

# Integrating Traffic Data and Model Predictive Control to Improve Fuel Economy

Nicholas J. Kohut,\* Professor J. Karl Hedrick,\*\*  
Professor Francesco Borrelli.\*\*\*

\*Mechanical Engineering Department, University of California-Berkeley, Berkeley, CA 94720  
USA (Tel: 510-642-6933; e-mail: kohut@berkeley.edu).

\*\* Mechanical Engineering Department, University of California-Berkeley, Berkeley, CA 94720  
USA (Tel: 510-642-2482; e-mail: khedrick@me.berkeley.edu).

\*\*\* Mechanical Engineering Department, University of California-Berkeley, Berkeley, CA 94720  
USA (Tel: 510-643-3871; e-mail: fborrelli@me.berkeley.edu)

---

**Abstract:** This paper presents a method for increasing fuel economy using traffic data and a model predictive controller. Using knowledge of the traffic ahead, a vehicle can react to changes in traffic density or speed before they happen, increasing the efficiency of a trip and providing valuable information to the driver. In particular, the traffic information is used to determine a time-varying velocity envelope that the vehicle must satisfy. Then, a vehicle model is used to compute the vehicle speed profile that minimizes fuel use and satisfies the velocity constraints. Simulation results show the feasibility of the proposed approach on a passenger vehicle with minor hardware modifications required for its implementation.

**Keywords:** Intelligent Transportation Systems, Model Predictive Control, Intelligent Vehicles, Engine Control, Adaptive Cruise Control

---

## 1. INTRODUCTION

Today's world sees an ever-increasing demand for more environmental and fuel-efficient technologies. These technologies often involve significant hardware modifications that improve the overall efficiency of a vehicle, but do not address how to drive a vehicle in the most efficient manner. Simply by driving a vehicle more efficiently, fuel economy can be improved and emissions can be reduced today, on today's vehicles, at little to no cost.

Past work has shown promising results in this area. Simulative results have predicted that less than 60 seconds of preview time of traffic conditions can yield fuel economy improvement equal to that of a hybrid vehicle using a conventional powertrain, without the heavy investment in new powertrain technology. Longer preview times of around 180 seconds have been shown to reduce fuel consumption by up to 33% [5]. Other experimental work has shown 3.5% improvement in fuel economy without an increase in trip time [4].

This paper formulates a predictive control problem in order to improve fuel economy by controlling the speed of the vehicle through the Adaptive Cruise Control, while fulfilling various constraints related to traffic and driver comfort. At each step information about the surrounding traffic is assumed to be known over a finite horizon, and a receding horizon controller computes the engine torques required to navigate traffic and minimize fuel use on a public highway.

In our first work [5] we have shown the early development of the longitudinal and fuel models, along with preliminary simulative results applied to the EPA Highway driving cycle. This work expands on that by performing a closer examination of the models involved, and using actual driving data to test the performance of the controller, with interpretation of these results.

The paper is structured as follows. First, the vehicle and fuel consumption models are presented. These models must be descriptive enough to accurately represent the longitudinal dynamics of the vehicle and the vehicle's fuel use, but also simple enough to be used in real-time control, where a complex model would pose an insurmountable computational burden. Then, an overview of the data sources and Model Predictive Control (MPC) scheme are presented.

Model Predictive Control theory [8] is an online optimization method that can incorporate knowledge of future conditions such as road grade changes and traffic flow variations as well as hard constraints on engine control variables. Recently, this method has been receiving a great deal of attention due to the speed and memory capability of modern microprocessors.

MPC uses a model of the plant to predict the future evolution of the system. Based on this prediction, at each step a performance index is optimized subject to linear and nonlinear constraints with respect to a sequence of future inputs. The first of these optimal moves is the control action applied to the plant at step  $k$ . At step  $k+1$ , a new optimization problem is solved over a shifted prediction horizon.

Advances in both theory and computing systems have expanded the range of applications where real-time MPC can be applied [9]. Yet, for a wide class of “fast” applications the computational burden of Nonlinear MPC is still a serious barrier for its implementation [1]. The paper concludes with some simulative results and their implementation.

### 1.1 Approach

Traffic data is most available and reliable on highways. In addition, highway traffic is more predictable, and does not include stop signs, stop lights, or other confounding factors. Due to these advantages, the proposed system is intended for highway use only, where adaptive cruise control can be used to regulate the speed of the vehicle.

We make use of a vehicle model that relates engine torque to vehicle speed and fuel consumption. Since the adaptive cruise control is used to control the vehicle, no low-level engine dynamics must be modelled. It is assumed that the engine will provide the torque requested of the ACC instantly. Real-time traffic data comes primarily from the California Freeway Performance Measurement System, or PeMS [7]. PeMS provides average traffic speed and density at a resolution of 0.3 to 3 miles every five minutes. This distance interval depends on the nature of the highway. If there are no exits on a stretch of highway (e.g. a bridge) only one loop will be used to measure traffic, regardless of the length of the link. Areas with more exits carry more loops, preserving accuracy. In addition, the use of probe data is anticipated. Probe vehicles are vehicles that carry special cell phones that are able to communicate their position and velocity in real-time. If there were probe vehicles near the controlled vehicle, this would provide a high fidelity measure of the traffic directly surrounding it.

Model Predictive Control is employed to compute an optimal engine torque every forty meters, using vehicle states and traffic measurements as inputs. When the models, data, and control strategy are combined, the optimal torque can be sent to the vehicle’s ACC to drive the car in the most efficient way possible. The system can be easily deactivated by the driver or by the ACC if the vehicle detects another car too close. In addition, because the controller requires an accurate representation of the traffic in front of the vehicle, this can be displayed to the driver, showing what to expect ahead.

## 2. MODELING

Two models are used for this work. A longitudinal model based on the work-energy principle is used to predict how the vehicle will travel down the road given its speed, torque, and gear. The formulation of this model is linear with respect to its state (vehicle velocity squared) and its input (torque). A nonlinear fuel model is used to predict how much fuel is consumed by the vehicle based on speed, torque, and gear. These two models allow the controller to accurately predict vehicle speed and fuel consumption.

### 2.1 Longitudinal Model

The discrete longitudinal model is based on the principle that the energy at the next step is a function of the energy at the current step plus the change in energy, caused by engine torque and air and road drag.

$$\frac{1}{2}mv^2(k+1) = \frac{1}{2}mv^2(k) - \frac{1}{2}\rho(C_a)(\Delta s)v^2(k) + \frac{R\Delta s}{r}(\eta)T(k) - \mu mg \Delta s \quad (1)$$

We use  $s(k)$  as the independent variable, representing position, and a fixed step size of  $\Delta s = s(k+1) - s(k)$  for all  $k$ .  $v(k)$  is the velocity of the vehicle at position  $s(k)$ ,  $C_a$  is the coefficient of drag multiplied by the car’s frontal area,  $m$  is the effective mass of the vehicle,  $r$  is the rolling radius of the tire,  $R$  is the ratio of wheel to engine speed, and  $g$  is the gravitational constant.  $R$  is assumed to be constant for this formulation.  $\eta$  is the driveline efficiency, which is considered constant here.  $T(k)$  is the torque output of the engine at position  $s(k)$ .  $\mu$  is the friction coefficient associated with the rolling resistance; it is assumed to be constant. It is assumed the tire slip is small, and is not taken into account.

Such an oversimplified model is used to insure the optimization problem solved at each step is tractable and can be solved in real-time on current automotive Electronic Control Units (ECUs). The model parameter can be identified and even adjusted in real time in order to better fit the measurements. The figure below shows a simple comparison between experimental data and model predictions with no tuning of model parameters.

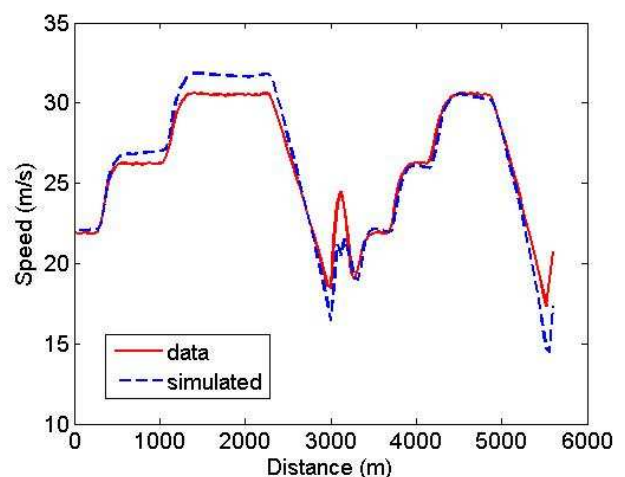


Fig. 1: Model Correlation with Audi A8L controlled by ACC

### 2.2 Fuel Model

The fuel model has been developed from static dynamometer test data. This data involves measuring the fuel consumption at various values of constant torque and rpm.

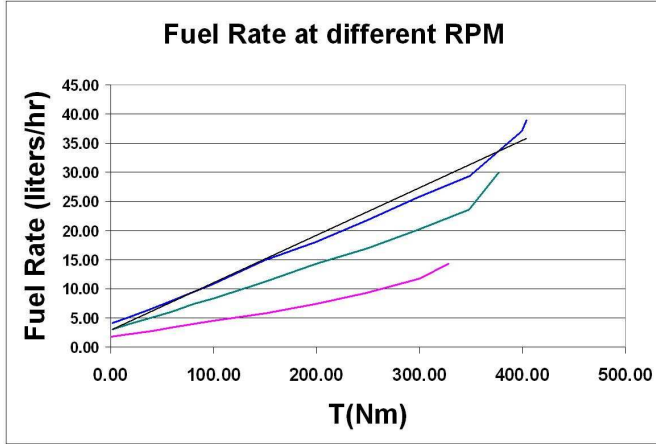


Fig. 2: Fuel Use for various RPM and Torque

The figure above shows a normalized fuel consumption for various values of constant RPM while the torque is varied. A linear fit was made for each line (not all lines are shown here for clarity) to determine the best approximation of this data. The average  $R^2$  value for these fits is 0.9769, with a worst  $R^2$  of 0.9708. This leads to the following equation

$$\dot{f}(k) = c_1 + c_2 T(k) \quad (2)$$

where  $c_1$  and  $c_2$  are constants and  $\dot{f}(k)$  is the fuel rate at position  $s(k)$ . We remark that  $\dot{f}$  is a symbol. In order to account for engine speed the coefficients  $c_1$  and  $c_2$  are parameterized as functions of engine speed.

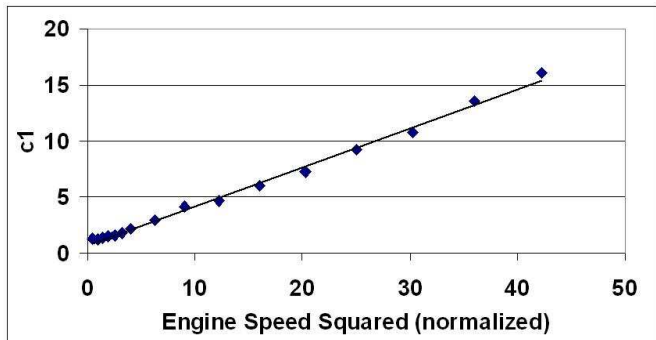


Fig. 3:  $c_1$  varies linearly with engine speed squared

When  $c_1$  and  $c_2$  are plotted against engine speed squared, a linear fit is very satisfactory, as shown in Figure 3 and 4.

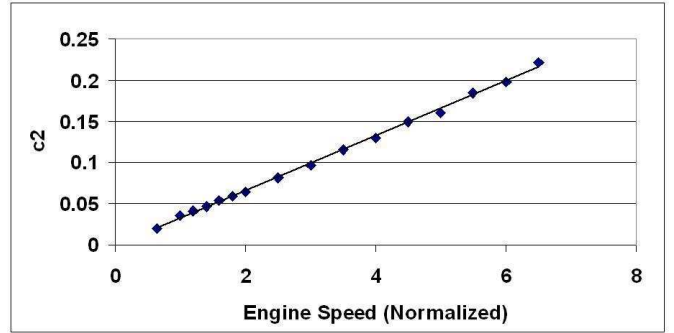


Fig. 4:  $c_2$  varies linearly with engine speed

The resulting equation is

$$\dot{f}(k) = c(\omega_e^2(k)) + d(T(k))(\omega_e(k)) \quad (3)$$

where  $\omega_e(k)$  is the engine speed at position  $s(k)$  and  $c$  and  $d$  are constants. Since we are interested in the total fuel use and not simply the rate, we multiply the fuel rate at position  $s(k)$  by the time it takes to reach position  $s(k+1)$ . This time can be found by dividing the step length by the velocity at position  $s(k)$ :

$$t(k) = \frac{\Delta s}{v(k)} \quad (4)$$

In order to use the model states, the engine speed is converted to vehicle speed through the ratio of engine to wheel speed and tire radius:

$$\omega_e(k) = \frac{v(k)}{r} R \quad (5)$$

This gives the final fuel model

$$f(k) = \left( c(v(k)) \left( \frac{R}{r} \right)^2 + d(T(k)) \left( \frac{R}{r} \right) \right) \Delta s \quad (6)$$

### 3. CONTROLS

#### 3.1 Data Sources

To gather information about traffic around the vehicle, two major data sources are used. The first source is used to determine the state of traffic far from the vehicle. This comes from the California Freeway Performance Measurement System, or PeMS. PeMS uses loop detectors embedded in California freeways to measure the speed and density of traffic every 30 seconds. This is aggregated into lane-by-lane data every 5 minutes. PeMS loops are positioned approximately every 0.3 to 3 miles along the road [6].

PeMS allows an accurate picture of the traffic ahead of the vehicle, but lacks the necessary resolution to be useful near the vehicle. This requires a different set of data. To create an accurate picture of the traffic directly around the vehicle,

other “probe vehicles” are used. These are cars that have been equipped with a cell phone that can broadcast the car’s position and velocity. Once this data is available and widespread, information about traffic flow almost anywhere can be easily used. At this point usage is fairly low, but for the purposes of this project it is assumed penetration will increase in the future and that this will be a viable data source.

When these data sets are combined, they create a picture of the behavior of the traffic ahead of the vehicle. To quantify this, a velocity profile is created of the expected traffic speed, which is then used in the controller. Combining the short distance probe data and long distance PeMS data creates the profile seen below.

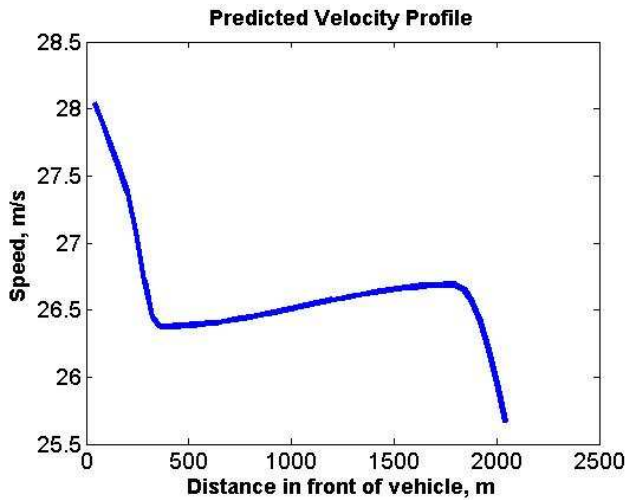


Fig. 5: An Example Velocity Profile

### 3.2 MPC

The data discussed above is used to design a feedback control scheme to control the vehicle and reduce fuel consumption. Every second, the velocity profile determined by the incoming data is fed to the controller, along with the current vehicle speed. Using this information, an optimal torque is determined and commanded to the ACC.

### 3.3 Problem Formulation

MPC is used to predict the future use of fuel for various choices of future torques. This fuel use is minimized, subject to constraints on maximum torque, vehicle velocity, and travel time. To minimize the fuel use subject to these constraints, the following nonlinear programming problem must be solved at each step:

$$\begin{aligned} & \min_{\overline{T(k)} \in \mathfrak{R}^n} J(\overline{v^2(k)}, \overline{T(k)}) \\ & \text{subject to } lb \leq \left\{ \begin{array}{l} \overline{T(k)} \\ \overline{AT(k)} \\ \overline{c(T(k))} \end{array} \right\} \leq ub \end{aligned} \quad (7)$$

where  $\overline{T(k)}$  is the set of torques over the finite horizon of size  $n$  at position  $s(k)$ . Similarly,  $\overline{v^2(k)}$  is the set of the velocities squared over the horizon. The function to be minimized is the fuel use over the horizon. The first constraint represents a bound on the torque, while the second constraint is a linear constraint restricting the vehicle’s velocity (a linear function of torque). The final constraint restricts the time spent on the horizon, a nonlinear function of torque. This optimization problem is solved at every step using the software package NPSOL [3].

The cost function to be minimized is the sum of the fuel use at each step over the horizon.

$$J = \sum_{i=1}^N f(k+i) \quad (8)$$

The bounds on the torque limit the maximum acceleration of the vehicle, for purposes of driver comfort. This is generally set so that the maximum acceleration is always below 0.4 g’s, in acceleration or braking. The linear constraints restrict the velocity the vehicle can travel. This is necessary in traffic, where, if traffic is flowing at 60 mph, speeds of 90 mph may be undesirable (and illegal), but so may be speeds of 30 mph. Hence, an upper and lower limit on speed is placed on the vehicle for safety reasons based on the traffic around it. This constraint is realized as

$$LSC^2 v_{avg}^2(k) \leq v^2(k) \leq USC^2 v_{avg}^2(k) \quad (9)$$

where LSC is the lower speed constraint factor, and USC is the upper speed constraint factor.

The final constraint faced by the controller is a constraint on time. To simply save fuel, going as slow as possible is often (though not always) the optimal choice. This strategy provides trivial results and little true benefit to the driver. To insure the trip is completed in a timely manner, an upper constraint is placed on the time. Here we consider the time over the entire horizon in our constraint, which we will call  $\tau(k)$ .

$$\tau(k) = \sum_{i=1}^N t(k+i) \quad (10)$$

It would seem best to constrain the trip time to a given value, for example, to travel from point A to point B in less than 20 minutes. In practice though, this is difficult. Predicting what traffic conditions will be far ahead is unreliable, and the computing power required to process an entire trip is not available. It would also be advantageous for the driver to choose to what degree he or she would like to balance the trip time and fuel economy. The solution chosen allows this.

To determine the balance of fuel economy and trip time, the control system uses the model propagation and the velocity constraints to determine the shortest time path and the longest

time path over a horizon. The driver supplies a “fuel rating,”  $\mathcal{K}$ , at the outset of the trip. This is a number from zero to 100. Then the time spent to traverse the horizon is constrained to a value between the shortest time and least fuel time according to the rating.

$$\tau(k) \leq \tau_{\min}(k) - \frac{\mathcal{K}}{100} (\tau_{\min}(k) - \tau_{\max}(k)) \quad (11)$$

Once the cost function and all the constraints have been established, a set of optimal torques can be established. At this point, the first optimal torque is applied, the horizon is re-established and re-evaluated, and the whole process repeats until the trip ends.

#### 4. RESULTS

To test this formulation, a trip was taken from Palo Alto, to San Jose, California, while recording average traffic speed. Using this data, simulations were run to determine how much fuel could have been saved if different driving choices had been made. The vehicle has a horizon length of 2000 meters. This means the traffic ahead of the vehicle up to 2000 meters at any time is assumed to be known.

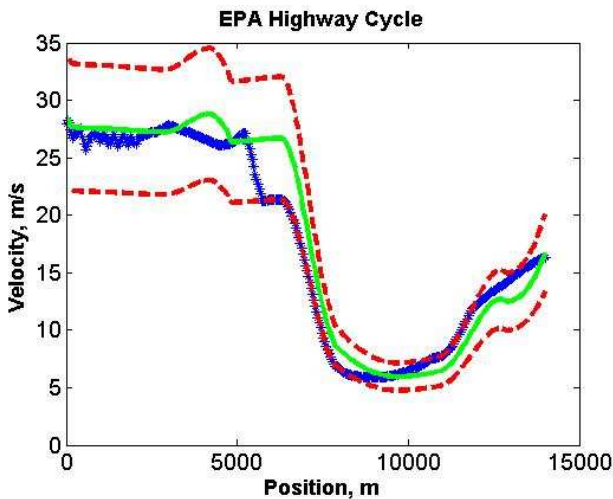


Fig. 6: A simulation run

The above graph shows the results of the simulation. The x-axis represents the distance along the trip while the y-axis represents the vehicle speed. The solid line represents the average traffic velocity, which serves as a baseline for comparison. The dashed lines represent the velocity constraints. The velocity must stay in between these lines at all times. Finally, the line of asterisks recommends the path that should be taken by the car in order to balance trip time and fuel economy best. This path takes the same amount of time to complete as the green path, but uses 5.4% less fuel. It is likely that the controller chooses this path for a few reasons. One is that if the vehicle is slowing down, but must speed up again in the future, it is advantageous to not slow down all the way, to reduce the amount of acceleration needed to keep up with traffic. This is seen between 8000 and 12000 meters on the trip. Similarly, at around 4000 meters, the vehicle sees a drop in speed and decides to slow down to preserve fuel. Another advantage of this path is that the

vehicle travels slowly relative to the traffic when the average speed is high, and relatively quickly when the average speed is low. To balance fuel use and trip time, it makes sense to slow down a bit at high speeds, when fuel consumption is high due to air drag, and make up for lost time by going a bit faster at low speeds, where the air drag penalty for doing so is much lower. (29)

#### 4.1 Optimal Horizon Length

Picking the horizon length is important for a problem of this nature. A long horizon generally offers an advantage in predictive control, because more data can be taken into account. However, if this data is unreliable, it may not be useful. Additionally, a long horizon means the solution takes much longer to compute, which can be critical in a real-time application such as this one.

Due to these considerations, it is desired to use a horizon that will be long enough to effectively improve fuel economy, while being short enough to process in real-time and provide reliable data. Testing for an ideal horizon can be difficult, because various traffic situations will have different optimal horizon lengths. This optimal length will be dependent on the traffic in front of the vehicle, and how quickly and to what degree the traffic speed changes. In the presented results we have chosen to use one trip as a benchmark, find the optimal horizon, and then use this as the horizon for all trips. Clearly the option is to change the horizon length in real-time as a function of environment and traffic conditions.

To find a suitable trip to determine the optimal horizon length, the EPA Highway fuel economy cycle was used as a baseline. This cycle is designed to represent a range of traffic conditions and will be what the vehicle is judged on in the marketplace for fuel economy.

A simulation similar to the results in Section 4 was run to determine the best horizon length. Again, the prescribed speed was used as a baseline, while the vehicle was constrained to travel no more than 20% faster or slower than the baseline speed at all times. Horizon lengths of 1200, 1600, 2000, 3000, 4000, and 5000 meters were tested.

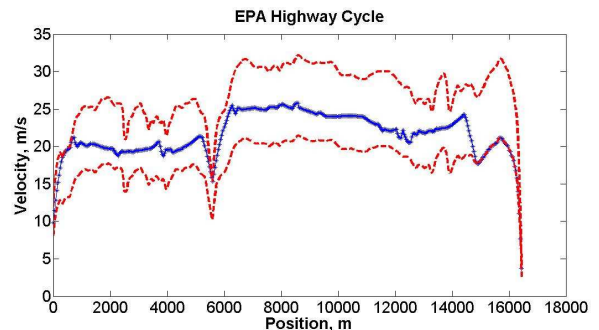


Fig. 7: EPA Highway Cycle Results

The figure above shows the results of the 2000 meter horizon simulation. In this case, 7.2% less fuel is used compared to the baseline case, while the trip takes 2.9% longer. The results of each horizon length are shown in the table below.

**Table 1: Fuel and Time savings vs. Horizon Length**

Horizon Length (m)	Fuel Saved (%)	Time Saved (%)
1200	5.2	-2.1
1600	6.4	-2.2
2000	7.2	-2.9
3000	8.7	-4.6
4000	9.6	-5.8
5000	10.3	-6.7

Positive percentages indicate improvements (less fuel or less time) while negative percentages indicate more fuel or time use. In this case, as the horizon length increases, the fuel savings increase as well. However, the trip time increases as the horizon length goes up. It should be kept in mind that the fuel use is minimized subject to constraints, while the trip time is constrained over the horizon to a certain value. This means that increasing horizon length gives more information to the controller which it can use to minimize fuel use, but since the torque and horizon are updated every step, there is no guarantee the total time will improve. To decide which of these situations is best, the percentage improvements for the fuel and time were added to create a “performance index.”

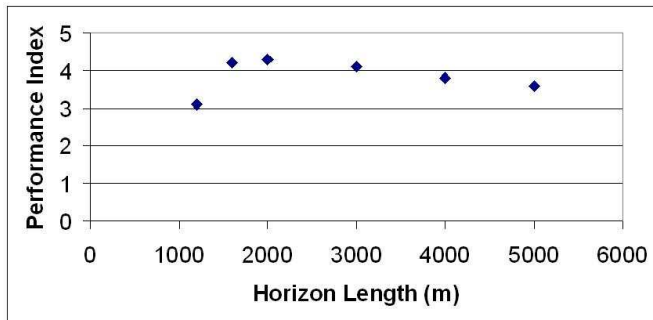


Fig. 8: Performance Index peaks at 2000 meters

It is clear here that 2000 meters is the best choice, and this is the horizon used for all simulations.

## 5. CONCLUSION

This work displays promising simulations that show reductions in fuel economy with little to no addition of hardware to the vehicle. A model has been developed that successfully reproduces vehicle behavior and can be used in a real-time control scheme. Model Predictive Control is used to minimize fuel use while maintaining realistic vehicle speeds and trip times. This leads to a fuel savings of 5 to 7 % for a trip time that changes 3% or less. The strategies presented here are close to implementation and could show an almost immediate affect on fuel economy for cars with access to traffic data similar to that used in our work.

## REFERENCES

P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, “A Hierarchical Model Predictive Control Framework for Autonomous Ground Vehicles,” *American Control Conference*, June 11-13, 2008, Seattle, Washington.

P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, “Predictive active steering control for autonomous vehicle systems,” *Accepted for publication in IEEE Trans. on Control System Technology*, 2006.

Phillip E. Gill, Walter Murray, Michael Sanders, Margaret Wright, “User’s Guide for NPSOL 5.0: A FORTRAN Package for Nonlinear Programming”, Technical Report SOL 86-1, July 1998

E. Hellstrom, M. Ivarsson, J. Aslund, L. Nielsen, “Look Ahead Control for Heavy Trucks to Minimize Trip Time and Fuel Consumption” *Fifth IFAC Symposium on Advances in Automotive Control*, August 20-22, 2007, Aptos, California.

N. Kohut, F. Borrelli, K. Hedrick (2008). “Utilization of Intelligent Transport Systems Information to Increase Fuel Economy through Engine Control.” *15<sup>th</sup> World Congress on Intelligent Transport Systems*, November 16-20, 2008, New York City, New York.

Chris Manzie, Harry Watson, Saman Halgamuge, “Fuel Economy Improvements for Urban Driving: Hybrid vs. Intelligent Vehicles”, *Transportation Research Part C*, pp. 1-16, 2007

California Freeway Performance Measurement System, <http://pems.eecs.berkeley.edu>

D.Q. Mayne, J.B. Rawlings, C.V. Rao, P.O.M. Scokaert, “Constrained Model Predictive Control: Stability and Optimality”, *Automatica* 36, November 1999, p. 789-814

V. M. Zavala, C. D. Laird, and L. T. Biegler. Fast solvers and rigorous models: Can both be accommodated in nmpc? *IFAC Workshop on Nonlinear Model Predictive Control for Fast Systems, plenary talk*, 2006.