Dynamic Model Overview

- Dynamic general equilibrium OLG model with heterogeneity
- Idiosyncratic productivity risk \( \Rightarrow \) distribution of earnings histories
- Detailed Social Security system, progressive taxes, immigration
- Evaluates unbalanced fiscal reform over long time horizons
- Considers open and closed economy frameworks
Household Labor Productivity ($z$)

- Deterministic dependence on age $j$
- Permanent shock drawn at birth
- Transitory and persistent (AR1) shocks
- Initial distribution of non-permanent shocks
Social Security benefit \( ss(b) \) depends on average lifetime labor earnings \( b \), issuance during retirement ages: \( j > Tr \).

Accidental bequests are collected by the government and redistributed evenly in lump-sum (\( beq \)) among all living households.
Let $q = (1 - e^{\tau_{cap}})$, where $e$ is the rate of investment expensing.

Following Altig, Auerbach, Kotlikoff, Smetters, and Walliser (2001), a no-arbitrage condition on old capital relative to new capital implies the net return to capital is $\frac{r_K-1}{q}$, where $r_K$ will be the gross rental rate of capital.
Household Taxation

Personal Income Tax

\[ y_{pit} = wzn \mathbb{1}_{\{j \leq Tr\}} + (1 - \phi_{ss}) ss(b) \mathbb{1}_{\{j > Tr\}} + \left( \phi_K \frac{r_K - 1}{q} (1 - \theta) + (1 - \phi_K)(r_G - 1) \right) a \]  
(1)

where \( wzn \) is labor income, \( (1 - \phi_{ss}) \) is the taxable share of Social Security benefits, \( \phi_K \) is the portfolio weight on physical capital, \( r_G \) is the return on government debt, \( \theta \) is the share of capital income subjected to the capital tax rate, and \( a \) is asset holdings.

\[ \tau_{pit}(y_{pit}) = \int_0^{y_{pit}} \xi(y) \, dy \]  
(2)

Tax function \( \tau_{pit}(y_{pit}) \) is cumulative tax liability from marginal tax rate function \( \xi(y) \). The function \( \xi(y) \) is a step-function (from TPC) which accounts for deductions and credits.
**Capital Tax**

\[ y_{\text{cap}} = \phi K \frac{r_k - 1}{q} \theta a \]  

(3)

\[ \tau_{\text{cap}}(y_{\text{cap}}) = \tau_{\text{cap}} y_{\text{cap}} \]  

(4)

**Payroll Tax**

\[ y_{\text{ss}} = \min\{wzn, y_{\text{taxmax}}\} \]  

(5)

\[ \tau_{\text{ss}}(y_{\text{ss}}) = \tau_{\text{ss}} y_{\text{ss}} \]  

(6)
Household Optimization: Working age

Working-age household Bellman’s equation:

\[
V_j(a, z, b) = \max_{a', n} \left\{ \left( \frac{c^\gamma(1 - n)^{1-\gamma}}{1 - \sigma} \right) + s_{j+1} \beta E_{\{z' | z\}} [V_{j+1}(a', z', b')] \right\}
\]

subject to:

\[
c = wzn + \left[ \phi_K \left( 1 + \frac{r_K - 1}{q} + \frac{q' - q}{q} \right) + (1 - \phi_K) r_G \right] a \\
- \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - \tau_{ss}(y_{ss}) - a' + beq
\]

(8)

\[
b_{j+1} = \frac{1}{j} \left( (j - 1)b_j + \min\{wzn, y_{\text{taxmax}}\} \right),
\]

(9)

where \(s_{j+1}\) is survival probability, (8) is the budget constraint, and (9) determines average earnings for SS benefit calculation. Households take \(\phi_K\) as exogenous.
Retired household Bellman’s equation:

$$V_j(a, b) = \max_{a'} \left\{ \left( \frac{c^\gamma(1)^{1-\gamma}1-\sigma}{1-\sigma} + s_{j+1}\beta V_{j+1}(a', b') \right) \right\} \quad (10)$$

subject to:

$$c = ss(b) + \left[ \phi_K \left( 1 + \frac{r_K - 1}{q} + \frac{q' - q}{q} \right) + (1 - \phi_k)r_G \right] a \quad (11)$$

$$- \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - a' + beq$$

$$b_{j+1} = b_j. \quad (12)$$
Production: Closed Economy

- Capital $K$ equals aggregate savings $A$ less government debt $D$ in capital terms: $K = \frac{A - D}{q}$.

- Output:
  
  $$ Y = K^\alpha L^{1-\alpha}, \quad (13) $$

  where $L$ is aggregate efficient labor.

- Firms’ problem:

  $$ \max_{K,L} \left\{ K^\alpha L^{1-\alpha} + (1 - \delta)K - r_K K - wL \right\}, \quad (14) $$

  where $\delta$ is depreciation and $r_K$ is the rental rate of capital faced by firms.

- Firms’ gross interest rates and wages are determined according to:

  $$ r_K = 1 + \alpha K^{\alpha-1} L^{1-\alpha} - \delta \quad (15) $$

  $$ w = (1 - \alpha)K^\alpha L^{-\alpha} \quad (16) $$
International capital flows in and out of the economy, but labor is immobile.

Let constant:

\[ \lambda \equiv \left( \frac{r_{K}^{ss} - 1}{q^{ss}} \right) (1 - \tau_{cap}^{ss}), \]  

(17)

denote the after-tax net capital return in the initial steady-state closed economy. We set \( \lambda \) as the international after-tax net capital return for the small open-economy.

After change in policy, territorial capital tax \( \Rightarrow \) capital flows until \( r_{K} \) clears international capital markets by matching the international after-tax net return on capital:

\[ \left( \frac{r_{K} - 1}{q} \right) (1 - \tau_{cap}) = \lambda, \]  

(18)

where \( q \) and \( \tau_{cap} \) represent the new transition path values.
Rearranging terms in (18) gives:

$$\frac{r_K - 1}{q} = \frac{\lambda}{(1 - \tau_{cap})} \quad (19)$$

Equation (15) implies a unique capital-to-labor ratio $\kappa = \frac{K}{L}$ given in the firm’s FOC:

$$r_K = 1 + \alpha \kappa^{\alpha - 1} - \delta, \quad (20)$$

or equivalently:

$$\frac{r_K - 1}{q} = \frac{\alpha \kappa^{\alpha - 1} - \delta}{q} \quad (21)$$

(19) and (21) give:

$$\kappa = \left[ \frac{1}{\alpha} \left( \frac{q \times \lambda}{1 - \tau_{cap}} + \delta \right) \right]^{\frac{1}{\alpha - 1}} \quad (22)$$
κ in (22) then gives wages:

$$w = (1 - \alpha)\kappa^\alpha$$  \hspace{1cm} (23)

Agents optimize over the transition path given (20) and (23), which results in an aggregate efficient labor sequence, $L$.

Then, the corresponding capital sequence is given by the product:

$$K = \kappa L$$  \hspace{1cm} (24)
Since the closed economy steady-state provides initial conditions for the open economy, we assume that initial foreign capital is zero.

Define $K_{domestic}$ to be the aggregate capital holdings of households in the economy. Then, foreign capital is

$$K_{foreign} = K - K_{domestic}$$  \hspace{1cm} (25)
Because of territorial capital tax, must account for taxation of foreign capital.

By assumption, capital tax share $\theta$ also applies to foreign capital. In other words, $1 - \theta$ of foreign capital income is not taxed in the US.

Capital tax revenue from foreign capital is then: $\theta \frac{r_K - 1}{q} K_f$. 
Household Portfolio

- Weighted average of return on capital and government interest rate:

\[
\phi_K = \frac{K}{K + D}
\]  

(26)

- Open economy: portfolio allocation fixed at initial steady state shares of capital and debt.

- Closed economy: portfolio allocation determined in general equilibrium throughout transition path ⇒ all debt is held by households.
Sequence of government interest rates $r_g$ is exogenous.

Government debt evolves according to:

$$D' + R = r_g D + G, \quad (27)$$

where $R$ is government revenue and $G$ is government expenditures.

$R$ and $G$ have explicit model components. For revenue, personal income taxes (PIT), capital taxes (CT) and payroll taxes (SSREV), and for expenditures, Social Security expenditures (SSEXP).

We can expand (27) as follows:

$$D' + \text{FIT} + \text{SSREV} + CT + \tilde{T} = r_g D + \text{SSEXP} + \tilde{G} \quad (28)$$

where $\tilde{G}$ is the non-interest government budget surplus not accounted by the explicit model revenue and expenditure components and $\tilde{T}$ is non-explicit tax revenue introduced only in counter-factual experiments (e.g., estate taxes, tariffs).
Simulating Debt Over the Transition Path

Process of matching CBO debt projection:

1. Choose $\tilde{G}$ to match CBO non-interest surplus in each year in the open economy.

2. Use CBO government interest rates to generate debt sequence (generates exactly the CBO debt projection).

3. Use resulting $\tilde{G}$ from open economy (no macroeconomic feedback from debt) to construct government budget in closed economy.

Key intuition: CBO debt projections correspond to our open economy (no feedback effects of debt).

Baseline closed economy accounts for macroeconomic feedback from this debt sequence.
Personal Income Tax is assessed on wage income and a portion of capital income:

\[ PIT_{BASE} = wL + (1 - \theta) \frac{rK - 1}{q} K_{domestic}, \]  

(29)

where \( L \) is aggregate efficient labor.

Capital Income is \( \frac{rK - 1}{q} K \)

- Portion taxed at PIT is \( (1 - \theta) \frac{rK - 1}{q} K \).
- Portion taxed at the single capital tax rate is \( \theta \frac{rK - 1}{q} K \).
- Capital tax is from corporate tax, tax on dividends, tax on capital gains, etc.

Complex tax structure

Simplified into the PIT and single rate structure as above.

PIT \( \Rightarrow \) residential, single rate \( \tau_{cap} \Rightarrow \) territorial
Since PIT is non-linear, it is not clear how to assign tax dollars from capital income portion:

\[ PIT_{REVENUE} = \tau_{pit}(PIT_{BASE}) \]  \hspace{1cm} (30)

We say taxes assigned by percent of income

\[ PIT_{CAP\_REV} = \frac{(1 - \theta)\frac{rK-1}{q}K_{domestic}}{PIT_{BASE}}\times PIT_{REVENUE} \]  \hspace{1cm} (31)

So CT is

\[ CT_{REVENUE} = \tau_{cap}\theta\frac{rK-1}{q}K + PIT_{CAP\_REV} \]  \hspace{1cm} (32)