WOBURN CHALLENGE

2016-17 Online Round 4

Sunday, April 9th, 2017

Senior Division Problems

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Problem S1: Parking Duty

14 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s1*

> *"Parking duty? You probably forgot, but I was top of my class at the academy."*
> *"Well then, writing one hundred tickets a day should be easy."*
> *"A hundred tickets… I'm not gonna write a hundred tickets. I'm gonna write two hundred tickets! Before noon!"*

Judy Hopps is not pleased about being assigned to parking duty on her first day as an officer of the Zootopia Police Department, but she's still going to give the task her all in order to prove herself.

There are N ($1 \le N \le 200{,}000$) parking meters within Judy's assigned area. Representing the area as a Cartesian plane, the i-th meter is located at coordinates (X_i, Y_i) ($-1{,}000{,}000 \le X_i, Y_i \le 1{,}000{,}000$), and is going to expire T_i ($1 \le T_i \le 1{,}000{,}000$) seconds after the start of Judy's shift. No two meters are at the same location, and the meters are given in strictly increasing order of expiration time ($T_1 < T_2 < \ldots < T_N$).

Judy suspects that none of the parked cars' owners will arrive before their meters expire, but they may move their cars shortly afterwards. As such, if she can be at a meter's location exactly when it expires, she'll be able to write a parking ticket for it! Writing a ticket can be done instantly, so if she's got a fast enough vehicle, she could drive around to visit all N meters at the appropriate points in time, and end up writing N parking tickets. However, she's going to take it a little bit easy on her first day – her goal is to write just $N - 1$ parking tickets, meaning that she may skip visiting any single meter of her choice.

At the start of the day, Judy can request a vehicle of her choice from the police department to use throughout the day. There are a variety of vehicles to choose from, with various top speeds, and Judy doesn't want to take a faster vehicle than she needs to get her job done. As such, she'd like to determine the minimum possible top speed of a vehicle which she'd need to be able to write $N - 1$ parking tickets throughout the day. Note that she'll have time before her shift starts to drive to any initial location of her choice.

In test cases worth 6/14 of the points, $N \le 1000$.

Input Format

The first line of input consists of a single integer N.
N lines follow, the i-th of which consists of three space-separated integers T_i, X_i, and Y_i (for $i = 1..N$).

Output Format

Output one line consisting of a single real number, the minimum possible top speed (in units/second) which would allow Judy to write $N - 1$ parking tickets.

Your answer must have no more than 10^{-5} absolute or relative error.

Sample Input

```
5
10 4 16
14 7 13
20 11 8
23 11 10
24 10 10
```

Sample Output

```
1.060660172
```

Sample Explanation

Using a vehicle with a top speed of $\sqrt{18} / 4 = 1.060660172$ units/second, Judy can drive from the first meter to the second one in exactly 4 seconds, allowing her to be present for each of their expiration times and write two parking tickets. She should then proceed directly to the fourth meter, and then to the fifth one, each with some time to spare. Using this strategy, she'll be able to write 4 parking tickets.

Problem S2: Pawpsicles

20 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s2*

> *"All right, slick Nick, you're under arrest."*
> *"Really, for what?"*
> *"Gee, I don't know. How about selling food without a permit, transporting undeclared commerce across borough lines, false advertising…"*
> *"Here: Permit, receipt of declared commerce, and I did not falsely advertise anything. Take care."*
> *"You told that mouse the pawpsicle sticks were redwood!"*
> *"That's right. Red wood. With a space in the middle. Wood that is red."*

Nick Wilde has got quite the clever money-making scheme going on. Every day, he…

1. Heads to an ice cream parlor, purchases a "Jumbo Pop" popsicle (unless he can hustle someone else into purchasing it for him), and melts it down into a sugary liquid.
2. Transports it to Zootopia's Tundratown district, makes paw prints in a snowy field, fills them with the popsicle liquid, places a popsicle stick in each one, and waits for the resulting "Pawpsicles" to solidify.
3. Sets up a sales booth outside a lemming bank just in time for the end of the workday, and sells his Pawpsicles to the lemmings as they leave the bank, collecting the discarded popsicle sticks.
4. Delivers the popsicle sticks to a miniature construction site in Little Rodentia, passing it off as lumber.

By following this sequence of steps, Nick is able to pull in a handsome profit of $200 every day. And the whole operation is perfectly legal! (Well, he may be failing to record his income on his tax returns, but that's besides the point.)

Still, time is money, so Nick's wondering if he's going about things as efficiently as possible. After all, Zootopia's a big place, so studying its map more carefully may suggest some improvements to his daily route.

Zootopia has N ($1 \le N \le 100{,}000$) key locations, with M ($0 \le M \le 100{,}000$) roads running amongst them. The i-th road runs between distinct locations A_i and B_i ($1 \le A_i, B_i \le N$), and can be travelled along in either direction in C_i ($1 \le C_i \le 100$) minutes. No pair of locations are directly connected by multiple roads.

For Nick's purposes, he's classified each location i with a type T_i ($0 \le T_i \le 4$). He has no interest in type-0 locations, but the other 4 location types correspond to the 4 steps of his Pawpsicle operation. Specifically, type-1 locations are ice cream parlors, type-2 locations are fields in Tundratown, type-3 locations are lemming banks, and type-4 locations are construction sites in Little Rodentia.

Nick starts each day in location 1, and needs to travel along a sequence of roads to visit a sequence of locations such that, at some point, he visits a type-1 location, then visits a type-2 location sometime later, then visits a type-3 location sometime after that, and finally visits a type-4 location sometime after that. He's considered to initially visit location 1, and on his way, he may travel through any locations as many times as he'd like.

In order to optimize the efficiency of his Pawpsicle operation, Nick would like to determine the minimum amount of time in which he can complete a route which visits the 4 useful types of locations in the required order. Unfortunately, it may also turn out that no such route exists, in which case Nick may need to consider making a more honest living…

In test cases worth 10/20 of the points, $N \le 200$.

Input Format

The first line of input consists of two space-separated integers N and M.
N lines follow, the i-th of which consists of a single integer T_i (for i = 1..N).
M lines follow, the i-th of which consists of three space-separated integers A_i, B_i, and C_i (for i = 1..M).

Output Format

Output one line consisting of a single integer – the minimum number of minutes required for Nick to complete his Pawpsicle operation, or −1 if it can't be done.

Sample Input

```
9 9
2 0 0 1 2 3 4 4 3
1 4 9
4 2 3
2 1 4
5 4 1
5 6 4
7 2 9
3 1 2
3 7 3
3 9 4
```

Sample Output

```
27
```

Sample Explanation

One optimal route which Nick can take is as follows (with the 4 locations at which he'll carry out the steps of his Pawpsicle operator indicated with an asterisk "*"):

1 → 2 → 4* → 2 → 1* → 3 → 9* → 3 → 7*

This route will take 4 + 3 + 3 + 4 + 2 + 4 + 4 + 3 = 27 minutes to complete.

Problem S3: Night Howlers

30 Points / Time Limit: 7.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s3*

> *"Ugh. Timber wolves. Look at these dum-dums. Bet ya a nickel one of them's gonna howl."*
> *"And there it is. I mean, what is it with wolves and the howling?"*

Judy and Nick's search for a group of kidnapped animals has led them to the abandoned Cliffside Asylum. The building is being guarded by a pack of N ($1 \le N \le 200{,}000$) wolves, who will all need to somehow be distracted if Judy and Nick are to sneak by and investigate the premises.

The wolves are all standing in a row, which can be represented as a number line. The i-th wolf is at position P_i ($1 \le P_i \le 10^9$) on the line, and has an alpha rank of A_i ($1 \le A_i \le 10^9$), with the pack's most respected members having the smallest ranks. All of the wolves are standing at distinct positions, and have distinct ranks.

Wolves sometimes like to howl out of the blue, and they sometimes like to howl when they hear other wolves howling. If wolf i decides to start a howl, then each wolf with a larger alpha rank standing within some howling radius R of wolf i will be obligated to join in (in other words, each wolf j such that $A_j > A_i$, $P_j \ge P_i - R$, and $P_j \le P_i + R$). Each wolf j which joins the howl in this fashion may in turn cause even more wolves to start howling (wolves k which have larger alpha ranks than wolf j and are standing close enough to him), and so on.

Judy and Nick figure that they can trick any K ($1 \le K < N$) wolves of their choice into starting to howl. They're hoping that this plan can result in all N wolves joining the howl, allowing them to sneak by. Unfortunately, they don't know the howling radius R, just that it's some positive integer, so they can't predict how things will go! Still, they'd like to get an idea of how likely their plan is to work, by determining the minimum howling radius R which would be necessary to allow them to get all N wolves to join a howl initially started by some K of the wolves.

In test cases worth 12/30 of the points, $N \le 2000$.

Input Format

The first line of input consists of two space-separated integers N and K.
N lines follow, the i-th of which consists of two space-separated integers P_i and A_i (for $i = 1..N$).

Output Format

Output a single integer – the minimum howling radius R necessary such that all N wolves can be made to join a howl started by K of them.

Sample Input

```
7 2
6 8
10 12
12 10
8 1
3 5
13 7
5 4
```

Sample Output

```
3
```

Sample Explanation

If $R = 3$ and wolves 4 and 6 start a howl, all 7 wolves will end up joining it.
On the other hand, if $R = 2$, then no 2 initial wolves can be chosen such that all 7 wolves will end up howling.

Problem S4: Replay Value

36 Points / Time Limit: 3.00s / Memory Limit: 64M *Submit online: wcipeg.com/problem/wc164s4*

> *"Hopps, Wilde… parking duty. Dismissed."*
> *"Just kidding! We have reports of a street racer tearing up Savannah Central. Find him. Shut him down."*

Nick has ended up joining the ZPD as Zootopia's first fox cop, and as Judy's partner, no less! They've been assigned a case to track down a dangerous street racing criminal, but it won't be easy – they'll need to do some investigative work before they can begin to assemble a list of suspects.

Zootopia has N ($2 \le N \le 200$) key locations, with $N - 1$ roads running amongst them. The i-th road runs between distinct locations A_i and B_i ($1 \le A_i, B_i \le N$), and can be traveled along in either direction in 1 minute. It's possible to reach every location from every other location by following a sequence of roads.

The ZPD headquarters are located in location 1, and aside from that, some other locations may be of significance to the investigation. For each location i, if $C_i = 1$, then Judy and Nick believe that it contains a clue, and will need to go check it out. Otherwise, if $C_i = 0$, then the location is of no interest to them. At least one location besides location 1 is guaranteed to contain a clue.

Judy and Nick will both start at the ZPD headquarters, and will then split up to each travel around Zootopia, inspecting all of the locations that contain clues. On the way, each of them may travel through any locations as many times as they'd like, and they may both be present in the same location at the same time. They've agreed to meet back at the headquarters once they're done. What's the minimum amount of time in which Judy and Nick can independently travel around by following sequences of roads, such that they both end up back at the ZPD headquarters, and such that every location with a clue ends up being visited by at least one of the two cops?

In test cases worth 10/36 of the points, $N \le 15$ and $C_i = 1$ for each i.
In test cases worth another 10/36 of the points, $N \le 50$ and $C_i = 1$ for each i.
In test cases worth another 8/36 of the points, $C_i = 1$ for each i.

Input Format

The first line of input consists of a single integer N.
N lines follow, the i-th of which consists of a single integer C_i (for $i = 1..N$).
$N - 1$ lines follow, the i-th of which consists of two space-separated integers A_i and B_i (for $i = 1..N - 1$).

Output Format

Output one line consisting of a single integer – the minimum number of minutes required for Judy and Nick to visit all locations with clues (between the two of them) and both return to the ZPD headquarters.

Sample Input

```
7
1
1
0
1
1
1
1
1 2
2 3
1 4
5 4
6 4
7 4
```

Sample Output

```
6
```

Sample Explanation

Judy can complete the following route in 6 minutes:

$1 \rightarrow 2 \rightarrow 1 \rightarrow 4 \rightarrow 5 \rightarrow 4 \rightarrow 1$

During the same 6 minutes, Nick can complete the following route:

$1 \rightarrow 4 \rightarrow 6 \rightarrow 4 \rightarrow 7 \rightarrow 4 \rightarrow 1$

All 6 locations with clues end up being visited by either Judy or Nick at least once.