WOBURN CHALLENGE

2016-17 Online Round 4

Sunday, April 9th, 2017

Junior Division Problems

Automated grading is available for these problems at:
wcipeg.com

For more problems from past contests, visit:
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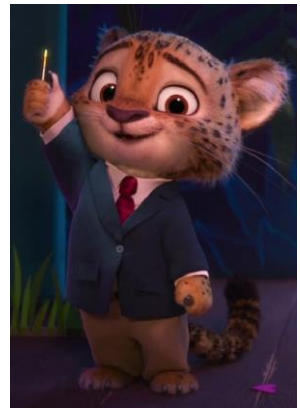
Problem J1: Anyone Can Be Anything

14 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc164j1*



> *"I don't have to cower in a herd anymore. Instead, I can be an astronaut!"*
> *"I don't have to be a lonely hunter anymore. Today I can hunt for tax exemptions; I'm gonna be an actuary!"*
> *"And I can make the world a better place, I am going to be… a police officer!"*

In an ever more progressive and open-minded society, Zootopia's slogan "Anyone can be Anything" is becoming a reality at last. Its animal inhabitants are no longer strictly bound to traditional professions based on their species, and are instead free to pursue any careers that they please.

There are 2 different categories of jobs in Zootopia, with jobs which have typically been performed by predators falling into category 1, and jobs traditionally associated with prey falling into category 2. There are also N ($1 \le N \le 15$) young animals who are preparing to join the workforce. Each animal will get assigned to one of the job categories. However, their assignment will no longer be dictated by their species – instead, the i-th animal will apply for their preferred job category P_i ($1 \le P_i \le 2$), and will have a fair shot of getting a spot in it, with the help of the city's strict anti-discrimination hiring laws.

Unfortunately, however, even the overcoming of deep-rooted social prejudices doesn't mean that everyone can end up happy. For example, even if everybody wants to become an astronaut, society can realistically only continue running smoothly if the majority of its members contribute in more practical roles, regardless of their wishes. To that end, the i-th job category has O_i ($0 \le O_i \le N$) openings, indicating that exactly O_i new animals must be assigned to it. The total number of openings $O_1 + O_2$ is equal to the number of animals N, meaning that once every animal has been assigned to a job category, both categories' openings should be exactly filled.

Perhaps everyone can't be anything, but at least some animals can be allowed to live out their dreams. Once all N animals have been assigned to job categories in some valid fashion, what's the maximum number of animals which can end up having been assigned to their preferred job category?

Input Format

The first line of input consists of a single integer N.
The second line consists of two space-separated integers O_1 and O_2.
The third line consists of N space-separated integers $P_1, \ldots, P_N$.

Output Format

Output a single line consisting of a single integer – the maximum number of animals who can be assigned to their preferred job category.

Sample Input

```
5
3 2
2 1 1 2 2
```

Sample Output

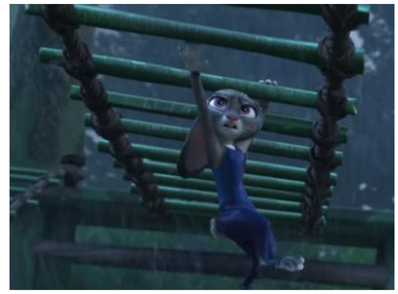
```
4
```

Sample Explanation

One optimal arrangement is to assign the first and last animals to the 2nd job category, and the remaining 3 animals to the 1st job category. In this situation, all of the animals will be assigned to their preferred job category besides the 4th animal, who will get stuck with job category 1 instead of 2.

Problem J2: You're Dead!

18 Points / Time Limit: 2.00s / Memory Limit: 16M *Submit online: wcipeg.com/problem/wc164j2*



> *"Scorching sandstorm! You're dead, bunny bumpkin."*
> *"One-thousand-foot fall! You're dead, carrot face!"*
> *"Frigid ice wall! You're dead, farm girl!"*
> *"Enormous criminal! You're dead!"*

Judy Hopps is determined to earn her place onto Zootopia's Police Department as the city's first rabbit cop, but it won't be easy. In order to graduate from the policy academy, she'll need to demonstrate her skills by completing a series of N ($1 \le N \le 1000$) physical challenges. The i-th challenge takes place in an environment with a temperature of T_i° Celsius ($-50 \le T_i \le 50$). Judy has decided to dedicate one training session to each challenge in advance to ensure that she'll then ace it in her graduation exam.

On every day leading up to the exam, Judy will have time for at most S ($1 \le S \le 1000$) training sessions. In each session, she can choose one challenge to train for. However, there's a limit to how quickly her body is able to adapt to wildly varying temperatures. As such, the temperatures of all of the challenges which Judy trains for on any single day must differ from one another by no more than L° ($1 \le L \le 100$). In other words, for each day, the difference between the maximum and the minimum temperatures of all of the challenges chosen on that day must be no greater than L.

Judy will only take her graduation exam once she's spent one training session on each of the N challenges, but she'd love for that to be the case as soon as possible – she needs to get out there and make the world a better place! Given that she optimally chooses which challenges to dedicate each day's training sessions to, what's the minimum number of days which she must spend training before she's sufficiently prepared?

Input Format

The first line of input consists of three space-separated integers N, S, and L.
N lines follow, the i-th of which consists of a single integer T_i (for i = 1..N).

Output Format

Output a single line consisting of a single integer – the minimum number of days which Judy must spend training.

Sample Input

```
9 3 5
34
24
20
26
23
25
28
28
29
```

Sample Output

```
4
```

Sample Explanation

One optimal training schedule is as follows:

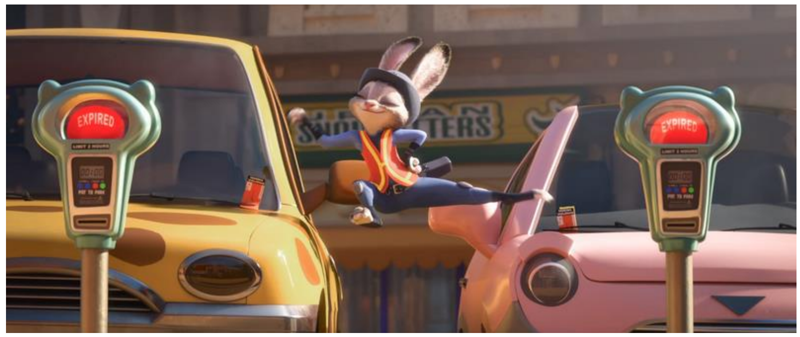
- Day 1: Challenges 1 (34°) and 9 (29°)
- Day 2: Challenges 2 (24°) and 4 (26°)
- Day 3: Challenges 3 (20°), 5 (23°), and 6 (25°)
- Day 4: Challenges 7 (28°) and 8 (28°)

Note that the temperatures of all of the challenges which Judy trains for on any single day differ by no more than 5°.

Problem J3: Parking Duty

28 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s1*

> *"Parking duty? You probably forgot, but I was top of my class at the academy."*
> *"Well then, writing one hundred tickets a day should be easy."*
> *"A hundred tickets… I'm not gonna write a hundred tickets. I'm gonna write two hundred tickets! Before noon!"*



Judy Hopps is not pleased about being assigned to parking duty on her first day as an officer of the Zootopia Police Department, but she's still going to give the task her all in order to prove herself.

There are N ($1 \le N \le 200{,}000$) parking meters within Judy's assigned area. Representing the area as a Cartesian plane, the i-th meter is located at coordinates (X_i, Y_i) ($-1{,}000{,}000 \le X_i$, $Y_i \le 1{,}000{,}000$), and is going to expire T_i ($1 \le T_i \le 1{,}000{,}000$) seconds after the start of Judy's shift. No two meters are at the same location, and the meters are given in strictly increasing order of expiration time ($T_1 < T_2 < \ldots < T_N$).

Judy suspects that none of the parked cars' owners will arrive before their meters expire, but they may move their cars shortly afterwards. As such, if she can be at a meter's location exactly when it expires, she'll be able to write a parking ticket for it! Writing a ticket can be done instantly, so if she's got a fast enough vehicle, she could drive around to visit all N meters at the appropriate points in time, and end up writing N parking tickets. However, she's going to take it a little bit easy on her first day – her goal is to write just $N - 1$ parking tickets, meaning that she may skip visiting any single meter of her choice.

At the start of the day, Judy can request a vehicle of her choice from the police department to use throughout the day. There are a variety of vehicles to choose from, with various top speeds, and Judy doesn't want to take a faster vehicle than she needs to get her job done. As such, she'd like to determine the minimum possible top speed of a vehicle which she'd need to be able to write $N - 1$ parking tickets throughout the day. Note that she'll have time before her shift starts to drive to any initial location of her choice.

In test cases worth 12/28 of the points, $N \le 1000$.

Input Format

The first line of input consists of a single integer N.
N lines follow, the i-th of which consists of three space-separated integers T_i, X_i, and Y_i (for $i = 1..N$).

Output Format

Output one line consisting of a single real number, the minimum possible top speed (in units/second) which would allow Judy to write $N - 1$ parking tickets.

Your answer must have no more than 10^{-5} absolute or relative error.

Sample Input

```
5
10 4 16
14 7 13
20 11 8
23 11 10
24 10 10
```

Sample Output

```
1.060660172
```

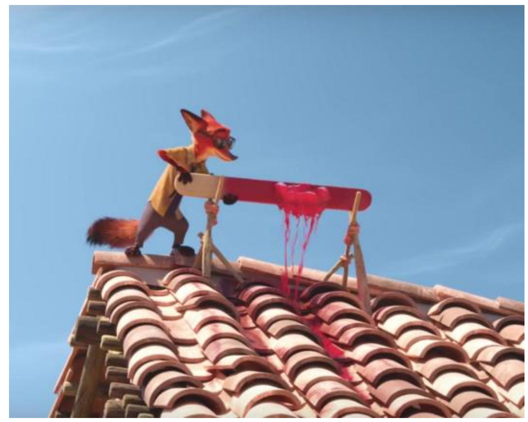
Sample Explanation

Using a vehicle with a top speed of $\sqrt{18} / 4 = 1.060660172$ units/second, Judy can drive from the first meter to the second one in exactly 4 seconds, allowing her to be present for each of their expiration times and write two parking tickets. She should then proceed directly to the fourth meter, and then to the fifth one, each with some time to spare. Using this strategy, she'll be able to write 4 parking tickets.

Problem J4: Pawpsicles

40 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s2*

> *"All right, slick Nick, you're under arrest."*
> *"Really, for what?"*
> *"Gee, I don't know. How about selling food without a permit, transporting undeclared commerce across borough lines, false advertising…"*
> *"Here: Permit, receipt of declared commerce, and I did not falsely advertise anything. Take care."*
> *"You told that mouse the pawpsicle sticks were redwood!"*
> *"That's right. Red wood. With a space in the middle. Wood that is red."*



Nick Wilde has got quite the clever money-making scheme going on. Every day, he…

1. Heads to an ice cream parlor, purchases a "Jumbo Pop" popsicle (unless he can hustle someone else into purchasing it for him), and melts it down into a sugary liquid.
2. Transports it to Zootopia's Tundratown district, makes paw prints in a snowy field, fills them with the popsicle liquid, places a popsicle stick in each one, and waits for the resulting "Pawpsicles" to solidify.
3. Sets up a sales booth outside a lemming bank just in time for the end of the workday, and sells his Pawpsicles to the lemmings as they leave the bank, collecting the discarded popsicle sticks.
4. Delivers the popsicle sticks to a miniature construction site in Little Rodentia, passing it off as lumber.

By following this sequence of steps, Nick is able to pull in a handsome profit of $200 every day. And the whole operation is perfectly legal! (Well, he may be failing to record his income on his tax returns, but that's besides the point.)

Still, time is money, so Nick's wondering if he's going about things as efficiently as possible. After all, Zootopia's a big place, so studying its map more carefully may suggest some improvements to his daily route.

Zootopia has N ($1 \le N \le 100{,}000$) key locations, with M ($0 \le M \le 100{,}000$) roads running amongst them. The i-th road runs between distinct locations A_i and B_i ($1 \le A_i, B_i \le N$), and can be travelled along in either direction in C_i ($1 \le C_i \le 100$) minutes. No pair of locations are directly connected by multiple roads.

For Nick's purposes, he's classified each location i with a type T_i ($0 \le T_i \le 4$). He has no interest in type-0 locations, but the other 4 location types correspond to the 4 steps of his Pawpsicle operation. Specifically, type-1 locations are ice cream parlors, type-2 locations are fields in Tundratown, type-3 locations are lemming banks, and type-4 locations are construction sites in Little Rodentia.

Nick starts each day in location 1, and needs to travel along a sequence of roads to visit a sequence of locations such that, at some point, he visits a type-1 location, then visits a type-2 location sometime later, then visits a type-3 location sometime after that, and finally visits a type-4 location sometime after that. He's considered to initially visit location 1, and on his way, he may travel through any locations as many times as he'd like.

In order to optimize the efficiency of his Pawpsicle operation, Nick would like to determine the minimum amount of time in which he can complete a route which visits the 4 useful types of locations in the required order. Unfortunately, it may also turn out that no such route exists, in which case Nick may need to consider making a more honest living…

In test cases worth 20/40 of the points, $N \leq 200$.

Input Format

The first line of input consists of two space-separated integers N and M.
N lines follow, the i-th of which consists of a single integer T_i (for i = 1..N).
M lines follow, the i-th of which consists of three space-separated integers A_i, B_i, and C_i (for i = 1..M).

Output Format

Output one line consisting of a single integer – the minimum number of minutes required for Nick to complete his Pawpsicle operation, or −1 if it can't be done.

Sample Input

```
9 9
2 0 0 1 2 3 4 4 3
1 4 9
4 2 3
2 1 4
5 4 1
5 6 4
7 2 9
3 1 2
3 7 3
3 9 4
```

Sample Output

```
27
```

Sample Explanation

One optimal route which Nick can take is as follows (with the 4 locations at which he'll carry out the steps of his Pawpsicle operator indicated with an asterisk "*"):

1 → 2 → 4* → 2 → 1* → 3 → 9* → 3 → 7*

This route will take 4 + 3 + 3 + 4 + 2 + 4 + 4 + 3 = 27 minutes to complete.