

# Penn Wharton Budget Model: Dynamics

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# Dynamic Model Overview

- Dynamic general equilibrium OLG model with heterogeneity
- Idiosyncratic productivity risk  $\Rightarrow$  distribution of earnings histories
- Detailed Social Security system, progressive taxes, immigration
- Evaluates unbalanced fiscal reform over long time horizons
- Considers open and closed economy frameworks

# Household Labor Productivity( $z$ )

- Deterministic dependence on age  $j$
- Permanent shock drawn at birth
- Transitory and persistent (AR1) shocks
- Initial distribution of non-permanent shocks

# Household Social Security and Bequests

- Social Security benefit  $ss(b)$  depends on average lifetime labor earnings  $b$ , issuance during retirement ages:  $j > Tr$ .
- Accidental bequests are collected by the government and redistributed evenly in lump-sum ( $beq$ ) among all living households

- Let  $q = (1 - e\tau_{cap})$ , where  $e$  is the rate of investment expensing.
- Following Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), a no-arbitrage condition on old capital relative to new capital implies the net return to capital is  $\frac{r_K - 1}{q}$ , where  $r_K$  will be the gross rental rate of capital.

## Personal Income Tax

$$y_{pit} = wzn1_{\{j \leq Tr\}} + (1 - \phi_{ss})ss(b)1_{\{j > Tr\}} + \left( \phi_K \frac{r_K - 1}{q} (1 - \theta) + (1 - \phi_K)(r_G - 1) \right) a \quad (1)$$

where  $wzn$  is labor income,  $(1 - \phi_{ss})$  is the taxable share of Social Security benefits,  $\phi_K$  is the portfolio weight on physical capital,  $r_G$  is the return on government debt,  $\theta$  is the share of capital income subjected to the capital tax rate, and  $a$  is asset holdings.

$$\tau_{pit}(y_{pit}) = \int_0^{y_{pit}} \xi(y) dy \quad (2)$$

Tax function  $\tau_{pit}(y_{pit})$  is cumulative tax liability from marginal tax rate function  $\xi(y)$ . The function  $\xi(y)$  is a step-function (from TPC) which accounts for deductions and credits.

## Capital Tax

$$y_{cap} = \phi_K \frac{r_K - 1}{q} \theta a \quad (3)$$

$$\tau_{cap}(y_{cap}) = \tau_{cap} y_{cap} \quad (4)$$

## Payroll Tax

$$y_{ss} = \min\{wzn, y_{\text{taxmax}}\} \quad (5)$$

$$\tau_{ss}(y_{ss}) = \tau_{ss} y_{ss} \quad (6)$$

Working-age household Bellman's equation:

$$V_j(a, z, b) = \max_{a', n} \left\{ \frac{(c^\gamma (1-n)^{1-\gamma})^{1-\sigma}}{1-\sigma} + s_{j+1} \beta E_{\{z'|z\}} [V_{j+1}(a', z', b')] \right\} \quad (7)$$

subject to:

$$c = wzn + \left[ \phi_K \left( 1 + \frac{r_K - 1}{q} + \frac{q' - q}{q} \right) + (1 - \phi_K) r_G \right] a - \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - \tau_{ss}(y_{ss}) - a' + beq \quad (8)$$

$$b_{j+1} = \frac{1}{j} \left( (j-1)b_j + \min\{wzn, y_{\text{taxmax}}\} \right), \quad (9)$$

where  $s_{j+1}$  is survival probability, (8) is the budget constraint, and (9) determines average earnings for SS benefit calculation.

Households take  $\phi_K$  as exogenous.



Retired household Bellman's equation:

$$V_j(a, b) = \max_{a'} \left\{ \frac{(c^\gamma(1)^{1-\gamma})^{1-\sigma}}{1-\sigma} + s_{j+1}\beta V_{j+1}(a', b') \right\} \quad (10)$$

subject to:

$$c = ss(b) + \left[ \phi_K \left( 1 + \frac{r_K - 1}{q} + \frac{q' - q}{q} \right) + (1 - \phi_K)r_G \right] a - \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - a' + beq \quad (11)$$

$$b_{j+1} = b_j. \quad (12)$$

# Production: Closed Economy

- Capital  $K$  equals aggregate savings  $A$  less government debt  $D$  in capital terms:  $K = \frac{A-D}{q}$ .

- Output:

$$Y = K^\alpha L^{1-\alpha}, \quad (13)$$

where  $L$  is aggregate efficient labor.

- Firms' problem:

$$\max_{K,L} \{ K^\alpha L^{1-\alpha} + (1-\delta)K - r_K K - wL \}, \quad (14)$$

where  $\delta$  is depreciation and  $r_K$  is the rental rate of capital faced by firms.

- Firms' gross interest rates and wages are determined according to:

$$r_K = 1 + \alpha K^{\alpha-1} L^{1-\alpha} - \delta \quad (15)$$

$$w = (1-\alpha) K^\alpha L^{-\alpha} \quad (16)$$

# Production: Open Economy

- International capital flows in and out of the economy, but labor is immobile.
- Let constant:

$$\lambda \equiv \left( \frac{r_K^{ss} - 1}{q^{ss}} \right) (1 - \tau_{cap}^{ss}), \quad (17)$$

denote the after-tax net capital return in the initial steady-state closed economy. We set  $\lambda$  as the international after-tax net capital return for the small open-economy.

- After change in policy, territorial capital tax  $\Rightarrow$  capital flows until  $r_K$  clears international capital markets by matching the international after-tax net return on capital:

$$\left( \frac{r_K - 1}{q} \right) (1 - \tau_{cap}) = \lambda, \quad (18)$$

where  $q$  and  $\tau_{cap}$  represent the new transition path values.

# Open Economy (cont.)

- Rearranging terms in (18) gives:

$$\frac{r_K - 1}{q} = \frac{\lambda}{(1 - \tau_{cap})} \quad (19)$$

- Equation (15) implies a unique capital-to-labor ratio  $\kappa = \frac{K}{L}$  given in the firm's FOC:

$$r_K = 1 + \alpha\kappa^{\alpha-1} - \delta, \quad (20)$$

or equivalently:

$$\frac{r_K - 1}{q} = \frac{\alpha\kappa^{\alpha-1} - \delta}{q} \quad (21)$$

- (19) and (21) give:

$$\kappa = \left[ \frac{1}{\alpha} \left( \frac{q \times \lambda}{1 - \tau_{cap}} + \delta \right) \right]^{\frac{1}{\alpha-1}} \quad (22)$$

- $\kappa$  in (22) then gives wages:

$$w = (1 - \alpha)\kappa^\alpha \quad (23)$$

- Agents optimize over the transition path given (20) and (23), which results in an aggregate efficient labor sequence,  $\mathbf{L}$ .
- Then, the corresponding capital sequence is given by the product:

$$\mathbf{K} = \kappa\mathbf{L} \quad (24)$$

- Since the closed economy steady-state provides initial conditions for the open economy, we assume that initial foreign capital is zero.
  
- Define  $\mathbf{K}_{domestic}$  to be the aggregate capital holdings of households in the economy. Then, foreign capital is

$$\mathbf{K}_{foreign} = \mathbf{K} - \mathbf{K}_{domestic} \quad (25)$$

- Because of territorial capital tax, must account for taxation of foreign capital.
- By assumption, capital tax share  $\theta$  also applies to foreign capital. In other words,  $1 - \theta$  of foreign capital income is not taxed in the US.
- Capital tax revenue from foreign capital is then:  $\theta \frac{r_K - 1}{q} K_f$ .

- Weighted average of return on capital and government interest rate:

$$\phi_K = \frac{K}{K + D} \quad (26)$$

- Open economy: portfolio allocation fixed at initial steady state shares of capital and debt.
- Closed economy: portfolio allocation determined in general equilibrium throughout transition path  $\Rightarrow$  all debt is held by households.



# Government Debt

- Sequence of government interest rates  $r_g$  is exogenous.
- Government debt evolves according to:

$$D' + R = r_g D + G, \quad (27)$$

where  $R$  is government revenue and  $G$  is government expenditures.

- $R$  and  $G$  have explicit model components. For revenue, personal income taxes (PIT), capital taxes (CT) and payroll taxes (SSREV), and for expenditures, Social Security expenditures (SSEXP).
- We can expand (27) as follows:

$$D' + FIT + SSREV + CT + \tilde{T} = r_g D + SSEXP + \tilde{G} \quad (28)$$

where  $\tilde{G}$  is the non-interest government budget surplus not accounted by the explicit model revenue and expenditure components and  $\tilde{T}$  is non-explicit tax revenue introduced only in counter-factual experiments (e.g., estate taxes, tariffs).

# Simulating Debt Over the Transition Path

- Process of matching CBO debt projection:
  - ① Choose  $\tilde{G}$  to match CBO non-interest surplus in each year in the open economy.
  - ② Use CBO government interest rates to generate debt sequence (generates exactly the CBO debt projection).
  - ③ Use resulting  $\tilde{G}$  from open economy (no macroeconomic feedback from debt) to construct government budget in closed economy.
- Key intuition: CBO debt projections correspond to our open economy (no feedback effects of debt).
- Baseline closed economy accounts for macroeconomic feedback from this debt sequence.

# Reported Tax Revenue Aggregates

- Personal Income Tax is assessed on wage income and a portion of capital income:

$$PIT_{BASE} = wL + (1 - \theta) \frac{r_K - 1}{q} K_{domestic}, \quad (29)$$

where  $L$  is aggregate efficient labor.

- Capital Income is  $\frac{r_K - 1}{q} K$ 
  - Portion taxed at PIT is  $(1 - \theta) \frac{r_K - 1}{q} K$ .
  - Portion taxed at the single capital tax rate is  $\theta \frac{r_K - 1}{q} K$ .
  - Capital tax is from corporate tax, tax on dividends, tax on capital gains, etc.
    - Complex tax structure
    - Simplified into the PIT and single rate structure as above.
    - PIT  $\Rightarrow$  residential, single rate  $\tau_{cap} \Rightarrow$  territorial

# Reported Tax Revenue Aggregates (cont.)

- Since PIT is non-linear, it is not clear how to assign tax dollars from capital income portion:

$$PIT_{REVENUE} = \tau_{pit}(PIT_{BASE}) \quad (30)$$

- We say taxes assigned by percent of income

$$PIT_{CAP\_REV} = \frac{(1 - \theta) \frac{r_K - 1}{q} K_{domestic}}{PIT_{BASE}} PIT_{REVENUE} \quad (31)$$

- So CT is

$$CT_{REVENUE} = \tau_{cap} \theta \frac{r_K - 1}{q} K + PIT_{CAP\_REV} \quad (32)$$