Inference and exactness


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Inference and exact numerical representation in early language development

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Abstract

How do children as young as 2 years of age know that numerals, like one, have exact interpretations, while quantifiers and words like a do not? Previous studies have argued that only numerals have exact lexical meanings. Children could not use scalar implicature to strengthen numeral meanings, it is argued, since they fail to do so for quantifiers (Papafragou & Musolino, 2003). Against this view, we present evidence that children’s early interpretation of numerals does rely on scalar implicature, and argue that differences between numerals and quantifiers are due to differences in the availability of the respective scales of which they are members. Evidence from previous studies establishes that (1) children can make scalar inferences when interpreting numerals, (2) children initially assign weak, non-exact interpretations to numerals when first acquiring their meanings, and (3) children can strengthen quantifier interpretations when scalar alternatives are made explicitly available.

Keywords: number; counting; exactness; scalar implicature; inference; quantifiers; language acquisition; semantics; pragmatics
When children learn to count and acquire labels for the positive integers (e.g., *one*, *two*, *three*, etc.), they harness a system for representing *exact* cardinalities. Humans are known to possess multiple non-linguistic systems for representing sets and quantities, none of which can both represent sets of unbounded size, and do so exactly (Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Laurence & Margolis, 2005). As a result, some researchers have argued that individuals who lack a counting system – e.g., young children, or members of indigenous groups like the Pirahã – may lack the capacity to represent large sets exactly (Carey, 2004; Spelke, 2003; Frank et al., 2008; Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). According to this view, language may be the source of exact numerical representations in human cognition.

Besides wondering whether language is *necessary* for distinguishing concepts like *fifty-four* from *fifty-five* (as opposed to representing it as *fifty-fourish*), researchers have also asked precisely how such meanings are represented by natural language. Given the possibility of exact meanings, how are they encoded by the lexical semantics and pragmatics of natural language? Are such meanings represented like those of other quantifiers in natural language (e.g., *some*, *many*, *several*, etc.) or do numerals require a distinct analysis, with qualitatively different lexical representations? This question – how language encodes exact numerical concepts – is the focus of this paper.

For the most part, the question of how language represents exactness has been investigated by linguists interested in the tradeoff between semantics and pragmatics. For example, linguists have debated whether words like *one*, *two*, and *three* represent exactness inherently (i.e., as part of their lexical semantics), or whether exact interpretations are derived pragmatically, and inferred from how numerals are used in context. Although numerals sometimes specify exact cardinalities, as in (1), they can also specify non-exact, “at least”, interpretations, as in (2):

(1) The student has read twelve books.
(2) Students must have read twelve books to enter the class.

In (1), the numeral *twelve* receives an exact interpretation, and would be inappropriate if the student had read twenty books. Example (2), on the other hand, receives an “at least” interpretation, such that students who have read twenty books (i.e., not exactly twelve) would still be admitted to the class. By one view, these examples are best described by a grammar in which numerals have lexically *exact* meanings. On this view, the exact interpretation of *twelve* in (1) is due to *twelve* having an exact lexical meaning. The “at least” interpretation in (2), however, must be derived via implicature (Breheny, 2008). A second view holds that numerals have weak, “at least”, meanings lexically, and that strong, exact, interpretations like in (1) are derived.

According to this second view, numerals belong to a larger class of words sometimes called “scalar items” (Horn, 1972). Scales provide sets of alternative meanings which are ordered according to their informational strength, and are implicitly contrasted during interpretation – e.g., *some / all; inclusive or / and; might / must*, etc. For example, it is normally assumed that the existential quantifier *some* is logically consistent with the universal quantifier *all*, since the use of *all* entails the use of *some* (i.e., *all* $\Rightarrow$ *some*; see Figure 1). Consider the examples in (3) through (5):

[Insert Figure 1 about here]

(3) To enter the class, students must have read some Shakespeare.

(4) The class admitted some students.

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1 For simplicity, we use the notation *all* $\Rightarrow$ *some* to stand in for complete expressions, recognizing that lexical items in isolation do not enter into entailment relations. Also, we note that these entailment relations are not absolute, and are modified by downward entailing environments (e.g., under one reading, it is not the case that “I don’t have all the money” entails “I don’t have some of the money”).

2 This is not true in downward entailing environments, in which the opposite entailment relation holds and the implicature is reversed (*all* $\Rightarrow$ $\neg$*some*).
(5) The class admitted all students.

In (3), the quantifier *some* receives a weak interpretation, such that students are admitted if they read one of Shakespeare’s plays or all thirty-six of them. However, in other contexts *some* can be strengthened by appeal to its scale mate *all*. For example, in (4) the use of *some* implies that not all students were admitted to the class (*some* → ¬*all*). Such inferences, which limit the interpretation of weak lexical items by appeal to stronger scalar alternatives, are typically called “scalar implicatures”. By most accounts (see Chierchia, Fox, & Spector, 2007; Grice, 1989; Gazdar, 1979; Horn, 2005; Levinson, 1983; van Rooij & Schulz 2006) computing scalar implicatures is thought to involve at least four distinct steps:

i. *Compute basic meaning* of a sentence S containing L, a scalar lexical item, where L has a weak (or basic) meaning.

ii. *Generate a set of alternatives* (a₁, a₂…aₙ) to S, called S<sub>alt</sub>. These are all the sentences that can be generated by replacing L with its scalar alternatives.

iii. *Restrict the alternatives in S<sub>alt</sub>* to the set containing only the stronger expressions, (i.e., such that A entails S, but S does not entail A), called S*.

iv. *Augment* (or “exhaustify”) the basic meaning of S (containing L) with the negation of all of the members of S*.

For a quantifier like *some*, the four steps would proceed as follows. First, the weak meaning of a sentence containing *some* would be represented. Second, its scalar alternatives would be generated (e.g., equivalent sentences containing scale mates like *all, many, most*, etc.). Third, this set of alternatives would be restricted to consider only those that are stronger, like *all* (we know *all* is stronger, since expressions containing *all* entail those containing *some*: (all ⇒ some) ∧ ¬(some ⇒ all)). Finally, the basic meaning of the sentence would be augmented by negating
the meanings of all equivalent sentences containing stronger scale mates (e.g., *some = some & \neg all*).

For linguists who claim that numeral meanings are also lexically weak and non-exact, these steps also apply to the interpretation of numerals: weak lexical meanings of numerals are represented, alternatives are generated and restricted to stronger scale mates, and the meaning of the sentence in which the numeral occurs is strengthened by negating the stronger alternatives. Thus, a numeral with a weak meaning, e.g., *two*, becomes strengthened by appeal to its stronger scale mate *three*, which imposes upon *two* an upper bound, resulting in an exact interpretation of “exactly two” (i.e., “at least two” and “at most two”). By this account, weak numeral meanings support “at least” interpretations by default. This is not because these meanings fail to specify cardinalities. Instead, it is because weak meanings are existential; they instantiate the existence of a quantity of individuals (e.g., there exist 2 cats) without excluding the possibility that additional individuals exist. Consider the logical forms in (6), described in Breheny (2008). In (6a), *two* is assigned a weak existential meaning, and thus licenses an “at least” interpretation (i.e., to say that there exist “two students” does not exclude the existence of three). In (6b), the generalized union operator (^{\bigcup}) is added to derive the exact interpretation. This operator takes all the sets that include only students who did well. If the union of these sets is a two-member set, then (6b) is true. However, if the set contains any amount greater or less than two, then (6b) is false.\(^3\)

(6) a. There exist two students who did well on the test.

\[ \exists X \left[ |X| = 2 \land S(X) \land W(X) \right] \]

\(^3\)Given two sets each containing one individual (e.g., call them \{a\} and \{b\}), where each individual both did well and was a student, then \(\bigcup\{a\}, \{b\}\} = \{a, b\}. In this case, (6b) is true. Given two sets each containing two individuals (e.g., \{a, b\}, \{c, d\}, where each is a student and did well, then \(\bigcup\{a, b\}, \{c, d\}\} = \{a, b, c, d\}. In this case, (6b) is false, since \(|\bigcup \{ X : S(X) \land W(X)\}| \neq 2\) (i.e., it is equal to 4).
b. There are exactly two students who did well on the test.

$$|\{X : S(X) \wedge W(X)\}| = 2$$

Unlike linguists, developmental psychologists have typically begun their investigations with the assumption that numeral meanings are lexically exact. Their research has instead focused on the other problem of exactness – i.e., how exact meanings originate in development, given the apparent lack of exact representations in pre-linguistic cognition (for examples see Gallistel & Gelman, 1992; Gelman & Gallistel, 1978; LeCorre, Van deWalle, Brannon, & Carey, 2006; LeCorre & Carey, 2007; Mix, Huttenlocher, & Levine, 2002; Schaeffer, Eggleston, & Scott, 1974; Wynn, 1990, 1992, etc.). The core problem for psychologists is how this special representational capacity, not found in other animals, emerges from the process of acquiring a linguistic counting system and mapping this system to non-linguistic representations.

Recently, studies of pragmatic development in young children have prompted psychologists to re-examine the status of numeral meanings, and whether children’s early number words are indeed lexically exact, or whether exact interpretations are derived via scalar implicature (the question of interest in this paper). The main conclusion of many such studies is that children’s numerals have exact meanings in the lexicon. This conclusion stems from the observation that children assign exact interpretations to numerals while they fail to strengthen quantifiers in similar contexts. For example, children have been found to experience difficulty computing implicatures for contrasts including *might vs. must* (Noveck, 2001), *a vs. some* (Barner et al., 2009), *some vs. all* (Huang, Snedeker, & Spelke, under review; Hurewitz, Papafragou, Gleitman, & Gelman, 2006; Noveck, 2001; Musolino, 2004; Papafragou & Musolino, 2003; Smith, 1980), and the distinction between inclusive and exclusive *or* (Chierchia, Crain, Guasti, Gualmini, & Meroni, L., 2001; Gualmini, Crain, Meroni, Chierchia, & Guasti, 2001).
A subset of these studies has directly contrasted children’s interpretation of numerals and quantifiers (Barner et al., 2009; Huang et al., under review; Hurewitz et al., 2006; Papafragou & Musolino, 2003; Musolino, 2004). These studies find that children exhibit exact interpretations of numerals in the same contexts in which they fail to strengthen other scalar items. For example, in a study by Papafragou and Musolino (2003), 5-year-old children were shown a scene including three horses, in which all three animals successfully jumped over a fence. When children were asked whether two of the horses jumped the fence, they said “no”, as did adult controls (because three horses jumped the fence). However, when they were asked whether some of the horses jumped the fence, they replied “yes”, unlike adults. Thus, like adults, children treated two as incompatible with sets of three, but, unlike adults, they treated some as compatible with sets containing all items in a context (i.e., as compatible with all).

Papafragou and Musolino (2003) and other studies since have concluded that children’s exact interpretation of numerals must be due to exact lexical meanings, since children fail to compute implicatures for quantifiers. If children cannot compute implicatures for quantifiers, the argument goes, then they must not use implicatures to derive exact interpretations of numerals; numerals must supply exact meanings lexically. However, other experiments raise doubts about the interpretation of these failures. On the one hand, some studies including Papafragou and Musolino’s indicate that children can compute implicatures for quantifiers under certain circumstances. For example, children do strengthen the quantifier some (to exclude all) when first trained to strengthen other utterances (e.g., to use “dog” in place of “small four legged animal”). On the other hand, experiments by Musolino (2004) show that, given a proper understanding of a context, 5-year-olds can use pragmatic inference to access multiple interpretations of numerals, including “at least”, “at most”, and exact interpretations. These studies raise serious questions about the precise nature of children’s failure to compute
implicatures when they do, and whether such failures with quantifiers should indeed license inferences about how children interpret numerals.

Relatively unexplored in this debate are the very first hypotheses that children make about the meanings of numerals. Although there is a rich literature investigating children’s early acquisition of numeral meanings, few studies have explicitly questioned whether these meanings are exact. Instead, most studies of very early acquisition either assume exactness, use methods of evaluation that presuppose it, or focus on other aspects of the acquisition problem. However, we believe that very early acquisition provides compelling clues about the origin of exactness in the numeral system. This is because early in acquisition numeral meanings are still being acquired, allowing us to distinguish their interpretation in isolation (e.g., in absence of meaningful successors) from their interpretation as part of a complete counting system.

In a study conducted in our labs, for example, we found that by 2 years of age English-speaking children assign an exact interpretation to the numeral *one*, and do so while apparently lacking knowledge of other numeral meanings, like *two* and *three*. At this same age, children nonetheless fail to assign strengthened interpretations to words like *a* and *some* (Barner et al., 2009). Thus, 2-year-old children make a striking distinction between *one*, which they treat as exact, and *a*, which they treat as weak and non-exact (to mean “at least one”).

In what follows, we argue that understanding this distinction between *a* and *one* yields deep insight into how children come to assign exact interpretations to numerals. Based on a review of the literature and a re-examination of data from various sources, we argue that children actually begin acquisition with weak, non-exact, meanings for numerals. Initially, *one* equals *a*. By our account, children derive exact interpretations of numerals by computing scalar implicatures, and

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4 Here, “*a*” stands in for indefinite singular NP’s.
therefore strengthen the interpretation of *one* (i.e., at least one) only after they have acquired its stronger scale mate *two* (i.e., at least two). Children fail to compute implicatures specifically for quantifiers due to the relative unavailability of quantifier alternatives, but readily access alternatives for numerals, as a result of learning the count list.

One Difference Between ‘A’ and ‘One’

The starting point for our investigation is the observation that, in addition to their difficulty computing implicatures for quantifiers like *some*, very young children also fail to assign exact interpretations to singular noun phrases like *a banana*, and do so despite treating *one* as exact. In a study by Barner et al. (2009), 2- to 5-year-old children were asked to make truth value judgments for both quantifiers and numerals. As in previous studies, children assigned a weak interpretation to *some*. When 8 of 8 objects in a context were placed in a circle, most children said “yes” when asked “Are some of the bananas in the circle?”. Also, 2-year-old children failed to assign *some* a lower bound of “two or more”, and agreed that there were *some bananas* in the circle when there was only one (despite denying that *all* of them were in the circle when there were only 3 out of 8).

[Insert Figure 2 about here]

Most striking was children’s distinction between *a* and *one*. When presented a context in which there were two objects in a circle (e.g., two bananas), 2- to 5-year-old children replied “yes” when asked “Is there a banana in the circle?” but said “no” when asked “Is there *one* banana in the circle?” (see Figure 2). A similar result was found in a second task, in which children were asked to place sets in a container. When asked to put *one banana* in the container,

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5 This result cannot be explained by young children’s failure to perceive the indefinite article or to notice bound plural morphology (or its absence). Test sentences contained singular-plural agreement on the verb (e.g., *is* vs. *are*) to which children are sensitive by 24-months (Kouider, Halberda, Wood, & Carey, 2006). Also, even 4-year-olds showed the same weak interpretation of singular nouns.
almost all 2-year-olds put in one object. However, when asked to give a banana, the majority of 2-year-olds gave more than one object (see Figure 3).

This set of results reveals a striking contrast: whereas children assigned an exact interpretation to one, they failed to do so for the indefinite article a. The fact that even 2-year-olds make this very subtle distinction raises the question: What information that is available to 2-year-old children could allow them to distinguish the semantics of a banana from one banana?

The sources of information that might distinguish these meanings for 2-year-olds are limited. One possibility is that children hear these words used to refer to different set sizes. For example, adult caregivers may be more likely to use singular NPs like a banana in contexts containing a plurality, relative to NPs like one banana. While evidence from our lab indicates that adults do assign an upper bounded interpretation to a under conditions in which children do not (see Spector, 2007, for evidence that this involves implicature of the form a ⇒ ¬some), it remains unknown how frequently adults use unstrengthened meanings when speaking to children (Barner et al., 2009). Thus, while it is possible that referential differences between a and one in the speech of caregivers cause 2-year-olds to interpret these words differently, there is currently no evidence to address this possibility. However, even if a and one were used differently, it’s not clear that this would license the inference that one is exact. Although reference to sets of one object (and not to larger sets) is consistent with an upper bounded meaning, it is also consistent with an at least interpretation. Such evidence would therefore leave open the question of how children make the induction that one is exact, whereas other words that refer to sets of one item are not.

A second conspicuous difference between a and one is that only one is part of the numeral system. One, unlike a, is part of an explicitly memorized count list that contains infinitely many
members, corresponding to the infinitely many cardinalities to which they apply. Learning this list – or at least a small portion of it – is the first problem that children confront when beginning to count (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1990). At 2 years of age, many children can recite some subset of the list (e.g., *one, two, three*). Though they make many errors (e.g., confusing the order of numerals, and sometimes repeating them within a single count), they begin to acquire the order of numerals well before they have figured out their meanings. This process takes many months, and continues after children have acquired various deeper properties of counting, but nonetheless is initiated early, and provides a crucial structure to acquiring numeral meanings.

In contrast, children clearly do not learn to recite quantifiers or determiners in a sequence, and many adults would be hard pressed to state scalar alternatives to a quantifier like *some*, if asked. Ultimately, scale mates become implicitly available because of their differences in informativeness or strength; quantifier scales are never explicitly learned or recited.

Based on this observation, it may not be valid to conclude that children *could not* compute implicatures for numerals, based on their treatment of quantifiers. Computing implicatures for numerals, we submit, may be substantially easier than for quantifiers, since children memorize and recite the numerals as an ordered list of alternatives from early in acquisition, but never do so for quantifiers. As noted earlier, a crucial prerequisite to computing implicatures is accessing relevant alternatives.

Following this logic, our suggestion is that children assign lexically weak meanings to both *a* and *one*, and that the strong, exact, interpretation of *one* is derived via implicature. Contrary to previous accounts (e.g., Papafragou & Musolino, 2003; Hurewitz et al., 2006), children may not have difficulty computing scalar implicatures *wholesale*, but may have selective difficulty
computing implicatures for quantifiers (such as *some* and *a*), due to the relative unavailability of their scalar alternatives.

Below, we offer a four-part argument in favor of this hypothesis that, for young children, numeral meanings are non-exact and strengthened via implicature. First, we review evidence from classic studies in developmental psychology that support the contention that children compute scalar inferences for numerals early in acquisition. Second, we provide an extensive re-analysis of behavioral data from multiple labs and multiple languages, and argue that such data support the hypothesis that children initially acquire weak, non-exact meanings for numerals. In doing so, we also present new data that support this analysis. Third, we describe how these weak meanings support scalar inference both for known and unknown numerals early in acquisition. Finally, we review evidence that children’s capacity to compute implicatures for numerals but not quantifiers is mediated by the explicit availability of scalar alternatives. We conclude that the core difference between quantifiers and numerals is the structure of the scales to which they belong.

Early Numeral Meanings and Implicature

The main argument that children do not compute implicatures for numerals comes from the observation that they have difficulty computing them for quantifiers like *some*. Since children have difficulty using pragmatics to constrain quantifier meanings, it is argued, they must not do so for numerals either (Papafragou & Musolino, 2003; Musolino, 2004; Hurewitz et al., 2006). This conclusion rests on the premise that children’s difficulty computing implicatures for quantifiers is general to all implicatures, and not specific to quantifiers. However, there is empirical evidence, beginning with Wynn (1990), which indicates that children may in fact have the ability to compute implicatures for numerals.
In her groundbreaking studies of children’s numeral acquisition, Wynn (1990, 1992) developed a system of “number knower” levels that has since been used extensively in the literature (see Barner, et al., 2009; Condry & Spelke, 2008; LeCorre & Carey, 2007; LeCorre, Brannon, Van deWalle, & Carey, 2006; Sarnecka et al., 2007; for a contrasting view, see Dehaene, 1997; Gallistel, 1990; Gallistel & Gelman, 1992). Wynn’s key contribution was to suggest that children’s acquisition of numeral meanings proceeds in distinct stages. Sometime around their second birthday, many children have acquired a subset of the count list (e.g., one, two, three, etc.), but have not yet figured out the meanings of these words. By Wynn’s account, children begin by acquiring the meaning of the word one, and remain “one-knowers” for anywhere between 6 and 12 months. Next, they acquire two, and then three, and by the time that they have figured out the meaning of four, most children can also correctly interpret all other numerals in the count sequence that they can recite (they are cardinal principle knowers).

To assess these knower levels, Wynn used the “Give-a-Number” task (GN). In this task, children are shown a set of objects (e.g., fish), and are asked to put a subset into a container (e.g., a pond): “Can you put one fish in the pond?” Typically, the experimenter starts with one, and moves to two, three, and upward as the child succeeds with each numeral. When the child fails to give a correct response for a number N, they are tested on N-1. To be categorized as an N-knower, the child must satisfy two criteria. First, the child must give N objects on 2/3 trials (e.g., a one-knower might give one object 2/3 times when asked for one). Second, the child must not give N objects when asked for quantities other than N. Thus, a child who gave one object when asked for one, but also when asked for two or three would be called a non-knower.

Wynn’s discovery of these discrete knower levels allowed her to notice that, before acquiring a numeral’s specific meaning (e.g., five), children draw on already known numerals (e.g., one) to constrain the interpretation of unknown numerals. Wynn observed two facts that suggest this
conclusion. First, she observed that although one-knowers systematically give one object when asked for one, they virtually never give one object when asked for two or three. Second, Wynn observed that when children are presented with two sets – one containing one individual, and the other containing five – one-knowers point systematically to the larger set when asked to point at the set with five (see also Condry & Spelke, 2008). To explain how children know that numbers like two, three, and five cannot refer to sets of one despite not knowing their meanings, Wynn (1992) suggested that they respect Clark’s (1988) principle of contrast – that all words must contrast in meaning: “Since all the children knew that the word ‘one’ refers to a single item, then if they knew that, for example, the word ‘five’ refers to a numerosity, they should infer that it does not refer to a single item since they already have a word for the numerosity one.” (p. 229)

However, as Wynn recognized, the principle of contrast in isolation (i.e., without further constraints) does not make the right predictions. For example, when children were shown two sets and asked, “Can you show me the blicket balloons?”, they showed no preference for the set of five. Not all word meanings are constrained by the meaning of one. Instead, Wynn (1992) argued, the difference between blicket and five is that only five denotes numerosities. Thus, children must restrict their inferences about numeral meaning via appeal to other numerals: “If children know that each of the number words refer to a specific, unique numerosity, then they will restrict the meanings of the number words so that no two refer to the same numerosity. For example, consider a child who knows the cardinal meanings of the words up to “two.” This child’s knowledge of the word “two” also gives her knowledge of the word “three”; she knows that “three” does not refer to 2. When shown pictures of two versus three items and asked to
point to the three items, she will therefore succeed even though she doesn’t know how many “three” does refer to.” (p. 237)⁶

Expanding on Wynn’s study, Condry and Spelke (2008) provide strong evidence that children draw on knowledge of known numerals when interpreting “unknown” number words. In their study, one- and two-knowers who could recite the numerals one through ten made numerous inferences about the meanings of unknown numerals. For example, having heard a set labeled as eight, children denied that this same unmodified set was four, despite failing to judge that eight is a better label for sets of eight objects than four. Thus, they recognized that unknown numerals contrast in meaning, without knowing what these specific meanings are. Further, children recognized that unknown numerals like eight denote larger sets than known numerals like one or two, despite not knowing whether unknown numerals denote greater sets than other unknowns (e.g., whether eight is greater than four). This suggests that children make inferences about unknown numeral meanings based on known meanings. Relevant to our later discussion, Condry and Spelke also provide evidence that although children think unknown numerals contrast in meaning (with known numerals and with each other), they do not think that these meanings are exact (see Sarnecka & Gelman, 2004, for evidence that children assume that unknown numbers contrast in meaning). Specifically, children in their study failed to recognize that a given numeral – e.g., eight – no longer applies to a set if an object is removed, or if the set is either doubled or halved (though see Sarnecka & Gelman, 2003, for conflicting results).⁷

⁶ Wynn actually skips a step, by assuming that other quantifiers do not restrict numeral meanings. This prediction is nonetheless confirmed by other studies, which show that numeral meanings do not constrain the interpretation of other quantifiers like some and a (Barner et al., 2009; Condry & Spelke, 2008). For example, when shown two sets, one of which is labeled with a numeral, children point at the previously labeled set half the time when asked to point to some. Also, younger 2-year-olds accept that some can apply to sets of one, in addition to a, suggesting that initially one does not constrain their interpretation (though older children are less likely to judge sets of one as compatible with one; Barner et al., 2009; Condry & Spelke, 2008).

⁷ This fact indicates that children do not think unknown meanings are exact, since an exact semantics would predict that for any numeral, N, there is an exclusive cardinality, N*. However, the opposite finding (e.g., if children judged that eight no longer applies when the cardinality changes) would not alone indicate exactness. Children might
To account for such observations, both Wynn (1990) and Condry and Spelke (2008) conclude that children assume that numerals contrast in meaning, that they contrast only with other numerals, and that the meanings of known numerals restrict the possible denotations of unknown numerals. Deploying this knowledge when interpreting an unknown numeral like *five* therefore involves at least four distinct steps:

1. **Compute basic meaning** of the sentence $S$ containing *five*, where *five* denotes some unspecified cardinality.
2. **Generate a set of alternatives** $(a_1, a_2, \ldots, a_n)$ to $S$, called $S^{alt}$. These are all the sentences that can be generated by replacing *five* with its scalar alternatives (*one*, *two*, *three*...).
3. **Restrict the alternatives in $S^{alt}$** to the set containing informative numerals called $S^*$ (i.e., for unknown numerals, informative alternatives are those numerals with known, upper-bounded, meanings).
4. **Augment** (or “exhaustify”) the basic meaning of $S$ (containing *five*) with the negation of all of the members of $S^*$ (containing $S^*$).

First, the word *five* is represented as a numeral that denotes some unspecified cardinality (*step i*). Knowing that *five* is a numeral, children automatically generate other numerals as scalar alternatives (*step ii*). Particularly relevant are numerals that have a known meaning, such as *one*. Numerals without known meanings do not constrain interpretation, since they fail to provide specific meanings to eliminate from consideration – i.e., they do not specify upper boundaries (*step iii*). As a result, unknown numerals are not constrained by other unknown numerals; despite being scale mates. Finally, based on the contrast between the unknown numeral and the known numeral(s), the child augments the interpretation of the unknown numeral by excluding known numerals that denote different sets (i.e., that numerals contrast with respect to the cardinalities that they denote) without thinking that they are *exact*. 
meanings (i.e., that impose an upper bound) from consideration in its interpretation (step iv; i.e., five = Some unknown cardinality but not one).

In short, to interpret five in the way that Wynn describes, one-knowers must restrict the numeral’s meaning by appeal to the meanings of known numerals, a process that closely resembles the computation of scalar implicatures. Recall the four steps by which weak meanings are augmented via appeal to stronger scale mates:

i. Compute basic meaning of a sentence S containing L, a scalar lexical item, where L has a weak (or basic) meaning.

ii. Generate a set of alternatives (a₁, a₂...aₙ) to S, called S_{alt}. These are all the sentences that can be generated by replacing L with its scalar alternatives.

iii. Restrict the alternatives in S_{alt} to the set containing only the stronger expressions (i.e. such that a entails S, but S does not entail A), called S*.

iv. Augment (or “exhaustify”) the basic meaning of s (containing L) with the negation of all of the members of S* (containing S*).

One possible difference between Wynn’s contrast algorithm and full-fledged implicature involves the third step. The appeal to relative strength or entailment relations (step iii) is a central feature of implicature calculation, beginning with Grice’s quantity maxim (Grice, 1989). However, it is unclear whether strength is considered when children interpret unknown numerals. On the one hand, it is possible that children assign highly specified (though weak) meanings to unknown numerals, equivalent to the existential quantifier “some” (e.g., five = “some cardinality”). In this case, five could be ordered with respect to one, and Wynn’s algorithm would be a special case of implicature. Alternatively children might treat unknown numerals as semantically underspecified. They may know that the unknown numeral should be assigned some specific cardinality, but not know which one. In this case, inferences based on strength
would not be possible, since the complete meaning of the numeral would be unavailable.\(^8\) In either case, this particular difference between Wynn’s view and full-fledged implicature disappears once numeral meanings are acquired. At this point, known numerals are eligible to be ordered according to strength, thereby permitting the calculation of entailment relations.

Putting aside the question of how unknown meanings are represented, the inferences that children make for unknown numerals otherwise share their central properties with true implicatures. This point has important consequences for how we think about children’s interpretation of known numerals, which clearly do support entailment relations. Specifically, if children can restrict the interpretation of unknown numerals via appeal to known ones, as Wynn shows, then it follows that they should also be capable of using known words to restrict other known words, as in required for scalar implicature. Any computational machinery that is available to the interpretation of unknown numerals should also be available to the interpretation of known words. Therefore, Wynn’s observations raise the possibility that children do in fact compute implicatures to arrive at exact interpretations of known numerals (and if not “true” implicatures, then inferences that are equivalently strong, for the purposes of interpreting and strengthening numerals)\(^9\). By extension, Wynn’s observations also raise the possibility that children’s difficulty computing implicatures is restricted to quantifiers. Later, in Section 6, we present evidence that children’s interpretation of quantifiers and numerals differs not because of

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\(^8\) In fact, there is evidence from Condry and Spelke (2008) that children think unknown numerals denote larger sets than known numerals. This, however, may not reflect the partial meanings of unknown words, but may be due to inferences based on contrast (i.e., the only cardinalities that remain unlabeled are greater than those denoted by known numerals). Future experiments should explore these alternatives.

\(^9\) So far as we can tell, the behaviors of children satisfy all known criteria of implicature once they have acquired numeral meanings. In fact, we believe that the evidence presented here for implicature is somewhat richer than elsewhere in the literature, where simply distinguishing two scalar items is normally sufficient. Still, as we note, whether or not the label “implicature” is appropriate is ultimately moot, since children can clearly make inferences using scale members to restrict the interpretation of numerals. Thus, children’s failure to make such inferences for quantifiers does not demonstrate that exactness must be lexical. Clearly, children have all of the knowledge necessary to strengthen numeral meanings, whether or not we choose to call these implicatures.
different lexical semantics (i.e., weak meanings for quantifiers and strong meanings for numerals), but because of differences in how children access and interpret scalar alternatives in each domain. Young children compute implicatures more easily for numerals than for quantifiers, we argue, because they readily access the alternative set for numerals (i.e., the count list) but have difficulty accessing the alternative set for quantifiers.

Evidence that N-Knowers have Non-Exact Knowledge of N+1

Thus far, we have argued that children can compute scalar inferences for numerals (though perhaps without appeal to strength when numeral meanings are unknown). To support our argument that children make full-fledged implicatures to derive exact interpretations for known words, the present section presents evidence that children initially assign non-exact meanings to numerals in acquisition.

In order to demonstrate this point, it is first necessary to contrast how each hypothesis explains the behavior of an N-knower in Wynn’s knower level system. Consider the behavior of a one-knower on the Give-a-Number task. Such a child gives one object (and only one object) when asked for one but never gives one when asked for other numbers. The exactness hypothesis explains this behavior by assuming that such a child knows only the meaning of one (i.e., exactly one). Based on Wynn’s criteria, children cannot be one-knowers if they give two objects when asked for one, or if they give one object when asked for two.

In contrast, according to the theory by which children acquire weak, non-exact, meanings, so-called “one-knowers” actually know both the meaning of one and of two. By our account, before the child can assign an exact interpretation to one, she first acquires a weak, non-exact meaning, which is equivalent to that of the indefinite article a. At this point, the child who knows one may give any amount when asked for one (i.e., they give “at least” one). This weak meaning of one is strengthened only when children acquire the weak, non-exact, meaning of two. At this
point, children assign an exact interpretation to *one* by computing a scalar implicature – i.e., they generate the weak meaning, *one*, generate the alternatives (*two, three, four,* etc.), restrict the alternatives to the only stronger item, *two*, and then augment *one* by eliminating the meaning of *two*, resulting in only “exactly one” as a possible interpretation. By this account, a one-knower knows *one* and *two*, but not the meaning of *three*, and therefore only *one* can be strengthened to receive an exact interpretation. As a result, *two* remains consistent with any set of two or more.

In many cases these two theories make overlapping predictions about children’s behavior on counting tasks. By almost any theory of number word acquisition, children must observe correspondences between numerals and cardinalities. *Two* corresponds most frequently to sets of two, and *three* to sets of *three*. Without such correspondences, nothing could prevent *ten* from meaning *one* and vice versa. The main point of departure between the theories is not these correspondences between numerals and cardinalities, but the logical boundaries that numeral meanings impose. By the non-exact semantics, children who treat *N* as exact (Wynn’s *N*-knowers) must also know the meaning of *N+1*, whereas by the exactness hypothesis *N*-knowers cannot know the meaning of *N+1* (i.e., they should not treat *N+1* as exact). Therefore, to decide between the exact and non-exact accounts we can ask whether there is evidence that *N*-knowers know something about the meaning of *N+1* (i.e., its correspondence to sets of *N+1*) despite failing to assign it an exact interpretation. Below, we test this prediction, and show that *N*-knowers treat *N+1* differently than higher numerals like *N+2, N+3*, etc. Specifically, we show that children respond correctly for *N+1* more often than they do with higher numerals despite failing to assign it an exact interpretation.

The relevant data come from two paradigms – Wynn’s Give-a-Number task (GN), and the “What’s on this Card” task. First, consider children’s behavior on GN. It has sometimes been informally observed by researchers who use the Give-a-Number task that children who are
classified as N-knowers frequently give correct amounts for N+1 (e.g., non-knowers frequently give one when asked for one). However, based on Wynn’s criteria, discussed earlier, these children cannot be called N+1 knowers because they also give N+1 frequently for higher numbers. That is, these children do not observe a boundary between N+1 and higher numerals. Thus, they are not called N+1 knowers because the knower level system presupposes an exact semantics for numeral meanings (i.e., that knowing N means assigning it both a lower and an upper bound). Typically, it is assumed that children’s tendency to give correct amounts for N+1 must be due to a response bias (e.g., to give N+1 by default, in absence of knowledge). This conclusion would perhaps be convincing if such cases were rare, or if N+1 responses were randomly distributed across unknown numerals. However, the behavior is very frequent and specific to requests for N+1 objects, consistent with the hypothesis that children have non-exact knowledge of N+1 when they are N-knowers.

Consider three data sets: Japanese and English data contributed by our own labs (from Barner, Libenson, Cheung, & Takasaki, in press; Barner, Chow, & Yang, 2009), Japanese, Russian, and English data from Sarnecka and colleagues (Sarnecka et al., 2007) and English data from LeCorre et al. (2006; see Appendix I for details about subjects and procedures for each study). For each data set, we performed an analysis in which we asked how often children who were non-knowers gave one object when asked for one, how often one-knowers gave two for two, and how often two-knowers gave three for three. Thus, how often did N-knowers, who only treat N as exact, give N+1 for N+1. Data are shown in Table 1.

Across the studies, N-knowers gave a correct amount for N+1 53% of the time. The lowest level of correct responding was 23% (by non-knowers in Barner et al.’s English-speaking children), while the highest level was 75% (by Sarnecka et al.’s English-speaking children; the
value for Russian non-knowers is 100%, but this reflects the data from only one child).\textsuperscript{10, 11}

Overall, children gave N+1 when asked for N+1 at a level significantly better than expected by chance ($p < .001$).\textsuperscript{12}

\begin{table}[h]
\centering
\caption{Average percentage of trials on which N-knowers gave N+1 when asked for N+1}
\begin{tabular}{llcccc}
\hline
Data source & Language & $\emptyset$-knower & 1-knower & 2-knower & Overall \\
\hline
LeCorre et al. & English & 62 & 47 & 27 & 43 \\
Sarnecka et al. & English & 75 & 73 & 49 & 66 \\
 & Japanese & 70 & 63 & 53 & 65 \\
 & Russian & 100 & 67 & 45 & 60 \\
Barner et al. & English & 23 & 37 & 24 & 29 \\
 & Japanese & 38 & 54 & 53 & 44 \\
\hline
Average & & 51 & 60 & 41 & 53 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{10} In Barner’s studies, children chose from among 8 objects, making the likelihood of choosing any given cardinality 1/8, or .125). Children in both LeCorre’s and Sarnecka’s studies chose from 15 objects (chance = .07).

\textsuperscript{11} N-knowers by definition always give correct responses at least 66.5% of the time when asked for N. In practice, they do so more frequently (e.g., 97% of the time in the Barner et al., 2009 study).

\textsuperscript{12} W=18962, N = 218, $p < .001$; Children’s number of correct responses was compared to the number of correct responses expected by chance (calculated individually for each child) using a Wilcoxon Signed-Ranks test. When tested with 15 objects the chance of giving a correct response is 1/15 (over three trials, the number of correct responses is 3/15). An analysis of variance examined effects of Knower Level (non-knowers vs. one-knowers vs. two-knowers), Language (English vs. Russian vs. Japanese), and Study (Barner vs. LeCorre vs. Sarnecka) on percentage correct for N+1 trials. There were no significant effects of Language, Knower Level (KL), or Study on performance. There were, however, significant interactions between KL and Study, KL and Language, and Study and Language (all $p$’s < .05). Such differences may be due, in part, to differences in age across studies. Children in Sarnecka et al’s study were both older than LeCorre’s and Barner’s (both $p$’s < .001) and more likely to give correct responses for N+1 (both $p$’s < .05), whereas children in the other two studies differed neither in age, nor in the rate at which they gave N+1 for N+1 ($p$’s > .5). Overall, age was significantly correlated with the rate at which children gave N+1 for N+1 ($r = .20, p < .005$) even when controlling for effects due to knower level ($p < .001$). Knower level, however, was not significantly correlated with giving N+1 for N+1 ($r = -.08, p > .2$).
By definition, we know that many of these children also gave N+1 for numerals greater than N+1, and that this is what led them to be categorized as N-knowers – i.e., they did not draw a boundary between N+1 and higher numerals, and thus did not treat N+1 as exact. One might suppose, by the exactness hypothesis, that children have some form of default response of giving N+1 objects for all numerals that they do not know (i.e., for N+1, N+2, etc.). This explanation fails, however, since children gave N+1 more often for N+1 than for other numerals. To assess this, we analyzed data from Sarnecka et al. (2007) since this study did not use the typical titration method, and thus collected data for one, two, three, five, six, and ten. Across English, Russian, and Japanese children in the study, N-knowers gave N+1 objects when asked for N+1 64% of the time, but gave N+1 only 51% of the time when asked for quantities greater than N+1 (t(117) = 4.19, p < .001). So, for example, non-knowers, by Wynn’s terminology, were more likely to give one object for one than for higher numerals. This suggests that children labeled as N-knowers (who only treat N as exact), actually treated N+1 as distinct from higher numerals. This fact is clearly inconsistent with the hypothesis that responses beyond N are random, or purely attributable to a default response or response bias.

A second possible explanation of children’s behavior for N+1 is that they have partial knowledge of exact numeral meanings. Children who are called N-knowers may be in the process of acquiring exact meanings for N+1, and therefore may access this exact meaning sporadically, resulting in better than chance, yet not quite perfect, performance.

Three facts militate against this hypothesis. First, as we have already noted, the reason why N-knowers are not called N+1 knowers is specifically that they fail to draw a boundary between N+1 and N+2 (e.g., by giving N+1 objects for higher numerals). The failure to contrast N+1 with higher numerals is predicted by a non-exact semantics, but not by the exactness hypothesis. Second, data from Wynn’s Point-to-N task indicate that children’s knowledge of N+1, if partial,
could not be exact. In Wynn’s study, when one-knowers were shown a set of 2 and a set of 3, for example, they responded precisely at chance when asked to point to *two* or to *three*. This result is consistent with thinking that either two or three are consistent with the word *two*, and with having no knowledge of *three*. It is not consistent with having partial exact knowledge of *two*. Partial knowledge would predict better than chance (though maybe not perfect) performance not only for *two* (based on its lexical meaning) but also for *three* (by contrast with a sporadically accessed exact meaning for *two*). However, neither result is reported by Wynn, at any knower level.

[Insert Figure 4 about here]

Finally, an experiment conducted with English-speaking adults in our labs suggests that children’s tendency to give N+1 for N+1 is consistent with an adult “at least” interpretation. In the experiment, we tested 16 adults using the Give-a-Number task, but asked participants to give “at least N” objects (they were also asked to give “exactly N”, “more than N”, “less than N”, or “at most N” objects; see Appendix III for detailed methods). When asked to give “at least N” objects (where N was equal to *one, two, three, or four*), adults did not give random amounts greater than N. Instead, their modal response was to give precisely N, which they did 58% of the time on average (see Figure 4). These data, and other data presented in Figure 4, suggest that when speakers are asked to give “at least N”, they give the quantity closest to N that satisfies the request. This, we suggest, is also how children behave, so long as they know the meaning of the word in question.

Independent evidence for a non-exact semantics in early acquisition comes from Wynn’s “What’s on this card?” task (WOC). In the WOC task children are shown large flashcards that depict different numbers of common objects such as bears, shoes, or balls, and are asked simply, “What’s on this card?”. If children fail to provide a numeral, then they are prompted to provide
more information. For example, the child might be asked, “How many?” and if they respond with only a numeral they might be asked, “So, what’s on this card?”, with the intention of eliciting a complete noun phrase (e.g., “three bears”).

### TABLE 3
Verbal responses to sets of 1, 2, 3, and 4 on the “What’s on this card” task, from LeCorre & Carey (2007) and current study (expressed in percent)

<table>
<thead>
<tr>
<th>Knower level</th>
<th>Verbal response</th>
<th>LeCorre &amp; Carey</th>
<th>Current</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-knowers</td>
<td>one for 1</td>
<td>53</td>
<td>50</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>one for 2</td>
<td>8</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>two for 2</td>
<td>55</td>
<td>69</td>
<td>63</td>
</tr>
<tr>
<td>one-knowers</td>
<td>two for 2</td>
<td>69</td>
<td>79</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>two for 3</td>
<td>76</td>
<td>46</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>three for 3</td>
<td>14</td>
<td>38</td>
<td>19</td>
</tr>
<tr>
<td>two-knowers</td>
<td>three for 3</td>
<td>44</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>three for 4</td>
<td>31</td>
<td>50</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>four for 4</td>
<td>16</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Overall</td>
<td>N+1 for N+1</td>
<td>58</td>
<td>73</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>N+1 for N+2</td>
<td>54</td>
<td>31</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td>N+2 for N+2</td>
<td>18</td>
<td>42</td>
<td>23</td>
</tr>
</tbody>
</table>

To assess this, we analyzed data from 48 children reported by LeCorre and Carey (2007), and data from 14 children collected in our labs (see Appendix II for details about subjects and procedures for each study). As shown in Table 3, when children were asked to name sets beyond their number knower level (as determined by Give-a-Number) their performance was far from random. For example, when non-knowers were asked to name sets of one object, they responded correctly 51% of the time, with the word *one*. Similarly, one-knowers labeled sets of two with *two* 72% of the time, and two-knowers labeled sets of three as *three* 50% of the time. Overall, N-knowers gave correct responses 62% of the time for N+1 and only 23% of the time for N+2 ($t(53) = 4.56, p < .001$). Also, and crucially, they said N+1 for sets of N+1 significantly more often.
than they said N+1 for sets of N+2 (62% vs. 49%; \( t (53) = 2.37, p < .05 \)). Thus, N-knowers were more likely to give correct responses for N+1 than for N+2, and were more likely to say N+1 for N+1 than for other numerals.

To salvage the claim that these children actually don’t know N+1, we might again try to explain correct usages via a response bias. Several studies have noted that, before becoming two-knowers, many children say *two* for all sets greater than one. This has led to suggestions that *two* may act as an early marker of plurality for children, before they acquire its precise meaning (Carey, 2004; Clark & Nikitina, 2009; Sarnecka et al., 2007). This proposal is supported by the very frequent use of *two* by children in many cases. For example, it might explain why one-knowers correctly say *two* for sets of two between 70% and 80% of the time. However, it cannot explain why non-knowers say *one* for sets of one (more than 50% of the time) but not for sets of two (less than 5% of the time), or why two-knowers say *three* for only three (not for four), and why they do not say *four* for four. Also, when all *two* responses are removed from the analysis (i.e., those responses in Table 3 that are circled by a dotted line), the pattern of results becomes even stronger than before. When *two* responses are not considered, N-knowers say N+1 for N+1 52% of the time, but 16% of the time for N+2. Similarly, N-knowers only say N+2 for N+2 about 22% of the time, on average.

In summary, data from children acquiring three different languages, collected by three independent labs, find evidence that N-knowers frequently provide correct responses when asked to give or name sets of N+1.

As argued above, such data cannot be naturally explained by existing accounts that posit exact numeral meanings. A simple response bias to give N+1 for unknown numerals cannot explain children’s behavior. Further, the results are inconsistent with the idea that N-knowers have “partial knowledge” of N+1, unless by partial knowledge we mean non-exact meanings.
Instead, the data presented here are consistent with the hypothesis that children’s initial numeral meanings are non-exact. By our hypothesis, so called one-knowers, who only treat one as exact, actually know the meaning of two. Likewise, two-knowers know the meaning of three, and three-knowers know the meaning of four. Thus, to become one-knowers, children must acquire the weak, non-exact meanings of both one and two. When interpreting one, children generate expressions containing two as a scalar alternative, and augment the interpretation of one by eliminating from consideration the meaning of two (i.e., stronger scale mates). By this account, two cannot receive an exact interpretation, since children have not yet acquired the meaning of three (i.e., “at least three”). Once children acquire the meaning of three, they become what Wynn calls two-knowers.

Given this analysis, children’s early differentiation of a and one can now be understood. Recall that when children become one-knowers they assign an exact interpretation to one but not to singular noun phrases like a banana. By the standard knower-level account, children assign an exact interpretation to one as soon as they acquire its meaning. This generates a puzzle, since it is unclear why children would assign distinct lexical meanings to a and one. Even if one were more strongly associated with sets of one than a, this would not explain how children know to treat alone one exactly (rather than assigning it a weak meaning, like for a and other quantifiers).

We propose that children do not initially assign an exact interpretation to one. Instead, evidence from tasks like “Give-a-Number”, “What’s on this Card?” and “Point-to-N” indicate that prior to becoming one-knowers, children acquire a weak meaning for one, at which time its meaning is identical to a. Our suggestion is that children become one-knowers only once they have acquired the weak meaning of two, at which time their interpretations of one and a part ways. By this account, assigning one an exact interpretation depends on first acquiring two, and deriving “exactly one” via implicature. Children fail to assign exact interpretations to singular
nouns (e.g., *a banana*) because they fail to access the plural as a scalar alternative (we will return to this point in section 7). As a result, *a* is initially assigned a weak interpretation whereas *one* receives a strong, exact interpretation.

**Informative Boundaries to Implicature**

We have now argued that children assign weak meanings to numerals and that they compute implicatures to strengthen their interpretations. To complete this argument, one final point must be addressed. In her analysis, Wynn proposed that known numeral meanings constrain the interpretation of unknown numerals. Knowing *one* prevents one-knowers from giving one object for *two*, and from pointing at one object when asked to point to *five*. We agreed with Wynn’s posit, but extended the logic of her argument to suggest that children should also be able to use known meanings to constrain not only the interpretation of unknown numerals, but also known ones – i.e., to compute scalar implicatures. However, to fully explain the data, we must be more precise. By our account, a one-knower knows the meanings of both *one* and *two* but, as Wynn observed, one-knowers fail to use *two* to constrain higher numbers. The question for the present account is why weak meanings (like *two* in this case) do not constrain the interpretation of unknown successors.

In fact, our answer to this question does not differ much from that of Wynn: by either an exact or a non-exact semantics, the one-knower’s meaning for *two* cannot constrain the interpretation of higher numerals because the one-knower’s lexical meaning for *two* is logically consistent with every cardinality of two or more (since it fails to specify an upper bound). To constrain the interpretation of unknown numerals, known words must specify an upper bound either lexically (e.g., via exact lexical meanings), or inferentially (e.g., via implicature). For the exactness hypothesis any known numeral can act as an alternative for implicature since all of these words impose exact lexical meanings. For the non-exact hypothesis, only strengthened
meanings can act as alternatives for unknown numerals. Thus, since *one* is strengthened and *two* is not, only *one* is an informative alternative to unknown numbers like *three*.

To clarify this from the perspective of the non-exact semantics, let us be explicit about the meaning that N-knowers assign to “unknown” numerals (i.e., those for which they have not yet acquired an adult meaning, such as N+2). By our account, a one-knower would assign weak meanings to both *one* and *two*. For all higher numerals, they would assume that the word denotes some unspecified cardinality, following Wynn. Thus, to interpret “*one* x”, the one-knower would compute an implicature as follows:

1. Compute a weak lexical meaning for *one*
2. Generate alternatives: *two, three, four*...
3. Restrict alternatives to stronger items: *two*
4. Augment: *one X \& \neg two X* (i.e., exactly one).

By this description, *one* is strengthened by appeal to its stronger scale mate *two*, and receives an exact interpretation. To interpret “*two* x” the one-knower would:

1. Compute a weak lexical meaning for *two*
2. Generate alternatives: *one, three, four*...
3. Restrict alternatives to those that are informative: *one* (strengthened via implicature)
4. Augment: *three X* (i.e., some unspecified cardinality, and not *one*).
According to this description, the only informative alternative to three is the strengthened meaning of one. This is true, since three fails to contrast with the basic, weak, meanings of one and two (since one is consistent with any cardinality of one or more, and two with any cardinality of two or more). All cardinalities are consistent with these meanings, and they therefore provide no informative boundaries to new numeral denotations. Thus, our suggestion is that the child uses augmented meanings to compute implicatures for unknown numerals (akin to what Spector, 2007, calls a “second-order implicature” in his discussion of the singular-plural distinction; for similar proposals extended to disjunction and other quantifiers, see Fox, 2007; Chierchia, 2004; Kratzer & Shimoyama, 2002; Chierchia, Spector, & Fox, 2007). Since the augmented meaning of one (exactly one) specifies an upper bound, it can meaningfully restrict the interpretation of three to “some cardinality greater than one”. However, since two is not upper-bounded, it cannot serve as a point of contrast. As a result, children sometimes give two when asked for three, and point to sets of two when asked to point to at three.

This account resembles Wynn’s in suggesting that only numerals with exact interpretations can constrain unknown numerals. However, it differs with respect to how these exact interpretations originate. For Wynn, exactness is lexical, whereas by our account the exact interpretation of one is derived via implicature.

The Availability of Scalar Alternatives and the Numeral / Quantifier Divide

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13 One possible interpretation of three that children might arrive at, in this hypothetical scenario, is, ironically, “exactly one”. Since “at least one” is no stronger than the existential “some x”, “at least one” by most accounts would not be generated as a scalar alternative to three. The only scalar alternative available would be “at least two”, such that children might interpret three to mean ~at least 2 X (i.e., exactly one).

14 Spector (2007), for example, makes use of second order implicatures to derive the interpretation of the singular/plural distinction. For Spector, the basic meanings of the plural and the singular overlap, as in 2-year-olds. The strengthened interpretation of the plural (at least 2) emerges as a second order implicature since it requires (as an alternative) the strengthened interpretation of the singular (exactly one). See Sauerland (2004) for a similar proposal. Spector also notes that higher-order implicatures are often unnecessary, since final interpretations are often obtained by 1st order implicatures. This is the case for known numerals, for example.
We have argued that the behaviors described by the studies of Wynn (1990, 1992) and Condry and Spelke (2008) involve a form of scalar inference, and that children may therefore make use of full-fledged implicatures for known numerals. By our account, children may not lack a general capacity to compute implicatures, despite the fact that they have difficulty doing so for other scalar items, like quantifiers. In this section we suggest that this quantifier-specific difficulty reflects a difference in the availability of alternatives rather than a wholesale pragmatic deficit.

Studies that find incomplete pragmatic competence in children, like Papafragou and Musolino (2003), are sometimes silent with respect to which of the four steps above causes children difficulty, and the computations involved in implicature remain undifferentiated. For example, Papafragou and Musolino consider two possible sources of children’s difficulty: “One possibility is that this failure reflects a genuine inability to engage in the computations required to derive scalar implicatures. Another possibility is that this failure is due to the demands imposed by the experimental task on an otherwise pragmatically savvy child.” (p. 261)

However, it is possible to be more specific in a way that can distinguish computations that are general to all implicatures from those that are specific to quantifiers (for example, see Tantalou & Papafragou, 2004; Papafragou, 2006). The evidence from Wynn (1990) indicates that children’s problem is not related to augmentation based on scale mates, since children apparently do augment when interpreting numerals that they have not yet mastered. To know that \( five \) cannot refer to sets of one, children must augment the interpretation of \( five \) by appeal to the meaning of \( one \). It is also not the case that children do not know the core meanings of quantifiers (see Barner et al., 2009, for review). The problem, we suggest, is that children have difficulty generating scalar alternatives for many scales, and thus are often unaware of which scalar alternatives are relevant in a given context. This difficulty, however, does not extend to
numerals, since children begin acquiring them as explicit alternatives early in acquisition (even before they assign meaning to these terms).

The evidence for this claim has two components. First, there is evidence that children can compute implicatures for quantifiers when the relevance of scalar alternatives is augmented contextually. Second, there is evidence that young children can compute implicatures when alternatives are mentioned explicitly in a context. Together, these points suggest that when scalar alternatives for quantifiers are available children can deploy them for computing implicatures.

First, Papafragou and Musolino (2003) show that, when trained to seek strong alternatives to weak expressions, children are more likely to compute scalar implicatures in a later task. In their study, children were introduced to a puppet who “sometimes says silly things”, and were encouraged to help her “say things better”. In a training phase, children were presented four scenarios in which the puppet either labeled objects in under-informative ways – e.g., calling a dog a “little animal with four legs” – or using stronger descriptions – e.g., “this is a dog”. For the weaker descriptions, children were asked to help the puppet say it better. When children failed to provide a stronger term, the word was provided explicitly by the experimenter. Thus, children were trained to detect the difference between strong and weak descriptions, and to strengthen descriptions when possible. Given this training, children were significantly more likely to compute implicatures relative to conditions that lacked training. This indicates that children can augment expressions when it is clearly required in a context, and when they have been prompted to generate scalar alternatives (see also Tantalou & Papafragou, 2004, and Papafragou, 2006; see Musolino, 2004, for evidence that 5-year-olds can access “at least” and “at most” interpretations for numerals when favored contextually).

The second piece of evidence that implicates difficulty generating alternatives comes from a study by Chierchia and colleagues (Chierchia et al., 2001; see also Gualmini et al., 2001). In
their study, Chierchia et al. investigated children’s interpretation of or, and whether they could
distinguish its weak, inclusive, interpretation from its strong, exclusive one. In contexts that
licensed a strong interpretation for adults – e.g., *Every boy chose a skateboard or a bike* –
children accepted weak interpretations of or (e.g., scenarios in which a boy chose both a
skateboard and a bike). However, Chierchia et al. also showed that when scalar alternatives were
made explicitly available to children, they had no difficulty choosing the stronger expression
when required by the context. For example, in a context in which all farmers had cleaned both a
horse and a rabbit, 3- to 6-year-old children preferred the sentence in (7) over the sentence in (8)
more than 90% of the time:

(7) Every farmer cleaned a horse or a rabbit
(8) Every farmer cleaned a horse and a rabbit

According to Chierchia et al., this result indicates that children have the knowledge relevant
to computing implicatures, and that children’s failure in absence of explicitly provided
alternatives is due to a problem accessing the relevant scalar alternatives. Similarly, Gualmini et
al. (2001) suggest that children are unable to “to construct the relevant alternatives on-line, such
that they fail to compute implicatures if the alternatives are not explicitly presented to them.”
Thus, by this account, children are able to represent the relative strength of quantifiers,
and are able to augment when alternatives are made available. What children specifically cannot
do is access the relevant alternatives for words like or, and we would argue, words like some.

Numerals, we suggest, do not cause this difficulty, since the first thing that children learn
about numerals is that they are members of an ordered list (Fuson, 1988), and thus that they are
alternatives to one another. Before acquiring their meanings, this knowledge leads children to

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15 The truth conditions of logical or are such that, for a proposition \( p \lor q \), the expression is T if \( p = T \), if \( q = T \), or if
\( p \land q \) is true. Thus: \( (p \land q) \Rightarrow (p \lor q) \). Exclusive or (i.e., \( \oplus \)) is stronger, such that \( (p \oplus q) = (p \land \neg q) \lor (q \land \neg p) \).
assume that numerals contrast in meaning (Condry & Spelke, 2008; Sarnecka & Gelman, 2004). Once numerals are mapped to meaning (and the ordered count list is mapped to an ordered set of cardinalities; LeCorre, under review), they provide boundaries to the interpretation of other numerals under the general principles of scalar implicature.

A Common Semantics for Quantifiers and Numerals in Language Acquisition

Based on results of previous studies and new data presented here, we have argued that children’s early representations of both numeral and quantifier meanings are weak and non-exact. Integer acquisition requires a logic like that needed for acquiring natural language quantifiers and determiners. Also, both systems draw on scalar inference for deriving strong interpretations. Integers differ from quantifiers, however, by providing an unbounded sequence of numerals, which children begin to acquire early in acquisition.

To support this view, we reviewed evidence from early number word acquisition. Following Wynn (1992), we argued that children draw on the meanings of scale mates when interpreting numerals, and that such behavior is consistent with the hypothesis that they compute (contrast-based) scalar inferences. This behavior demonstrates a capacity to augment numeral meanings, indicating that children’s difficulty with other instances of scalar terms does not lie with augmentation (which is general to all forms of implicature), but with generating scalar alternatives, which may vary from scale to scale, and may affect quantifiers while not affecting numerals. Since children’s first knowledge of numerals is of their organization in an ordered list, the structure of this list may guide their inferences about meaning in a way that is not possible for other scalar items. In support of this, we reviewed evidence that children are capable of computing implicatures when trained to seek strong interpretations of words, or when explicitly provided with scalar alternatives. Finally, we presented evidence from new and existing data that children actually acquire weak meanings for numerals before they can interpret them exactly.
A question left untouched by our study is whether integer acquisition draws on a special hypothesis space, distinct from that of quantifiers. Our study assumes that both quantifier and numeral meanings can be expressed by a common semantic language, and argues that, in either case, the interpretation of lexical meanings can be augmented via scalar implicature. This view, however, is neutral with respect to how particular lexical meanings are acquired, and what conceptual distinctions they draw on. It therefore does not address the question of whether integer acquisition requires mapping numerals to a special hypothesis space, or if such meanings can be constructed from non-linguistic systems of numerical representation (e.g., for review, see Carey, 2004; Gallistel, Gelman, & Cordes, 2005; LeCorre & Carey, 2007; Spelke & Tsivkin 2001; see Laurence & Margolis, 2005).

By some accounts, no special hypothesis space is required for integer acquisition. For example, Carey (2004) has suggested that the initial meaning of one may be borrowed from children’s knowledge of the singular-plural distinction, and that initially one equals a: “the partial meanings of number words seem to be organized initially by the semantics of quantifiers - the singular-plural distinction and the meanings of words like ‘some’ and ‘a’.” (p. 64). Echoing this, our argument has been that children’s very first numeral meaning – one – does not initially differ in interpretation from the indefinite article a. However, the two views differ in the meanings that are assigned to a and one. By Carey’s account, both meanings are exact from the get-go; simply being associated with singleton sets, by this view, is equivalent to having an exact lexical meaning (see also Le Corre & Carey, 2007; Sarnecka et al., 2007). Our assertion, in contrast, is that neither a nor one is initially exact, and that exactness cannot be gotten from reference alone. Instead, each meaning is weak and has an existential meaning (e.g., there exists a / one x), which does not exclude reference to sets greater than one (since the existence of one thing does not rule out the existence of two). Children’s interpretations of a and one part ways
when they acquire the meaning of two and thereby assign an upper bound to one, becoming one-knowers (by the criteria of Wynn’s knower level system). By this account, one is strengthened via appeal to its scale mate two, and thus receives an exact interpretation via scalar implicature (e.g., there exists only one x).

Given the evidence that children assign a weak interpretation to a when they treat one as exact (Barner et al., 2009), Carey’s initial hypothesis cannot be right. However, if we assume, following our argument, that both a and one have weak lexical meanings, then the hypothesis that numeral meanings are bootstrapped from quantifier meanings may yet be salvaged. Other non-linguistic number systems (e.g., parallel individuation and approximate number; Feigenson, Dehaene, & Spelke, 2004) may be relevant to verifying the meanings of numerals, but may not actually define these meanings. Instead, meanings may be defined by the same logic that defines natural language quantifiers like a and some.

To summarize, we have argued that young children can compute scalar inferences early in acquisition, that these inferences support the strengthening of numeral meanings, and that children begin acquisition with weak meanings. Future studies should explore whether weak meanings, a count list, and scalar inference are sufficient to support the development of exact numeral meanings in early acquisition.
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Inference and exactness


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LeCorre, M. (under review). Disorder in the count list.


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We would like to thank Mathieu Le Corre and Barbara Sarnecka for very generously sharing their raw data for analysis. Without this gesture, this paper would have been impossible. Thanks also to the following people for their comments, which greatly strengthened an initially weak paper (implicature intended): Alan Bale, Neon Brooks, Ivano Caponigro, Susan Carey, Stan Dehaene, Danny Fox, Mathieu Le Corre, Amanda Libenson, Anna Papafragou, Barbara Sarnecka, Jesse Snedeker, Benjamin Spector, Liz Spelke, Josh Tenenbaum, members of Ladlab, and three anonymous reviewers.
Appendix I – Summary of methods for Give-a-Number task

In each study cited, children were asked to give N number of objects or put N objects into a container (i.e. “Can you put two fish into the red circle?”). Children were called N-knowers (e.g., two-knowers) if they correctly gave N fish 2 out of 3 times when they were asked for N, but failed to give the correct number 2 out of 3 times for N+1. As well, in order to be an N-kower children were required not to give N when asked for other numbers. Children were credited as CP-knowers (Cardinal Principle knowers) if they could correctly give six fish at least two out of three times.

Two differences in methods existed between the studies. First, in Barner et al. (2009) children were presented 8 objects, whereas children tested by Le Corre et al. (2006) and Sarnecka et al. (2007) were presented 15 objects. Second, whereas Barner et al. and Le Corre et al. used the titration method (Wynn, 1992), Sarnecka did not. Using the titration method, when children successfully gave N fish (e.g., 3), they were then asked to give N+1 fish (e.g., 4). If they gave an incorrect response, even after being asked to correct their response, they were then tested with N-1 (e.g., 2). Testing ended when children had given the incorrect number 2 out of 3 times for a given number. In contrast, Sarnecka conducted 3 blocks of 5 trials. In each block, children were asked for one, two and three in a counterbalanced order and then five and six, in a counterbalanced order. Therefore, children were always tested on all numerals even if they failed to comprehend smaller numbers.

In Barner et al.’s study, participants were 45 Japanese-speaking children (mean age = 33.04 months) recruited from childcare centers in Fukuoka, Japan and 29 English-speaking children (mean age = 35.14) from the greater Boston and Toronto areas tested at local childcare centers or in the laboratory. In Le Corre et al.’s study participants were 28 English-speaking children (mean age = 33.85 months) recruited from the Boston area and tested at the laboratory or local
daycares. Finally, in Sarnecka et al., participants were 43 English-speaking children (mean age = 37.21 months) recruited from preschools in Ann Arbor Michigan, 43 Japanese-speaking children recruited from preschools in Kobe, Japan (mean age = 37.32) and 32 Russian-speaking children recruited from preschools in St. Petersburg, Russian (mean age = 37.49).

Table 1. Distribution of 0-, 1- and 2-knowers in the Give-a-Number task

<table>
<thead>
<tr>
<th>Data set</th>
<th>0-knowers</th>
<th>1-knowers</th>
<th>2-knowers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barner- English</td>
<td>8</td>
<td>14</td>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>Barner – Japanese</td>
<td>28</td>
<td>8</td>
<td>9</td>
<td>45</td>
</tr>
<tr>
<td>Sarnecka- English</td>
<td>4</td>
<td>26</td>
<td>13</td>
<td>43</td>
</tr>
<tr>
<td>Sarnecka- Japanese</td>
<td>22</td>
<td>16</td>
<td>5</td>
<td>43</td>
</tr>
<tr>
<td>Sarnecka- Russian</td>
<td>1</td>
<td>20</td>
<td>11</td>
<td>32</td>
</tr>
<tr>
<td>Le Corre - English</td>
<td>8</td>
<td>9</td>
<td>11</td>
<td>28</td>
</tr>
</tbody>
</table>
Appendix II: Summary of methods for *What’s on this Card?* task

*Le Corre & Carey* (2007). Participants were 44 English-speaking children (range = 2;6 – 3;11; *mean age* = 33.6 months) recruited from the New York area and Greater Boston area and tested in the laboratory or local daycare centers. On each trial the child was presented with a card that depicted anywhere between 1 and 8 objects (from among 8 different kinds of object, for a total of 64 cards). On each trial, the child was asked, “What’s on this card?” On the first trial, the experiment modeled the use of a single numeral to describe the set (e.g., “that’s right, that’s one ball”). The task then proceeded quasi-sequentially such that trials were randomized within increasingly large subsets (2 and 3, 4 and 5, and 6, 7, and 8).

*Data from our labs.* Participants were 14 English-speaking children including 5 girls (mean age = 31.5 months; range = 24;6 – 37;21) recruited from the greater Toronto area and tested in the laboratory or at local childcare centers.

*Stimuli.* Stimuli were flashcards that presented photographs of four types of inanimate objects on a white background: cars, balls, shoes, or keys. These objects were chosen based on the presence of the corresponding words in early speech, as determined by the MacArthur Communicative Development Inventory (MCDI). Each word was present in the speech of at least 80% of children over the age of 23 months. For each type of item (car, key, shoe, ball), 6 cards were created, each card presenting a different number of objects (1, 2, 3, 4, 6 or 8). This resulted in a total of 24 cards. Cards that depicted more than one instance of an item included multiple different photos (e.g., four distinct keys).

*Procedure.* Children were seated at a child-sized table and were told that they would play a game with the experimenter. To begin, they were familiarized with the object types using flashcards that each depicted a single object. Children were asked to label each item: “Do you know what this is?” If the child correctly labeled an item, the experimenter would move on to the
next item. If the child said they did not know or said something other than the target word (e.g., labeling a shoe as “a boot”), the experimenter would say “Can you say shoe?” and encourage the child to repeat the target word. If a child had incorrectly labeled an item or did not know the name of it, the experimenter returned to the item after all other items had been labeled to ensure that the child remembered its name.

Test trials were modeled after the procedure of Gelman (1993) and Le Corre and Carey (2007). The experimenter presented the child with cards one at a time, and asked, “What’s on this card?” for each. The first card that the child saw was always of a single item, (e.g., a single ball). On subsequent trials the cardinality depicted was pseudo-randomized, such that the same cardinality was never presented on two consecutive trials. On the first trial (with one object), if the child said “a ball” the experimenter replied, “That’s right, that’s one ball,” and asked the child to repeat “one ball” if they had not spontaneously produced a numeral. Data from first trials were excluded from analysis when children required prompting to produce a cardinal response. For all subsequent trials, if the child did not produce a numeral, the experimenter asked “How many?” in an attempt to elicit a numeral response. If the child counted but did not give a cardinal response (e.g., said “two” instead of “two balls”), the experimenter asked, “So, what’s on this card?” in an attempt to elicit a cardinal response. The experimenters refrained from uttering the target nouns or from using words that conveyed number information (e.g., a, these, those). If the child still could not provide a cardinal response, the experimenter continued with the next trial, and repeated the procedure until all 24 trials were completed.

Following the What’s on this Card? task, children’s knower levels were assessed using the Give-a-Number task, adapted from Wynn (1992). Stimuli consisted of a red plastic circle and a set of eight plastic fish. To begin, the experimenter presented the fish to the child and asked, “Do you like to count? Could you count the fish for me?” Then, the experimenter presented the
red plastic circle and asked the child to put a certain number of fish into the circle, starting with one (e.g., “Could you put one fish into the red circle?”). Following Wynn (1992) we used a titration method. When children successfully gave N fish (e.g., 3) they were then asked to give N + 1 fish (e.g., 4). When they failed, they were given a chance to count and correct their response. If they continued to provide an incorrect response, they were then tested with N-1 (e.g., 2).

Testing ended when children had given the incorrect number 2 out of 3 times for a given number. Children were called N-knowers (e.g., two-knowers) if they correctly gave N fish 2 out of 3 times when they were asked for N, but failed to give the correct number 2 out of 3 times for N + 1. Children were credited as CP-knowers (Cardinal Principle knowers) if they could correctly give six and seven fish at least 2 out of 3 times for each. Otherwise, their knower level was identified only up to the number that they could give correctly at least 2 out of 3 times (e.g., one-knower, two-knower, three-knower, or four-knower).

Table 2. Distribution of 0-, 1-, and 2-knowers in the What’s on this Card task

<table>
<thead>
<tr>
<th>Data set</th>
<th>0-knowers</th>
<th>1-knowers</th>
<th>2-knowers</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barner</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>Le Corre</td>
<td>3</td>
<td>24</td>
<td>18</td>
<td>45</td>
</tr>
</tbody>
</table>
Appendix III – Methods for Give-at-least-N task

Participants

Sixteen undergraduate students at the University of California, San Diego (10 females and 6 males) participated in this study for course credit.

Stimuli and Procedures

Participants were tested individually in the laboratory. Testing sessions lasted approximately 5 minutes. To begin, participants were presented with three sets of plastic fruit: 6 bananas, 6 oranges, and 6 strawberries. On each trial, participants were asked to put some quantity of a particular fruit into a red plastic circle. For each of the numbers two, three, and four, the experimenter requested “less than n”, “at most n”, “exactly n”, “at least n”, and “more than n” objects – e.g., “Can you put at least two bananas into the red circle?” In addition to these trials, participants were also asked for “exactly”, “at least”, and “more than” one (i.e., there were no “one” trials for “less than n” nor for “at most n”, since responses would be either impossible or meaningless). Requests were presented in one of two quasi-random orders, such that the same request was never made on two consecutive trials. If a participant asked for clarification, the original instruction (e.g. “put more than four oranges in the circle”) was repeated.
Figure 1. Quantifier scale representing gradations from weak *some* to strong *all* (based on Sapir, 1944; see Horn, 1989).
Figure 2. The percentage of yes responses for *a* and *one* for one or two objects, when 2-year-olds were asked – e.g., *Is there a banana in the circle?* or *Is there one banana in the circle?*
Figure 3. Number of 1-knowers who gave either one object or more than one when asked for “a” thing in the Give-a-Set task.
Figure 4. Percentage of trials on which adults gave $N$ objects, $N-1$ objects, $N+1$ objects (or some amount more or less than these quantities) when asked for “at least $N$,” “more than $N$,” “less than $N$,” “at most $N$,” or “exactly $N$” objects – e.g., “Can you put at least two bananas in the circle?”
Figure Captions

Figure 1. Quantifier scale representing gradations from weak some to strong all (based on Sapir, 1944; see Horn, 1989).

Figure 2. The percentage of yes responses for a and one for one or two objects, when 2-year-olds were asked – e.g., Is there a banana in the circle? or Is there one banana in the circle?

Figure 3. Number of 1-knowers who gave either one object or two or more objects when asked for “a” thing in the give-a-set task

Figure 4. Percentage of trials on which adults gave N objects, N-1 objects, N+1 objects (or some amount more or less than these quantities) when asked for “at least N,” “more than N,” “less than N,” “at most N,” or “exactly N” objects.