The Origins of Firm Heterogeneity: A Production Network Approach

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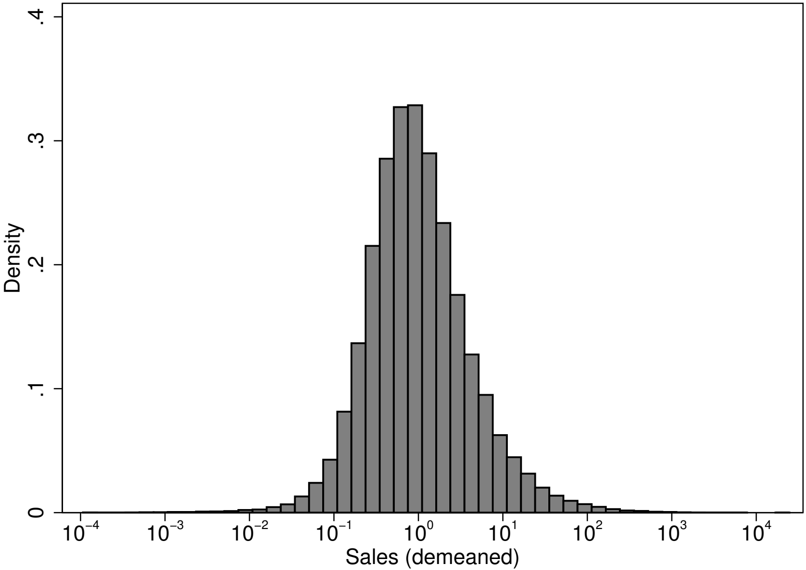
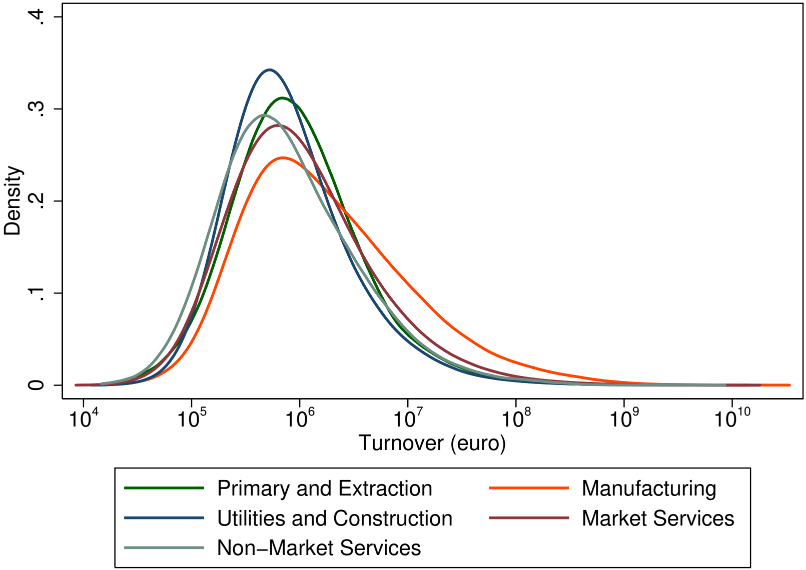
Preliminary

The Firm Size Distribution



The Firm Size Distribution



Motivation

Firm size distribution fundamental for

- Trade: who exports what & where, welfare gains.
- Macro: granularity, aggregate productivity, creative destruction.
- Labor: wages & income inequality (Autor et al, 2017).

Research Question

- Firms are part of production networks:
  - buy inputs from other firms.
  - sell to other firms and possibly to final consumers.
  - production network is sparse & most goods lack centralized exchanges.

- What is the role of the production network in explaining the firm size distribution?
  - Number of customers/suppliers.
  - Characteristics of customers/suppliers.
  - Match quality.

What We Do

- Model: Firms are both buyers and sellers
    - own unit costs depend on suppliers' unit costs.
    - own sales depend on customers' sales.
- Use model & production network data to identify firm-level fundamentals.
- Exact model-based decomposition of firm-level sales.
    - Demand (# customers, capability of customers).
    - Supply (production capability, # suppliers, capability of suppliers).
    - Match.
- Use estimates + embed model in GE to perform counterfactuals.
- Links are pre-determined.

Related Literature

- Firm size distribution:
  - Gibrat (1931), Sutton (1997).
- Firm size and productivity:
  - Jovanovic (1982), Hopenhayn (1992), Melitz (2003), Luttmer (2007), Arkolakis (2016).
- Firm size and demand:
  - Foster, Haltiwanger and Syverson (2015), Fitzgerald, Haller and Yedid-Levi (2016).
- Firm size variance decomposition:
  - Hottman, Redding and Weinstein (2016).
- Firm-to-firm trade:
  - Bernard et al (forthcoming), Eaton et al. (2016), Kramarz et al (2016), Lim (2016).
- Firm-employee matching:
  - Abowd, Kramarz and Margolis (1999), Card et al. (2015).

Today

- Data.
- Model & Exact decomposition.
- Estimation.
- Results.
- GE.
- Sensitivity.
- Conclusions.

Data : Relationships

- Panel 2002-2014 with annual sales relationships between all VAT reporting firms in Belgium.
  - Firm i selling to firm j for x euro in a given year within Belgium.

- Coverage (close to) universal,
  - All sectors except financial services.
  - All annual sales $m_{ij} >$ €250 included.

Data: Relationships

- 16.8 million B2B sales relationships in 2012.
- 812,543 firms (555,722 sellers and 797,595 buyers).
- Some firms only sell to final demand (buyers but not sellers).
- Top 10% relationships account for 92% of value.

Relationship	N	Median	Mean	Top 100	Top 10
value (€)	16,800,368	1,414	30,884	253 mill	1,310 mill

Table: Distribution of bilateral relationships (€s) (2012).

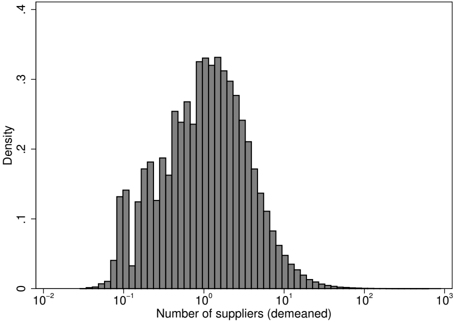
Data : Balance sheet

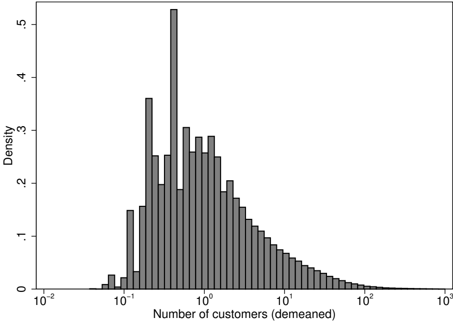
- In main decomposition, we merge with balance sheet to get data on:
  - Total sales S_i, input purchases M_i, labor cost share α_j, 2-digit industry (NACE).
- We retain 87% of aggregate gross turnover.

Industry	N	Mean	Median
All	111,581	7,361	788
Primary and Extraction	1,630	2,776	788
Manufacturing	11,052	21,400	1,276
Utilities	557	61,000	2,371
Market Services	93,449	5,610	775
Non-Market Services	4,893	4,553	419

Table: Sales (€000s) (2012).

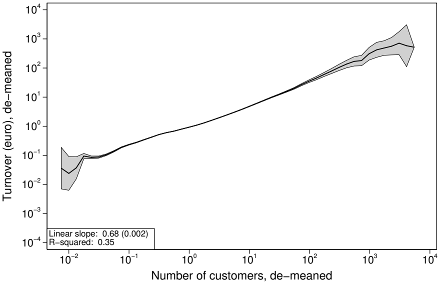
Heterogeneity in relationships

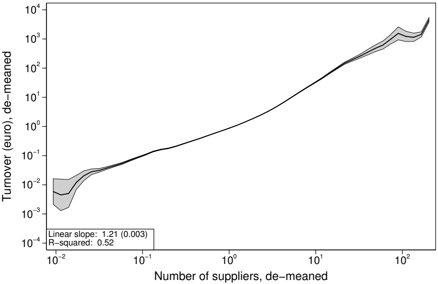




Median/mean indegree (outdegree) 9/21 (2/21).
Top 1% transact with more than 400 buyers and 177 sellers.

Relationships & Size





Bigger firms have more buyers and suppliers (within industry).
Average sales decreasing in # customers.

The Model

- Firms use labor and a CES bundle of inputs.
- Network is pre-determined.
- Heterogeneity in productivity/quality.
- Minimal set of assumptions.
    - We impose a few more when solving the GE later on.

The Model

Assumption

The production function is

$$y_i = z_i l_i^{\alpha} v_i^{1-\alpha}$$
$$v_i = \left(\sum_{k \in \mathcal{S}_i} (\phi_{ki} \nu_{ki})^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} .$$

- Marginal costs and prices

$$c_i = \kappa \frac{w^{\alpha} P_i^{1-\alpha}}{z_i} \qquad \text{and} \qquad p_{ij} = \tau_{ij} c_i .$$

- Firm-to-firm sales

$$m_{ij} = \left(\frac{\phi_{ij}}{p_{ij}} \right)^{\sigma-1} P_j^{\sigma-1} M_j .$$

The Model

Assumption

Demand & supply (mark-up) shifters are

$$\phi_{ij} = \phi_i \tilde{\phi}_{ij}$$
$$\tau_{ij} = \tau_i \tilde{\tau}_{ij}.$$

- Rewrite firm-to-firm sales to $m_{ij} = \psi_i \theta_j \omega_{ij}$ with

$$\psi_i \equiv \left(\frac{\phi_i}{\tau_i c_i} \right)^{\sigma-1}, \qquad \theta_j \equiv P_j^{\sigma-1} M_j, \qquad \omega_{ij} \equiv \left(\frac{\tilde{\phi}_{ij}}{\tilde{\tau}_{ij}} \right)^{\sigma-1}.$$

- We discuss identification of $\Psi = \{\psi_i, \theta_j, \omega_{ij}\}$ soon.

The Exact Decomposition

- Fraction of sales/purchases within the network: β_i^c and β_i^s.
  - E.g., $1-\beta_i^c$ is exports.
- Total sales can be written:

$$\ln S_i = \ln \psi_i + \ln \xi_i - \ln \beta_i^c$$

$$\xi_i \equiv \sum_{j \in \mathcal{C}_i} \theta_j \omega_{ij}$$

- Variance decomposition, using Ψ and data on β_i^c:
  - Regress each component on $\ln S_i$.
  - Each coefficient gives share of variation in sales explained by margin.
  - All components demeaned at 2-digit industry level.

The Demand Side

- The demand parameter ξ_i can be written:

$$\ln \xi_i = \ln n_i^c + \ln \bar{\theta}_i + \ln \Omega_i^c,$$

$$\bar{\theta}_i \equiv \left(\prod_{j \in \mathcal{C}_i} \theta_j \right)^{1/n_i^c}$$

$$\Omega_i^c \equiv \frac{1}{n_i^c} \sum_{j \in \mathcal{C}_i} \omega_{ij} \frac{\theta_j}{\bar{\theta}_i}$$

- Variance decomposition, using Ψ and data on n_i^c:
  - Regress each component on $\ln \xi_i$.

The Supply Side : The Reflection Problem

- *Reflection problem*: Seller effect ψ_i embodies technology of suppliers.
- Given estimates Ψ and data on (M_i, α), back out own production capability:

$$\tilde{z}_i \equiv \left(\frac{\phi_i z_i}{\kappa \tau_i w^\alpha} \right)^{\sigma-1} = \psi_i \left(\frac{\theta_i}{M_i} \right)^{1-\alpha}$$

- Intuition:
  - Cheap inputs $\longrightarrow \theta_i / M_i = P_i^{\sigma-1}$ low.
  - To get $\tilde{z}_i$, seller effect ψ_i must be adjusted down.

The Supply Side

- The seller effect ψ_i can be written

$$\ln \psi_i = \ln \tilde{z}_i + (1 - \alpha) \left[\ln n_i^s + \ln \bar{\psi}_i + \ln \Omega_i^s - \ln \beta_i^s \right],$$

$$\bar{\psi}_i \equiv \left(\prod_{k \in \mathcal{S}_i} \psi_k \right)^{1/n_i^s}$$

$$\Omega_i^s \equiv \frac{1}{n_i^s} \sum_{k \in \mathcal{S}_i} \omega_{ki} \frac{\psi_k}{\bar{\psi}_i}$$

- Variance decomposition, using Ψ and data on $\{M_i, \alpha, n_i^s, \beta_i^s\}$:
  - Regress each component on $\ln \psi_i$.

Estimation

- Step 1: Estimate Ψ.
- Step 2: Calculate variables for decomposition: $\ln \xi_i$, $\ln \tilde{z}_i$, $\ln \bar{\theta}_i$, $\ln \bar{\psi}_i$, $\ln \Omega_i^s$ and $\ln \Omega_i^c$.
- Step 3: Exact firm size decomposition.

Estimation : Step 1

- The aim is to estimate

$$\ln m_{ij} = \ln \psi_i + \ln \theta_j + \ln \omega_{ij}.$$

- $\ln \psi_i \approx$ average market share of i among her customers.
- $\ln \theta_j \approx$ average market share of j among her suppliers.
- We get unbiased estimates when

$$E\left[\ln \psi_i \ln \omega_{ij}\right] = E\left[\ln \theta_j \ln \omega_{ij}\right] = 0.$$

- Conditional exogenous mobility:
  - Assignment of suppliers to customers exogenous wrt ω_{ij}.

Estimation : Step 1

- Identifying assumption holds if firms match based on..
  - $\ln \psi_i$ and $\ln \theta_j$.
  - Fixed costs (e.g., Lim, 2016, Bernard et al, forthcoming).
  - Pair-wise shocks unrelated to ω_{ij} (e.g., Eaton et al, 2015).
- Robustness 1: Asymmetry test a la Card et al (2015).
- Robustness 2: Include covariates for ω_{ij}.

Estimation : Step 1 : Avalanching

- ≥ 2 customers $\longrightarrow$ seller effect $\ln \psi_i$.
- ≥ 2 suppliers $\longrightarrow$ buyer effect $\ln \theta_i$.
- Avalanching: firm A dropped $\longrightarrow$ firm B too few links $\longrightarrow$ B dropped as well.
- Only main component used.
- 99% of links retained, 95% of value
- 75% of sellers retained, and 89% of buyers

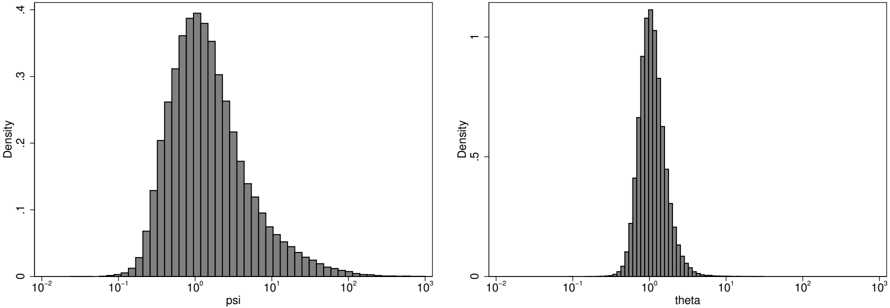
Initial			Baseline, share			
# Links	# Sellers	# Buyers	Links	Value	Sellers	Buyers
16,800,368	555,722	797,595	.99	.95	.75	.89

Table: Avalanching (2012).

Estimation : Steps 2 & 3

- Step 2:
  - Use estimates Ψ with observables ($\ln S_i$, $\ln M_i$, $\ln \beta_i^c$, $\ln \beta_i^s$, $\ln n_i^c$, $\ln n_i^s$, $\mathcal{C}_i$ and $\mathcal{S}_i$) to get ($\ln \xi_i$, $\ln \tilde{z}_i$, $\ln \bar{\psi}_i$, $\ln \bar{\theta}_i$, $\ln \Omega_i^s$ and $\ln \Omega_i^c$).
  - Firms with missing S_i and M_i still part of 1st stage, but not main decomposition.
  - α is total wage bill relative to production costs by 2-digit industry.
- Step 3:
  - Demean variables by NACE 2-digit industry average.
  - Regress each component on demeaned $\ln S_i$ / $\ln \psi_i$ / $\ln \xi_i$.

Results : 1st Stage



$$R^2 = .43,\ sd\left(\ln \psi_i\right) = 1.15,\ sd\left(\ln \theta_i\right) = .50,\ corr\left(\ln \psi_i, \frac{\ln \theta_i}{M_i}\right) = -.10$$

Results : Main Decomposition

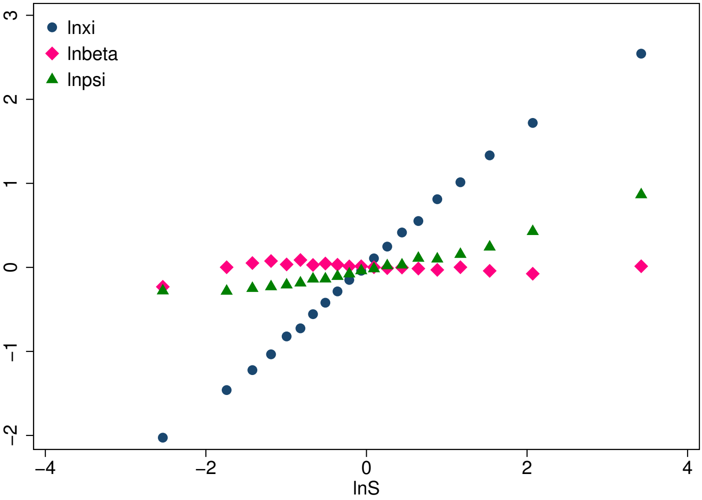
	N	Final Demand $\ln\beta_i^c$	Supply $\ln\psi_i$	Demand $\ln\xi_i$
$\ln S_i$	95,529	.00	.15***	.85***

Notes: Significance: * < 5%, ** < 1%, *** <0.1%.

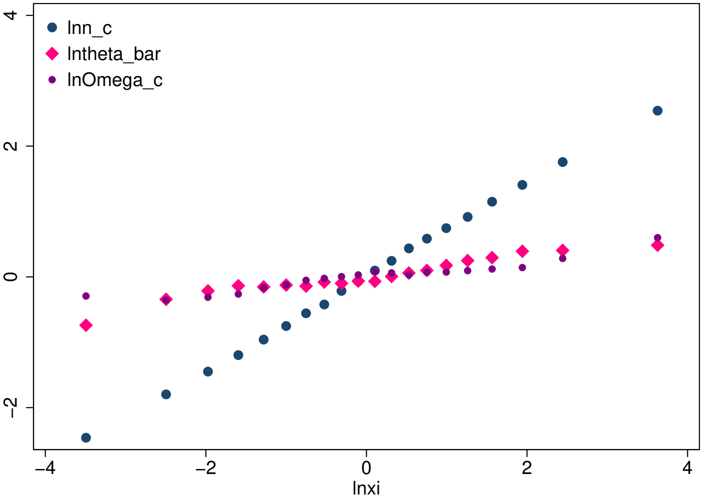
Table: Overall decomposition (2012).

- Variance in sales mostly explained by the demand component.
- Firms are not big because they have large market shares among their customers.

$$\ln S_i = \ln \psi_i + \ln \xi_i - \ln \beta_i^c$$

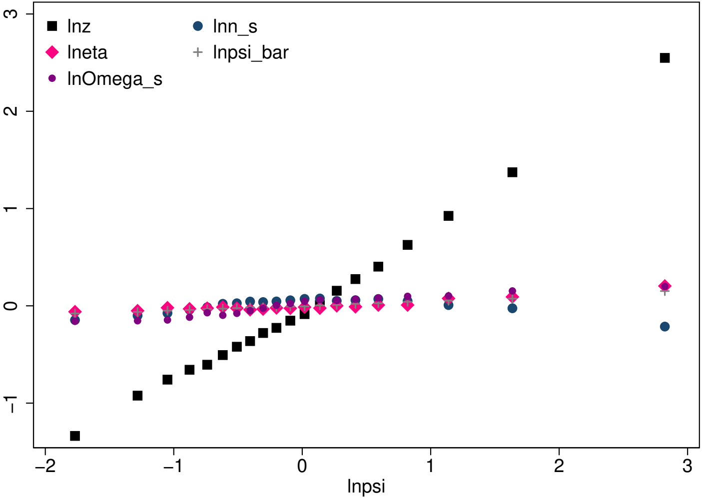


$$\underset{.69}{\ln \xi_i = \ln n_i^c} \underset{.14}{+ \ln \bar{\theta}_i} \underset{.17}{+ \ln \Omega_i^c}$$



Results : Supply

$$\ln \psi_i = \underset{.96}{\ln \tilde{z}_i} + (1-\alpha) \; (\underset{.02}{\ln n_i^s} + \underset{.00}{\ln \bar{\psi}_i} + \underset{.02}{\ln \Omega_i^s} - \underset{.00}{\ln \beta_i^s})$$



General Equilibrium

Close the model and perform counterfactual analyses.

- CES final demand with same elasticity of substitution σ.
  - Consumers supply labor inelastically.
  - They are shareholders of the firms
  - Income is $X = wL + \Pi$, where Π is aggregate profits.
- Constant (but potentially heterogeneous) mark-ups.
  - Consequently, $\mu_i \equiv M_i/S_i = (1-\alpha)\,/Markup_i$ is constant.

Solution Method

Solution method:

1. Use estimates of $\tilde{z}_i$ and ω_{ij} in *backward fixed point*:

$$\tilde{P}_j = \sum_{i \in \mathcal{S}_i} \left(\frac{p_{ij}}{\phi_{ij}} \right)^{1-\sigma} = \sum_{i \in \mathcal{S}_i} \tilde{P}_i^{1-\alpha} \tilde{z}_i \omega_{ij},$$

   with $\tilde{P}_j = P_j^{1-\sigma}$.

2. Use (i) estimates of $\tilde{z}_i$ and ω_{ij}, (ii) data on μ_i and (iii) solution for $\tilde{P}_j$ in *forward fixed point*:

$$S_i = \tilde{z}_i \tilde{P}_i^{1-\alpha} \left(\frac{X}{\tilde{\mathcal{P}}} + \sum_{j \in \mathcal{C}_i} \frac{\mu_j S_j}{\tilde{P}_j} \omega_{ij} \right),$$

   where $\tilde{\mathcal{P}}$ is the final demand price index.

Next steps: Counterfactuals, e.g. shocks to z_i, ω_{ij}, network structure.

Extensions & Robustness

- Exogenous mobility test.
- Observables for ω_{ij} : Distance.
- Firm growth.
- Across 2-digit industries.

Exogenous Mobility Test

- Empirical test in the spirit of Card et al (2015).
- Under exogenous mobility, we have

$$E\left[\ln m_{ij} - \ln m_{ik}\right] = -E\left[\ln m_{ik} - \ln m_{ij}\right] = \ln \theta_j - \ln \theta_k.$$

- Under endogenous mobility, moves from big to small may not lead to (a large) decline if moves are driven by unobserved shocks.

Exogenous Mobility Test

Procedure:

- Estimate Ψ for the 2005 cross-section ($t = 0$).
- Group firms into quartiles based on $\ln \theta_j$, q_k, $k = 1, 2, 3, 4$.
- *Upgraders*: Firms that have at least one q_1 buyer in $t = 0$ and add at least one q_4 buyer in $t = 1$.
  - Calculate $\ln m_{ij(q_4),t=1} - \ln m_{ij(q_1),t=0}$, where $j(q_k)$ denotes a customer in the q_k'th quartile.
  - Form the average of all possible combinations: $\bar{\Delta}_i^{Up}$.
  - Take the average of $\bar{\Delta}_i^{Up}$ across all upgraders.
- Calculate $\bar{\Delta}_i^{Down}$ for *downgraders.*
- Prelim results: $\left|\bar{\Delta}_i^{Down}\right| > \bar{\Delta}_i^{Up}$.

Extensions : Growth

- Perform decomposition on growth rates.
- Estimate Ψ in 2002 ($t = 0$) and 2014 ($t = 1$).
- Calculate change in every variable, e.g. $\Delta \ln S_i$.
- Sample:
  - Decomposition: Only survivors, with non-missing θ_i and ψ_i in both years.
  - But adding/dropping of relationships OK, e.g. the terms $\Delta \ln \psi_i$ and $\Delta \ln \xi_i$ change because of extensive margin.

Extensions : Growth

		Sales $\ln S_i$	Demand $\ln\xi_i$	Supply $\ln\psi_i$
Final Demand	$-\ln\beta_i^c$	$.11^{***}$		
Supply	$\ln\psi_i$	$.04^{***}$		
Demand	$\ln\xi_i$	$.85^{***}$		
# customers	n_i^c		$.57^{***}$	
Avg. customer capability	$\bar{\theta}_i$		$.08^{***}$	
Covariance	Ω_i^c		$.35^{***}$	
Production capability	$\tilde{z}_i$			1.03^{***}
# suppliers	n_i^s			$-.02^{***}$
Avg. supplier capability	$\underline{\psi}_i$			.00
Covariance	Ω_i^c			.00
Outside network supply	β_i^s			-.01
Obs.		41,923	41,923	41,923

Notes: Significance: * < 5%, ** < 1%, *** <0.1%.

Table: Growth decomposition (2012).

Extensions : Across Industries

NACE	Industry	N	Final Demand $-\ln\beta_i$	Supply $\ln\psi_i$	Demand $\ln\xi_i$
01-09	Primary and Extraction	2,836	-.03	.24	.79
10-33	Manufacturing	16,886	-.01	.26	.75
35-39	Utilities	849	.04	.15	.81
41-43	Construction	18,976	-.10	.11	.99
45-82	Market Services	52,908	.05	.18	.77
84-96	Non-Market Services	1,134	.02	.13	.85

Table: Across industries (2014).

Conclusions

- Exact decomposition framework yields new insights:
  - Variance in sales mostly driven by customer component.
  - The supply component mostly driven by own productivity.
- This begs the question: How do firms attract customers?

Appendix : Correlation Matrix

	$\ln S_i$	$\ln \psi_i$	$\ln \xi_i$	$\ln \beta_i^c$
$\ln S_i$		.14	.61	.00
$\ln \psi_i$			-.22	.47
$\ln \xi_i$				.49
$\ln \beta_i^c$				

Table: Correlation Matrix (2012).