Computation of the Temperature spike and Temperature diffusion within the Earth's atmosphere immediately following the impact of the comet Eliza I

J.D. Carnes
L.M. Hachadoorian

A. Introduction

The effects of a hypothetical comet strike into the Pacific Ocean have been examined in a series of articles by Anderson, 1996. This comet, called Eliza I, is the first in a string of comets similar to the Shoemaker-Levy comet that struck Jupiter in July of 1994.

The Jupiter impact is the only large-scale event observed while still in progress. In light of the scarcity of observations of large-scale impact phenomena, and because of the diversity of models available, we model only the impact process whose effects were most readily observable from the Shoemaker-Levy 9 comet: the plume of super-heated atmospheric and cometary material that migrated to fill the space evacuated by the comet along its trajectory. In particular, we are concerned with the plume's temperature profile. The aim of these calculations is to present a novel thermal calculation using simple physical relationships to model thermal phenomena when there is too little information for full constraint.

B. The Temperature Spike

Though the post-impact explosion products are distributed horizontally and vertically from ground zero, here we are concerned only with the thermal evolution of the plume (the vertical portion) from the time it reaches its maximum height until its dissipation to pre-impact temperatures. According to Takata et al. (1994), this plume formation is the most significant atmospheric phenomenon at this time, and can be thought of as rising, accelerating, atmospheric gases whose peak altitude goes to infinity if gravitational influence neglected, and whose temperature is about 10^4 K. The existence of a high-altitude plume for the Shoemaker-Levy 9 impact was predicted by numerical simulations, but impact plumes observed by HST (Hubble Space Telescope) were four times higher than some models (Shoemaker, 1995).

We model the bounds of a temperature "spike" along the comet's trajectory by using a functional relationship commonly used to model the Dirac delta function, as the height of the spike is much larger than its width. At the spatial boundary, the temperature rises discontinuously from the initial atmospheric temperature to the spike temperature. The initial temperature distribution inside the spike is described as being proportional to the width of the "delta function" in the vertical direction (with the temperature at the geometric half-height at 10^4 K) and isothermal in the horizontal direction. Here we assume that no thermal conduction has occurred during the formation of the spike, as it is formed in the short time before the vacuum created by the comet's passage is back-filled by the outlying atmosphere.

The thermal evolution of the spike is modeled by considering the flow of heat from a three dimension interpretation of a Dirac-delta function, whose diameter correspond to the width of the spike's bounding function and whose heights are constant. The temperature is calculated at the geometric center of each of the thermal spike boundary, such that the temperature is instantaneously 10^4 K throughout the spike in the vertical direction as a function of time after spike formation.

C. Thermal Diffusion

We interpret the diffusion process as a separate model, and make the assumption that this model will follow instantaneously after the thermal spike. The model consists of a finite cylinder, with initial radius of 5 km. (the radius of the comet), and a maximum height of 200 km., the minimum height of the thermal spike.

D. Summary of Equations

The functional relationship used to model the thermal spike boundary is:

$$\delta_n(x,y) = \frac{n}{\sqrt{\pi}} * e^{-n^2(x^2+y^2)} \qquad (1)$$

The diffusion of the temperature immediately after is then revised to the relationship:

$$C\left(\frac{\alpha t}{r^2}\right) \times S\left(\frac{\alpha t}{l^2}\right) \qquad (2)$$

Where **α** is the thermal diffusivity, **t** is time **r** is the radius of the initial strike, and l is the vertical distance from the center to the upper bound of the atmosphere; and where **C(x)** models the heat flow in the radial direction and is given by

$$C(x) \equiv 2\left[\frac{e^{-xz_1^2}}{z_1 J(z_1)} + \frac{e^{xz_2^2}}{z_2 J(z_2)}\right] \qquad (3)$$

And **S(x)** models the heat flow along the cylindrical axis and is given by

$$S(x) \equiv \frac{4}{\pi}\left[e^{-\pi^2 x} - \frac{1}{3}e^{-9\pi^2 x} + \frac{1}{5}e^{-25\pi^2 x}\right] \quad . \qquad (4)$$

E. Results and Conclusions

We model the initial state of the thermal spike and the subsequent diffusion of heat into the surrounding atmosphere assuming the atmospheric temperature surrounding the spike remains constant and that heat transfer is due solely to conduction. In actuality, of course, the temperature of the surrounding air will rise, decreasing the cooling rate. Opposing this effect would be an increased cooling rate due to adiabatic expansion of the superheated gasses. Therefore, the underlying assumption of this study is that the magnitudes of these two effects are equal.

We utilize the software program *Mathematica* for both the plotting of the three dimensional Dirac-delta function *(Eqn. 1)* and the temperature equation for a finite cylinder *(Eqn. 2)*. The Dirac-delta is used for the initial spike, with an initial diameter of 10 km. across. The results indicate that the thermal spike may have a minimum height of 200 km.*(see fig. 1)*, which is beyond the earth's upper atmospheric boundary.

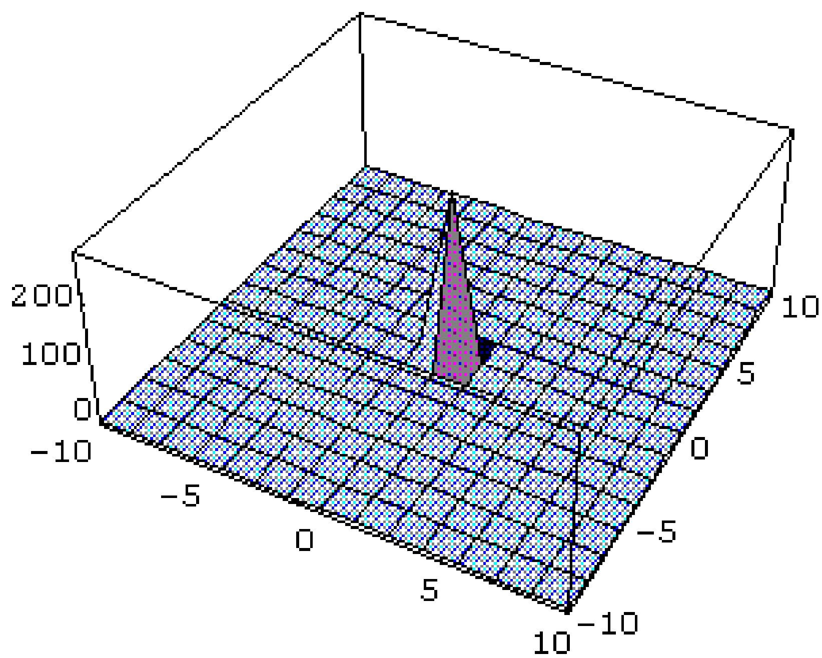
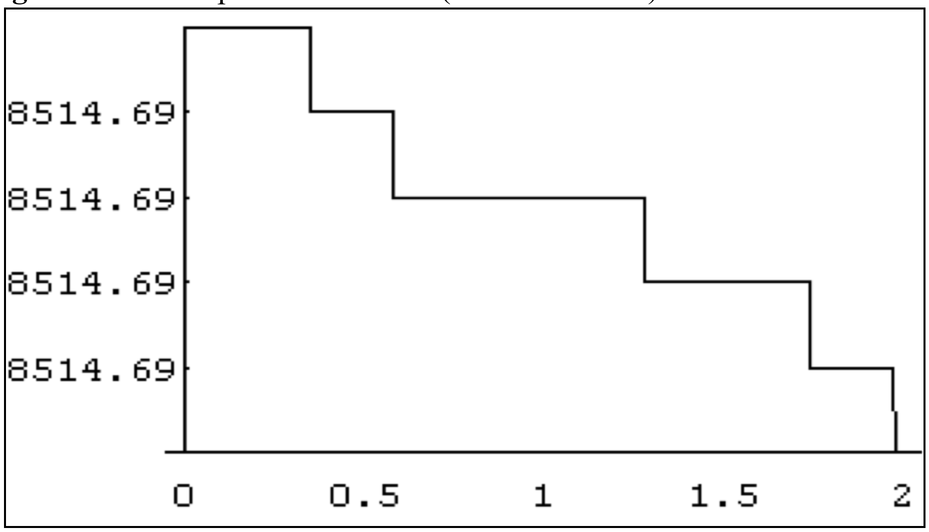


Fig. 1 3-D Spatial plot of the temperature spike using the Dirac-delta function

Immediately following the thermal spike, the heat flow will slowly diffuse into the atmosphere. We assume this diffusive process to be similar to that of a finite cylinder, and therefore governed by Eqn. 2. The two plots which follow are at **t** from zero to two seconds, and **t** from zero to 1000. This shows the temperature dissipation as an extremely gradual process, once the temperature reaches 8514.69 K (See Figs. 2 & 3).

Fig. 2 Plot of temperature vs. time (time: 0-2.0 sec.)



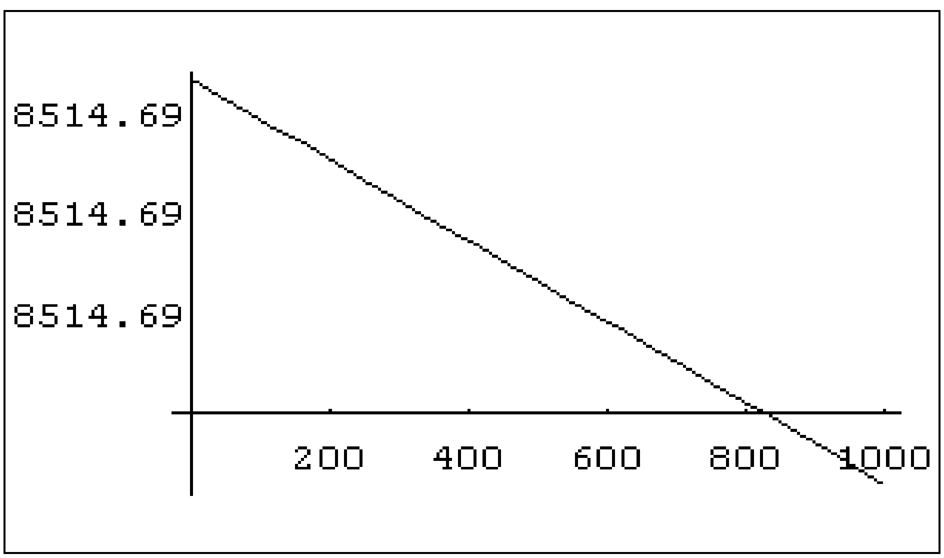


Fig. 3 Plot of temperature vs. time (time: 0-1000 sec.)

F. Future Work

Here we present the strategy of a novel calculation. Its utility could be demonstrated by varying initial conditions and comparing with the results of more detailed numerical simulations. The initial conditions of the calculation (initial spike height, etc.) could be refined by considering laboratory impact simulations and using simple scaling arguments (Newman et al, 1996).

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