A Summary of Actuarial Formulae and Tables

Curtin Student Actuarial Society

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Contents

Introduction

This document is designed to provide information on the formulae which are relevant to actuarial science units at Curtin university. Actuarial students are provided with selected formulae prepared by the UK Institute and Faculty of Actuaries for use in all examinations that follow their curriculum, which Curtin (and Australia as a whole) adhere to. Students are typically provided with a pdf document including the required formulae on Blackboard in any units where they are provided in the exam but this is simply an abridged version a publication which is available for purchase (Curtin students need not purchase it) known as the "orange book" (for the fact that it has an orange cover). In addition to excerpts from the orange book, Curtin students are often also provided with a copy of separate publication called 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams as an alternative to the statistical tables that appear in the orange book. This book is often used in first year statistics units so having a copy and being familiar with it is recommended. Also note that in spite of the orange book being created in 2002, the actuaries institute are **not** introducing a new formula and tables for the new curriculum (at least for now).

These formulae are not provided in all units. The units for which these formulae are provided in the exam are those for which actuarial exam conditions are adhered to. These conditions include:

- 3 hour exams
- Exams weighted higher than 50% (typically 60% or 70%)
- Actuarial tables and formulae provided
- Students are **not** permitted to bring any notes of their own into actuarial exams
- Only approved calculators allowed
- Content for these units follows closely with the actuarial curriculum and thus the official ActEd CT notes are often the prescribed textbook.

The units for which these conditions are adhered to are always CT exemption units, but the converse is not always true. This is because some of the earlier units are not exclusively part of the actuarial science program. The nature of each CT exemption unit is displayed in the table below.

CT subject	Curtin Exemption Unit	Actuarial conditions	Actuarial formulae	Separate statistical tables	Run by
CT1	MATH2004 Theory of Interest	✓	✓	✓	Maths and Stats (Actuarial)
CT2	ACCT1000 Accounting - The Language of Business				Business School
CT2	FNCE2000 Introduction to Finance Principles				Business School
CT3	STAT1000 Regression and Non-Parametric Inference			✓	Maths and Stats
CT3	STAT1001 Statistical Probability			✓	Maths and Stats
CT3	STAT1002 Statistical Data Analysis			✓	Maths and Stats
CT3	STAT2001 Mathematical Statistics	(p)	✓	✓	Maths and Stats
CT4	STAT3005 Stochastic Processes	✓	✓	✓	Maths and Stats (Actuarial)
CT4	MATH3005 Survival Analysis	✓	✓	✓	Maths and Stats (Actuarial)
CT5	MATH3006 Life Contingencies 1	✓	✓	✓	Maths and Stats (Actuarial)
CT5	MATH3007 Life Contingencies 2	✓	✓	✓	Maths and Stats (Actuarial)
CT6	STAT3001 Statistical Modelling	✓	✓	✓	Maths and Stats (Actuarial)
CT6	STAT3002 Risk Analysis and Credibility Theory	✓	✓	✓	Maths and Stats (Actuarial)
CT7	ECON1000 Introductory Economics				Business School
CT7	ECON1001 Actuarial Economics	(p)			Maths and Stats (Actuarial)
CT7	ECON2001 Macroeconomics Principles				Business School
CT8	STAT3006 Investment Science 1	✓	✓	✓	Maths and Stats (Actuarial)
CT8	STAT3007 Investment Science 2	✓	✓	✓	Maths and Stats (Actuarial)

✓ Completely satisfied

(p) Partially satisfied (students are sort of eased into it in earlier units).

Document conventions

- In orange boxes, are the formulae and tables which are actually provided in actuarial exams. *The color orange was chosen because these are the formulae that appear in the 'orange' book* (see Introduction).
- In the green boxes are some extensions made to the formulae and tables you will be provided with that you may wish to remember. That is to say anything in the green boxes is not in the formula sheet you are given in the exam but they are formulae you might be expected to draw upon in the exam.
- Red text is used when a formula or table is described instead of being explicitly given in this document.
- Blue text will be used to indicate the actuarial units in which a formula or table is used.
- An asterisk* after a unit name indicates that the actuarial tables and formulae are not provided in the unit but they are still involved in this document in some way (usually because they are a unit in which a formula is introduced or explained).

Disclaimer

The validity of the information in this document cannot be guaranteed. Curtin units and the actuarial curriculum change over time.

This document was typeset in LaTeX in an attempt to make all the formulae appear as similarly as possible to the ones in the orange book. Subtle and superficial differences may exist.

Should you find any errors in this document please let us know via our website. Link: `https://www.curtinactuary.com/actuarial-formulae/corrections`.

Acknowledgments

This document was devised and prepared as an extension to the SAS Guide by Tim Gummer, our Honours Representative for 2019. Assistance and suggestions for revisions were provided for versions 1 and 2 of this document by 2018 secretary Tom Marshall.

This document was created using document preparation and typesetting language LaTeX by way of the online editor Overleaf.

A Actuarial Formulae

1 Mathematical Methods

1.1 Series

Formula 1.1.1: Exponential Function

$$\exp(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

This formula may be familiar to some students from high school but it is also taught in MATH1016 Calculus 1* and MATH1018 Accelerated Mathematics 2* under the topic of Taylor series.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Extension 1.1.1.1: Exponential Function Using Sigma Notation

A more precise version of formula 1.1.1 would be to use $\sum$ notation.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Formula 1.1.2: Natural log Function

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 1.1.3: Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + b^n$$

where n is a positive integer

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

1.2 Calculus

Formula 1.2.1: Taylor series (one variable)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \cdots$$

Taylor series are covered in MATH1016 Calculus 1* or MATH1018 Accelerated Mathematics 2*. You will have likely seen it in a different form however.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory
- STAT3006 Investment Science 1

Formula 1.2.2: Taylor series (two variables)

$$f(x+h, y+k) = f(x,y) + hf'_x(x,y) + kf'_y(x,y) + \frac{1}{2!}\Big(h^2 f''_{xx}(x,y) + 2hkf''_{xy}(x,y) + k^2 f''_{yy}(x,y)\Big) + \cdots$$

This formula is covered in MATH2009 Calculus 2*.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 1.2.3: Integration by parts

$$\int_a^b u\tfrac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = [uv]_a^b - \int_a^b v\tfrac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Integration by parts is covered in MATH1016 Calculus 1* or MATH1017 Accelerated Mathematics 1*. It is a common integration technique. Most students will likely have remembered this formula by this time but ...
You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2
- STAT3002 Risk Analysis and Credibility Theory

Formula 1.2.4: Double integrals (changing the order of integration)

$$\int_a^b \left(\int_a^x f(x,y)\,\mathrm{d}y\right)\mathrm{d}x = \int_a^b \left(\int_y^b f(x,y)\,\mathrm{d}x\right)\mathrm{d}y \text{ or}$$
$$\int_a^b \mathrm{d}x \int_a^x \mathrm{d}y\, f(x,y) = \int_a^b \mathrm{d}y \int_a^x \mathrm{d}x\, f(x,y)$$

The domain of integration here is the set of values (x,y) for which $a \leq y \leq x \leq b$

These are covered for the first time in STAT1001 Statistical Probability, but are given more detail in MATH2009 Calculus 2*.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- MATH3007 Life Contingencies 2

Formula 1.2.5: Differentiating an integral

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

1.3 Solving Equations

Formula 1.3.1: Newton-Raphson method

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 1.3.2: Integrating factors

The integrating factor for solving the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + P(x)y = Q(x)$ is:

$$\exp\left(\int P(x)\,\mathrm{d}x\right)$$

This formula is explained in MATH1016 Calculus 1* or MATH1018 Accelerated Mathematics 2*.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3005 Stochastic Processes
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 1.3.3: Second-order difference equations

The general solution of the difference equation $ax_{n+2} + bx_{n+1} + cx_n = 0$ is:

if $b^2 - 4ac > 0:\ x_n = A\lambda_1^n + B\lambda_2^n$ (distinct real roots, $\lambda_1 \neq \lambda_2$)

if $b^2 - 4ac = 0:\ x_n = (A + Bn)\lambda^n$ (equal real roots, $\lambda_1 = \lambda_2 = \lambda$)

if $b^2 - 4ac < 0:\ x_n = r^n(A\cos n\theta + B\sin n\theta)$ (complex roots, $\lambda_1 = \overline{\lambda}_2 = re^{i\theta}$)

where λ_1 and λ_2 are the roots of the quadratic equation $a\lambda^2 + b\lambda = 0$.

This formula is derived in MATH1017 Accelerated Mathematics 1*, but simply taken as a fact in STAT3001 Statistical Modelling.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

1.4 Gamma Function

Formula 1.4.1: Definition

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,\mathrm{d}t,\ x > 0$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 1.4.2: Properties

$$\Gamma(x) = (x-1)\Gamma(x-1)$$
$$\Gamma(n) = (n-1)!,\ n = 1, 2, 3, \ldots$$
$$\Gamma(1/2) = \sqrt{\pi}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

1.5 Bayes' Formula

Formula 1.5.1: Bayes' formula

Let $A_1, A_2, \ldots, A_n$ be a collection of mutually exclusive and exhaustive events with $P(A_i) \neq 0$, $i = 1, 2, \ldots, n$.

For any event B such that $P(B) \neq 0$:

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{j=1}^{n} P(B|A_j)P(A_j)}, i = 1, 2, \ldots, n.$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

2 Statistical Distributions

Formula 2.0.1: Notation

$$\begin{aligned}
\text{PF} &= \text{Probability function}, p(x) \\
\text{PDF} &= \text{Probability density function}, f(x) \\
\text{DF} &= \text{Distribution function}, F(x) \\
\text{PGF} &= \text{Probability generating function}, G(s) \\
\text{MGF} &= \text{Moment generating function}, M(t)
\end{aligned}$$

Note. Where formulae have been omitted below, this indicates that (a) there is no simple formula or (b) the function does not have a finite value or (c) the function equals zero.

These conventions will have been first encountered in STAT1001 Statistical Probability.
This notation is used throughout 2.1 and 2.2

Extension 2.0.1.1: Common alternative conventions

- Referring to the "probability function" in 2.0.1 as a "probability **mass** function" (PMF).
- Referring to the "distribution function" in 2.0.1 as a "**cumulative** distribution function" (CDF).
- A subscript is often used to indicate which random variable the function corresponds to. E.g. $f_X(x)$ denotes the PDF of some random variable X.

Extension 2.0.1.2: Definitions that correspond to these conventions

- The definition of a PF is $p(x) := P(X = x)$.
- The definition of a PDF is $f(x) := \frac{\mathrm{d}}{\mathrm{d}x}F(x)$.
- The definition of a DF is $F(x) := P(X \leq x)$.
- The definition of a PGF is $G(s) := E(s^X)$.
- The definition of a MGF is $M(t) := E(e^{tX})$.

Extension 2.0.1.3: Moment generation using the MGF

If $M(t)$ is r times differentiable at $t = 0$, then

$$M_X^{(r)}(0) = E(X^r)$$

Extension 2.0.1.4: Cumulant Generating Function

The cumulant generating function (CGF) is defined as

$$C(t) := \log M(t)$$

This is a commonly used transformation for the following properties which follow from 2.0.1.3 and some basic differentiation techniques:

$$C'_X(0) = E(X)$$

$$C''_X(0) = \mathrm{var}(X)$$

$$C'''_X(0) = E\left[\left(X - E(X)\right)^3\right]$$

(note that higher order derivatives do not continue this nice pattern).

2.1 Discrete Distributions

Formula 2.1.1: Binomial distribution

Parameters: n, p ($n =$ positive integer, $0 < p < 1$ with $q = 1 - p$)

PF: $p(x) = \binom{n}{x} p^x q^{n-x}$, $x = 0, 1, 2, \ldots, n$

DF: The distribution function is tabulated in the statistical tables section.

PGF: $G(s) = (q + ps)^n$

MGF: $M(t) = (q + pe^t)^n$

Moments: $E(X) = np$, $\mathrm{var}(X) = npq$

Coefficient of skewness: $\dfrac{q - p}{\sqrt{npq}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

- STAT3007 Investment Science 2

Formula 2.1.2: Bernoulli distribution

The Bernoulli distribution is the same as the binomial distribution with parameter $n = 1$.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3007 Investment Science 2

Formula 2.1.3: Poisson distribution

Parameter: μ $(\mu > 0)$

PF: $p(x) = \dfrac{e^{-\mu}\mu^x}{x!}$, $x = 0, 1, 2, \ldots$

DF: The distribution function is tabulated in the statistical tables section.

PGF: $G(s) = e^{\mu(s-1)}$

MGF: $M(t) = e^{\mu(e^t-1)}$

Moments: $E(X) = \mu$, $\text{var}(X) = \mu$

Coefficient of skewness: $\dfrac{1}{\sqrt{\mu}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3005 Stochastic Processes
- MATH3005 Survival Analysis

Formula 2.1.4: Negative binomial distribution – Type 1

Parameters: $k,\, p\, (k = \text{positive integer}, 0 < p < 1 \text{ with } q = 1 - p)$

PF: $p(x) = \binom{x-1}{k-1} p^k q^{x-k},\ x = k, k+1, k+2, \ldots$

PGF: $G(s) = \left(\frac{ps}{1-qs}\right)^k$

MGF: $M(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$

Moments: $E(X) = \frac{k}{p},\ \text{var}(X) = \frac{kq}{p^2}$

Coefficient of skewness: $\frac{2-p}{\sqrt{kq}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.1.5: Negative binomial distribution – Type 2

Parameters: $k,\, p\, (k = \text{positive integer}, 0 < p < 1 \text{ with } q = 1 - p)$

PF: $p(x) = \frac{\Gamma(k+x)}{\Gamma(x+1)\Gamma(k)} p^k q^x,\ x = k, k+1, k+2, \ldots$

PGF: $G(s) = \left(\frac{ps}{1-qs}\right)^k$

MGF: $M(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$

Moments: $E(X) = \frac{k}{p},\ \text{var}(X) = \frac{kq}{p^2}$

Coefficient of skewness: $\frac{2-p}{\sqrt{kq}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.1.6: Geometric distribution

The geometric distribution is the same as the negative binomial distribution with parameter $k = 1$.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.1.7: Uniform distribution (discrete)

Parameters: a, b, h ($a < b$, $h > 0$, $b - a$ is a multiple of h)

PF: $p(x) = \dfrac{h}{b-a+h}$, $x = a, a+h, a+2h, \dots, b-h, b$

PGF: $G(s) = \dfrac{h}{b-a+h}\left(\dfrac{s^{b+h} - s^a}{s^h - 1}\right)$

MGF: $M(t) = \dfrac{h}{b-a+h}\left(\dfrac{e^{(b+h)t} - e^{at}}{e^{ht} - 1}\right)$

Moments: $E(X) = \dfrac{1}{2}(a+b)$, $\text{var}(X) = \dfrac{1}{12}(b-a)(b-a+2h)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics

2.2 Continuous distributions

Formula 2.2.1: Standard normal distribution – $N(0,1)$

Parameters: none

PDF: $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$, $-\infty < x < \infty$

DF: The distribution function is tabulated in the statistical tables section.

MGF: $M(t) = e^{\frac{1}{2}t^2}$

Moments: $E(X) = 0$, $\text{var}(X) = 1$

$$E(X^r) = \frac{1}{2^{r/2}} \frac{\Gamma(1+r)}{\Gamma(1+\frac{r}{2})}, \; r = 2, 4, 6, \dots$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2
- MATH3005 Survival Analysis

Formula 2.2.2: Normal (Gaussian) distribution – $N(\mu, \sigma^2)$

Parameters: μ, σ^2 ($\sigma > 0$)

PDF: $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}, \ -\infty < x < \infty$$

MGF: $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Moments: $E(X) = \mu$, $\text{var}(X) = \sigma^2$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2
- MATH3005 Survival Analysis

Formula 2.2.3: Exponential distribution

Parameters: λ ($\lambda > 0$)

PDF: $f(x) = \lambda e^{-\lambda x}$, $x > 0$

DF: $F(x) = 1 - e^{-\lambda x}$

MGF: $M(t) = \left(1 - \dfrac{t}{\lambda}\right)^{-1}$, $t < \lambda$

Moments: $E(X) = \dfrac{1}{\lambda}$, $\mathrm{var}(X) = \dfrac{1}{\lambda^2}$

$E(X^r) = \dfrac{\Gamma(1+r)}{\lambda^r}$, $r = 1, 2, 3, \ldots$

Coefficient of skewness: 2

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3005 Stochastic Processes
- MATH3005 Survival Analysis

Formula 2.2.4: Gamma distribution

Parameters: α, λ ($\alpha > 0$, $\lambda > 0$)

PDF: $f(x) = \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$, $x > 0$

DF: When 2α is an integer, probabilities for the gamma distribution can be found using the relationship:

$2\lambda X \sim \chi^2_{2\alpha}$

MGF: $M(t) = \left(1 - \dfrac{t}{\lambda}\right)^{-\alpha}$, $t < \lambda$

Moments: $E(X) = \dfrac{\alpha}{\lambda}$, $\text{var}(X) = \dfrac{\alpha}{\lambda^2}$

$E(X^r) = \dfrac{\Gamma(\alpha + r)}{\Gamma(\alpha)\lambda^r}$, $r = 1, 2, 3, \ldots$

Coefficient of skewness: $\dfrac{2}{\sqrt{\alpha}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.2.5

The chi-square distribution with ν degrees of freedom is the same as the gamma distribution with parameters $\alpha = \frac{\nu}{2}$ and $\lambda = \frac{1}{2}$.

The distribution function for the chi-square distribution is tabulated in the statistical tables section.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- MATH3005 Survival Analysis

Formula 2.2.6: Uniform distribution (continuous) – $U(a, b)$

Parameters: a, b $a < b$

PDF: $f(x) = \frac{1}{b-a}$, $a < x < b$

DF: $F(x) = \frac{x-a}{b-a}$

MGF: $M(t) = \frac{1}{(b-a)}\frac{1}{t}(e^{bt} - e^{at})$

Moments: $E(X) = \frac{1}{2}(a+b)$, $\text{var}(X) = \frac{1}{12}(b-a)^2$

$$E(X^r) = \frac{1}{(b-a)}\frac{1}{r+1}(b^{r+1} - a^{r+1}),\ r = 1, 2, 3\ldots$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.2.7: Beta distribution

Parameters: α, β $(\alpha > 0,\ \beta > 0)$

PDF: $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$

Moments: $E(X) = \frac{\alpha}{\alpha+\beta}$, $\text{var}(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$$E(X^r) = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\alpha+\beta+r)},\ r = 1, 2, 3, \ldots$$

Coefficient of skewness: $\frac{2(\beta-\alpha)}{\alpha+\beta+2}\sqrt{\frac{\alpha+\beta+1}{\alpha\beta}}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.2.8: Lognormal distribution

Parameters: μ, σ^2 $(\sigma > 0)$

PDF: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\dfrac{1}{x}\exp\left\{-\dfrac{1}{2}\left(\dfrac{\log x - \mu}{\sigma}\right)\right\}$, $x > 0$

Moments: $E(X) = e^{\mu+\frac{1}{2}\sigma^2}$, $\text{var}(X) = e^{2\mu+\sigma^2}\left(e^{\sigma^2} - 1\right)$

$E(X^r) = e^{r\mu+\frac{1}{2}r^2\sigma^2}$, $r = 1, 2, 3, \ldots$

Coefficient of skewness: $\left(e^{\sigma^2} + 2\right)\sqrt{e^{\sigma^2} - 1}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2
- MATH2004 Theory of Interest

Extension 2.2.8.1: Variance of the Lognormal distribution

It can be useful identify the following relationship between the mean and variance of the lognormal distribution:

$$\text{var}(X) = \left[E(X)\right]^2\left(e^{\sigma^2} - 1\right)$$

This is useful whenever the parameters of a lognormal distribution are estimated by way of the method of moments. This is done in MATH2004 Theory of Interest and STAT3002 Risk Analysis and Credibility Theory.

Formula 2.2.9: Pareto distribution (two parameter version)

Parameters: α, λ ($\alpha > 0$, $\lambda > 0$)

PDF: $f(x) = \dfrac{\alpha\lambda^\alpha}{(\lambda + x)^{\alpha+1}}$, $x > 0$

DF: $F(x) = 1 - \left(\dfrac{\lambda}{\lambda + x}\right)^\alpha$

Moments: $E(X) = \dfrac{\lambda}{\alpha - 1}(\alpha > 1)$, $\text{var}(X) = \dfrac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)}(\alpha > 2)$

$$E(X^r) = \frac{\Gamma(\alpha - r)\Gamma(1 + r)}{\Gamma(\alpha)}\lambda^r,\ r = 1, 2, 3, \ldots,\ r < \alpha$$

Coefficient of skewness: $\dfrac{2(\alpha + 1)}{(\alpha - 3)}\sqrt{\dfrac{\alpha - 2}{\alpha}}\ (\alpha > 3)$

This is also known simply as the *Pareto distribution*.
This is mentioned as an example but not actually assessed in STAT2001 Mathematical Statistics.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Extension 2.2.9.1: Variance of the Pareto distribution

It can be useful identify the following relationship between the mean and variance of the Pareto distribution:

$$\text{var}(X) = \left[E(X)\right]^2 \frac{\alpha}{\alpha - 2}$$

Formula 2.2.10: Pareto distribution (three parameter version)

Parameters: α, λ, k ($\alpha > 0$, $\lambda > 0$, $k > 0$)

PDF: $f(x) = \dfrac{\Gamma(\alpha + k)\lambda^\alpha x^{k-1}}{\Gamma(\alpha)\Gamma(k)(\lambda + x)^{\alpha+k}}$, $x > 0$

Moments: $E(X) = \dfrac{k\lambda}{\alpha - 1}(\alpha > 1)$, $\text{var}(X) = \dfrac{k(k + \alpha - 1)\lambda^2}{(\alpha-1)^2(\alpha-2)}(\alpha > 2)$

$$E(X^r) = \frac{\Gamma(\alpha - r)\Gamma(k + r)}{\Gamma(\alpha)\Gamma(k)}\lambda^r,\ r = 1, 2, 3, \ldots,\ r < \alpha$$

This is also known as the *generalised Pareto distribution*.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.2.11: Weibull distribution

Parameters: c, γ ($c > 0$, $\gamma > 0$)

PDF: $f(x) = c\gamma x^{\gamma-1} e^{-cx^{\gamma}}$, $x > 0$

DF $F(x) = 1 - e^{-cx^{\gamma}}$

Moments: $E(X^r) = \Gamma\left(1 + \frac{r}{\gamma}\right)\frac{1}{c^{r/\gamma}}\, r = 1, 2, 3, \ldots$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- MATH3005 Survival Analysis

Formula 2.2.12: Burr distribution

Parameters: α, λ, γ ($\alpha > 0, \lambda > 0$, $\gamma > 0$)

PDF: $f(x) = \frac{\alpha\gamma\lambda^{\alpha}x^{\gamma-1}}{(\lambda + x^{\gamma})^{\alpha+1}}$, $x > 0$

DF $F(x) = 1 - \left(\frac{\lambda}{\lambda + x^{\gamma}}\right)^{\alpha}$

Moments: $E(X^r) = \Gamma\left(\alpha - \frac{r}{\gamma}\right)\Gamma\left(1 + \frac{r}{\gamma}\right)\frac{\lambda^{r/\gamma}}{\Gamma(\alpha)}\, r = 1, 2, 3, \ldots, r < \alpha\gamma$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

2.3 Compound Distributions

Formula 2.3.1: Conditional expectation and variance

$$E(Y) = E[E(Y|X)]$$

$$\mathrm{var}(Y) = \mathrm{var}[E(Y|X)] + E[\mathrm{var}(Y|X)]$$

This is also known as the "*tower property*".
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3002 Risk Analysis and Credibility Theory

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 2.3.2: Moments of a compound distribution

If $X_1, X_2, \ldots$ are IID random variables with MGF $M_X(t)$ and N is an independent nonnegative integer-valued random variable, then $S = X_1 + \cdots + X_N$ (with $S = 0$ when $N = 0$) has the following properties:

Mean: $E(S) = E(N)E(X)$

Variance: $\text{var}(S) = E(N)\text{var}(X) + \text{var}(N)[E(X)]^2$

MGF: $M_S(t) = M_N[\log M_X(t)]$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.3.3: Compound Poisson distribution

Mean: λm_1

Variance: λm_2

Third central moment: λm_3

where $\lambda = E(N)$ and $m_r = E(X^r)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 2.3.4: Recursive formulae for integer-valued distributions

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

2.4 Truncated Moments

Extension 2.4.0.1: Notation used in truncated moments formulae

It is not stated anywhere in the formula section of the orange book (but it is in the Standard Normal probabilities table, which Curtin students aren't actually provided with in exams since they are replaced with 'Mathematical Formulae and Statistical Tables for Tertiary Institutions') but it is common to use the following notation for a standard normal random variable ($Z \sim N(0,1)$):

$$\text{PDF: } \phi(z) \equiv f_Z(z)$$
$$\text{DF: } \Phi(z) \equiv F_Z(z)$$

Note that the ϕ and Φ are the lowercase and uppercase versions respectively of the greek letter 'phi'.

Formula 2.4.1: Normal distribution

If $f(x)$ is the PDF of the $N(\mu, \sigma^2)$ distribution, then

$$\int_L^U xf(x)dx = \mu[\Phi(U') - \Phi(L')] - \sigma[\phi(U') - \phi(L')]$$

where $L' = \dfrac{L-\mu}{\sigma}$ and $U' = \dfrac{U-\mu}{\sigma}$

This formula is derived in STAT3002 Risk Analysis and Credibility Theory.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 2.4.2: Lognormal distribution

If $f(x)$ is the PDF of the lognormal distribution, then

$$\int_L^U x^k f(x)dx = e^{k\mu + 1/2k^2\sigma^2}[\Phi(U_k) - \Phi(L_k)]$$

where $L_k = \dfrac{\log L - \mu}{\sigma} - k\sigma$ and $U_k = \dfrac{\log U - \mu}{\sigma} - k\sigma$

This formula is derived in STAT3002 Risk Analysis and Credibility Theory.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory
- STAT3007 Investment Science 2

Relationships between statistical distributions

Formula 2.4.3: Statistical distributions diagram

This diagram shows the relationships between all the Discrete Distributions and Continuous distributions above in the form of a flowchart. It has been omitted due to its cumbersome nature.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 2.4.4: Explanation of the distribution diagram

A description of the conventions used in 2.4.3. It has been omitted due to its cumbersome nature.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

3 Statistical Methods

3.1 Sample Mean and Variance

Formula 3.1.1: Sample mean and variance

The random sample $(x_1, x_2, \ldots, x_n)$ has the following sample moments:

Sample mean: $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

Sample variance: $s^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right\}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics

3.2 Parametric Inference (Normal Model)

Formula 3.2.1: One sample

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.2.2: Two samples

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

3.3 Maximum Likelihood Estimators

Formula 3.3.1: Asymptotic distribution

If $\hat{\theta}$ is the maximum likelihood estimator of a parameter θ based on a sample $\underline{X}$, then $\hat{\theta}$ is asymptotically normally distributed with mean θ and variance equal to the Cramér-Rao lower bound

$$\text{CRLB}(\theta) = -1 \Big/ E\left[\frac{\partial^2}{\partial\theta^2}\log L(\theta, \underline{X})\right]$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 3.3.2: Likelihood ratio test

$$-2(\ell_p - \ell_{p+q}) = -2\log\left(\frac{\max\limits_{H_0} L}{\max\limits_{H_0 \cup H_1} L}\right) \sim \chi_q^2 \text{ approximately (under } H_0\text{)}$$

where $\ell_p = \max\limits_{H_0} \log L$ is the maximum log-likelihood for the model under H_0 (in which there are p free parameters)

and $\ell_{p+q} = \max\limits_{H_0 \cup H_1} \log L$ is the maximum log-likelihood for the model under $H_0 \cup H_1$ (in which there are $p+q$ free parameters)

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- STAT3001 Statistical Modelling

3.4 Linear Regression Model With Normal Errors

All of the below formulae are explained/derived in STAT1000 Regression and Non-Parametric Inference.

Formula 3.4.1: Model

$$Y_i \sim N(\alpha + \beta x_i, \sigma^2),\ i = 1, 2, \ldots, n$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 3.4.2: Intermediate calculations

$$s_{xx} = \sum_{i=1}^{n}(x_i - \overline{x})^2 = \sum_{i=1}^{n} x_i^2 - n\overline{x}^2$$
$$s_{yy} = \sum_{i=1}^{n}(y_i - \overline{y})^2 = \sum_{i=1}^{n} y_i^2 - n\overline{y}^2$$
$$s_{xy} = \sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y}) = \sum_{i=1}^{n} x_i y_i - n\overline{x}\,\overline{y}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 3.4.3: Parameter estimates

$$\hat{\alpha} = \overline{y} - \hat{\beta}\overline{x},\ \hat{\beta} = \frac{s_{xy}}{s_{xx}}$$
$$\hat{\sigma}^2 = \frac{1}{n-2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \frac{1}{n-2}\left(s_{yy} - \frac{s_{xy}^2}{s_{xx}}\right)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 3.4.4: Distribution of $\hat{\beta}$

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.4.5: Variance of predicted mean response

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.4.6: Testing the correlation coefficient

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.4.7: Fisher Z transformation

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.4.8: Sum of squares relationship

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

3.5 Analysis of Variance

Analysis of variance is covered in STAT1002 Statistical Data Analysis but it is not revisited in later CT subjects which adhere to actuarial exam conditions at Curtin.

Formula 3.5.1: Single factor normal model

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.5.2: Intermediate calculations (sums of squares)

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.5.3: Variance estimate

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 3.5.4: Statistical test

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

3.6 Generalised Linear Models

Formula 3.6.1: Exponential Family

For a random variable Y from the exponential family, with natural parameter θ and scale parameter ϕ:

Probability (density) function: $f_Y(y;\theta,\phi) = \exp\left[\dfrac{y\theta - b(\theta)}{a(\phi)} + c(y,\phi)\right]$

Mean: $E(Y) = b'(\theta)$

Variance: $\mathrm{var}(Y) = a(\phi)b''(\theta)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 3.6.2: Canonical Link Functions

Binomial: $g(\mu) = \log\dfrac{\mu}{1-\mu}$

Poisson: $g(\mu) = \log\mu$

Normal: $g(\mu) = \mu$

Gamma: $g(\mu) = \dfrac{1}{\mu}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

3.7 Bayesian Methods

Formula 3.7.1: Relationship between posterior and prior distributions

$$\textit{Posterior} \propto \textit{Prior} \times \textit{Likelihood}$$

The posterior distribution $f(\theta|\underline{x})$ for the parameter θ is related to the prior distribution $f(\theta)$ via the likelihood function $f(\underline{x}|\theta)$:

$$f(\theta|\underline{x}) \propto f(\theta) \times f(\underline{x}|\theta)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

Formula 3.7.2: Normal / normal model

If $\underline{x}$ us a random sample of size n from a $N(\mu,\sigma^2)$ distribution, where σ^2 is known, and the prior distribution for the parameter μ is $N(\mu_0,\sigma_0^2)$, then the posterior distribution for μ is:

$$\mu|\underline{x} \sim N(\mu_*,\sigma_*^2)$$

where $\mu_* = \left(\dfrac{n\overline{x}}{\sigma^2} + \dfrac{\mu_0}{\sigma_0^2}\right) \Bigg/ \left(\dfrac{n}{\sigma^2} + \dfrac{1}{\sigma_0^2}\right)$ and $\sigma_*^2 = 1 \Bigg/ \left(\dfrac{n}{\sigma^2} + \dfrac{1}{\sigma_0^2}\right)$

This result is derived in STAT3001 Statistical Modelling.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

3.8 Empirical Bayes Credibility – Model 1

Formula 3.8.1: Data requirements

$$\{X_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\}$$

X_{ij} represents the aggregate claims in the j th year from the i th risk.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 3.8.2: Intermediate calculations

$$\overline{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}, \quad \overline{X} = \frac{1}{N}\sum_{i=1}^{N} \overline{X}_i$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 3.8.3: Parameter estimation

Quantity	*Estimator*
$E[m(\theta)]$	$\overline{X}$
$E[s^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2\right\}$
$\text{var}[m(\theta)]$	$\frac{1}{N-1}\sum_{i=1}^{N}(\overline{X}_i-\overline{X})^2 - \frac{1}{Nn}\sum_{i=1}^{N}\left\{\frac{1}{n-1}\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2\right\}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Extension 3.8.3.1: Simplification of parameter estimation

If the following additional intermediate calculations are performed:

$$S_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_{ij} - \overline{X}_i)^2,\ \overline{S}^2 = \frac{1}{N}\sum_{i=1}^{N}(\overline{X}_i - \overline{X})^2$$

These calculations as well as those for the the means in 3.8.2 can be performed quickly and accurately by making use of the statistics functionality of most scientific calculators.

We can then make the following simplifications to 3.8.3:

Quantity	*Estimator*
$E[s^2(\theta)]$	$\frac{1}{N}\sum_{i=1}^{N} S_i^2$
$\mathrm{var}[m(\theta)]$	$\overline{S}^2 - \frac{1}{n}E[s^2(\theta)]$

Formula 3.8.4: Credibility Factor

$$Z = \frac{n}{n + \frac{E[s^2(\theta)]}{\mathrm{var}[m(\theta)]}}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Extension 3.8.4.1: Credibility formula

When applying formula 3.8.4 (and 3.9.4), it is important to remember that the credibility premium is:

$$Z\overline{x} + (1 - Z)\overline{X}$$

Where

$\overline{x}$ is an estimate of the credibility premium based on **direct** data. If we are interested in the k th risk, then in the notation above, we would set $\overline{x} = \overline{X}_k$.

$\overline{X}$ is an estimate of the credibility premium based on **collateral** data. Note that this is the same as $\overline{X}$ in the formulae above.

This is usually presented as $Z\overline{X} + (1 - Z)\mu$ in the CT notes, however this is inconsistent with the notation used in the formulae above, so the notation has been adjusted accordingly.

3.9 Empirical Bayes Credibility – Model 2

Formula 3.9.1: Data requirements

$$\{Y_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\},\ \{P_{ij}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n\}$$

Y_{ij} represents the aggregate claims in the j th year from the i th risk;
P_{ij} is the corresponding risk volume.

Note that Y_{ij} here is analogous to X_{ij} in 3.8.1
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 3.9.2: Intermediate calculations

$$\overline{P}_i = \sum_{j=1}^{n} P_{ij}, \quad \overline{P} = \sum_{i=1}^{N} \overline{P}_i, \quad P^* = \frac{1}{Nn-1} \sum_{i=1}^{N} \overline{P}_i \left(1 - \frac{\overline{P}_i}{\overline{P}}\right)$$

$$X_{ij} = \frac{Y_{ij}}{P_{ij}}, \quad \overline{X}_i = \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\overline{P}_i}, \quad \overline{X} = \sum_{i=1}^{N} \sum_{j=1}^{n} \frac{P_{ij} X_{ij}}{\overline{P}}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Extension 3.9.2.1: Quantities of interest

Here X_{ij} represents number of claims per year **per risk volume**. Whether we are more estimating X or Y depends on the particular problem. We need to use the relationship in 3.9.2 if we want to recover an estimate for Y.

Formula 3.9.3: Parameter estimation

Quantity	*Estimator*
$E[m(\theta)]$	$\overline{X}$
$E[s^2(\theta)]$	$\frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{n-1} \sum_{j=1}^{n} P_{ij}(X_{ij} - \overline{X}_i)^2 \right\}$
$\text{var}[m(\theta)]$	$\frac{1}{P^*} \left(\frac{1}{Nn-1} \sum_{i=1}^{N} \sum_{j=1}^{n} P_{ij}(X_{ij} - \overline{X})^2 - \frac{1}{N} \sum_{i=1}^{N} \left\{ \frac{1}{n-1} \sum_{j=1}^{n} P_{ij}(X_{ij} - \overline{X}_i)^2 \right\} \right)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Formula 3.9.4: Credibility Factor

$$Z_i = \frac{\sum_{j=1}^{n} P_{ij}}{\sum_{j=1}^{n} P_{ij} + \frac{E[s^2(\theta)]}{\text{var}[m(\theta)]}}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

4 Compound Interest

Formula 4.0.1: Increasing/decreasing annuity functions

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}, \quad (Da)_{\overline{n}|} = \frac{n - a_{\overline{n}|}}{i}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH2004 Theory of Interest

Formula 4.0.2: Accumulation factor for variable interest rates

$$A(t_1, t_2) = \exp\left(\int_{t_1}^{t_2} \delta(t)\,\mathrm{d}t\right)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH2004 Theory of Interest

5 Survival Models

5.1 Mortality "Laws"

Formula 5.1.1: Survival Probabilities

$${}_tp_x = \exp\left(\int_0^t \mu_{x+s}\,\mathrm{d}s\right)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

Formula 5.1.2: Gompertz' Law

$$\mu_x = Bc^x, \quad {}_tp_x = g^{c^x(c^t-1)} \text{ where } g = e^{-B/\log c}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Formula 5.1.3: Makeham's Law

$$\mu_x = A + Bc^x, \quad {}_tp_x = s^t g^{c^x(c^t-1)} \text{ where } s = e^{-A}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Formula 5.1.4: Gompertz-Makeham formula

The Gompertz-Makeham graduation formula, denoted by $\mathrm{GM}(r, s)$, states that

$$\mu_x = \mathit{poly}_1(t) + \exp[\mathit{poly}_2(t)]$$

where t us a linear function of x and $\mathit{poly}_1(t)$ and $\mathit{poly}_2(t)$ are polynomials of degree r and s respectively.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

5.2 Empirical Estimation

Formula 5.2.1: Greenwood's formula for the variance of the Kaplan-Meier estimator

$$\text{var}[\tilde{F}(t)] = \left[1 - \hat{F}(t)\right]^2 \sum_{t_j \leq t} \frac{d_j}{n_j(n_j - d_j)}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Formula 5.2.2: Variance of the Nelson-Aalen estimate of the integrated hazard

$$\text{var}[\tilde{\Lambda}_t] = \sum_{t_j \leq t} \frac{d_j(n_j - d_j)}{n_j^3}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

5.3 Mortality Assumptions

Formula 5.3.1: Balducci Assumption

$$_{1-t}q_{x+t} = (1-t)q_x \ \ (x \text{ is an integer, } 0 \leq t \leq 1)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- MATH3007 Life Contingencies 2

5.4 General Markov Model

Formula 5.4.1: Kolmogorov forward differential equation

$$\frac{\partial}{\partial t} {}_tp_x^{gh} = \sum_{j \neq h} \left({}_tp_x^{gj} \mu_{x+t}^{jh} - {}_tp_x^{gh} \mu_{x+t}^{hj} \right)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3005 Stochastic Processes

5.5 Graduation Tests

Formula 5.5.1: Grouping of signs test

If there are n_1 positive signs and n_2 negative signs and G denotes the observed number of positive runs, then:

$$P(G = t) = \frac{\binom{n_1 - 1}{t - 1}\binom{n_2 + 1}{t}}{\binom{n_1 + n_2}{n_1}} \text{ and, approximately,}$$

$$G \sim N\left(\frac{n_1(n_2+1)}{n_1+n_2}, \frac{(n_1 n_2)^2}{(n_1+n_2)^3}\right)$$

Critical values for the grouping of signs test are tabulated in the statistical tables section for small values of n_1 and n_2. For larger values of n_1 and n_2 the normal approximation can be used.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Formula 5.5.2: Serial correlation test

$$r_j \approx \frac{\frac{1}{m-j}\sum_{i=1}^{m-j}(z_i - \overline{z})(z_{i+j} - \overline{z})}{\frac{1}{m}\sum_{i=1}^{m}(z_i - \overline{z})^2} \text{ where } \overline{z} = \frac{1}{m}\sum_{i=1}^{m} z_i$$

$r_j \times \sqrt{m} \sim N(0,1)$ approximately.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Formula 5.5.3: Variance adjustment factor

$$r_x = \frac{\sum_i i^2 \pi_i}{\sum_i i \pi_i}$$

where π_i is the proportion of lives at age x who have exactly i policies.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

5.6 Multiple Decrement Tables

Formula 5.6.1

For a multiple decrement table with three decrements α, β and γ, each uniform over the year of age $(x, x+1)$ in its single decrement table, then

$$(aq)_x^\alpha = q_x^\alpha \left[1 - \frac{1}{2}(q_x^\beta + q_x^\gamma + \frac{1}{3} q_x^\beta q_x^\gamma)\right]$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3007 Life Contingencies 2

5.7 Population Projection Models

Formula 5.7.1: Logistic model

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

6 Annuities and Assurances

6.1 Approximations for Non Annual Annuities

Formula 6.1.1: Approximations for Non Annual Annuities

$$\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m}$$

$$\ddot{a}_{x:\overline{n}|}^{(m)} \approx \ddot{a}_{x:\overline{n}|} - \frac{m-1}{2m}\left(1 - \frac{D_{x+n}}{D_x}\right)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

6.2 Moments of Annuities and Assurances

Formula 6.2.1

Let K_x and T_x denote the curtate and complete future lifetimes (respectively) of a life aged exactly x.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

Formula 6.2.2: Whole life assurances

$$E[v^{K_x+1}] = A_x,\ \text{var}[v^{K_x+1}] = {}^2A_x - (A_x)^2$$

$$E[v^{T_x}] = \bar{A}_x,\ \text{var}[v^{T_x}] = {}^2\bar{A}_x - (\bar{A}_x)^2$$

Similar relationships hold for endowment assurances (with status $\cdots_{x:\overline{n}|}$), pure endowments (with status $x:\overset{1}{\overline{n}|}$), term assurances (with status $\overset{1}{x}:\overline{n}|$) and deferred whole life assurances (with status ${}_{m|}\cdots_x$).

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

Formula 6.2.3: Whole life annuities

$$E[\ddot{a}_{\overline{K_x+1}|}] = \ddot{a}_x,\ \text{var}[\ddot{a}_{\overline{K_x+1}|}] = \frac{{}^2A_x - (A_x)^2}{d^2}$$

$$E[\bar{a}_{\overline{T_x}|}] = \bar{a}_x,\ \text{var}[\bar{a}_{\overline{T_x}|}] = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{\delta^2}$$

Similar relationships hold for temporary annuities (with status $\cdots_{x:\overline{n}|}$).

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

6.3 Premiums and Reserves

Formula 6.3.1: Premium conversion relationship between annuities and assurances

$$A_x = 1 - d\ddot{a}_x, \ \bar{A}_x = 1 - \delta\bar{a}_x$$

Similar relationships hold for endowment assurance policies (with status $\cdots_{x:\overline{n}|}$).

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1

Formula 6.3.2: Net premium reserve

$${}_tV_x = 1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}, \ {}_t\overline{V}_x = 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}$$

Similar formulae hold for endowment assurance policies (with statuses $\cdots_{x:\overline{n}|}$ and $\cdots_{x+t:\overline{n-t}|}$).

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

6.4 Thiele's Differential Equation

Formula 6.4.1: Whole life assurance

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 6.4.2: Multiple state model

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

7 Stochastic Processes

7.1 Markov "Jump" Processes

Formula 7.1.1: Kolmogorov differential equations

Forward equation: $\frac{\partial}{\partial t} p_{ij}(s,t) = \sum_{k \in S} p_{ik}(s,t)\sigma_{kj}(t)$

Backward equation: $\frac{\partial}{\partial s} p_{ij}(s,t) = -\sum_{k \in S} \sigma_{ik}(s) p_{kj}(s,t)$

where $\sigma_{ij}(t)$ is the transition rate from state i to state j $(j \neq i)$ at time t, and $\sigma_{ii} = -\sum_{j \neq i} \sigma_{ij}$.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3005 Stochastic Processes

Formula 7.1.2: Expected time to reach a subsequent state k

$$m_i = \frac{1}{\lambda_i} + \sum_{j\neq i, j\neq k} \frac{\sigma_{ij}}{\lambda_i} m_j, \quad \text{where } \lambda_i = \sum_{j\neq i} \sigma_{ij}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3005 Stochastic Processes

7.2 Brownian Motion and Related Processes

Formula 7.2.1: Martingales for standard Brownian motion

If $\{B_t, t \geq 0\}$ is a standard Brownian motion, then the following processes are martingales:

$$B_t, \ B_t^2 - t \text{ and } \exp(\lambda B_t - \frac{1}{2}\lambda^2 t)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 7.2.2: Distribution of the maximum value

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 7.2.3: Hitting times

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 7.2.4: Ornstein-Uhlenbeck process

$$\mathrm{d}X_t = -\gamma X_t \,\mathrm{d}t + \sigma \,\mathrm{d}B_t, \quad \gamma > 0$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

7.3 Monte Carlo Methods

Formula 7.3.1: Box-Muller formulae

If U_1 and U_2 are independent random variables from the $U(0,1)$ distribution then

$$Z_1 = \sqrt{-2\log U_1}\cos(2\pi U_2) \text{ and } \sqrt{-2\log U_1}\sin(2\pi U_2)$$

are independent standard normal variables.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 7.3.2: Polar method

If V_1 and V_2 are independent random variables from the $U(-1,1)$ distribution and $S = V_1^2 + V_2^2$ then, conditional on $0 < S \leq 1$,

$$Z_1 = V_1\sqrt{\frac{-2\log S}{S}} \text{ and } Z_2 = V_2\sqrt{\frac{-2\log S}{S}}$$

are independent standard normal variables.

Pseudorandom values from the $U(0,1)$ distribution and the $N(0,1)$ distribution are included in the statistical tables section.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

8 Time Series

8.1 Time Series – Time Domain

Formula 8.1.1: Sample autocovariance and autocorrelation function

Autocovariance: $\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(x_t - \hat{\mu})(x_{t-k} - \hat{\mu})$, where $\hat{\mu} = \frac{1}{n}\sum_{t=1}^{n} x_t$

Autocorrelation: $\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 8.1.2: Autocorrelation function for $\mathrm{ARMA}(1,1)$

For the process $X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$:

$$\rho_k = \frac{(1+\alpha\beta)(\alpha+\beta)}{(1+\beta^2+2\alpha\beta)}\alpha^{k-1},\ k = 1, 2, 3, \ldots$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 8.1.3: Partial autocorrelation function

$$\phi_1 = \rho_1,\ \phi_2 = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}$$

$$\phi_k = \frac{\det P_k^*}{\det P_k},\ k = 2, 3, \ldots,$$

where $P_k = \begin{pmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{k-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{k-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{k-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{k-1} & \rho_{k-2} & \rho_{k-3} & \cdots & 1 \end{pmatrix}$

and P_k^* equals P_k, but with the last column replaced with $(\rho_1, \rho_2, \rho_3, \ldots, \rho_k)^\top$.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 8.1.4: Partial autocorrelation function for MA(1)

For the process $X_t = \mu + e_t + \beta e_{t-1}$:

$$\phi_k = (-1)^{k+1} \frac{(1-\beta^2)\beta^k}{1 - \beta^{2(k+1)}},\ k = 1, 2, 3, \ldots$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

8.2 Time Series – Time Frequency

Formula 8.2.1: Spectral density function

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 8.2.2: Inversion formula

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Formula 8.2.3: Linear filters

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

8.3 Time Series – Box-Jenkins Methodology

Formula 8.3.1: Ljung and Box "portmanteau" test of the residuals for an ARMA(p, q) model

$$n(n+2) \sum_{k=1}^{m} \frac{r_k^2}{n-k} \sim \chi^2_{m-(p+q)}$$

where r_k $(k = 1, 2, \ldots, m)$ is the estimated value of the k th autocorrelation coefficient of the residuals and n is the number of data values used in the ARMA(p, q) series.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

Formula 8.3.2: Turning point test

In a sequence of n independent random variables the number of turning points T is such that:

$$E(T) = \frac{2}{3}(n-2) \text{ and } \operatorname{var}(T) = \frac{16n-29}{90}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3001 Statistical Modelling

9 Economic Models

9.1 Utility Theory

Formula 9.1.1: Utility functions

Exponential: $U(w) = e^{-aw},\ a > 0$

Logarithmic: $U(w) = \log w$

Power: $U(w) = \gamma^{-1}(w^{\gamma} - 1),\ \gamma \leq 1,\ \gamma \neq 0$

Power: $U(w) = w + dw^2,\ d < 0$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 9.1.2: Measures of risk aversion

Absolute risk aversion: $A(w) = -\dfrac{U''(w)}{U'(w)}$

Relative risk aversion: $R(w) = wA(w)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

9.2 Capital Asset Pricing Model (CAPM)

Formula 9.2.1: Security market line

$$E_i - r = \beta_i(E_M - r) \text{ where } \beta_i = \frac{\operatorname{cov}(R_i, R_M)}{\operatorname{var}(R_M)}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

Formula 9.2.2: Capital market line (for efficient portfolios)

$$E_p - r = (E_M - r)\frac{\sigma_p}{\sigma_M}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1

9.3 Interest Rate Models

Formula 9.3.1: Spot rates and forward rates for zero-coupon bonds

Let $P(\tau)$ be the price at time 0 of a zero-coupon bond that pays 1 unit at time τ.

Let $s(\tau)$ be the spot rate for the period $(0, \tau)$.

Let $f(\tau)$ be the instantaneous forward rate at time 0 for time τ.

Spot rate

$$P(\tau) = e^{-\tau s(\tau)} \text{ or } s(\tau) = -\frac{1}{\tau}\log P(\tau)$$

Instantaneous forward rate

$$P(\tau) = \exp\left(-\int_0^\tau f(s)\,\mathrm{d}s\right) \text{ or } f(\tau) = -\frac{\mathrm{d}}{\mathrm{d}\tau}\log P(\tau)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

Formula 9.3.2: Vasicek model

Instantaneous forward rate

$$f(\tau) = e^{-\alpha\tau}R + (1 - e^{-\alpha\tau})L + \frac{\beta}{\alpha}e^{-\alpha\tau}(1 - e^{-\alpha\tau})$$

Price of a zero-coupon bond

$$P(\tau) = \exp\left[-D(\tau)R - (\tau - D(\tau))L - \frac{\beta}{2}D(\tau)^2\right]$$

where $D(\tau) = \dfrac{1 - e^{-\alpha\tau}}{\alpha}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

Extension 9.3.2.1: Alternative parameterisation of the Vasicek model

9.3.2 is not the way in which this model is presented in the CT8 notes (also the formula students are expected to used in the STAT3007 exam).

Price of a zero-coupon bond

$$B(t,T) = e^{a(\tau) - b(\tau)r(t)}$$

Instantaneous forward rate

$$f(\tau) = e^{-\alpha\tau}r(t) + (\mu - \frac{\sigma^2}{2\alpha^2})(1 - e^{-\alpha\tau}) + \frac{\sigma^2}{2\alpha^2}e^{-\alpha\tau}(1 - e^{-\alpha\tau})$$

where:

$$b(\tau) = \frac{1 - e^{-\alpha t}}{\alpha}$$

$$a(\tau) = (b(\tau) - \tau)\left[\mu - \frac{\sigma^2}{2\alpha^2}\right] - \frac{\sigma^2}{4\alpha}b(\tau)^2$$

We can get to this from 9.3.2 by knowing that:

$$\tau = T - t$$

$$R = r(t)$$

$$D(\tau) = b(\tau)$$

$$\beta = \frac{\sigma^2}{2\alpha}$$

$$L = \mu - \beta$$

Also, 9.3.2 implicitly assumes that $t = 0$.

10 Financial Derivatives

Formula 10.0.1

Note. In this section, q denotes the (continuously-payable) dividend rate.

10.1 Price of a Forward or Futures Contract

Formula 10.1.1

For an asset with fixed income of present value I:

$$F = (S_0 - I)e^{rT}$$

For an asset with dividends:

$$F = S_0 e^{(r-q)T}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH2004 Theory of Interest
- STAT3007 Investment Science 2

10.2 Binomial Pricing ("Tree" Model)

Formula 10.2.1: Risk-neutral probabilities

$$\text{Up-step probability } = \frac{e^{r\Delta t} - d}{u - d}$$

where $u \approx e^{\sigma\sqrt{\Delta t} + q\Delta t}$

and $\quad u \approx e^{-\sigma\sqrt{\Delta t} + q\Delta t}$

Also known as 'calibration' of the binomial model.
You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

10.3 Stochastic Differential Equations

Formula 10.3.1: Generalised Wiener process

$$\mathrm{d}x = a\,\mathrm{d}t + b\,\mathrm{d}z$$

where a and b are constant and $\mathrm{d}z$ is the increment for a Wiener process (standard Brownian motion).

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 10.3.2: Ito process

$$\mathrm{d}x = a(x,t)\,\mathrm{d}t + b(x,t)\,\mathrm{d}z$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 10.3.3: Ito's lemma for a function $G(x,t)$

$$\mathrm{d}G = \left(a\tfrac{\partial G}{\partial x} + \frac{1}{2}b^2\tfrac{\partial^2 G}{\partial x^2} + b\tfrac{\partial G}{\partial t}\right)\mathrm{d}t + b\frac{\partial G}{\partial x}\,\mathrm{d}z$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 10.3.4: Models for the short rate r_t

Ho-Lee: $\mathrm{d}r = \theta(t)\,\mathrm{d}t + \sigma\,\mathrm{d}z$

Hull-White: $\mathrm{d}r = [\theta(t) - ar]\,\mathrm{d}t + \sigma\,\mathrm{d}z$

Vasicek: $\mathrm{d}r = a(b - r)\,\mathrm{d}t + \sigma\,\mathrm{d}z$

Cox-Ingersoll-Ross: $\mathrm{d}r = a(b - r)\,\mathrm{d}t + \sigma\sqrt{r}\,\mathrm{d}z$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

10.4 Black-Scholes Formulae for European Options

Formula 10.4.1: Geometric Brownian motion model for a stock price S_t

$$\mathrm{d}S_t = S_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}z)$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Formula 10.4.2: Black-Scholes partial differential equation

$$\frac{\partial f}{\partial t} + (r - q)S_t\frac{\partial f}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 f}{\partial S_t^2} = rf$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

Formula 10.4.3: Garman-Kohlhagen formulae for the price of call and put options

Call: $c_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2)$

Put: $p_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t e^{-q(T-t)}\Phi(-d_1)$

where $d_1 = \dfrac{\log(S_t/K) + (r - q + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$

and $d_2 = \dfrac{\log(S_t/K) + (r - q - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

10.5 Put-Call Parity Relationship

Formula 10.5.1

$$c_t + Ke^{-r(T-t)} = p_t + S_t e^{-q(T-t)}$$

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

B Non-Statistical (Mostly Life) Tables

Compound Interest Table

(Mostly) Life Table 1: Compound Interest

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

This table is relevant to, but never used in MATH2004 Theory of Interest.

Population Mortality Table

(Mostly) Life Table 2: ELT15 (Males)

The following quantities are tabulated for ages (x) 0 to 109:

- ℓ_x
- d_x
- q_x
- μ_x
- $\mathring{e}_x$

You may find some use for this table in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

(Mostly) Life Table 3: ELT15 (Females)

The following quantities are tabulated for ages (x) 0 to 112:

- ℓ_x
- d_x
- q_x
- μ_x
- $\mathring{e}_x$

You may find some use for this table in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

Assured Lives Mortality Table

(Mostly) Life Table 4: AM92

The following quantities are tabulated for ages (x) 17 to 120:

- $\ell_{[x]}$, $\ell_{[x+1]-1}$, ℓ_x
- $d_{[x]}$, $d_{[x+1]-1}$, d_x
- $q_{[x]}$, $q_{[x+1]-1}$, q_x
- $\mu_{[x]}$, $\mu_{[x+1]-1}$, μ_x
- $e_{[x]}$, $e_{[x+1]-1}$, e_x

The following commutation factors are tabulated for ages (x) 17 to 110, using a rate of interest of 4%:

- $D_{[x]}$, $D_{[x+1]-1}$, D_x
- $N_{[x]}$, $N_{[x+1]-1}$, N_x
- $S_{[x]}$, $S_{[x+1]-1}$, S_x
- $C_{[x]}$, $C_{[x+1]-1}$, C_x
- $M_{[x]}$, $M_{[x+1]-1}$, M_x
- $R_{[x]}$, $R_{[x+1]-1}$, R_x

The following quantities are tabulated for ages (x) 17 to 120, using a rate of interest of 4% and 6%:

- $\ddot{a}_{[x]}$, $\ddot{a}_x$
- $A_{[x]}$, A_x
- ${}^2A_{[x]}$, ${}^2A_x$
- $(I\ddot{a})_{[x]}$, $(I\ddot{a})_x$
- $(IA)_{[x]}$, $(IA)_x$
- The following for $n = 60 - x$ and $n = 65 - x$:
  - $\ddot{a}_{[x]:\overline{n}|}$, $\ddot{a}_{x:\overline{n}|}$
  - $A_{[x]:\overline{n}|}$, $A_{x:\overline{n}|}$

You may find some use for this table in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

Pensioner Mortality Table

(Mostly) Life Table 5: PMA92Base and PFA92base

The following quantities are tabulated for ages (x) 50 to 105:

- q_x

You may find some use for this table in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

(Mostly) Life Table 6: PMA92C20 and PFA92C20

The following quantities are tabulated for ages (x) 50 to 105:

- ℓ_x
- d_x
- q_x
- μ_x
- $\mathring{e}_x$

The following quantities are tabulated for ages (x) 50 to 105 using a rate of interest of 4%:

- $\ddot{a}_x$
- ${}^2A_x$
- For male (x), female (y), let $d = y - x$.
  For integer d between -5 and 5 (inclusive) as well as ± 10 and ± 20:
  $\ddot{a}_{xy}$

You may find some use for this table in the exam for the following actuarial unit(s):

- MATH3006 Life Contingencies 1
- MATH3007 Life Contingencies 2

International Actuarial Notation

This component of the orange book is for some reason never provided to students in actuarial examinations at Curtin. Perhaps students should request it given it is in the official orange book (and could come in handy during exams for example it contains the meaning of standard commutation functions). The knowledge in this section is taught and used throughout MATH2004 Theory of Interest, MATH3006 Life Contingencies 1, MATH3007 Life Contingencies 2.

Sickness Table

(Mostly) Life Table 7: S(ID)

This part of the formulae and tables is <u>not</u> used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Example Pension Scheme Table

(Mostly) Life Table 8: PEN

The following **service table and relative salary scale** quantities are tabulated for ages (x) 16 to 65:

- ℓ_x
- w_x
- d_x
- i_x
- r_x
- s_x^*
- $s_x = (1.02)^x s_x^*$
- $z_x = \frac{1}{3}(s_{x-3} + s_{x-2} + s_{x-1})$
- $z_{x+\frac{1}{2}} = \frac{1}{2}(z_x + z_{x+1})$

The following **contribution functions** quantities are tabulated for ages (x) 16 to 65 at an interest rate of 4%:

- $D_x = v^x \ell_x$
- $\overline{D}_x = \frac{1}{2}(D_x + D_{x+1})$
- $\overline{N}_x = \sum \overline{D}_x$
- ${}^s\overline{D}_x = s_x \overline{D}_x$
- ${}^s\overline{N}_x = \sum {}^s\overline{D}_x$
- ${}^sD_x = s_x D_x$

The following **ill health retirement functions** quantities are tabulated for ages (x) 16 to 65 at an interest rate of 4%:

- $\bar{a}^i_{x+\frac{1}{2}}$
- $C_x^i = v^{x+\frac{1}{2}} i_x$
- $M_x^i = \sum C_x^i$
- $\overline{R}_x^i = \sum (M_x^i - \frac{1}{2} C_x^i)$
- $C_x^{ia} = C_x^i \bar{a}^i_{x+\frac{1}{2}}$
- $M_x^{ia} = \sum C_x^{ia}$
- $\overline{R}_x^{ia} = \sum (M_x^{ia} - \frac{1}{2} C_x^{ia})$
- ${}^s\overline{M}_x^{ia} = s_x (M_x^{ia} - \frac{1}{2} C_x^{ia})$
- ${}^s\overline{R}_x^{ia} = \sum {}^s\overline{M}_x^{ia}$
- ${}^zC_x^{ia} = z_{x+\frac{1}{2}} C_x^{ia}$
- ${}^zM_x^{ia} = \sum {}^zC_x^{ia}$
- ${}^z\overline{R}_x^{ia} = \sum ({}^zM_x^{ia} - \frac{1}{2} {}^zC_x^{ia})$

(Mostly) Life Table 9: PEN *cont.*

The following **age retirement functions** quantities are tabulated for ages (x) 16 to 65 at an interest rate of 4%:

- $\bar{a}^r_{x+\frac{1}{2}}$ (a^r_{65} at 65)
- $C^r_x = v^{x+\frac{1}{2}} r_x$ ($v^{65} r_{65}$ at 65)
- $M^r_x = \sum C^r_x$
- $\overline{R}^r_x = \sum (M^r_x - \frac{1}{2} C^r_x)$
- $C^{ra}_x = C^r_x \bar{a}^r_{x+\frac{1}{2}}$ ($v^{65} r^{65} \bar{a}^r_{65}$ at 65)
- $M^{ra}_x = \sum C^{ra}_x$
- $\overline{R}^{ra}_x = \sum (M^{ra}_x - \frac{1}{2} C^{ra}_x)$
- ${}^s\overline{M}^{ra}_x = s_x (M^{ra}_x - \frac{1}{2} C^{ra}_x)$
- ${}^s\overline{R}^{ra}_x = \sum {}^s\overline{M}^{ra}_x$
- ${}^zC^{ra}_x = z_{x+\frac{1}{2}} C^{ra}_x$ ($z_{65} C^{ra}_{65}$ at 65)
- ${}^zM^{ra}_x = \sum {}^zC^{ra}_x$
- ${}^z\overline{R}^{ra}_x = \sum ({}^zM^{ra}_x - \frac{1}{2} {}^zC^{ra}_x)$

The following **Functions for return of contributions, accumulated with interest at 2% p.a., on death** quantities are tabulated for ages (x) 16 to 65 at an interest rate of 4%:

- ${}^jC^d_x = v^{x+\frac{1}{2}} (1+j)^{x+\frac{1}{2}} d_x$
- ${}^jM^d_x = \sum {}^jC^d_x$
- ${}^j\overline{R}^d_x = \sum \left(\frac{{}^jM^d_x - \frac{1}{2} {}^jC^d_x}{(1+j)^{x+\frac{1}{2}}} \right)$
- ${}^{sj}\overline{R}^d_x = \sum s_x \left(\frac{{}^jM^d_x - \frac{1}{2} {}^jC^d_x}{(1+j)^{x+\frac{1}{2}}} \right)$
- ${}^jC^w_x = v^{x+\frac{1}{2}} (1+j)^{x+\frac{1}{2}} w_x$
- ${}^jM^w_x = \sum {}^jC^w_x$
- ${}^j\overline{R}^w_x = \sum \left(\frac{{}^jM^w_x - \frac{1}{2} {}^jC^w_x}{(1+j)^{x+\frac{1}{2}}} \right)$
- ${}^{sj}\overline{R}^w_x = \sum s_x \left(\frac{{}^jM^w_x - \frac{1}{2} {}^jC^w_x}{(1+j)^{x+\frac{1}{2}}} \right)$

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3007 Life Contingencies 2

Sample Time Series

(Mostly) Life Table 10

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

This table is relevant to, but never used in STAT3001 Statistical Modelling.

C Statistical tables

Note that this part of the orange book is not provided to Curtin students in examinations. In this section we will indicate which statistical tables are provided to students in another form in various units.

Standard Normal probabilities

Statistical Table 1

Instead of this table, curtin students are provided with "Cumulative probabilities for the standard normal distribution" from 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams which includes: Probabilities for values of Z between -3.49 and 3.39 at increments of 0.01.

You may find some use for this formula in the exam for the following actuarial unit(s):

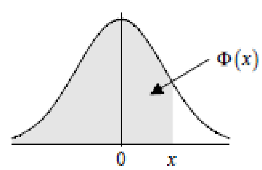
- MATH2004 Theory of Interest MATH3005 Survival Analysis
- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory
- STAT3005 Stochastic Processes
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

Statistical Table Extension 1.1: Notation that appears in the orange book version

The following appears above the standard normal probabilities table in the orange book:

The distribution function is denoted by $\Phi(x)$, and the probability density function is denoted by $\phi(x)$.

$$\Phi(x) = \int_{-\infty}^{x} \phi(t)\,dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$$



This is not given to students in exams despite the fact that formulae 2.4.1 and 2.4.2 use this notation. This is indicated in extension 2.4.0.1.

Standard Normal percentage points

Statistical Table 2

Instead of this table, curtin students are provided with "Cumulative probabilities for the standard normal distribution" from 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams which can be used to obtain the percentage points (critical values) in reverse. The method of doing this is first taught in STAT1002 Statistical Data Analysis.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH2004 Theory of Interest MATH3005 Survival Analysis
- STAT2001 Mathematical Statistics
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

- STAT3005 Stochastic Processes
- STAT3006 Investment Science 1
- STAT3007 Investment Science 2

t percentage points

Statistical Table 3

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

χ^2 probabilities

Statistical Table 4

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Since this distribution is only really used as a distribution for a test statistic in certain hypothesis tests, it's probabilities are redundant information if we have access to critical values. Indeed 'Mathematical Formulae and Statistical Tables for Tertiary Institutions', does not contain a probabilities table for the chi-squared distribution.

χ^2 percentage points

Statistical Table 5

Instead of this table, curtin students are provided with "Critical points of the chi-squared distribution" from 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams which includes:
Critical values for degrees of freedom between 1 and 100 and for right tail probabilities of 0.995, 0.99, 0.95, 0.9, 0.1, 0.05, 0.025, 0.01, 0.005.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis
- STAT3001 Statistical Modelling
- STAT3002 Risk Analysis and Credibility Theory

F percentage points

Statistical Table 6

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

Piosson Probabilities

Statistical Table 7

Instead of this table, curtin students are provided with "Cumulative probabilities for the Poisson distribution" from 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams which includes:
Probabilities for parameter values between 0.1 and 3 at increments of 0.1, between 3.5 and 10 at increments of 0.5 and x values over a range for which significant changes occur.
In addition, "Individual probabilities for the Poisson distribution" has the same values available.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3002 Risk Analysis and Credibility Theory

Binomial Probabilities

Statistical Table 8

Instead of this table, curtin students are provided with "Cumulative probabilities for the binomial distribution" from 'Mathematical Formulae and Statistical Tables for Tertiary Institutions' in exams which includes: Probabilities for n values between 2 and 15, as well as n values of 20, 25 and 30, p values between 0.1 and 0.9 with increments of 0.1 and x values between 0 and n.
In addition, "Individual probabilities for the binomial distribution" has the same values available.

You may find some use for this formula in the exam for the following actuarial unit(s):

- STAT3007 Investment Science 2

Critical values for the grouping of signs test

Statistical Table 9

This table contains critical values of the grouping of signs test for n_1 and n_2 between 1 and 25.

This is the only table which is actually provided to students directly from the orange book.

You may find some use for this formula in the exam for the following actuarial unit(s):

- MATH3005 Survival Analysis

Pseudorandom values from $U(0, 1)$ and from $N(0, 1)$

Statistical Table 10

This part of the formulae and tables is not used in any Curtin units in which actuarial formulae and tables are provided in the exam and has thus been **omitted**.

D Reference by particular units

ACCT1000 Accounting - The Language of Business

This is a CT exemption unit for which no actuarial formulae are provided.

ECON1000 Introductory Economics

This is a CT exemption unit for which no actuarial formulae are provided.

ECON1001 Actuarial Economics

This is a CT exemption unit for which no actuarial formulae are provided.

ECON2001 Macroeconomics Principles

This is a CT exemption unit for which no actuarial formulae are provided.

FNCE2000 Introduction to Finance Principles

This is a CT exemption unit for which no actuarial formulae are provided.

MATH1016 Calculus 1*

Although this unit does not provide the actuarial tables and formulae in the exam, it does introduce/ derive formulae from the following sections:

- 1.1 Series
- 1.2 Calculus
- 1.3 Solving Equations

MATH1017 Accelerated Mathematics 1*

Although this unit does not provide the actuarial tables and formulae in the exam, it does introduce/ derive formulae from the following sections:

- 1.1 Series
- 1.2 Calculus
- 1.3 Solving Equations

MATH1018 Accelerated Mathematics 2*

Although this unit does not provide the actuarial tables and formulae in the exam, it does introduce/ derive formulae from the following sections:

- 1.1 Series
- 1.3 Solving Equations

MATH2004 Theory of Interest

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
4 Compound Interest	Interest rates, Annuities

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points

MATH2009 Calculus 2*

Although this unit does not provide the actuarial tables and formulae in the exam, it does introduce/ derive formulae from the following sections:

- 1.2 Calculus

MATH3005 Survival Analysis

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
2.1 Discrete Distributions	The Binomial and Poisson models
2.2 Continuous distributions	Survival models, Graduation
3.3 Maximum Likelihood Estimators	Estimating the lifetime distribution function, Proportional hazards models
5.1 Mortality "Laws"	Survival models
5.2 Empirical Estimation	Estimating the lifetime distribution function
5.3 Mortality Assumptions	Survival models, The Binomial and Poisson models
5.5 Graduation Tests	Graduation

The following life tables are useful in exam for this unit:

- Population Mortality Table

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points
- χ^2 percentage points
- Critical values for the grouping of signs test

MATH3006 Life Contingencies 1

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.2 Calculus	Life assurance contracts, Life annuity contracts
5.1 Mortality "Laws"	ALL
6.1 Approximations for Non Annual Annuities	Evaluation of assurances and annuities
6.2 Moments of Annuities and Assurances	ALL
6.3 Premiums and Reserves	Net and gross premiums and reserves

The following life tables are useful in exam for this unit:

- Population Mortality Table
- Assured Lives Mortality Table

MATH3007 Life Contingencies 2

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.2 Calculus	Payments involving two lives
5.1 Mortality "Laws"	Payments involving two lives, Contingent and reversionary benefits
5.6 Multiple Decrement Tables	Competing risks
6.1 Approximations for Non Annual Annuities	Payments involving two lives, Contingent and reversionary benefits
6.2 Moments of Annuities and Assurances	ALL

The following life tables are useful in exam for this unit:

- Population Mortality Table
- Assured Lives Mortality Table
- Pensioner Mortality Table
- Example Pension Scheme Table

STAT1000 Regression and Non-Parametric Inference

This is a CT exemption unit for which no actuarial formulae are provided.

STAT1001 Statistical Probability

This is a CT exemption unit for which no actuarial formulae are provided.

Although this unit does not provide the actuarial tables and formulae in the exam, it does introduce/ derive formulae from the following sections:

- 1.2 Calculus
- 2.1 Discrete Distributions
- 2.2 Continuous distributions

STAT1002 Statistical Data Analysis

This is a CT exemption unit for which no actuarial formulae are provided.

STAT2001 Mathematical Statistics

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.4 Gamma Function	Probability distributions
2.1 Discrete Distributions	Probability distributions, Expectation, Generating functions
2.2 Continuous distributions	Probability distributions, Expectation, Generating functions
2.3 Compound Distributions	Conditional probability, Functions of random variables
3.1 Sample Mean and Variance	Estimation
3.3 Maximum Likelihood Estimators	Estimation

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points

STAT3001 Statistical Modelling

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.1 Series	Time Series
1.3 Solving Equations	Time Series
1.4 Gamma Function	Bayesian statistics
1.5 Bayes' Formula	Bayesian statistics
2.1 Discrete Distributions	Bayesian statistics, GLMs
2.2 Continuous distributions	Bayesian statistics, GLMs
3.3 Maximum Likelihood Estimators	Bayesian statistics, GLMs
3.6 Generalised Linear Models	GLMs
3.7 Bayesian Methods	Bayesian statistics
7.3 Monte Carlo Methods	Monte Carlo simulation

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points
- χ^2 percentage points

STAT3002 Risk Analysis and Credibility Theory

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.1 Series	Ruin theory
1.2 Calculus	Ruin theory, Reinsurance
1.4 Gamma Function	Loss distributions, Reinsurance, Credibility theory
2.1 Discrete Distributions	Credibility theory, Risk Models, Ruin theory
2.2 Continuous distributions	Loss distributions, Reinsurance, Credibility theory, Risk models, Ruin theory
2.3 Compound Distributions	Risk models, Ruin theory
2.4 Truncated Moments	Loss distributions, Reinsurance
3.3 Maximum Likelihood Estimators	Loss distributions, Credibility theory
3.7 Bayesian Methods	Credibility theory
3.8 Empirical Bayes Credibility – Model 1	EBCT
3.9 Empirical Bayes Credibility – Model 2	EBCT

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points
- χ^2 percentage points
- Piosson Probabilities

STAT3005 Stochastic Processes

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
2.1 Discrete Distributions	Stochastic processes
2.2 Continuous distributions	Stochastic processes
5.4 General Markov Model	Markov Jump Processes
7.1 Markov "Jump" Processes	Markov Jump Processes

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points

STAT3006 Investment Science 1

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.2 Calculus	Stochastic calculus
1.3 Solving Equations	Stochastic calculus
2.2 Continuous distributions	Stochastic models of security prices
2.3 Compound Distributions	Brownian motion and martingales
3.4 Linear Regression Model With Normal Errors	Asset pricing models
7.2 Brownian Motion and Related Processes	Brownian motion and martingales, Stochastic calculus
9.1 Utility Theory	Utility theory
9.2 Capital Asset Pricing Model (CAPM)	Asset pricing models
10.3 Stochastic Differential Equations	Stochastic calculus
10.4 Black-Scholes Formulae for European Options	Stochastic models of security prices

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points

STAT3007 Investment Science 2

Formula from the following sections are useful in the exam of this unit.

Section	Topic(s)
1.3 Solving Equations	The term structure of interest rates
2.1 Discrete Distributions	The Binomial model
2.2 Continuous distributions	Black-Scholes, 5-step Method
2.3 Compound Distributions	5-step Method
2.4 Truncated Moments	5-step Method
7.2 Brownian Motion and Related Processes	The term structure of interest rates, 5-step Method
9.3 Interest Rate Models	The term structure of interest rates
10.1 Price of a Forward or Futures Contract	Derivative securities
10.2 Binomial Pricing ("Tree" Model)	The binomial model
10.3 Stochastic Differential Equations	Black-Scholes, 5-step Method, Term structure of interest rates
10.4 Black-Scholes Formulae for European Options	Black-Scholes, Credit risk
10.5 Put-Call Parity Relationship	ALL

The following statistical tables are useful in exam for this unit:

- Standard Normal probabilities
- Standard Normal percentage points
- Binomial Probabilities