion)—we predict that the possibility presupposition is canceled.

Going back to McKay and van Inwagen’s example, the general idea is that (11) carries two preferences in virtue of its presupposed content: that the context supports the possibility of Spain’s fighting with the Axis ($\Diamond Ax$) and that it supports the possibility of Spain’s fighting with the Allies ($\Diamond A1$). The latter presupposition—but not the former—is cancelled since it conflicts with the implicature-based constraint that the context rejects the possibility of Spain fighting with the Allies. Let me explain how the implicature may be derived. Take any hyperstate π such that $\pi \Vdash \Diamond A1$ and consider the result of updating π with the asserted content of (11), that is ‘ $\Box((Ax \vee A1) \supset Ax)$ ’: clearly, $\pi + \Box((Ax \vee A1) \supset A1) \Vdash \Diamond(Ax \wedge A1)$, that is, an update of π with the asserted content of (11) results in a state according to which Spain might have fought with the Allies *and* the Axis. But ordinary speakers believe—and are commonly believed to believe—that Spain would not have fought on both sides of the war. Hence an utterance of (11) communicates, in virtue of its asserted content, that the speaker does not admit the possibility of Spain’s fighting with Allies. Note here that a parallel line of reasoning does not go through if we start with a hyperstate π such that $\pi \Vdash \Diamond Ax$ —updating π with the asserted content of (11) does not lead to implausible possibility commitments—and so we correctly predict that the possibility presupposition of Spain’s fighting with the Axis remains unconflicted.

I thus conclude that there is a principled pragmatic explanation for why certain counterfactuals resist simplification. The reason, in brief, is that presuppositions in general, and possibility presuppositions in particular, are cancelable in case of a conflict with other discourse constraints like conversational implicatures. Such cases do not undermine the validity of simplification but create contexts in which this rule of inference has no purchase. While this account taps into pragmatic resources to account for the problematic data, the needed assumptions appear to be fairly modest and well-motivated.

Let me briefly anticipate one concern before concluding the discussion. The fact that I have appealed to conversational implicatures in handling McKay and van Inwagen’s case raises the question what happens if the counterfactual is embedded in belief contexts.

- (13) Jones believes that if Spain had fought for the Axis or the Allies, she would have fought for the Axis.
- a. \rightsquigarrow Jones believes that if Spain had fought for the Axis, she would have fought for the Axis.
 - b. ??? Jones believes that if Spain had fought for the Allies, she would have fought for the Axis.

(13a) is not unreasonable while (13b) strikes us as odd. However, I do not take this fact to be very problematic for the explanatory strategy pursued here: what needs to be said is that certain implicatures may arise under embeddings. This is commonly admitted to be the case when it comes to scalar implicatures—though it is a matter of current controversy why this is so—and I suggest that it is also true of the implicature at play here.

5 Conclusion

A dynamic strict analysis that exploits insights from inquisitive semantics predicts why counterfactuals simplify in the way they do. The explanation is compatible with the consistency of Sobel sequences and with the fact that certain counterfactuals resist simplification for principled pragmatic reasons.

Let me conclude the discussion by pointing to a few remaining tasks left for another day. First, simplification is not a feature particular to counterfactual conditionals: indicative conditionals are just as amenable to simplification as are their counterfactual cousins. What is needed then is a story about how the framework developed here can be generalized so that it covers other conditional constructions as well. While this is not a trivial task, the basic idea is clear: all conditionals are strict and come with a possibility presupposition; their differences amount to differences in the domain of quantification.

Second, free choice effects go beyond possibility modals scoping over disjunction, and so there is the general question what the framework can say about these. It would go far beyond the scope of this paper to address the full range of free choice effects in natural language, but let me highlight one important aspect of the story told here. What explains the free choice effect in formulas of the form $\ulcorner \diamond(\phi \vee \psi) \urcorner$ is the interaction between an issue raising operation with a test that effectively asks whether each alternative generated under its scope is consistent. There is no reason to think that the operations at play exclusively manifest in natural language in the form of disjunction and possibility modals, respectively. In fact, the inquisitive proposal in Coppock and Brochhagen (2013) for scalar modifiers allows us to expand the proposal developed here so that it explains why, for instance, ‘You may take at most two apples’ grants permission to take fewer than two apples.

Let me briefly highlight the details of one issue that can be handled by the framework developed here given certain minimal assumptions. Santorio (2014) observes that simplification survives embeddings in downward entailing environments like the following one:

- (14) Jones doubts that if Alice or Bert had come to the party, it would have been fun.
- a. \rightsquigarrow Jones doubts that if Alice had come to the party, it would have been fun.
 - b. \rightsquigarrow Jones doubts that if Bert had come to the party, it would have been fun.

This observation can be accommodated as long as we find a way to bring the suggested analysis for negation into play. Specifically, assume that Jones’s doubting that ϕ amounts to there being some distinguished hyperstate π supporting $\ulcorner \neg\phi \urcorner$. Observe that $\pi \models \neg((\phi \vee \psi) \Box \rightarrow \chi)$ if and only if $\pi \models (\phi \vee \psi) \Diamond \rightarrow \neg\chi$ if and only if $\pi \models (\phi \Diamond \rightarrow \neg\chi) \wedge (\psi \Diamond \rightarrow \neg\chi)$ iff $\pi \models \neg(\phi \Box \rightarrow \chi)$ and

$\pi \models \neg(\psi \Box \rightarrow \chi)$, and hence the inference of (14a) and (14b) from (14) is just what we expect. So here we have a case where a plausible bridging principle gives the framework some scope beyond its original focus.

Third, the attempt to combine a dynamic treatment of modals with an inquisitive treatment of disjunction brings to mind the question, discussed by Ciardelli et al. (2009) and Roelofsen (2013), of how to distinguish between informative, inquisitive, and attentive content. Here I want to briefly observe that the notion of a hyperstate is fine-grained enough to keep track of various kinds of discourse information. First, there is the *informational content* of a hyperstate π , understood as the possible worlds compatible with what is taken for granted: $\mathbf{Info}(\pi) = \bigcup \{\mathbf{info}(s) : s \in \pi\}$. Second, we may associate with π an *issue* understood as the set of its maximal alternatives: $\mathbf{Issue}(\pi) = \{\sigma : \sigma \in \mathbf{Alt}(\pi) \ \& \ \neg \exists \tau \in \mathbf{Alt}(\pi). \sigma \subset \tau\}$, where $\mathbf{Alt}(\pi) = \bigcup \{s : s \in \pi\}$. Looking at the initial hyperstate π_0 we can then say that an atomic sentence p has $\llbracket p \rrbracket$ as its *informational content* in the sense that the informational content of $\pi_0 + p$ is just $\llbracket p \rrbracket$. For parallel reasons we can say that ‘ $p \vee q$ ’ has $\llbracket p \rrbracket \cup \llbracket q \rrbracket$ as its informational content and $\{\llbracket p \rrbracket, \llbracket q \rrbracket\}$ as its *inquisitive content* since $\llbracket p \rrbracket$ and $\llbracket q \rrbracket$ are in the issue of $\pi_0 + p \vee q$. The distinction between the informativeness of a formula and its inquisitiveness is thus analyzed in terms of the potential to eliminate possibilities and to raise issues in discourse. And this strategy also allows us to look at hyperstates in a way that identifies yet another kind of content. Let me explain.

Earlier I said that each element of a hyperstate is a potential domain of quantification. If we think of a domain of quantification as a candidate for the region of logical space that is *relevant* for the modal discourse under consideration, it makes sense to think of a hyperstate π as identifying what I have called elsewhere a set of *serious* or *live possibilities* (see Willer (2013a)), that is, the possibilities compatible with every state in π : $\mathbf{Live}(\pi) = \{\sigma : \forall s \in \pi \exists w \in \mathbf{info}(s). w \in \sigma\}$. We can then say that a sentence has *attentive content* in virtue of its potential to bring hitherto ignored possibilities into view: ‘ $\Diamond p$ ’, for instance, has $\{\llbracket p \rrbracket\}$ as its attentive content since $\pi_0 + \Diamond p$ treats $\llbracket p \rrbracket$ as a live possibility, and for parallel reasons ‘ $\Diamond(p \vee q)$ ’ has $\{\llbracket p \rrbracket, \llbracket q \rrbracket\}$ as its attentive content.

A comprehensive discussion would explore in more detail what predictions the setup sketched here makes about the interaction between informational, inquisitive, and attentive content, and how its predictions differ from those of other frameworks. For now, let me just conclude that the framework is of general semantic interest beyond its capacity to handle some empirical challenges pertaining to inferences licensed by counterfactual conditionals: it combines a very attractive dynamic semantic treatment of possibility modals as highlighting the significance of certain possibilities in discourse with a—no less attractive—inquisitive treatment of disjunction as refining issues in discourse. The fact that combining these treatments allows us to make substantial progress toward a better understanding of the free choice effect gives us all the more reason to think that the dynamic inquisitive story told here deserves further exploration.

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