WOBURN CHALLENGE

2016-17 On-Site Finals

Saturday, May 6th, 2017

Junior Division Problems

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wcipeg.com

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Problem J1: Fencing

11 Points / Time Limit: 2.00s / Memory Limit: 16M

Bo Vine, beloved leader of the peace-loving cows of Scarberia, has gotten wind that his land may be in danger. It seems that his long-time nemesis, the Head Monkey, may be planning to end their year-long armistice and lead her troops to invade Scarberia! To confirm this intelligence, Bo Vine will need to send in a spy to infiltrate the monkeys' ranks.

There are N ($2 \le N \le 100$) trained cow spies, numbered from 1 to N, any of whom would surely be able to complete the mission successfully. As such, Bo Vine will have them engage in a round robin fencing tournament, with the victor earning the honour of being sent on the mission. As it turns out, the cows are quite lazy, and none of them actually want to be chosen. As such, they'll all try their best to lose (without being too obvious about it), but at the end of the day, one of them is sure to win the largest number of fencing matches and be forced to go on the mission.

Over the course of the tournament, each of the N cows will partake in one match against each of the remaining $N-1$ cows. During the match between each pair of distinct cows i and j, cow i will score $S_{i,j}$ points, while cow j will score $S_{j,i}$ points ($0 \le S_{i,j}, S_{j,i} \le 10$, $S_{i,j} \ne S_{j,i}$). Whichever of them scores more points than the other will be declared the winner of that match. Note that there are no ties. Also note that cows don't play against themselves, so $S_{i,i}$ is given to be 0 for each i.

At the conclusion of the tournament, the cow who has won the largest number of their $N-1$ matches will be crowned the champion. It's guaranteed that there will be a unique cow with strictly the largest number of wins. Given the results of all of the matches, can you help Bo Vine determine the winner?

Input Format

The first line of input consists of a single integer N.
N lines follow, the i-th of which consists of N space-separated integers $S_{i,1}, \ldots, S_{i,N}$ (for $i = 1..N$).

Output Format

Output one line consisting of a single integer – the number of the cow who will be sent on the spy mission.

Sample Input

```
5
0 4 0 9 2
5 0 9 2 2
9 0 0 0 0
1 4 9 0 2
1 6 9 1 0
```

Sample Output

```
4
```

Sample Explanation

The 5 cows won 2, 2, 1, 3, and 2 matches, respectively. As such, cow 4 won the largest number of matches.

Problem J2: Enigmoo

13 Points / Time Limit: 2.00s / Memory Limit: 16M

Well, this is suspicious… after a year of living in begrudging peace after having been evicted from their homeland of Scarberia, the monkeys have just intercepted a secret message from the cows. The Head Monkey is no fool – this can only mean that her long-time nemesis, Bo Vine, is looking for trouble once again!

The message is a single word, a string W consisting of N ($1 \le N \le 100$) lowercase letters. Knowledge of this word may give the monkeys valuable insight into the cows' military plans, but there are two problems.

The first problem is that the string has been encrypted! Fortunately, the Head Monkey is well aware of the naive Enigmoo encryption algorithm which Bo Vine likes to use when transmitting information. Bo Vine has surely chosen some integral shift value S ($1 \le S \le 25$), and then cyclically shifted each letter in the string forward in the alphabet by S spots. For example, if $S = 2$, then the original message "`hey`" would get encrypted into the string "`jga`". Note that even Bo Vine wouldn't have been stupid enough to choose $S = 0$, as that would have resulted in the encrypted string being equal to the original one.

The second problem is that the message may have been damaged, rendering some of its letters unreadable. If the monkeys can't identify the i-th letter in the encrypted string, then W_i = "`?`". It's possible that all of the letters are still readable, or none of them, or anything in between.

The Head Monkey has dug out her comprehensive dictionary of cow words, which contains M ($1 \le M \le 100$) distinct words of length N. The i-th of these words is D_i, and consists of N lowercase letters. Given that the cows may have chosen any possible shift value, and that each unreadable letter in W might actually be equal to any lowercase letter, can you help the monkeys narrow their search by counting the number of different words in the dictionary which, when encrypted, might correspond to W? It's possible that the original message is actually some gibberish which doesn't match anything in the dictionary.

In test cases worth 4/13 of the points, none of the characters in W are equal to "`?`".
In test cases worth another 4/13 of the points, exactly one of the characters in W is equal to "`?`".

Input Format

The first line of input consists of two space-separated integers N and M. The second line consists of the string W. M lines follow, the i-th of which consists of D_i (for i = 1..M).

Output Format

Output one line consisting of a single integer – the number of different words in the dictionary which the encrypted string might match.

Sample Input

```
6 7
??zz?k
attack
cattle
treats
farmer
missed
battle
sleepy
```

Sample Output

```
3
```

Sample Explanation

The original message might have been "`cattle`" – if the cows had chosen $S = 6$, then the encrypted string would've become "`igzzrk`", which is consistent with the message found by the monkeys. However, the original message might've also been "`battle`" ($S = 6$) or "`missed`" ($S = 7$).

Problem J3: Cow-Bot Construction

18 Points / Time Limit: 3.00s / Memory Limit: 32M

It would seem that war is inevitable. Bo Vine's communication with his cow spy has been intercepted by the devious monkeys, confirming his suspicions that the Head Monkey is up to no good! In preparation for the impending conflict with the monkeys, Bo Vine has ordered the construction of a new, state-of-the-art Cow-Bot model from scratch.

The schematics call for N ($1 \le N \le 200{,}000$) different modules to be installed, in any order. Bo Vine's cow engineers require E ($1 \le E \le 10{,}000$) minutes to install any single module. However, the Cow-Bot (being a sophisticated, artificially intelligent piece of hardware) may also be able to help out! If at least M_i ($0 \le M_i \le N$) different modules have already been installed into the Cow-Bot up to a certain point in time, then it becomes able to install the i-th module into itself in B ($1 \le B \le 10{,}000$) minutes. Only one module may be in the process of being installed into the Cow-Bot at any time, meaning that the engineers and Cow-Bot may not simultaneously install two different modules at once.

Please help the cows determine the minimum amount of time required for all N modules to be installed into the Cow-Bot.

In test cases worth 14/18 of the points, $N \le 2000$.

Input Format

The first line of input consists of three space-separated integers N, E, and B.
N lines follow, the i-th of which consists of a single integer M_i (for $i = 1..N$).

Output Format

Output one line consisting of a single integer – the minimum number of minutes required for all N modules to be installed into the Cow-Bot.

Sample Input

```
7 7 4
4
0
4
2
6
4
4
```

Sample Output

```
34
```

Sample Explanation

One optimal sequence of module installations is as follows (with each step's completion time indicated):

- Module 2 by the Cow-Bot (4 minutes)
- Module 3 by the engineers (11 minutes)
- Module 7 by the engineers (18 minutes)
- Module 4 by the Cow-Bot (22 minutes)
- Module 6 by the Cow-Bot (26 minutes)
- Module 1 by the Cow-Bot (30 minutes)
- Module 5 by the Cow-Bot (34 minutes)

Problem J4: Rational Recipes

24 Points / Time Limit: 2.00s / Memory Limit: 16M

The worst has come to pass, with the war between the monkeys and cows now officially in full swing. In fact, the first major battle has been scheduled for next week! As an experienced military leader, the Head Monkey is well aware of the importance of an often under-appreciated aspect of war – the nutritional well-being of the soldiers. She's taken it upon herself to personally prepare the upcoming battle's rations.

The Head Monkey has N ($1 \le N \le 100$) different types of fruit ingredients to work with, numbered from 1 to N. Fruit type 1 corresponds to the monkeys' beloved bananas, of course, while the other fruit types are less tasty but have their own nutritional benefits. There are F_i ($1 \le F_i \le 1{,}000{,}000{,}000$) fruits of the i-th type available.

The Head Monkey intends to come up with a smoothie recipe, which will call for combining a certain set of fruits together to create a single smoothie. The recipe R will dictate that exactly R_i fruits of type i must go into the smoothie, where R_i is a strictly positive integer for each i between 1 and N. It's unclear exactly what recipe she'll come up with, though – we can only hope that it will be edible, for the monkeys' sake.

It's also unclear how many monkeys will actually be participating in the upcoming battle, though the number of monkeys will surely be some positive integer. However, it's imperative that each of them receive a single smoothie, with all of the smoothies having been created using the same recipe as one another.

Of course, the number of available fruits is a serious limiting factor. If there are m monkeys, and some recipe R is chosen, then $m \times R_i$ fruits of each type i will be required in total, and this quantity cannot exceed F_i. However, there's an additional restriction – due to the high value placed on bananas, it's important that there be no leftover bananas, as they'd go to waste. Therefore, it must be the case that $m \times R_1 = F_1$.

The Head Monkey has lots of great recipes in mind, but she's willing to accept that some of them might not work out in terms of producing a valid set of smoothies for 1 or more monkeys. That being said, she's wondering exactly how many different possible recipes she could validly choose. This quantity may be quite large, so she's only interested in its value when taken modulo 10,007.

As a hint, the following properties of modular arithmetic may be useful:

$(A + B) \bmod M = ((A \bmod M) + (B \bmod M)) \bmod M$
$(A \times B) \bmod M = ((A \bmod M) \times (B \bmod M)) \bmod M$

In test cases worth 4/24 of the points, $N \le 3$ and $F_i \le 100$ for each i.
In test cases worth another 6/24 of the points, $F_i \le 100$ for each i.
In test cases worth another 4/24 of the points, $F_i \le 10{,}000$ for each i.

Input Format

The first line of input consists of a single integer N.
The second line of input consists of N space-separated integers $F_1, \ldots, F_N$.

Output Format

Output one line consisting of a single integer – the number of different possible recipes which might be used (modulo 10,007).

Sample Input

```
3
4 2 4
```

Sample Output

```
10
```

Sample Explanation

3 possible recipes are as follows:

- *R* = [4, 1, 3] (serving 1 monkey)
- *R* = [4, 2, 1] (serving 1 monkey)
- *R* = [2, 1, 2] (serving 2 monkeys)

There are 7 other possible recipes.

Problem J5: Privacy

34 Points / Time Limit: 2.00s / Memory Limit: 64M

It's almost time to head into battle, but Bo Vine's cow soldiers insist on taking a nice long drink of water first. All N ($1 \le N \le 400$) of his army's cows have lined up at a trough to drink water. However, the cows like their privacy when drinking, and the i-th cow insists that they must drink from a trough from which at most C_i ($0 \le C_i < N$) other cows are also drinking.

The cows refuse to budge until they get hydrated, so to help make that possible, Bo Vine is prepared to install at most K ($0 \le K < N$) dividers at various points along the trough, effectively dividing it into multiple troughs as far as the cows are concerned. For example, if he installs a single divider between cows i and $i + 1$, then cows $1..i$ will be considered to drink from one trough, while cows $(i + 1)..N$ will be considered to drink from a different trough.

It may turn out that the cows' demands can't all be met even after the installation of K dividers. As such, Bo Vine may also need to "encourage" some of them to relax their requirements. Each cow is willing to increase their C value by 1 in exchange for 1 treat. Bo Vine may bribe any cow as many times as he'd like.

What's the minimum total number of treats which Bo Vine must give to the cows such that, once at most K dividers are installed, each cow will be willing to drink from its trough?

In test cases worth 8/34 of the points, $N \le 50$ and $K \le 1$.
In test cases worth another 16/34 of the points, $N \le 50$.

Input Format

The first line of input consists of two space-separated integers N and K.
N lines follow, the i-th of which consists of a single integer C_i (for $i = 1..N$).

Output Format

Output one line consisting of a single integer – the minimum total number of treats required such that the cows can all be satisfied after the installation of K dividers.

Sample Input

```
7 2
2 0 5 0 1 3 1
```

Sample Output

```
3
```

Sample Explanation

One optimal strategy is to give the second cow 2 treats to increase its C value from 0 to 2, and the fourth cow 1 treat to increase its C value from 0 to 1. Then, Bo Vine can install one divider between cows 3 and 4, and one more between cows 5 and 6, in order to yield the following valid set of troughs:

```
[ 2 2 5 | 1 1 | 3 1 ]
```