2015-16 Online Round 3

Friday, February 12th, 2016

Senior Division Problems

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Problem S1: Energy Absorption

15 Points / Time Limit: 2.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc153s1*

What is a savior but an emblem of hope – but a monument for the people to look up to? Should he be anything that the world needs him to be – or should he be none of it? What does a savior owe the world? The time has come for the man of steel to finally face these difficult questions. And while Superman is nothing short of a god amongst mortals, Batman is… well, Batman.

Superman knows that fighting the dark knight is going to be no easy task. He knows that Batman will enter the battle with a heavily armored mech-suit with a multitude of unpredictable special abilities. It is very likely that Batman's armor will already be prepped to take on the stupendously powerful blows that Superman can deal. From collaborating with the U.S. government, Superman has received intel saying that Batman's suit has the ability to absorb the kinetic energy from any strike dealt onto it and store it for later use to power Batman's own offensive blows. This is an incredibly sneaky technology, and seems to give Batman a huge edge in battle. However, he has also learned this advanced technology has not yet been perfected. That is, the armor itself may fail at capturing the kinetic energy of the blows, resulting in actual damage being done. Luckily, Superman found out about it beforehand and now has a chance to plan out his attack strategy.

In particular, Superman plans to land N ($1 \le N \le 10^6$) punches onto Batman's armor. The punches are numbered from 1 to N, and he will deal them in order. By analyzing the stolen blueprint of the mech-suit, Superman knows that the i-th punch he lands on Batman will deal D_i ($-1000 \le D_i \le 1000$) units of damage to his suit. Specifically, the damage will be negative if Batman's suit happens to absorb $|D_i|$ units of Superman's kinetic energy, or it will be positive if the absorption fails and Superman successfully deals D_i units of damage.

Superman is no fool. In the battle, he will continue to perform punches 1 to N in order, but will stop as soon as the next punch would cause the net damage on Batman's suit to become negative. The government will do its best to help Superman by sabotaging Batman's suit beforehand, so when the battle starts, the amount of initial damage sustained by Batman's suit will be positive. Still, Superman would like to land as many hits onto Batman as possible because it will nonetheless exhaust the aging hero beneath the armor.

Since the only piece of information that Superman doesn't have is the initial damage to Batman's suit when he enters the fight, he would like your help in writing a program that can quickly compute how many punches he'll throw based on the original damage to Batman's suit. In particular, your program must answer M ($1 \le M \le 500{,}000$) queries where the i-th query asks: "if Batman's suit were to have Q_i ($1 \le Q_i \le 10^9$) units of damage done to it when he enters the battle, what is the number punches Superman will throw before making the net damage to Batman's suit negative?"

In test cases worth 6/15 of the points, $N \le 1000$ and $M \le 1000$.

Input Format

The first line of input contains two space-separated integers N and M.
The second line of input contains N space-separated integers $D_1, \ldots, D_N$.
The third line of input contains M space-separated integers $Q_1, \ldots, Q_M$.

Output Format

M lines with a single integer on each line, the answers to queries 1..M.

Sample Input

```
4 3
-3 5 -3 -3
2 3 4
```

Sample Output

```
0
3
4
```

Explanation

If Batman's suit starts with 2 units of damage sustained from the sabotage, then punching him would give him energy and immediately lower the damage done to −1, so Superman won't do it.

If the suit starts with 3 units of damage, it will be lowered to 0 units of damage after receiving the first blow, but raised to 5 units of damage after the second blow, and lowered to 2 units of damage after the third blow. By then, Superman will have realized that it's not worthwhile to make the fourth blow (which will make the net damage −1).

If the suit starts with 4 units of damage, it will have sustained 3 units of damage going into the final blow. Thus Superman will be able to deal the last punch and still have a nonnegative damage of 0 sustained by the suit. Although this last punch seems to be suboptimal in Superman's favor, he will still deal it nonetheless because he wants the battle to be as long as possible to fatigue Batman.

Problem S2: Detective Work

20 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc153s2*

While Superman was preparing in secrecy for the fight to come, feelings of fever, rage and powerlessness were festering within the dark knight of Gotham. He can do nothing but watch as the planet praise an alien capable of annihilating it. The peace of Gotham he fought so long to protect is now compromised by a godly figure that has unworthily gained the world's reverence.

To take care of this threat for good, the masked billionaire will have to elaborate a plan like no other he's ever concocted. In preparation, Batman plans on doing some heavy-duty detective work to figure out Superman's weaknesses. The logical place to start is by determining his secret identity. After compiling an abundance of data on Superman's movements and past battles, as well as newspaper articles written about him, he will need a program that can efficiently cross-reference them to fill in the missing pieces and hopefully uncover the man behind Superman. As the new intern of the batcave, your first task is to write a cryptanalysis program that can process the compiled data to help Batman figure out who Superman really is.

The data that Batman has compiled consists of N ($1 \le N \le 1000$) uppercase, alphabetical strings $S_1, \ldots, S_N$, each of length M ($1 \le M \le 500$). A good detective knows that no data from the real world is ever fully complete or accurate, so Batman has decided to represent unknown information in the strings with wildcard letters denoted by question marks. Specifically, each of these N strings will only consist of uppercase letters from "`A`" to "`Z`", as well as special wildcard characters ("`?`"). The instincts of the world's greatest detective tell him that it will be possible to transform all of these strings into the same string (any string X of Batman's choice, made up of M letters), and hopefully this will be the real name of Superman. To accomplish this, some of the letters in the strings may be replaced with other letters at a cost of 1 each. But for this method to be effective in actually extracting useful data, the number of letters changed must be as small as possible! The lower the total cost, the more likely it is for the result to be accurate. Each wildcard character can also be changed into any letter of Batman's choice, at no cost.

But measuring this cost is not definitive, so Batman would also like to know the "error" of the process. In particular, this error is defined as the number of different strings X that Batman can choose, such that all N strings can be transformed into X with this minimal cost. Knowing this error is important, because the larger the total number of possibilities, the less likely that X is the actual name of Superman. Since this error value can actually get very large, Batman doesn't care any more than the last 3 digits (if the error is in the thousands or more, the data is unreliable anyway). So you are only required to compute the error modulo 1000. For example, if the error is 2016, simply output 16. Fortunately, Batman is also a genius-level number theorist so he has made you aware of the following two identities, which may prove useful:

- $(A + B) \bmod M = ((A \bmod M) + (B \bmod M)) \bmod M$
- $(A \times B) \bmod M = ((A \bmod M) \times (B \bmod M)) \bmod M$

Will Batman be able to solve one of the greatest-kept secrets in the world to help him defeat the most dangerous being in the world? His fate rests in your hands.

Input Format

The first line of input will contain two space-separated integers N and M.
The next N lines will each contain a single string, where line i contains S_i, for $i = 1..N$.

Output Format

The output should be a single line consisting of two space-separated integers. The first integer should be the cost, i.e. the minimum possible number of letters which must be changed to allow all *N* words to be turned into the same word. The second integer should be the error, i.e. the number of ways in which this can be accomplished (modulo 1000).

Sample Input 1

```
4 9
??ARKKANT
CL?RK?ENT
SUPERGIRL
???RK?UNT
```

Sample Output 1

```
11 64
```

Explanation 1

All four strings can be turned into the word "`CLARKKENT`" with minimal cost. The second string can be transformed to that with no extra cost, since the wildcard characters line up. The last "`A`" is wrong in the first string, so it will take a cost of 1 to make it an "`E`". The third string shares no matching letters whatsoever with "`CLARKKENT`", and therefore all 9 letters of it should be swapped. Finally, the "`U`" in the last string is incorrect, and should be switched for an "`E`" at a cost of 1. Similarly, there are 64 other words that may be reached by making exactly 11 replacements.

Sample Input 2

```
1 3
???
```

Sample Output 2

```
0 576
```

Explanation 2

With only one string, any assignment of the wildcard characters to letters will suffice. Each one can independently be turned into any of the 26 letters, so there are $26^3 = 17576$ possibilities. This set of data is clearly not that helpful for Batman.

Problem S3: Rescue Mission

25 Points / Time Limit: 2.00s / Memory Limit: 256M *Submit online: wcipeg.com/problem/wc153s3*

While the dust settles in the battle between the dark knight and the last son of Krypton, the two have suddenly come to recognize the true mastermind behind it all. After General Zod's invasion on Earth, LexCorp was contracted by the U.S. government to carefully reverse-engineer leftover Kryptonian technology. As it turns out, the battle between the heroes had simply been Lex Luthor's test in his greater conspiracy to craft a powerful biotechnological weapon against Superman. At that point, it became clear to both that the world is in much graver danger than ever before. Now, the heroes have decided to team up and together, track down Luthor to end the evil mastermind's ploys once and for all.

They have tracked down Luthor's activities preceding their fight to Cadmus, a LexCorp subsidiary dedicated to genetic research on the outskirts of Metropolis. Unbeknownst to even the government, Cadmus has been conducting dangerous human experimentation for a secret biotechnology project. Infiltrating the facility, the heroes have discovered a huge number of human subjects that are held prisoner by Luthor. As they plan to secretly rescue all of them, Lex has already discovered their presence. Immediately, he sets his men, armed with advanced weaponry, into action to capture Batman and Superman. With a limited amount of time before they can escape, the duo must try to rescue as many prisoners as possible.

The Cadmus facility can be represented as a rectangular grid with N rows of M cells each ($2 \le N, M \le 2000$), with the rows numbered 1..N from north to south, and the columns numbered 1..M from west to east. The j-th cell in the i-th row can be denoted as cell (i, j). Each cell (i, j) happens to contain $P_{i,j}$ ($0 \le P_{i,j} \le 10^5$) prisoners. Batman and Superman must work together to rescue as many as they can – but with the clock ticking, they have to make their rescue plan as efficient as possible.

Thus, Batman and Superman will each decide on their own "rescue zone". Each rescue zone is defined by a central point (r, c), a non-negative height h, and a non-negative width w. A rescue zone consists of $h + w + 1$ cells – the central cell itself, the h cells directly above it, and the w cells directly to the left of it. This zone is only valid if all of its cells are within the grid – that is, $0 \le h < r \le N$ and $0 \le w < c \le M$ must hold true. However, to avoid getting in each other's way, their chosen rescue zones should not overlap with one another. In other words, no cell in the Cadmus facility may be part of both rescue zones. Following this, all prisoners in each cell which are either part of Batman or Superman's attack zones will be rescued.

Please write a program to help the Earth's greatest heroes determine the maximum number of prisoners that can be rescued this way.

Input Format

The first line contains two space-separated integers N and M.
The next N lines each contain M integers, where line i contains integers $P_{i,1}, \ldots, P_{i,M}$, for i = 1..N.

Output Format

Output a single integer, the maximum number of prisoners that can be rescued by Batman and Superman.

Sample Input

```
4 6
2 0 3 0 0 0
0 0 0 0 4 0
0 1 1 2 0 0
2 0 3 0 0 1
```

Sample Output

```
15
```

Explanation

One optimal pair of rescue zones is as follows:

- r = 3, c = 5, h = 1, w = 1 (containing 4 + 2 = 6 prisoners)
- r = 4, c = 3, h = 3, w = 2 (containing 3 + 1 + 3 + 2 = 9 prisoners)

Problem S4: Lex Luthor's Landmines

40 Points / Time Limit: 4.00s / Memory Limit: 512M *Submit online: wcipeg.com/problem/wc153s4*

Lex Luthor's international masquerade as a humanitarian has attracted the attention of Princess Diana of Themyscira, also known as Wonder Woman. Thanks to her unexpected addition to team Batman and Superman, all the prisoners from Cadmus have fortunately been rescued. But it's too late. Luthor has already unleashed his dangerous bio-weapon of unimaginable power – Doomsday. To destroy the reputation of Superman, Doomsday has been wreaking havoc and causing wanton destruction upon Metropolis. Doomsday was originally a deadly monster born from the depths of ancient Krypton. Having worked so hard to reanimate the abominable legend from his very own laboratories on Earth, Luthor might as well make this moment count.

The mastermind knows for a fact that while Doomsday is trampling the city, the heroes will be intently focused on minimizing civilian casualties. To further tie up their hands during their clash with Doomsday, he has decided to throw in the age-old weapon of landmines. The current street of the fight can be represented as a number line of integer positions numbered from 0 to 10^9, inclusive.

Luthor has planted N ($1 \le N \le 10^6$) landmines at (not necessarily distinct) positions on the street, where each landmine is numbered uniquely from 1 to N. The i-th landmine is located at position X_i ($0 \le X_i \le 10^9$) on the street. But there is a catch – these landmines have been specially engineered to explode in an asymmetrical fashion. When the i-th landmine goes off, its explosion reaches L_i ($1 \le L_i \le 10^9$) units to the left (in the negative direction), and R_i ($1 \le R_i \le 10^9$) units to the right (in the positive direction). As such, its explosion reaches all positions in the inclusive range $[X_i - L_i, X_i + R_i]$ on the street.

Lex Luthor will control Doomsday to set off exactly one landmine of his choice – however, this may start a chain reaction! If any other landmines are within the initial landmine's explosion range, they'll also go off. Their explosions may in turn set off even more landmines, and so forth. As if fighting the deadly beast isn't enough on their plates, Batman, Superman, and Wonder Woman will need to evacuate the citizens of the street to safety from the landmines. To do so, they will have to evaluate how dangerous certain positions on the street are.

They know that M ($1 \le M \le 300{,}000$) civilians numbered from 1 to M are currently located at possibly non-distinct points on the street, where the i-th civilian is located at position C_i ($0 \le C_i \le 10^9$). For each civilian i, they'd like to know the number of different initial landmines that Doomsday could set off which would cause that civilian to be engulfed in at least one of the resulting explosions. Please write a program to help the trio answer these questions. Superman's reputation is dwindling with every moment of Metropolis's destruction and countless civilian lives are at stake, so you'd better code fast!

For cases worth 30% of the points, $N \le 500$ and $M \le 500$.

Input Format

The first line of input will contain two space-separated integers N and M.
The next N lines will each contain three space-separated integers X_i, L_i, and R_i, for $i = 1..N$.
The next M lines will each contain a single integer C_i, for $i = 1..M$.

Output Format

Output M lines, each containing a single integer – the number of initial landmines which would cause the civilian at position C_i to be engulfed by an explosion, for i = 1..M.

Sample Input

```
4 4
10 5 12
4 5 6
15 1 1
20 10 80
101
100
0
14
```

Sample Output

```
0
3
1
4
```

Explanation

Position 101 is too far to be within any possible explosion.
Position 100 will be engulfed by the 4th landmine's explosion if either the 1st, 2nd, or 4th landmine is initially set off. If the 2nd one is set off, for example, its explosion will set off the 1st landmine, which will set off the 4th one, which will just barely reach the point.
Position 0 will only be engulfed if the 2nd landmine is initially set off.
No matter which landmine is initially set off, position 14 will always be engulfed.