Abstract

The federal budget is one of the most important documents produced by our government and provides a roadmap for our nation’s priorities. However, there is currently a limited opportunity for policymakers to understand how a policy change will impact our nation’s economy and budget while they are writing legislation. Penn Wharton Budget Model (PWBM) seeks to inform policy discussions before bills are finalized and before lobbying and reputational stakes are put into the ground. PWBM combines modern advances in economic modeling, big data science, cloud computing and visualization tools to provide a “sandbox” in which different policy ideas can be tested before legislation is drafted. PWBM’s model consists of multiple integrated model components. This document describes the current version of PWBM’s Dynamic OLG Model, a dynamic general equilibrium overlapping generations model.
1 Model

The economy consists of a large number of overlapping-generations of households, a perfectly competitive representative firm with constant-returns-to-scale technology and a government with a commitment technology. Time is discrete and one model period corresponds to one year. For simplicity, we omit the time subscript when it does not compromise the understanding of the model. In a steady-state (stationary) equilibrium, the model economy is assumed to be on a balanced-growth path with a population growth rate of $g_{pop}$.1

1.1 Households

Households are heterogeneous with respect to age ($j$), assets ($a$), labor productivity ($z$) and average lifetime labor earnings ($b$). Let $s = (j, a, z, b)$ denote a household state and $S = (x(s), D)$ denote the aggregate state of the economy, where $x(s)$ is the period population density function of households in the economy and $D$ is the government’s debt in the beginning of period.

Labor productivity $z$ has four components:

$$z = z_{age} + z_{perm} + z_{trans} + z_{pers}. \quad (1)$$

When households enter the economy, they draw a permanent component of their labor productivity, $z_{perm}$. With probability $p_{perm}$, a household has a high permanent component, and with probability $(1 - p_{perm})$, a low one. There is a deterministic lifecycle component of labor productivity, $z_{age}$, that varies with age. And, finally, there are two idiosyncratic shocks on households’ labor productivity received each period: $z_{trans} \sim N(0, \sigma_{trans}^2)$ is a transitory shock and $z_{pers}$ is a persistent shock, which follows a first-order autoregression:

$$z_{pers}' = \rho z_{pers} + \eta'; \quad \eta \sim N(0, \sigma_{\eta}^2). \quad (2)$$

In every period, households are endowed one unit of time that can be allocated to work and leisure. However, at age $J_r$, households are forced to retire and start receiving Social Security benefits, $ss(b)$, that depend on their average lifetime labor earnings, $b$. Labor supply, $n$, is a continuous choice variable. In addition to leisure, households also derive utility from consumption $c$. The period-by-period return function is given by:

$$U(c, n) = \left(\frac{c^\gamma (1-n)^{1-\gamma}}{1-\gamma}\right)^{1-\sigma}. \quad (3)$$

1In the following model description, aggregate variables are not adjusted by population growth.
At each period households aged $j$ survive to the next period with probability $s_{j+1}$ unless $j = J$, in which case they die with probability 1. In the event of death, accidental bequests are collected by the government and uniformly distributed among the living population by means of lump-sum transfers, $beq$.

Households can accumulate positive assets. A unit of asset $a$ is a portfolio that combines a share $\phi_K$ of physical capital and a share $(1 - \phi_K)$ of government debt. Households take $\phi_K$ as exogenous. This assumption is required in order to generate a positive demand for both types of assets while imposing a spread between their return rates. The sequence of government interest rates, $r_G$, is exogenous to the model, based on projections by the CBO. Hence, for a given share $\phi_K$, the return rate on the portfolio is a weighted average of return on physical capital and the government interest rate:

$$R = \phi_K \tilde{r}_K + (1 - \phi_k)r_G, \quad (4)$$

where $\tilde{r}_K$ is the after-tax return to physical capital, including capital gains and losses. But before understanding how $\tilde{r}_K$ is determined, one must learn about capital income taxation in this economy.

Following Auerbach and Kotlikoff (1982) and Altig et al. (2001) let $\tau_{cap}$ denote the tax rate on physical capital income and $e$ be the rate of investment expensing. Normalizing output price to 1, equation (5) expresses Tobin’s $q$:

$$q = 1 - e\tau_{cap}. \quad (5)$$

For new capital the net acquisition cost is 1, the price of new capital, less the tax rebate from expensing $e\tau_{cap}$. And since old and new capital are perfect substitutes in production, their net acquisition costs must be identical in equilibrium; hence, old capital sells for $e\tau_{cap}$ less than new capital because the purchaser of new capital receives $e\tau_{cap}$ from the government, while the purchaser of old capital receives no tax rebate.

Let $r_K$ denote the net return to physical capital. Then the net return to physical capital, accounting for capital gains and losses, is given by:

$$\tilde{r}_K = \frac{r_K}{q} + \frac{q' - q}{q}. \quad (6)$$

We can then rewrite the return rate on the portfolio of equation (4) as:

$$R = \phi_K \left( \frac{r_K}{q} + \frac{q' - q}{q} \right) + (1 - \phi_k)r_G. \quad (7)$$

Finally, the amount paid on capital income taxes, $\tau_{cap}(y_{cap})$, depends on capital in-
come, \( y_{\text{cap}} \), as follows:

\[
\tau_{\text{cap}}(y_{\text{cap}}) = \theta \tau_{\text{cap}} y_{\text{cap}}.
\]

where \( \theta \) is the share of capital income subjected to the capital tax rate. The remaining \((1 - \theta)\) share is subjected to the personal income tax rate.

In addition to capital income taxes, there are two other taxes that households must pay the government. Payroll (Social Security) tax on labor income is defined as:

\[
\tau_{\text{ss}}(y_{\text{ss}}) = \tau_{\text{ss}} y_{\text{ss}},
\]

where \( y_{\text{ss}} \) is defined as

\[
y_{\text{ss}} = \min\{wzn, y_{\text{taxmax}}\},
\]

\( w \) is the economy wage, and \( y_{\text{taxmax}} \) is the maximum labor income subject to payroll taxation.

Personal income tax, \( \tau_{\text{pit}}(y_{\text{pit}}) \), depends on personal income, \( y_{\text{pit}} \), as follows:

\[
\tau_{\text{pit}}(y_{\text{pit}}) = \int_{0}^{y_{\text{pit}}} \xi(y) \, dy,
\]

where the tax function \( \tau_{\text{pit}}(y_{\text{pit}}) \) is a cumulative tax liability from the marginal tax rate function \( \xi(y) \). The function \( \xi(y) \) is a step-function imported from the PWBM Microsimulation module (henceforth PWBMsim). This effective marginal tax rate accounts for deductions, credits and other features not explicitly modeled in the dynamic model. Personal income is defined for each household age group in the next subsections.

1.1.1 Working-age Households

Recall that a household’s type is given by \( s = (j, a, z, b) \), where \( j \) denotes age, \( a \) denotes assets, \( z \) denotes labor productivity and \( b \) denotes average lifetime labor earnings. Let \( V^w(s, S; \Psi) \) denote the value of type \( s \) working-age households for a given government policy schedule, \( \Psi \), at the beginning of the period. The working-age household Bellman’s equation is given by:

\[
V^w(s, S; \Psi) = \max_{c, a', n} \{ U(c, n) + s_{j+1} \beta E[V^w(s', S'; \Psi')] \}
\]

s.t. \( c = wzn + (1 + R)a - \tau_{\text{pit}}(y_{\text{pit}}) - \tau_{\text{cap}}(y_{\text{cap}}) - \tau_{\text{ss}}(y_{\text{ss}}) - a' + beq \)

\[
b' = \frac{1}{j} \left( (j - 1)b + \min\{wzn, y_{\text{taxmax}}\} \right)
\]

\( c, a', n \geq 0, \)

where \( (13) \) is the budget constraint, \( (14) \) determines average earnings for Social Security benefit calculation and \( (15) \) is the standard non-negativity constraints.
Personal income for a working-age household is defined as:

\[ y_{pit} = wzn + \left( \phi_K \frac{r_K}{q} (1 - \theta) + (1 - \phi_K)(r_G - 1) \right) a. \] (16)

Notice that

\[ y_{pit} \neq wzn + (1 + R) a, \] (17)

because only a fraction \((1 - \theta)\) of the physical capital income is subjected to personal income tax. The remaining fraction is subjected to the capital tax rate \(\tau_{cap}\). Also, capital gains and losses due to changes in the price of capital are not considered capital income in our specification.

### 1.1.2 Retired Households

Let \(V^r(s, S; \Psi)\) denote the value of type \(s\) retired households. The retired household Bellman’s equation is given by:

\[
V^r(s, S; \Psi) = \max_{c, a'} \{ U(c, 0) + s_{j+1} \beta [V^r(s', S'; \Psi')] \} \tag{18}
\]

s.t. \(c = ss(b) + (1 + R)a - \tau_{pit}(y_{pit}) - \tau_{cap}(y_{cap}) - a' + beq \) \tag{19}

\(b' = b \) \tag{20}

\(c, \ a' \geq 0. \) \tag{21}

where (19) is the budget constraint, (20) determines average earnings for Social Security benefit calculation and (21) is the standard non-negativity constraints.

Personal income for a retired household is defined as:

\[ y_{pit} = (1 - \phi_{ss})ss(b) + \left( \phi_K \frac{r_K}{q} (1 - \theta) + (1 - \phi_K)(r_G - 1) \right) a, \] (22)

where \(\phi_{ss}\) is the fraction of Social Security benefits deductible from federal income taxation. Again, notice that only a fraction \((1 - \theta)\) of the physical capital income is subjected to personal income tax and that capital gains and losses due to changes in the price of capital are not considered capital income in our specification.

### 1.2 Production

There is a representative firm that demands labor services and rents physical capital. The firm also bears the depreciation costs of capital. Total physical capital in the economy, \(K\), equals aggregate savings, \(A\), less government debt, \(D\), in units of physical
capital (as opposed to dollar units):

$$K = \frac{A - D}{q}. \quad (23)$$

The problem of the representative firm is:

$$\max_{K,L} \left\{ K^{\alpha}L^{1-\alpha} - (\delta + r_K)K - wL \right\}, \quad (24)$$

where $\delta$ is capital depreciation rate and $r_K$ is the rental rate of capital faced by firms (or the net rental rate of physical capital). Capital and labor demand are determined according to FOC:

$$r_K = \alpha K^{\alpha-1}L^{1-\alpha} - \delta, \quad (25)$$

$$w = (1 - \alpha)K^{\alpha}L^{-\alpha}. \quad (26)$$

### 1.2.1 Closed Economy

In the closed economy model, the rental rate of capital $r_K$ is determined in equilibrium so that factor markets clear:

$$K = \int a dX(s) - D, \quad \text{and} \quad (27)$$

$$L_t = \int zn(s, S_t; \Psi_t) dX_t(s). \quad (28)$$

where $X_t(s_t)$ is the cumulative distribution function associated with $x_t(s_t)$. Therefore, gross domestic product, which is identical to gross national product, $Y_t$, is determined by:

$$Y = K^{\alpha}L^{1-\alpha} = (r_K + \delta)K + wL. \quad (29)$$

### 1.2.2 Open Economy

In the open economy model, there is trade in international capital flows, which implies that foreigners hold some of the total physical capital of the economy, that is:

$$K = K_d + K_f, \quad (30)$$

where $K_d$ denotes physical capital held by domestic households and $K_f$ denotes physical capital held by foreigners. Notice that $K_f > 0$ implies foreigners hold domestic capital, while $K_f < 0$ implies domestic households hold foreign capital abroad in net terms.

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2For ease of notation, we omit the time subscript in the firms’ problem.
Given the territorial capital tax assumption, capital tax on the fraction $\theta$ of total assets in the economy also applies to foreign capital. However foreigners’ personal income cannot be taxed, in other words, a fraction $(1 - \theta)$ of $K_f$ income is not taxed in the US.

We assume that in the open economy model the rental rate of capital $r_K$ is such that the after-tax net capital return equals the international after-tax net capital return, $\lambda$, that is:

$$\frac{r_K}{q}(1 - \tau_{cap}) = \lambda. \quad (31)$$

Under the small open economy assumption, $\lambda$ is set to a constant by the international capital market. Assuming a time invariant $\tau_{cap}$, equations (25) and (31) imply a unique capital-to-labor ratio $\kappa = \frac{K}{L}$ given in the firm’s FOC:

$$r_K = \alpha\kappa^\alpha - 1 - \delta, \quad (32)$$

Then, equations (31) and (32) give:

$$\kappa = \left[\frac{1}{\alpha} \left(\frac{\lambda q}{1 - \tau_{cap}} + \delta\right)\right]^{\frac{1}{\alpha - 1}}. \quad (33)$$

Hence, (26) and (33) imply wages:

$$w = (1 - \alpha)\kappa^\alpha. \quad (34)$$

Lastly, gross domestic product is determined by equation (29), where $K$ is given by (30).

### 1.3 Government

Government issues debt, $D$, that pays a return $r_G$. Government debt evolves according to:

$$D' + R = r_G D + G, \quad (35)$$

where $R$ is government revenue and $G$ is government expenditures.

Government expenditures have two components:

$$G = SSEXP + \tilde{G}, \quad (36)$$

where $SSEXP$ denotes Social Security expenditures and $\tilde{G}$ is an exogenous variable that denotes the non-interest government budget surplus not accounted by the explicit model revenue and expenditure components. In the model, Social Security expenditures are given by:

$$SSEXP = \int_{\forall s: j \in T_R, \ldots, l} ss(b)X(s)ds. \quad (37)$$
Government revenue is composed of all tax revenues:

\[ R = PIT + CIT + SSREV, \]  

(38)

where \( PIT \) denotes personal income taxes, \( CIT \) denotes capital taxes, and \( SSREV \) denotes payroll taxes. Those tax revenues are respectively defined as:

\[
PIT = \int \tau_{pit} y_{pit}(s) X(s) ds
\]

\[
CIT = \tau_{cap} \phi_K ((r_K - 1) \theta K
\]

(39)

\[
SSREV = \int \tau_{ss} y_{ss}(s) X(s) ds.
\]

(40)

To match the model’s revenue to projected revenue from PWBMsim, we include an exogenous component \( \tilde{T} \) to denote a non-explicit tax revenue residual.

### 1.4 Recursive Competitive Equilibrium

We define the recursive competitive equilibrium of this model economy as in Nishiyama and Smetters (2013).

Let \( s_t = (j, a, z, b) \) be the individual state of households, let \( S_t = (x_t(s), D_t) \) be the aggregate state of the economy and let \( \Psi \) be the government policy schedule committed at the beginning of period \( t = 0 \),

\[
\Psi = \{ r_{G,t}, \tau_{cap,t}, e_t, \tau_{ss,t}, y_{taxmax,t}, \tau_{pit,t}(\cdot), \tau_{ss,t}(\cdot), D_{t+1}, G_t, \tilde{T}, \beta q_t \}_{t=0}^\infty.
\]

(41)

A time series of factor prices and the government policy variables,

\[
\Omega = \{ r_{K,t}, w_t, \Psi \}_{t=0}^\infty,
\]

(42)

the value function of households, \( \{ V(s_t, S_t; \Psi) \}_{t=0}^\infty \), the decision rules of households,

\[
\{ d(s_t, S_t; \Psi) \}_{t=0}^\infty = \{ c(s_t, S_t; \Psi), n(s_t, S_t; \Psi), a'(s_t, S_t; \Psi), b'(s_t, S_t; \Psi) \}_{t=0}^\infty
\]

(43)

and the distribution of households, \( \{ x_t(s_t) \}_{t=0}^\infty \), are in a recursive competitive equilibrium if, for all \( t = 0, \ldots, \infty \), each working-age household solves the optimization problem (12)-(15) and each retired household solves the optimization problem (18)-(21), taking \( S_t \) and \( \Omega \) as given; the firm solves its profit maximization problem (24); the government policy schedule satisfies conditions (35)-(40), and the factor markets are cleared as shown in Equations (27)-(28). The economy is in a steady-state equilibrium, and thus on the balanced-growth path, if, in addition, \( S_{t+1} = S_t \) for all \( t = 0, \ldots, \infty \) and the government policy schedule is time-invariant.
In the above competitive equilibrium, the resource (feasibility) constraint is satisfied – the goods market clears – in each period by Walras’ law following equation (29).

2 Calibration

This section describes how we map the model to the data. We start by presenting the demographic data and the idiosyncratic earnings process used in this paper. Next, we present calibration of the preference and technology parameters. Then we discuss the government parameters, that is, taxes, debt, government debt interest rate, $\tilde{G}$ and Social Security structure. Table (1) summarizes the main parameters with their corresponding sources and/or targets.

2.1 Demographic Parameters

One period in our model is associated with one year of calendar time. Households enter the model at age 21. We set $T_r$ to 47 so that households retire at age 67. The maximum life span is set to 100 years.

Our demographic parameters come from the output of PWBMsim. Mortality rates are chosen to match those of the U.S. population in 2016. We set the annual birth rate of the national population to 1.9% to match the growth rate of the 21-year-olds cohort in 2016. Lastly, we use the distribution of immigrants by age and the annual growth rates of legal and illegal immigration to build the measures of immigrants by age. The legal immigration rate equals 0.0016, while the illegal one equals 0.0024.

2.2 Labor Earnings Process

Our labor productivity structure mimics the Storesletten et al. (2004) labor earnings process. We use numbers from Conesa et al. (2017) to estimate the deterministic lifecycle labor productivity profiles, $z_{age}$. For the idiosyncratic shock process we use some point estimates from Storesletten et al. (2004) and recalibrate others. Specifically, we use their numbers for the permanent and persistent variances, $\sigma^2_{perm} = 0.2105$, $\sigma^2_{trans} = 0.0630$, respectively. We apply Tauchen’s discretization method and obtain a first order Markov process with realizations $\{\pm 0.3661\}$ for the permanent shock. For the transitory shocks, we use an i.i.d. two-state Markov chain, with realizations $\{\pm 0.2003\}$. Innovations to the persistent component are assumed to be i.i.d. with realizations $\{\pm 0.4638\}$ derived from a Tauchen discretization process where $\sigma_q = 0.018$ and the persistence parameter is set to 0.990.

In addition, we include a “Michael Jordan” productivity shock a la Castaneda et al. (2003), which consists in a state of very high labor productivity with low persistence.
In a survey on quantitative methods of wealth inequality, De Nardi (2015) points out that standard OLG models do not generate enough wealth concentration without adding complementary forces. The inclusion of high earnings risk for the top earners allows us to better match earnings and wealth distribution moments of the U.S. economy. In particular, we calibrate our model economy to match the 0.48 Gini coefficient on labor earnings, measured as pre-tax labor income by PWBMsim. In addition, the Gini coefficient on wealth implied by our model is 0.76, while that measured using PWBMsim data is 0.86. This feature is an intermediate step. In the future, we will implement a labor earnings calibration entirely derived from PWBMsim projections.

2.3 Preference and Technology Parameters

Preferences parameters are jointly calibrated within the model. In particular, we target a capital-output ratio equal to three, a Frisch labor supply elasticity equal to 0.5, and an elasticity of intertemporal substitution equal to 0.5 to determine the discount factor ($\beta$), the consumption share in intratemporal utility ($\gamma$) and the risk aversion parameter ($\sigma$). Also, we set model dollars so that the value of GDP per adult is 79,800 dollars, as observed by PWBMsim for the year of 2017.

We normalize total factor productivity to one. Capital’s share of income is set to 0.34 as derived from the National Income and Product Accounts (NIPAs). We set the depreciation rate to 0.056 based on data from the Bureau of Labor Statistics (BLS) multifactor productivity program.

In addition, we consider two rates of return to capital: (a) the rate implied by the marginal return to capital and (b) a lower adjusted rate. In acknowledging that our model does not account for aggregate uncertainty nor an insurance provided by the capital tax loss carry-forwards, we experiment with a wedge to lower the capital return rate to a level closer to the risk-free rate. Admittedly, this is an ad hoc method.

2.4 Government Parameters

We use PWBM Tax module outputs for setting the model’s tax structure. Each tax plan provides a step-wise effective marginal tax rate function for individuals, a capital tax rate $\tau_{\text{cap}}$, a capital tax share $\theta$, an expensing rate $e$, and the time series of revenue-to-GDP targets which are used to construct $\tilde{T}$. Capital share in total assets, $\phi_K$, is calculated as the ratio of capital to total assets.

In our baseline, we currently have $\tau_{\text{cap}}$ to 18.6%, $\theta$ to 100% and $e$ to 58%.

Payroll (Social Security) tax $\tau_{\text{ss}}$ is set to 12.4% and it falls upon a maximum annual labor income of $118,500. Given an average lifetime labor earning $b$, we set Social Security benefits as follows. First, we identify in which earnings bracket containing $b$. There are three monthly earnings brackets: below $856$, in between $856$ and $5,157$, and above
We then use bracket-specific discounting constants and replacement rates as follows:

\[ ss(b) = (b - \text{discounting}_b) \times \text{replacement}_b, \]  

(44)

where

\begin{align*}
\text{discounting} &= [0, 856, 5157], \text{ and} \\
\text{replacement} &= [0.9, 0.32, 0.15].
\end{align*}

(45)  

(46)

Personal income taxation has a cumulative tax liability structure with currently 14 income brackets in the baseline, each with a corresponding tax rate. For instance, if a household’s personal income falls within the tenth bracket, her personal income tax equals the sum of the tax burden of the first nine brackets plus the product between the marginal rate for bracket number ten and her income subtracted by the income level starting bracket number ten. We set the fraction of deductible Social Security earnings, \( \phi_{ss} \), to 0.85.

We get series for government debt interest rate, government expenditures, government revenue and debt from the PWBMSim. We construct the exogenous non-interest government budget surplus \( \tilde{G} \) by multiplying the total tax revenue net of federal government interest surplus as a percentage of GDP by GDP and then subtracting Social Security benefits. Similarly, we compute \( \tilde{T} \) by multiplying the total tax revenue as a percentage of GDP by GDP and subtracting the revenue sources already accounted for by the model, that is, personal income taxes, Social Security taxes and capital income taxes.

3 Conclusion

The PWBM Dynamic OLG model is a general equilibrium, overlapping generations, incomplete markets model which is computed through backwards value function iteration. Households in the model are distinguished by age, idiosyncratic labor productivity shocks, asset holdings and earnings history. Two economic pricing scenarios are allowed: (a) a “closed” economy where wages and return to physical capital investment are computed by iterating over the capital-labor ratio towards a fixed point and (b) an “open” economy where the after-tax return to capital is fixed and wages are derived from the resulting capital-labor ratio.

The model obtains inputs from PWBMSim to drive the dynamic model’s robust taxation system, demographics and immigration flows and Social Security system. Government expenditures and tax revenues from these components generate changes in government debt. This debt feeds back into the model as an asset which can crowd-out investment into productive capital in the model economy.
Because of the large set of policy features and the calibration of the model from rich demographics projections of PWBMsim, the PWBM Dynamic OLG model produces policy experiment projections which (a) take account of interactions between different areas (e.g., immigration’s effect on Social Security) and (b) provide a match to detailed demographics and policy nuances which are ordinarily not explicitly available in a dynamic model due to state-space limitation. These features make the Penn Wharton Budget Model a best-in-class economic model and give policymakers and researchers a high degree of confidence in using it for projections and analysis.

References


Table 1: Main Parameter Values

<table>
<thead>
<tr>
<th><strong>Demographics</strong></th>
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</thead>
<tbody>
<tr>
<td>$J$ Maximum life span</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td>$J_r$ Retirement age</td>
<td>47</td>
<td>67</td>
</tr>
<tr>
<td>$g_{pop}$ Annual birth rate</td>
<td>0.0200</td>
<td>MicroSim</td>
</tr>
<tr>
<td>$g_{pop,legal}$ Annual legal immigration rate</td>
<td>0.0016</td>
<td>MicroSim</td>
</tr>
<tr>
<td>$g_{pop,illegal}$ Annual illegal immigration rate</td>
<td>0.0024</td>
<td>MicroSim</td>
</tr>
<tr>
<td>$s_j$ Survival probability</td>
<td></td>
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</tbody>
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| **Preferences - low capital return**                  |       |       |
| $\beta$ Discount factor                               | 1.00  | $K/Y = 3.0$ (baseline) |
| $\gamma$ Consumption share                            | 0.68  | Frisch elasticity = 0.5 |
| $\sigma$ Risk aversion                                | 1.50  | Savings elasticity = 0.5 |
| Scale adjustment parameter                            | $4.13 \times 10^{-5}$ | Avg HH income = $79,800$ |

| **Preferences - high capital return**                 |       |       |
| $\beta$ Discount factor                               | 0.98  | $K/Y = 3.0$ (baseline) |
| $\gamma$ Consumption share                            | 0.75  | Frisch elasticity = 0.5 |
| $\sigma$ Risk aversion                                | 1.24  | Savings elasticity = 0.5 |
| Scale adjustment parameter                            | $4.35 \times 10^{-5}$ | Avg HH income = $79,800$ |

| **Output**                                            |       |       |
| $A$ Total factor productivity                         | 1     | Normalization |
| $\alpha$ Capital share of output                      | 0.34  | NIPA     |
| $d$ Depreciation rate                                 | 0.056 | BLS      |

| **Labor productivity**                                |       |       |
| $p_{perm H}$ High permanent prod. probability         | 0.500 | Storesletten et al. (2004) |
| $\sigma_{perm}$ Permanent productivity variance       | 0.2105| Storesletten et al. (2004) |
| $\sigma_{trans}$ Transitory productivity variance     | 0.063 | Storesletten et al. (2004) |
| $\sigma_{pers}$ Persistent productivity variance      | 0.018 | Storesletten et al. (2004) |
| $\rho$ Persistent prod. autocorrelation               | 0.990 | Storesletten et al. (2004) |