Incentives or Disincentives?*

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Abstract

If a policymaker wishes to encourage members of a population to take a socially beneficial action, should she reward those who take the desired behavior or punish those who do not? For example, should government subsidize the purchase of health insurance or fine the failure to purchase health insurance? This question lies, often implicitly, at the heart of many policy debates, yet there does not exist a widely applicable framework with which to understand the optimal policy instrument. This paper develops a model that facilitates both normative and positive perspectives on the use of incentives and disincentives in public policy. Members of a population each choose between two actions, one of which benefits (or does less harm to) society at large. The model reveals an asymmetry in the way that costs accrue from incentive and disincentive policies. Incentives become more expensive to administer as they get larger, because more of the population earns the reward. Disincentives become less costly to administer as they get larger, because fewer people fail to take the desired action. This asymmetry should lead policymakers to induce high (resp. low) shares of the population to take the desired behavior using disincentives (resp. incentives). The redistributive implications for members of the population only amplify this tendency. As such, the majority preference tends to be for larger-than-optimal punishments and smaller-than-optimal rewards. Policies that tax socially beneficial behavior emerge as an extreme example of the latter.

Keywords: Rewards, punishments, subsidies, fines, redistribution, policy instruments

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1 Introduction

In 2017, Congressional Republicans made numerous attempts to fulfill a seven-year promise to repeal the Affordable Care Act (ACA). Central to conservatives' complaints about the ACA was the individual mandate, which requires individuals to purchase health insurance or else pay a fine. It followed, then, that a key element of all of the Republican proposals was replacing the individual mandate with subsidies. Certainly other issues were on the table, including the possibilities of significant cuts to medicaid, an erosion of minimum essential coverage standards, and insurers being able to reject those with pre-existing conditions. The heart of the debate, though, was the question of whether government ought to incentivize the purchase of insurance or disincentivize the failure to insure oneself.

The decision between using incentives or disincentives to achieve a given policy aim is hardly limited to healthcare. In the realm of environmental policy, government discourages polluting activities through taxes, while initiatives such as Cash for Clunkers rewards the retirement of less efficient vehicles to subsidize the purchase of more efficient vehicles. National education policy has featured both disincentive- and incentive-based policies in recent years in the form of No Child Left Behind and Race to the Top, respectively (Howell 2004, Howell & Magazinnik 2017). The domain of agricultural and food policy abounds with examples: the Conservation Reserve Program incentivizes farmers to take land out of agricultural production,[1] the infamous "sugary drink taxes" are a disincentive for unhealthy consumption,[2] and in many cities individuals receive double the value of food stamps when used on fruits and vegetables, a reward for healthy consumption choices.

Broadly, for a policymaker seeking to encourage members of a population to take a behavior, is it better to reward those who take the behavior with incentives, or punish those who do not with disincentives? The examples above illustrate that this is not merely a thought exercise. In practice, policymakers turn to both incentives and disincentives even within a given policy domain, often to achieve similar aims.

This paper uses a model to offer normative and positive perspectives on this question. The setting is kept as simple as possible while still emphasizing distinguishing features of a policymaking environment. Each member of a population will choose one of two actions, one of which is more socially beneficial than the other. The propensity to choose this action – in the absence of any policy inducements – varies across members of the population. Two sets of questions follow. The first set of questions surrounds the conditions under which a public-interested social planner would turn to incentives or disincentives to encourage the socially

[1]`https://www.fsa.usda.gov/programs-and-services/conservation-programs/`, accessed January 2017.

[2]Neuman, William. Sept. 16, 2009. "Proposed Tax on Sugary Beverages Debated." *The New York Times*, accessed at `http://www.nytimes.com/2009/09/17/business/17soda.html`.

beneficial action. The second set of questions ask what policy a majority of the population would support given the redistributive implications, in terms of both the type of policy and whether it serves to encourage the socially beneficial action or the socially-harmful action. Comparing the social planner's preferred policy to the majority-preferred policy will characterize the way in which the policy outcome of a democratic process is likely to differ from the socially-optimal policy.

Throughout, the model assumes incentives and disincentives influence the behavior of members of the population identically. Adopting such a stark rational choice perspective challenges the model to disprove the null hypothesis that incentives and disincentives are simply two sides of the same enforcement coin. It allows the analysis to uncover only those asymmetries between the two policies that emerge when considering the constraints of policymaking. Section 2 discusses the reasons for and implications of this approach in greater depth, as well as the way in which the paper's focus differs from a number of related literatures. One of the main implications is limiting the analysis to only pecuniary policies: fines and taxes on one hand, subsidies and other financial rewards on the other.

In modeling incentives and disincentives symmetrically, it is essential that the various costs and benefits associated with a policy intervention accrue similarly under each type of policy. Section 3 specifies the social planner's considerations, namely, the external benefit, which rises in the share of the population taking the socially beneficial action; the loss in well-being incurred by inducing some members of the population taking their less preferred action, which deepens as the policy intervention grows larger; and the cost of administering an incentive or a disincentive, which increases in the share of the population that to which a given policy is administered. While the costs entailed in government intervention always deserve attention, the costs of administering policy interventions play an especially central role here.

Generally, all those who take an incentivized behavior are entitled to any reward being offered and at the same level, while all those who do not may receive any punishment on the books, again at the same level. An important asymmetry follows when combined with the premise that the administrative cost associated with a given policy grows in the share of the population receiving that policy. Increasing the size of an incentive induces a larger share of the population to take the desired action, which in turn increases the share of the population who must receive the reward, causing administrative cost to rise. Increasing the size of a punishment also induces a larger share of the population to take the desired action, but this reduces the share of the population who must receive the punishment, causing administrative costs to fall. Thus, inducing higher shares of the population to take the beneficial behavior and the use of punishments rather than rewards are complements to one another.

Policies apply to entire populations, and this leads the administrative cost of disincentives (resp. incentives) to fall (resp. rise) as a larger share of the population takes the desired action. In light of this asymmetry, the type of policy (incentive or disincentive?) as well as the size of the intervention (how big is the reward/punishment, how much compliance does it induce?) are of interest. Section 4 explores the implications of this asymmetry for the type and size of intervention a social planner would endorse. Comparative statics explore how these answers change in response to changes in the rate at which benefit accrues to society from its members taking the socially beneficial action, the distribution of preferences for taking that action across the population, and the costs of administering each type of policy intervention. Broadly, conditions that favor inducing high shares of the population to take the beneficial action also favor the use of disincentives, and vice versa for incentives.

The analysis then turns to the political economy of incentives and disincentives. Section 5 models the political environment. Specifically, the redistribution of revenue generated from fines and the financing of subsidies both occur through lump-sum transfers, with each member of the population receiving and contributing an equal share, respectively. A member of the population's utility under a given policy then depends upon the action the policy induces that member to take, the benefit to society produced under the policy, the redistributive benefits net of the redistributive costs, and the administrative cost which is a loss that cuts into redistribution of revenues and raises the amount taxpayers must finance. If a member of the population most prefers a given policy, then all other members of the population that choose the same action under the policy also prefer that policy the most. This implies that the majority preference is identical to preferences of the member of the population with the median valuation for taking the beneficial action.

The complementarity between using disincentives and inducing most of the population to take the beneficial action grows even stronger in the context of the majority preference than it was for the social planner. The larger the incentive or disincentive, the more likely the policy is to induce the median member of the population to take the desired behavior. Furthermore, if a member of the population takes the beneficial action, the redistributive implications lead her to prefer a larger policy intervention. A larger intervention, though, is one that induces a larger share to take the beneficial action and which in turn makes disincentives relatively more cost-effective than incentives. Section 6 lays out this logic in greater detail, but the significance is that the median voter, and thus a majority of the population, will tend to prefer larger-than-optimal disincentives and smaller-than-optimal incentives.

Section 7 explores an extension in which policies may be targeted at a sub-population of interest. The rest of the population is neither confronted with the action nor eligible for incentives or disincentives, but revenue from fines is distributed across the entire population, and all members of the population must contribute to the financing of subsidies. This analysis asks which policy a member of this only-indirectly-affected, at-large population would prefer. If the only effects of a policy on a member of the population are redistributive, such a member receives the same utility under a disincentive policy as a member of the subpopulation of interest who takes the beneficial behavior, and the same utility under an incentive policy as a member of the subpopulation of interest who takes the socially-harmful behavior. The tendency for the majority to prefer excessively large punishments and insufficiently large rewards then becomes the rule. Specifically, if the preferred policy entails a disincentive, it will induce more of the beneficial behavior than the social planner would prefer, and if the preferred policy uses incentives, it will induce less of the beneficial behavior than the social planner would prefer.

While a social planner would never seek to increase the share of the population choosing the less beneficial (more harmful) action, this need not be the case under the majority-preferred policy. Indeed, policies that encourage the harmful behavior, usually through a tax on the beneficial behavior, arise as an example of insufficiently large incentive policies. The majority-preferred incentive for the desirable behavior may be so small as to be negative, actually constituting a fine or tax. The suggestion that policies may run counter to societal benefit is not surprising. Of interest, however, is that policies encouraging and discouraging the beneficial behavior may have highly similar implications for the utility of median voter, despite their drastically different intentions. Section 8 concludes with a brief summary and directions in which this modeling framework might be expanded.

2 Representing the Realities of Policy

> . . . the only fair way to begin must be with the tenet that there is no basic or universal rationale for having a general predisposition toward one control mode or the other. . . Even on an abstract level, it would be useful to know how to identify a situation where employing one mode is relatively advantageous, other things being equal.
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> Weitzman (1974)

Two principles receive priority in modeling the use of incentives and disincentives as policy instruments. The first, and as per Weitzman, is to set up the two types of policies to be as similar as possible.[3] Accordingly, members of the population will respond identically to incentives and disincentives. This entails setting aside behavioral effects in order to adopt a strictly rational choice approach.[4]

[3]Weitzman wrote on price vs. quantity policies; both incentives and disincentives constitute variants of price policies. The difference between price and quantity policies in this and follow-on papers hinges on uncertainty about the benefits and costs of regulation, while the model herein relies only on the heterogeneity of actors within a population (Pizer 1997, Grodecka & Kuralbayeva 2015).

[4]A sizable body of behavioral work has studied differences likely to arise in individual-level responses to

The second principle in constructing the model is to embody the core realities of policymaking. These two aims may at times be at odds with one another. When this occurs, the Weitzman critique takes precedent and dictates modeling incentives and disincentives under the same assumptions. For example, only pecuniary rewards and punishments receive attention and wealth constraints do not limit the effectiveness of fines. Features of policymaking environments that receive prominent consideration in the model include the heterogeneous propensities across the population for choosing one behavior over another and the costliness of policy implementation. Specifically, costs are increasing in the share of the population to which a given policy must be administered.

Rational choice analyses of policymaking have traditionally, if implicitly, assumed incentive and disincentive policies are equivalent enforcement mechanisms. Political scientists have instead focused on aspects of the policymaking process surrounding the choice of policy instrument. The large literature on bureaucracy has devoted significant attention to the decision of a political principal to delegate decisions, such as those over the policy instruments, to other actors within the government (Gailmard & Patty 2013). When it comes to the implementation of policy decisions, even Pressman & Wildavsky's (1984) landmark study focuses on the target of policies, rather than the choice of instrument with which to pursue a given target. This paper not only seeks to remedy that omission but also to address the nature of the policies likely to emerge from a democratic political process.

In contrast to the work that treats incentive and disincentive policies as entirely interchangeable, public administration and legal scholars exploring alternative approaches to regulation have explicitly weighed "punishment" against "persuasion" (see Gunningham (2012), Baldwin, Cave & Lodge (2012, ch. 7), Lodge & Wegrich (2012, pp. 76-80), and De Geest & Dari-Mattiacci (2013)). While they highlight a variety of potential asymmetries between the regulatory strategies of incentives and disincentives, it is the embrace of these dissimilarities from the outset that prevents this work from speaking to more fundamental, institutional differences. To serve as an effective counterpoint to the notion that incentives and disincentives constitute essentially the same enforcement technology, one must begin more agnostically, treating the two approaches identically. Further, without formally modeling the salient features common across policy domains, it is difficult to extend those analyses to speak to the policy that would emerge from the democratic process, much less to compare such predictions to the socially optimal policy.

Law and economics has long appreciated the ostensible symmetry underlying the assignment of liability, asking whether to lay fault with the injurer (e.g., a firm) or the injured (e.g., a consumer) in the event of an accident (Calabresi & Melamed 1972, Miceli 2004, Posner 2005). The questions in this paper emphasize

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positive and negative inducements (Kaplow & Shavell 2007, Benabou & Tirole 2011).

a different set of transactions costs than Coase (1960), but they can be placed in the Coasian framework. Should the government retain "property rights" over a domain and exact payment from those that choose the socially-costly action, or should the government cede these rights and offer to pay any individuals who choose the socially beneficial action?

Studies of the political economy of regulation, inaugurated by Stigler (1971) and Posner (1974), sought to replace the long-standing assumption that regulation served the public interest with the understanding that regulation was the outcome of strategic interaction among competing interest groups (Peltzman 1989, Shleifer 2005). Indeed, this view informs the latter half of this project. This understanding may, however, have somewhat shifted attention away from the evaluation of alternative policy instruments. While Becker's (1983, 1985) pathbreaking work on interest groups does consider taxes and subsidies, they are purely redistributive, separated from any policy seeking to affect behavior. The literature on public enforcement is almost entirely concerned with the deterrence of undesirable actions through punishment (see Polinsky & Shavell (2000) for a thorough review of the literature). Again, a notable exception comes from Becker (1968), who does consider the use of rewards, as well as punishments, and who appreciates the way that the size and scale of policies affect costs differently in the two types of policies. He stops short, however, of asking why beneficial behaviors are sometimes induced with incentives and at other times with disincentives.

Finally, moral hazard and other principal-agent models deal directly with behavior change through incentives and disincentives (Holmstrom 1979), but the principals are usually profit-seeking rather than public-interested or office-motivated. Further, because much of this literature considers a single agent (Banks & Sundaram 1998, p. 299), this leaves no room for a true choice between incentives and disincentives. For example, Dal Bo, Dal Bo & Di Tella (2006) study the use of bribes and/or threats (physical and political) by a group looking to gain influence over government officials. Because the politician will either comply or not, the group will never have to undertake both costly endeavors and may put both on the table. This is no longer the case when applying incentives or disincentives to an entire population or subpopulation, and a crucial feature of any public policy is that it applies to a population.

3 A Model of Incentives and Disincentives

The setting consists of a single period in which all members of a population choose $a \in \{0, 1\}$ exactly once. The population consists of a unit mass of individuals. Denote a member of the population by i, and denote i's *ex ante* valuation for taking the desired action by v^i (i.e., in the absence of any incentives or disincentives).[5] Assume the valuations are distributed according to a continuously differentiable cdf $F(\cdot)$, with $\underline{v} \leq v^i \leq \overline{v}, \forall i$.

[5] This term captures the net benefit of taking $a = 1$ or $a = 0$. Indeed, set $u^i(0) = 0$, so $\forall i, v^i = u^i(1)$.

Note that v^i may be negative, indicating a latent propensity to take $a = 0$, or positive, indicating that i would choose $a = 1$ without any further inducement. Indeed, assume $\underline{v} < 0 < \overline{v}$, such that there are *ex ante* compliers and non-compliers in the population.[6]

When members of the population choose $a = 1$ instead of $a = 0$, it imparts social benefit (or, equivalently, reduces social harm). As such, the social planner – a public-interested policymaker – would only consider policies that encourage $a = 1$ or $a = 0$. In principle, however, a policy may encourage either $a = 1$ or $a = 0$, and indeed redistributive implications may lead to majority support for a policy of incentives for $a = 0$ or disincentives for $a = 1$. The social planner's optimal policy receives attention first, though, because it serves as a benchmark for the subsequent analyses and is interesting in its own right as a normative point of view. Further, the model considers only monetary incentives and disincentives, namely subsidies and taxes, though the analysis is amenable to other kinds of incentive and disincentive policies.[7]

A social planner wishing to encourage citizens to take action $a = 1$ rather than $a = 0$ may reward those who take the beneficial action, punish those who do not, both, or neither. To capture the constraints of policymaking, assume that the policy cannot impose different levels of reward or punishment across the population. This requires that any individual in the population receiving a reward receives the same level of reward, and similarly for punishments.[8] Suppose at first that all members of the population are subject to the policy. The final section considers the presence of an "unaffected" subpopulation – members of the population not subject to a choice of $a \in \{0, 1\}$ or the corresponding rewards or punishments. Suppose further that the policymaker has full information, sidestepping issues of probabilistic enforcement.

Incentives, R, and disincentives, P, enter additively and linearly into the utility functions of individuals

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[6]An example of the concept of heterogenous valuations is the use of incentives or disincentives to grow certain crops. The border between growing regions ought not to be thought of as entirely rigid nor entirely malleable. Between two regions, farmers can switch between crops more easily than those firmly inside a given growing region. How far into an area the margin is found depends on the size of the policy intervention.

[7]Taxes connote fines here, as in "carbon tax," where the tax is a penalty, not redistributive in purpose.

[8]Consider the Environmental Quality Incentives Program (EQIP). This is a reward-based initiative encouraging a variety of sustainable practices in agriculture. A central problem of enforcement of this program has been larger operations receiving funds for practices they would have already undertaken, and which do not represent additional efforts taken towards sustainability (Wilde 2013, pp. 50-51). For example, large-scale animal feeding operations have applied and received funding for setting up waste storage lagoons – a necessary endeavor for these facilities and one that need not have been incentivized. Yet that is the nature of rewards. They must be applied to all who comply and are in the eligible population.

`https://www.nrcs.usda.gov/wps/portal/nrcs/main/national/programs/financial/eqip/` accessed January 2017.

so as to abstract from behavioral concerns.[9] Individuals choose $a = 1$ in the case of indifference. It follows immediately that i chooses $a = 1$ if and only if the utility she derives from doing so plus any rewards offered for choosing $a = 1$ is at least as large as the utility she receives from choosing $a = 0$ minus any threatened punishments, i.e., $v^i + R \geq -P$. The following result derives the share of the population choosing $a = 1$ given disincentives of size P and incentives of size R.

Fact 1. *A member of the population i chooses $a = 1$ iff $v^i \geq -R - P$. The share of the population choosing $a = 1$ is then $1 - F(-R - P)$, while the share of the population choosing $a = 0$ is $F(-R - P)$.*

The social planner takes into account any positive externalities from members of the population choosing $a = 1$ and/or negative externalities from those choosing $a = 0$, the utility that members of the population derive from their chosen action, and the deadweight costs of administering policies. Let the function $W : [0, 1] \to \mathbb{R}$ represent the global external benefit to society from a given share of the population choosing $a = 1$. Let $W(\cdot)$ be continuously differentiable, with $W' > 0$. It is assumed that members of a population do not impose heterogeneous external costs and/or benefits through their actions.

The internalized effects of individuals' actions on their utility across the population are given by the "sum" of the valuations of all those who choosing $a = 1$. As our population is a continuum, this is given by $\int_{-R-P}^{\bar{v}} vf(v)dv$, where $\{i | v^i \in [-R - P, 0)\}$ constitutes the set of those choosing $a = 1$ *ex post* that would not have chose $a = 1$ without the inducement of any incentives or disincentives. Recall that v^i accounts for the benefits and costs of choosing $a = 1$ to members of the population. As such, foregone utility or profit, or the cost of adopting a new technology, would be included in v^i. This quantity is decreasing in compliance, exerting downward pressure on the social planner's utility as more individuals switch from their *ex ante* preferred choice of behavior.

Total rewards are given by $R \cdot [1 - F(-R - P)]$, as the entire share of those choosing $a = 1$ must be given the reward, which is of size R. This represents an addition to the utility of those receiving the reward but must be financed through taxation. Transfers have no net effect on the social planner's utility. Total punishments equal $P \cdot F(-R - P)$, as the share of those choosing $a = 0$ must receive the punishment. This represents a subtraction from the utility of those receiving the punishment but is then redistributed to the population as a whole, so it is also not taken into account by the social planner.

Such transfers, however, are not administered without costs. Let $C_p : [0, 1] \to \mathbb{R}$ and $C_r : [0, 1] \to \mathbb{R}$ be differentiable functions giving the administrative cost incurred to apply disincentives or incentives, respectively, to a given share of the population. As such, C_p takes as its argument the share of the population choosing $a = 0$, F, while C_r takes as its argument the share of the population choosing $a = 1$, $1 - F$. To

[9] Receiving R will always be contingent on $a = 1$, paying P will always be contingent on $a = 0$.

capture the notion that the costs of administering a policy are increasing in the measure of individuals to whom the policy is applied (those receiving rewards and/or those receiving punishments), $C_p'(\cdot) > 0$ and $C_r'(\cdot) > 0$.[10] Such costs are only incurred if the policy entails a nonzero amount of punishment or reward, such that $C_p(F(0)) = 0$ and $C_r(1 - F(0)) = 0$.

The two types of policy share many costs in common. In many instances, most monitoring costs are incurred across the entire population and would be necessary for either type of policy. It is likely, though, that certain forms of rewards, such as tax credits, require a smaller administrative apparatus than punishments. In any case, to allow the fixed costs to differ from one another, let $C_p(0) \geq 0$ and $C_r(0) \geq 0$, where strict inequalities at 0 imply some fixed cost. Note this assumption also implies that $\forall F \neq F(0), C_p(F) > 0$ and $C_r(1 - F) > 0$.

Example The figures illustrating key results that appear throughout the paper all employ the same simple functional forms. The equations below specify these functional forms and also serve to ground the descriptions above of the components of the model.

Letting $v^i \sim U(\underline{v}, \overline{v})$, then $F(v) = (v - \underline{v})/(\overline{v} - \underline{v})$. Imposing $\underline{v} < 0 < \overline{v}$, *ex ante* compliance is then $1-F(0) = \frac{\overline{v}}{\overline{v}-\underline{v}} \in (0,1)$. The social cost from individuals changing their behavior is given by $\int_{-R-P}^{\overline{v}} v f(v) dv = \frac{\overline{v}^2-(-R-P)^2}{2(\overline{v}-\underline{v})}$ Further, suppose:

$$\begin{aligned} W(1 - F(-R-P)) &= g \cdot [1 - F(-R-P)] = g \cdot \left(\frac{\overline{v} + R + P}{\overline{v} - \underline{v}} \right), \ g > 0 \\ C_p(F(-R-P)) &= c_p \cdot F(-R-P) = c_p \cdot \frac{-R - P - \underline{v}}{\overline{v} - \underline{v}}, \ c_p > 0 \\ C_r(1 - F(-R-P)) &= c_r \cdot [1 - F(-R-P)] = c_r \cdot \frac{\overline{v} + R + P}{\overline{v} - \underline{v}}, \ c_r > 0 \end{aligned}$$

In this example, there is no fixed administrative cost to either type of policy, i.e., $C_r(0) = C_p(0) = 0$.

[10]It is likely that administrative costs also increase in the size of the reward and/or punishment, as well. The formulation above provides the most straightforward analytical approach to drawing out the central insights of the model, however, so the analysis proceeds as though administrative costs of a policy depend only on the measure of individuals to whom that policy is applied. Importantly, allowing administrative costs to increase in the size of the policy as well as in the share of the population recieving the reward or punishment does not alter any of the substantive conclusions presented below.

4 The Social Planner's Optimal Policy

The social planner – a public-interested policymaker – thus faces the following problem:

$$\max_{(P,R)\in\mathbb{R}^2_+} W(1-F(-R-P)) + \int_{-R-P}^{\infty} vf(v)dv - \mathbb{I}_{P>0}\cdot C_p(F(-R-P)) - \mathbb{I}_{R>0}\cdot C_r(1-F(-R-P)), \quad (1)$$

subject to the constraint that this be greater than social welfare under the status quo, which is given by $W(1-F(0)) + \int_0^{\overline{v}} vf(v)dv$. Lemma 1 establishes that the policymaker will use only punishments or only rewards, if she intervenes at all. The proof may be found in Section A.1 of the Appendix.

Lemma 1. *It is never optimal to use strictly positive levels of both punishments and rewards.*

Recall that increasing the share of the population choosing $a = 1$ raises the administrative cost of rewards but lowers on the administrative costs of punishment policies. It follows then that there exists a threshold share of the population choosing $a = 1$ above which punishment policies are cheaper and below which reward policies are cheaper. When indifferent, the policymaker would still never want to use both types of policy to induce that share of the population to choose $a = 1$, as this would double up on the costs.[11]

The social planner's problem may now be stated as follows: determine the optimal size of reward, R^*, if restricted to only use rewards; determine the optimal size of punishment, P^*, if restricted to only use punishments; having found the optimal size of each type of policy, compare social welfare under P^* and R^* to find the optimal type (and size) of policy. Equations 2 and 3 characterize the solutions to the problems of choosing an optimal P and an optimal R as individual policies.

The social planner maximizes $W(1-F(-P)) + \int_{-P}^{\overline{v}} vf(f)dv - C_p(F(-P))$ with respect to P to find the best choice for the size of punishment to obtain:

$$[P] : \left[W'\left(1-F(-P^*)\right) - P^* + C_p'\left(F(-P^*)\right)\right]\cdot f(-P^*) = 0. \quad (2)$$

The social planner maximizes $W(1-F(-R)) + \int_{-R}^{\overline{v}} vf(v)dv - C_r(1-F(-R))$ with respect to R to find the best choice for the size of reward to obtain:

$$[R] : [W'\left(1-F(-R^*)\right) - R^* + C_r'\left(1-F(-R^*)\right)]\cdot f(-R^*) = 0. \quad (3)$$

These two equations implicitly characterize the best choice of P and the best choice of R, were the policymaker

[11] As remarked in the Appendix, this result is robust to expanding the influences on administrative cost to include the size of the policy itself (i.e., P, R), not just the share of the population to which it is applied.

restricted to each type of policy, respectively. Results from the theory of monotone comparative statics help characterize the social planner's most preferred policy (Ashworth & Bueno de Mesquita 2006).

In this approach, the type of policy is taken to be one choice variable, and the share of the population choosing $a = 1$ *ex post* is taken to be the other.[12] The choice over the type of policy compares the attractiveness of using punishments to the attractiveness of using rewards to induce a given share of the population to choose $a = 1$. The choice over the share of the population choosing the socially beneficial action holds the type of policy fixed. The share of the population choosing $a = 1$ is a more meaningful measure of the "size" of a policy intervention than the value of P or R. The decision to deploy a policy intervention or to remain at the status quo share of the population choosing $a = 1$ (when $P, R = 0$) is a subsequent decision in which the optimal policy intervention (defined below) is compared to the status quo.

Definition 1 (Optimal Policy Intervention)**.** *The social planner's most-preferred policy intervention, characterized by an optimal share of the population choosing $a = 1$ and an optimal type of policy that together specify $P^* > 0$ or $R^* > 0$.*

The ultimate goal is to understand how the optimal choices of type and size of policy change as a function of changes in the exogenously given elements of the model. In a problem with two choice (i.e., endogenous) variables, the first step is to characterize the relationship between the two. The next lemma establishes that the *ex post* share of the population choosing $a = 1$ and the use of disincentives (as opposed to incentives) are complementary to one another. This is a direct result of the asymmetric way in which administrative costs accrue under incentive and disincentive policies.[13]

Lemma 2. *An increase in the share of the population choosing $a = 1$ ex post (given by $1 - F$), is more attractive under the use of disincentives than under the use of incentives. Equivalently, the use of disincentives is increasingly attractive relative to the use of incentives as the share choosing $a = 1$ increases.*

Any exogenous change which leads to an increase (resp. decrease) in the optimal value of at least one of the choice variables will indirectly lead to an increase (resp. decrease) in the other. All increases/decreases are weak, and the choices of policy are ordered $p > r$. An exogenous change that increased the optimal share choosing $a = 1$ would make the use of punishments increasingly attractive relative to the use of rewards, though rewards may still be the optimal policy. An exogenous change that made punishments more attractive relative to rewards would indirectly lead to (a weak) increase in the optimal share of the population choosing $a = 1$. This approach ensures that any indirect effects that occur among the choice variables reenforce

[12]The equivalence of this approach is established, and the approach fully explicated, in Appendix A.1.

[13]Equation 7 and the surrounding discussion in Appendix A.1 suffice as a proof of the assertion.

the direct effect of the exogenous variable on the optimal policy. The crucial condition is complementarity between the parameter and both choice variables. Only if a change in an exogenous variable leads to an increase in one choice variable but a decrease in the other will it be impossible to characterize the overall effect of a change in the variable on the optimal policy.[14]

The parameters of interest are the functions $W(\cdot)$, $F(\cdot)$, $C_p(\cdot)$, and $C_r(\cdot)$. Proposition 1 appears in three parts to facilitate the exposition and explanation of the result for each parameter.[15] The proposition makes clear that complementarity between the exogenous element and both of the choice variables exists for some of the parameters but not all.

Proposition 1 (a)**.** *As the added value to society from an increase in the share of the population choosing $a = 1$ rather than $a = 0$ increases at all levels: the share choosing $a = 1$ in the optimal policy intervention increases; this indirectly makes the use of disincentives increasingly attractive relative to the use of incentives; and the optimal policy intervention becomes more attractive relative to the status quo.*

Part (a) of the proposition regards the externality at the heart of the social planner's problem. Recall that $W(\cdot)$ denotes either the societal benefit that accrues from a share of the population choosing $a = 1$ and/or the social cost from members of the population choosing $a = 0$. Increasing the added benefit from a larger share choosing $a = 1$ (for all $1 - F \in [0, 1]$) leads the social planner to want higher shares choosing $a = 1$ *ex post*. Even though there is no direct effect on the optimal type of policy, because of the complementarity between higher shares choosing $a = 1$ and the use of punishments rather than rewards, using disincentives grows more attractive relative to using incentives. Finally, while increasing the benefit from a larger share choosing $a = 1$ increases the social planner's utility from the status quo, it increases the social planner's utility from the optimal policy intervention by at least as much. Policy interventions become more attractive as the added benefit from inducing more of the population to choose $a = 1$ increases..

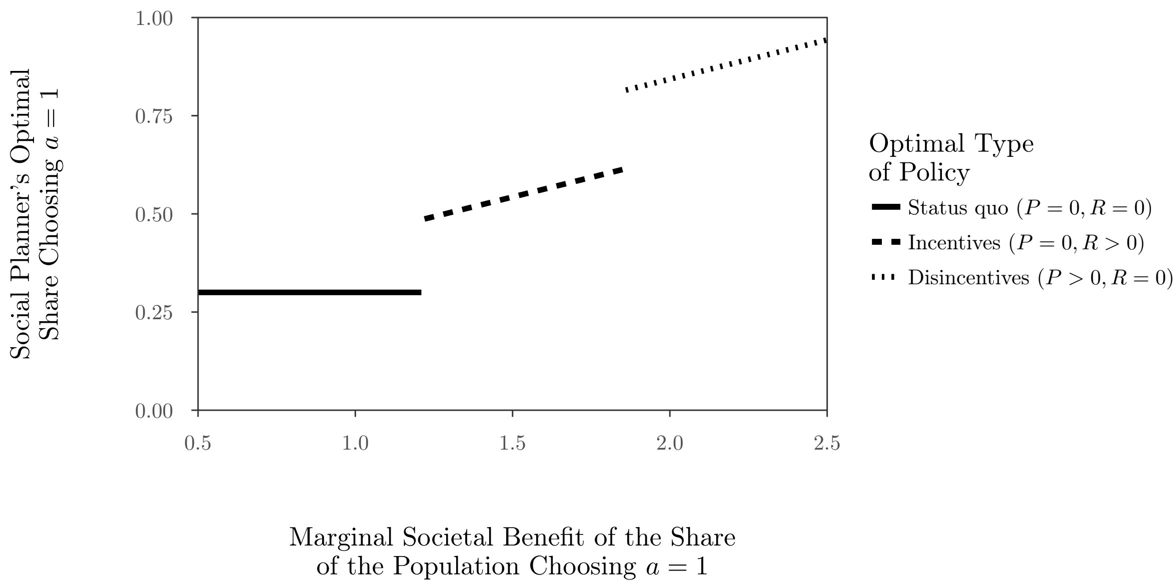
Figure 1 depicts the comparative statics from part (a). As the marginal societal benefit from more of the population choosing $a = 1$ increases (at all levels), the optimal share choosing $a = 1$ is weakly increasing. Accordingly, the optimal type of policy moves from rewards to punishments. Further, once a policy intervention is optimal (vis-à-vis the status quo), policy intervention remains optimal as the marginal societal benefit of higher shares choosing $a = 1$ continues to increase.

A useful characterization of the result, albeit not entirely precise, goes as follows. Policy domains in

[14]The implication of such a "non-result" is that a comparative static derived from a given choice of functional form would not be robust to the choice of funcational forms from among those that satisfy the broad conditions stated above.

[15]Each part follows immediately from a more formal, corresponding statement given in Appendix A.2.

Figure 1: The effect of increasing the marginal benefit to society of the share of the population choosing $a = 1$ on the social planner's preferred share of the population choosing $a = 1$ and type of policy



Notes: The dotted line represents the status quo share of the population choosing $a = 1$. As per the functional forms given in the example above, g is the marginal societal benefit, W' in the notation of the model.

which small increases in the share of the population choosing $a = 1$ relative to the status quo yield large but rapidly diminishing increases in societal benefit (or reduction of societal cost) favor the use of rewards. Policy domains in which a small share of the population not choosing $a = 1$ bears high societal cost (or forfeits large societal benefits) favor the use of punishments. This comprises the most direct answer to the normative aspect of the question motivating this paper.

Consider the rewards in the form of monopolistic market power that are granted to patent and copyright holders. When inducing innovation of any given product or even artistic creation, most benefit accrues at low levels of "compliance" but levels off at higher levels of compliance. This suggests that rewards are likely to be optimal. The seeming absurdity of the notion that government might penalize people for not innovating is in fact a reflection of the inefficiency of punishing a large segment of the population in order to spur innovation by those with the highest *ex ante* valuation for innovating (likely those best positioned to do so). For actions in which the order and well-functioning of society depend on nearly full compliance (e.g., safe driving, respecting property rights, not committing violent acts), small levels of non-compliance achieved with a (possibly large) punishment attain the external benefit of compliance without incurring excessive administrative costs.

The next part of the proposition considers the distribution of valuations across the population for choosing

$a = 1$ rather than $a = 0$ in the absence of additional inducements, positive or negative. Specifically, the statement invokes an "upward shift" in the distribution of valuations for compliance, F.[16]

Definition 2 (Upward Shift in the Distribution of Valuations)**.** *$\hat{F}$ represents an upward shift of F if for $v \sim F$ and $\hat{v} \sim \hat{F}$, $\hat{v} = \mu + v$ for some $\mu \geq 0$.*

Proposition 1 (b)**.** *As the distribution of the population's valuations for choosing $a = 1$ rather than $a = 0$ shifts upwards: the optimal share choosing $a = 1$ increases. This indirectly makes the use of disincentives increasingly attractive relative to the use of incentives. The optimal policy intervention and the status quo both provide the social planner greater utility, neither at an unambiguously higher rate than the other.*

This part of the proposition asks how the social planner's optimal policy would change if the population were *ex ante* more prone to choosing the socially-desirable behavior, $a = 1$. In a population more prone to choosing the beneficial behavior, fewer individuals must change their behavior in order to achieve a desired share choosing $a = 1$. The policy intervention will be less distortionary, so the negative valuations incurred weigh less heavily on the social planner's calculus. The optimal share of the population choosing $a = 1$ will increase.

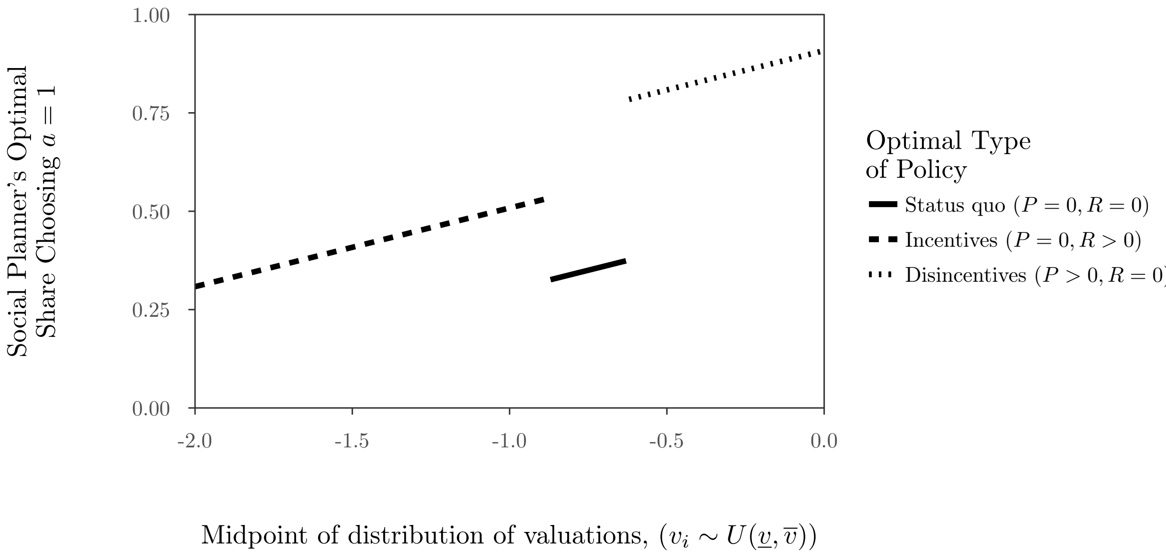
Similar to the analysis of W, an upward shift in F has no direct effect on the attractiveness of using punishments instead of rewards. Its effect on the optimal share of the population choosing $a = 1$, however, an upward shift in F indirectly favors the use of disincentives rather than incentives. A larger share of the population choosing $a = 1$ drives the administrative cost of using punishments down while driving the administrative cost of using rewards up.

An upward shift in F affects the comparison to the status quo utility more intricately than the change considered in part (a). The social planner's utility from the optimal policy intervention increases, but so does her utility from the status quo through $W(1 - F(0))$. An upward shift will not always make a policy intervention increasingly attractive relative to the status quo.

Figure 2 illustrates these results. Moving to the right along the horizontal axis, the distribution of valuations, F, shifts upwards. The example entails a uniform distribution of valuations, and the running variable is the midpoint of the distribution. The optimal share choosing $a = 1$ is increasing when a policy intervention is optimal, and this ultimately favors the use of punishments instead of the use of rewards. It may be, however, that the optimal level of compliance and type of policy "regress" to the status quo even as F continues to shift upwards.

[16] This is a stronger assumption than necessary at this point, but the restriction to upward shifts is necessary in later sections. For Proposition 1(b), the results hold for any first-order stochastic increase in the distribution of valuations.

Figure 2: The effect of increasing the *ex ante* valuations for choosing $a = 1$ rather than $a = 0$ on the social planner's preferred share of the population choosing $a = 1$ and type of policy



Notes: The dotted line represents the status quo share choosing $a = 1$, given by $1 - F(0)$. As in the example, $v_i \sim U(\underline{v}, \overline{v})$, with the horizontal axis tracking $(\overline{v} + \underline{v})/2$. An increase in this quantity corresponds to an upward shift in the distribution of valuations ($F(\cdot)$).

The result pertaining to F will play an important role below in considering a subpopulation for which the policy intervention is not intended. Incorporating uncertainty in enforcement into the model would manifest through the distribution of valuations, though with little substantive effect on the results. The administrative costs of the different types of policy receive attention next.

Proposition 1 (c)**.** *A decrease in the marginal cost of using disincentives to induce more of the population to choose $a = 1$ rather than $a = 0$ with no change in fixed costs has competing effects on the level of compliance and type of policy in the optimal policy intervention. The same holds for increases in the marginal cost of using incentives to induce more compliance with $a = 1$.*

The final result in Proposition 1 is surprising precisely because of the ambiguity it highlights. Adjusting the marginal costs of the types of policies would seem to be the most straightforward way to favor one type of policy over the other, but this is not the case. The changes have direct effects on both the optimal share of the population choosing $a = 1$ and the optimal type of policy, but in such ways that the indirect effects find themselves at odds. It is impossible to state – without invoking additional assumptions – that reducing the per-unit administrative costs of a type of policy always favors the use of that policy.[17]

[17]Specifically, the additional assumptions – likely related to the fixed cost – would also need to ensure that

Lowering the marginal cost of either type of policy encourages larger policy interventions. The other direct effect in each case is to favor the use of rewards rather than punishments: lowering the marginal cost of rewards makes the use of incentives cheaper at all levels; lowering the marginal cost of punishments, while holding fixed costs constant, makes disincentives more costly at all levels. The indirect effect of making higher levels of compliance cheaper to achieve is to favor the use of punishments, but the indirect effect of making rewards relatively more attractive is to favor lower levels of compliance. The direct effects thus lead to opposing indirect effects.

Figure 3 shows exactly these effects. The running variable is the ratio of the coefficients on the cost terms (see the example functional forms given above) representing the marginal costs. To demonstrate changes in both variables, the coefficients sum to one, although this forces the marginal cost of rewards to increase as the marginal cost of punishment decreases. The optimal level of compliance decreases even as the marginal cost of a disincentive-based policy falls relative to the marginal cost of incentive-based policy. While the optimal type of policy does not switch back to rewards, as the proposition suggests could occur, this is likely because a decrease in C_p is tied to an increase in C_r. Furthermore, the competing effects that occur when marginal costs change allow the status quo to be more attractive than either type of policy intervention at intermediate levels.

5 Modeling the Political Environment

In contrast to a social planner, an office-motivated politician seeks a policy that will garner the support of a majority of the population. This section characterizes the majority-preferred policy and its relationship to the social-planner's optimal policy. This task requires deeper engagement with the redistributive consequences of different types of policies than was necessary above.

The Redistributive Implications of Incentives and Disincentives

Lump-sum transfers applied uniformly across the population accomplish both the disbursement of revenues from fines as well as the financing of subsidies, as in Meltzer & Richard (1981). All members of the population thus receive an equal share of revenues collected from fines and carry an equal burden in financing of subsidies. Fines are still only administered to those who choose $a = 0$, while subsidies are only given to those who choose $a = 1$. Given a fine of size P and a subsidy of size R, revenues from fines available for redistribution are given by $P \cdot F(-P - R) - C_p(F(-P - R))$, while the required financing for a subsidy is given by

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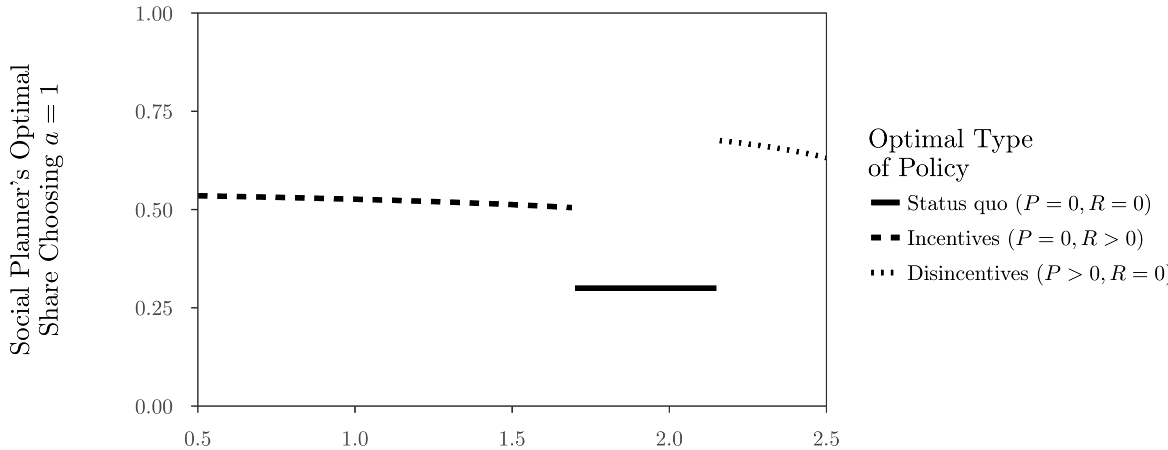
the average cost of a given policy decreased at all levels of compliance.

Figure 3: The effect of decreasing the marginal cost of disincentives and increasing the marginal cost of incentives on the social planner's preferred share of the population choosing $a = 1$ and type of policy



Marginal Cost of Reward/Marginal Cost of Punishment (where $C_p + C_r = 1$)

Notes: The dotted line represents the status quo share of the population choosing $a = 1$, $(1 - F(0))$. The ratio of marginal cost coefficients, C_r/C_p, is increasing along the horizontal axis. For purposes of illustration, $C_p + C_r = 1$, though of course this need not be the case more generally.

$R \cdot [1 - F(-P - R)] + C_r(1 - F(-P - R))$. Every citizen receives/pays these quantities, which account for the deadweight administrative costs entailed in each type of policy. Similarly, societal benefit from the share of the population choosing $a = 1$, $W(1 - F(-P - R))$, accrues to all individuals, regardless of whether they choose the desired behavior.

It is helpful to clarify the utility a given member of the population, i, receives from a policy (P, R). The subscript 1 indicates that i chose $a = 1$, while 0 indicates that i chose $a = 0$. Fact 1 from above determines when an individual will and will not comply, for which the utility function below accounts. Indeed, $U^i(P, R)$ is the upper envelope of $U^i_1(P, R)$ and $U^i_0(P, R)$. Recall $1 - F(-P - R)$ is the proportion of the population that chooses the desired behavior, $a = 1$, given punishment of size P and reward of size R.

$$U^i(P,R) = \begin{cases} U_1^i(P,R) = \\ W(1-F(-P-R)) + F(-P-R)(P+R) - C_p(F(-P-R)) - C_r(1-F(-P-R)) + v^i \\ \text{ if } v^i \geq -P-R \\ U_0^i(P,R) = \\ W(1-F(-P-R)) - [1-F(-P-R)](P+R) - C_p(F(-P-R)) - C_r(1-F(-P-R)) \\ \text{ if } v^i < -P-R \end{cases} \tag{4}$$

Remark 1. *The policy that maximizes a member of the population's utility will maximize the utility of all those who made the same choice.*

If $(P_1^{**}, R_1^{**}) = \arg\max_{(P,R)} U_1^i(P,R)$, then (P_1^{**}, R_1^{**}) is also the policy that maximizes utility for members of the population that choose $a = 1$. If $(P_0^{**}, R_0^{**}) = \arg\max_{(P,R)} U_0^i(P,R)$, then (P_0^{**}, R_0^{**}) is the policy that maximizes utility for all who chose $a = 0$. These cases include $(P,R) = (0,0)$.

Remark 2. *No restriction that $P, R \geq 0$ appears here, as it did above.*

The quantity P continues to refer to a policy applied to those who choose $a = 0$, but negative values of P correspond to a policy rewarding those that choose $a = 0$. Similarly, R continues to refer to a policy applied to those who choose $a = 1$, but negative values of R correspond to a policy punishing those that choose $a = 1$. These policies would encourage socially harmful actions or discourage socially beneficial behavior. The possibility of $P, R < 0$ receives further attention below.

Preferences over Incentive and Disincentive Policies

The next result establishes that the preference of the individual with the median valuation for compliance is equivalent to the majority preference. Equivalently, support of the individual with the median valuation, henceforth the "median voter" (MV), is necessary and sufficient for majority support. The median voter's most preferred policy would be the winning policy in an election between two office-motivated candidates in which all members of the population cast exactly one vote according to the weakly dominant strategy of voting for the candidate whose proposal offers them greatest utility. Denote this policy $(P^{**}, R^{**}) := \arg\max U^{MV}(P,R)$, where $v^{MV} = F^{-1}(1/2)$.

Lemma 3. *There does not exist a policy more preferred by a majority of individuals than (P^{**}, R^{**}).*

For the majority preference relation to be equivalent to the preference relation of the individual with the median valuation, v^{MV}, the profile of preferences across the population must satisfy monotonicity with respect to the valuations. Remark 1 above implies that, for all i, $\arg\max_{(P,R)} U^i(P,R) \in \{(P_1^{**}, R_1^{**}), (P_0^{**}, R_0^{**})\}$. The proof then demonstrates the existence of a threshold valuation such that all members of the population with valuations above the threshold prefer (P_1^{**}, R_1^{**}), while those with valuations below the threshold prefer (P_0^{**}, R_0^{**}). This demonstrates the monotonicity of preferences in the valuations necessary to invoke the median voter theorem (Gans & Smart 1996).

This result establishes that if $(P^{**}, R^{**}) = (P_1^{**}, R_1^{**})$ then the median voter along with a majority of the population will choose $a = 1$ under the policy that receives majority support. Conversely, if $(P^{**}, R^{**}) = (P_0^{**}, R_0^{**})$, then the median voter along with a majority of the population will choose $a = 0$ under the policy that receives majority support. If the median voter prefers no policy intervention, such that $(P^{**}, R^{**}) = (0, 0)$, then so, too, will the majority of the population that chooses the same action without any inducements to behave differently.

6 The Majority-Preferred Policy

To understand the winning policy from Lemma 3, (P^{**}, R^{**}), the next task is to characterize (P_1^{**}, R_1^{**}) and (P_0^{**}, R_0^{**}). As with the social planner, no member of the population would employ both types of policy to the same end. It may still be that no policy is the most preferred.

Lemma 4. *All members of the population prefer the use of either an incentive or a disincentive policy – but not both – to achieve any given level of compliance.*

This insight follows from precisely the same logic as it did for the social planner in the previous section. Further, it permits an approach to analyze the majority-preferred policy that closely mirrors the approach taken to analyze the optimal policy from the social planner's perspective. Indeed, Proposition 3 in Appendix Section B.2 states that the comparative statics of the majority-preferred policy are highly similar to the comparative statics of the social planner's optimal policy (see Proposition 1). This is especially true when comparing the majority-preferred policy intervention, formally defined below, and the social planner's optimal policy intervention, as defined above.[18]

[18] The possibility remains that for some exogenous change, the status quo (i.e., $P = 0, R = 0$) becomes increasingly attractive as incentive- or disincentive-based policies also become more attractive relative to the other. As such, and similar to the social planner's optimal policy, characterizing the overall majority-preferred policy requires some care.

Definition 3 (Majority-Preferred Policy Intervention). *The most preferred policy intervention by the member of the population with the median valuation for choosing* $a = 1$ *rather than* $a = 0$*, characterized by a majority-preferred level of compliance and a majority-preferred type of policy that together specify* $P^{**} \neq 0$ *or* $R^{**} \neq 0$*.*

A complementarity again exists between pursuing high shares of the population choosing $a = 1$ and the use of policies applied to those choosing $a = 0$ instead of policies applied to those choosing $a = 1$. Proposition 3 in Appendix B.2 demonstrates that majority-preferred policy intervention responds similarly to increases in the marginal societal benefit from additional compliance, upward shifts in the distribution of valuations, and decreases in the marginal cost of each type of policy. The majority-preferred policy and the social planner's optimal policy are not the same, however. The more interesting characterization of the majority-preferred policy is in relation to the social planner's optimal policy, and this is the focus of the next proposition. It states that the median voter's preferred policy intervention "sandwiches" the social planner's optimal policy intervention first from above and then from below, and the two only coincide when each prefers the status quo to their preferred policy intervention.

Proposition 2. *Comparing the majority-preferred policy intervention (*$P^{**} \neq 0$ *or* $R^{**} \neq 0$*) to the social planner's optimal policy intervention (*$P^{*} > 0$ *or* $R^{*} > 0$*):*

A majority will neither punish itself for choosing $a = 0$ *nor reward a minority who choose* $a = 1$ *sufficiently to achieve the social planner's optimal share of the population choosing* $a = 1$*.*

A majority will either reward itself for choosing $a = 1$ *or punish a minority who choose* $a = 0$ *excessively, achieving a higher share of the population choosing* $a = 1$ *than the social planner would.*

The complementarity between a majority of the population choosing $a = 1$ *and the use of policies applied to those choosing* $a = 0$ *favor two of these outcomes: a majority choosing* $a = 1$ *and imposing a larger-than-optimal disincentive on the minority choosing* $a = 0$*; a majority choosing* $a = 0$ *and instituting a smaller-than-optimal incentive for the minority choosing* $a = 1$*.*

The implications regarding majority behavior follow immediately from the fact that the median voter chooses $a = 1$ if and only if a majority of individuals are also choosing $a = 1$. The median voter's choice of $a = 1$ is a monotonically increasing step function of the share of the population choosing $a = 1$. Parameter changes that increase the share of the population choosing $a = 1$ then naturally make it more likely that the median will choose $a = 1$.[19]

[19]More accurately, the *ex post* action of the median voter, the overall level of compliance, and the type of policy are jointly determined in finding the majority-preferred policy, as the latter two were jointly determined in finding the social planner's optimal policy. The three decisions display weak pairwise complementary.

A member of the population choosing $a = 1$ *ex post* has a marginal utility function that is the same as the social planner's, but with a higher marginal net benefit at all levels of compliance. A member of the population choosing $a = 0$ *ex post* has a marginal utility function that is the same as the policymaker's, but with a lower marginal net benefit at all levels of compliance. Proposition 1 (a) then provides the results necessary to compare the median voter's preferred policy and the social planner's. Maximizing utility given a choice of $a = 1$ will lead to greater levels of compliance than the social planner's optimal level and, indirectly, favor the use of punishments over rewards. Maximizing utility given a choice of $a = 0$ will lead to lower levels of compliance than the social planner's optimal level and, indirectly, favor the use of reward-based policies over the use of punishment-based policies. Relative to those choosing $a = 0$, members of the population choosing $a = 1$ will desire higher levels of compliance and favor the use of punishment-based policies.

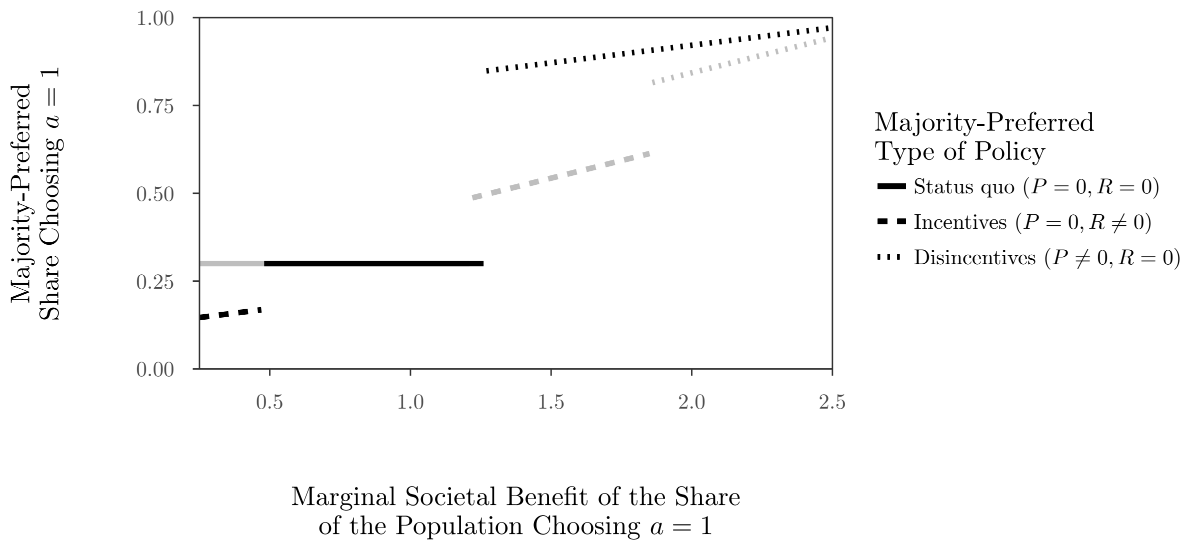
Proposition 2 highlights the intuitive nature of these comparisons. A majority may excessively reward itself for choosing $a = 1$ or excessively punish a minority for choosing $a = 0$, or a majority may insufficiently reward a minority choosing $a = 1$ or insufficiently punish itself for choosing $a = 0$. That a majority would ever choose to punish its own harmful behavior or reward its own beneficial behavior is perhaps counterintuitive. In fact, these two outcomes are less likely in a sense. Conditions that lead a majority to choose $a = 1$ (resp. $a = 0$) increase the attractiveness of punishments (resp. rewards).

Figures 4-6 illustrate these results.[20] The black lines correspond to the majority-preferred share of the population choosing $a = 1$ and the line types indicate the type of policy. The gray lines correspond to the social planner's optimal share choosing $a = 1$ and the line types again indicate the type of policy. The gray lines lie above and then below the black lines, switching when the policy intervention would induce the median voter to choose $a = 1$ instead of $a = 0$. The median voter's preferred policy will not achieve the social planner's preferred share of the population choosing $a = 1$, unless both prefer the absence of a policy intervention, the status quo. Because conditions that favor more of the population choosing $a = 1$ also favor the use of punishments, the majority-preferred policy intervention will tend to use punishments when larger than the social planner's and rewards when smaller than the social planner's.[21]

[20]Figures 5-6 appear in Appendix B.2.

[21]Of course, examples do exist of insufficiently large punishments that the majority levies on itself for choosing $a = 0$. The proliferating taxes on sugary drink purchases – almost always quite small – appear to be such a policy.

Figure 4: The effect of increasing the marginal benefit to society of the share of the population choosing $a = 1$ on the majority-preferred share of the population choosing $a = 1$ and type of policy



Notes: The dotted line represents the status quo share of the population choosing $a = 1$, $1 - F(0)$. The gray lines correspond to the social planner's optimal share choosing $a = 1$ and type of policy. The horizontal axis represents increases in the marginal benefit to society of members of the population choosing $a = 1$, W', or g in the context of the example functional forms that underlie the figures.

Encouraging the Socially Harmful Action

A particularly surprising feature of the above results is the possibility that members of the population choosing $a = 1$ might be punished or that those choosing $a = 0$ might be rewarded. It would never have been optimal for the social planner to implement $P, R < 0$. In the context of popular support, however, encouraging the socially desirable action garners support among the population only in as much as the social benefit contributes to utility enough to outweigh redistributive concerns. The median voter need not take into account any valuations other than her own, save for the role the valuations play in determining compliance.

Any of the cases in Proposition 2 may involve "negative rewards" for choosing $a = 1$ (fine for choosing $a = 1$) or "negative punishments" for choosing $a = 0$ (subsidy for choosing $a = 0$). When will $1 - F(-P^{**} - R^{**}) \leq 1 - F(0)$? A majority-preferred policy that deters the choice of $a = 1$ will likely entail $R^{**} < 0$. It involves low shares of the population choosing $a = 1$ (with the median voter likely choosing $a = 0$), so a policy that applies to those choosing $a = 1$ is the least costly.

Corollary 1. *Conditions that favor low shares of the population choosing $a = 1$ favor a majority-preferred policy that induces lower shares of the population to choose $a = 1$ ex post than would have ex ante. Such conditions also favor the use of policies that apply to members of the population choosing $a = 1$, including*

$P^{**} = 0, R^{**} < 0$, *a fine ("negative reward") administered to those choosing the beneficial action.*

The majority-preferred policy induces smaller shares of the population to choose $a = 1$ than the social planner's optimal policy when the median voter chooses $a = 0$ *ex post*. At the extreme, these "rewards" are so small as to become fines for those choosing $a = 1$ (see Figure 4). While such policies appear radically different than policies encouraging the socially beneficial behavior, it is important to note that a policy barely encouraging the beneficial behavior and a policy barely discouraging the beneficial behavior provide the median voter nearly equivalent utility.

7 Accounting for Unaffected Subpopulations

The model has thus far set aside the possibility that the policy applies only to a subpopulation of interest, while the population at-large is not confronted with a choice between $a = 1$ and $a = 0$. This may take one of two forms: 1) it is not possible to apply the policy only to the subpopulation of interest, 2) it is possible to administer the policy only to the subpopulation of interest.

In the presence of an unaffected subpopulation that tacitly chooses $a = 1$ (e.g., non-drivers not speeding), it is as though the distribution of valuations for the whole population is an upward shift of the distribution of valuations for the subpopulation of interest (drivers). Invoking Proposition 1(b), this favors the use of disincentives. In the example of discouraging speeding, this formalizes the intuition that rewarding a substantial segment of the population (non-drivers) for not speeding (when they were at no risk of doing so anyway) would be incredibly inefficient. From the social planner's perspective, these conditions favor the use of fines and a high share of the population choosing to drive safely (or not at all), and the fines would likely be even larger if majority-preference dictated the choice of policy.

In the context of copyright for artistic works, or patents for inventions, government wishes to encourage innovation, but it is unable to target the subpopulation of possible innovators, artistic or otherwise. This constitutes the presence of a large subpopulation that will never "comply" with the behavior government wishes to encourage. The distribution of valuations of the population as a whole is less prone to choose $a = 1$ than the distribution of valuations within the subpopulation of interest. Referencing Proposition 1(b) again, these conditions favor a social planner using rewards to spur innovation by a small share of the overall population. That majority-preference would likely lead to a smaller-than-optimal reward foots with oft-heard complaints from innovators across fields, namely, that the reward is insufficient compensation for their creative effort.

In many circumstances, however, it is easy to differentiate those in a subpopulation of interest from those who are not. For instance, a policymaker may wish to target an industry. It is usually straightforward to identify firms from individuals and, further, firms in a certain industry from firms in other industries. Those who do not own cars would not be penalized for failure to possess vehicle registration, those without cropland would be ineligible for farm subsides. Call the portion of the population that would not receive either a reward or a punishment under a given policy the "unaffected subpopulation." The "subpopulation of interest" refers to the portion of the population to whom any incentive or disincentive would apply.

If enforcement is able to discriminate between the subpopulation of interest and the rest of the population, then the analyses of the social planner's optimal policy hold without further modification. The policymaker may ignore redistributive implications for subpopulations not directly affected by the policy, as she could with redistributive implications for individuals in the subpopulation affected by the policy. In the analysis of the popular support for incentive and disincentive policies, however, the presence of an unaffected subpopulation will materially affect the analysis. This population certainly benefits from any additional compliance with the desired behavior from the population of interest. Furthermore, such individuals must also contribute to the financing of subsidies, but they may likewise benefit from the redistribution of fines or taxes collected.

Let the size of the subpopulation not directly affected by the policy, i.e., not eligible for a reward or punishment, be given by $\lambda \leq 1$. Denote an arbitrary member of this group by ℓ. The entire population is still of mass 1, so the size of the subpopulation of interest is $1-\lambda$. Under a policy that involves the use of rewards (as well as potentially punishments), ℓ would have to contribute $(1-\lambda)R(1-F(-P-R))+C_r((1-\lambda)(1-F(-P-R)))$ to finance the subsidy, but receive no compensation for her behavior. Under a punishment-based policy, ℓ will not receive any fine, but she will receive $(1-\lambda)PF(-P-R)-C_p((1-\lambda)F(-P-R))$.

With regards to policies applied to those choosing $a=1$, ℓ's utility function takes the same form as a member of the subpopulation of interest who chooses $a=0$. With regards to policies applied to those choosing $a=0$, however, ℓ shares the same utility function as a member of the subpopulation of interest who chooses $a=1$. Accordingly, maximizing ℓ's utility entails comparing the most-preferred punishment-based policy for a member of the sub-population of interest given a choice of $a=1$ to the most-preferred reward-based policy for a member of the sub-population of interest given a choice of $a=0$, and then comparing the best of those to the utility ℓ receives in the absence of any further policy intervention, namely $W((1-\lambda)(1-F(0)))$.

The final result characterizes the preferences of a member of an unaffected subpopulation, focusing on the social planner's optimal policy intervention and the majority-preferred policy intervention. Assume that the share of the population that is unaffected by the policy is given by $\lambda \geq \frac{1}{2}$, such that a majority of the population will receive neither incentive nor disincentive under a policy intervention. Let $P^{**} \neq 0$ denote the best disincentive policy intervention from the perspective of a member of the subpopulation of interest (where $v^i > -P^{**}$ such that i chooses $a=1$). Let $R^{**} \neq 0$ denote the best policy intervention from the perspective of a member of the subpopulation of interest (where $v^i < -R^{**}$ such that i chooses $a=0$).

Corollary 2. *The majority-preferred policy will entail either a larger-than-optimal disincentive applied to those choosing $a = 0$ or a smaller-than-optimal incentive applied to those choosing $a = 1$ (potentially a "negative reward," punishing those who choose $a = 1$). An increase in the marginal benefit to society of members of the affected subpopulation choosing $a = 1$ rather than $a = 0$ increases the utility ℓ receives from P^{**} and decreases the utility ℓ receives from R^{**}. The same holds for an upwards shift in the distribution of valuations among the affected subpopulation for choosing $a = 1$ rather than $a = 0$.*[22]

When a subpopulation not directly affected by incentives or disincentives in a given policy domain is sufficiently large so as to decide the policy for the entire population, it will always be the case that rewards will achieve smaller than the socially optimal share of the population choosing $a = 1$ and punishments will achieve larger than the socially optimal share of the population choosing $a = 1$. This was the tendency uncovered in Proposition 2, but incorporating the unaffected subpopulation makes this a certainty.

As above, the downward pressure on rewards may drive them to become negative, constituting a disincentive for choosing $a = 1$. The onerous fines that farms incur to receive organic certification are an example of a negative reward applied to those in a small subpopulation of interest that take a socially beneficial behavior. The low marginal benefit of a given (usually small) farm choosing to adopt organic practices favors the use of policies applied to those choosing $a = 1$, which further leads to smaller-than-optimal (even negative) incentives. That such a policy would generate revenue to be redistributed among the population at large would only help to overcome the loss of social benefit for a member of the unaffected subpopulation.

8 Conclusion

This paper began with a set of straightforward yet previously neglected questions: Should policy seeking to encourage a behavior offer incentives to those who take it or threaten those who do not with disincentives? What type of policy would a majority of the population support, and how will this differ from what a public-interested policymaker would choose? How do these answers change in response to the different conditions across policy domains?

The complementarity between the use of disincentives (vis-à-vis incentives) and inducing large shares of the population to take the beneficial behavior was central to the analysis. Inducing larger shares of the population to take the beneficial action becomes more attractive as the incremental benefit to doing so increases or as the population becomes more prone to take the beneficial behavior absent any policy interventions. In turn, this favors the use of disincentives. The complementarity driving these results only grew stronger

[22] Corollary 2' in Appendix C provides a more formal statement.

when accounting for the redistributive consequences of incentives and disincentives across members of the population, especially for those not directly affected by the policy. A majority will tend to prefer larger punishments for those choosing the harmful action and smaller rewards for those choosing the beneficial action than a social planner would prefer. Policies that entail a disincentive for taking the beneficial action emerge as an example of a smaller-than-optimal reward for those taking the beneficial action; the reward is so small as to be negative. The spirit of a policy encouraging the beneficial action and a policy discouraging it could not be more opposed, but the model demonstrates the highly similar redistributive implications of the two for the median voter, in whose eyes the policies are not so different.

The monotone comparative statics techniques used to derive these results emphasize generality and transparency. The insights are thus robust to any specific modeling choices that satisfy the broad assumptions made about the policymaking environment. The applicability of the model also holds great promise for building upon the insights above and extending the analysis in any of a number of directions.

Future work would do well to focus on inequality. Characterizing the implications of incentives and disincentives for inequality would be a first step. Additionally, a better understanding of policy choice based on the correlation of one's wealth and one's valuation for taking socially beneficial/harmful actions could produce an array of new insights about the use of incentives and disincentives in public policy.

Incorporating population-based behavioral effects, such as crowding in/out, norms, or coordination in interactions among the population would offer a bridge between the rather this paper's rational choice account and the large extant behavioral literature that studies the ways in which people and groups respond to policy interventions. Would the complementarities highlighted above persist in such settings? This remains an open question.

Another means of engaging with the behavioral literature would involve the political economy of "scaling up." Many interventions begin with pilot initiatives that entail incentives, such as those rewarding children to make better choices over school lunch options (Just & Price 2013, List & Samek 2017). While discussions of expansion usually focus on the extensive margin, rolling the program out to other venues, a clear implication of small-scale success would be an expansion along the intensive margin, scaling up the size of the intervention at all venues. The model warns, however, that as larger shares of the population are induced to choose a desired action, the optimal policy instrument would shift to disincentives, and popular support for rewards would wane dramatically.

Indeed, this was likely a factor in the failed attempts at healthcare reform. Granting subsidies across the board – including to segments of the population that required subsidies neither to pay for insurance nor to nudge them into insuring themselves – was innately inefficient. Incentives would only be optimal if small shares of the population purchased insurance, but insurance markets are precisely the type of setting for which the shared benefit of additional participation remains high even when only a small share of the population is not participating. This feature of the health policy domain further favors the use of disincentives, such as those implicit in the mandate. The veracity of popular support for the ACA, at least in the face of any of the suggested reforms, was particularly strong, as the model would predict.

This case suggests a key feature of the political environment that would likely enrich future work using this model a great deal. Ideologies vary across the population, and this may manifest itself in the way in which different groups evaluate the benefit or cost of actions others take across the population. If political parties group members of the population that systematically differ from the rest of the population in their evaluations of the externalities prompting policy intervention, and if there exists a correlation between these groupings, wealth, and the propensity to take the beneficial action, then it is almost certain that the parties will support vastly different policy interventions. This direction seems particularly worth exploring going forward.

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Appendix

A Proofs from Section 3-4: Social Planner

A.1 Reframing the social planner's problem

Lemma 1. *It is never optimal to use strictly positive levels of both punishments and rewards.*

Proof of Lemma 1. Recall that $C_p(\cdot)$ and $C_r(\cdot)$ are increasing in their arguments, F and $1-F$, respectively. Additionally, the cost of using punishments is zero when the whole population takes $a = 1$ ($1 - F = 1$), and the cost of using rewards is zero at full non-compliance ($F = 1$). As such, there must exist a share of the population taking $a = 1$, denote it $1 - \tilde{F}$, such that $C_p(\tilde{F}) = C_r(1 - \tilde{F})$.

For all lower shares taking $a = 1$ such that $1 - F < 1 - \tilde{F}$, $C_r(1 - F) < C_p(F)$, so rewards are the cheaper type of policy with which to attain a given level of compliance. Conversely, for higher shares of the population taking $a = 1$ such that $1 - F > 1 - \tilde{F}$, $C_p(F) > C_r(1 - F)$, so punishments are the cheaper type of policy with which to attain a given level of compliance. At $1 - \tilde{F}$, either type of policy entails the same administrative cost, but only one should be used so that costs are not incurred twice to achieve compliance of $1 - \tilde{F}$. To induce any share of the population to choose $a = 1$, then, only one type of policy should be employed. ∎

Remark 3. *It is worth clarifying the generality with which this result holds. Given the model's somewhat simplistic assumptions, especially with regards to the accrual of costs, it emerges rather starkly that it is never optimal to employ strictly positive levels of punishment and reward to achieve precisely the same end. The finding, however, does not rely on such assumptions. The result and even the same approach are still valid even if both of the administrative cost terms increase in the size of the intervention (i.e., P, R) in addition to increasing in the measure of the population to which they are applied (i.e., $F, 1-F$). In that case, the marginal cost of administering punishment-based policies overtakes the marginal cost of administering reward-based policies. As such, there exists a share choosing $a = 1$ at which one would not only be indifferent between using incentive and disincentive policies, but at which one would be willing to use any mixture of the two policies to induce that share of the population to choose $a = 1$. Yet this will never be the share choosing $a = 1$ at which the marginal administrative costs are equal, so it will still be true that the optimal policy using only punishments or the optimal policy using on rewards achieves higher social welfare and entails a different level of compliance.*

To reframe the social planner's problem as discussed in text, we denote a type of policy by $\theta \in \{p, r, \phi\}$, with p denoting the use of punishments, r denoting the use of rewards, and ϕ denoting the absence of a policy intervention. We adopt the ordering $p > r$ for the subset $\{p, r\}$. We leave ϕ out of the ordering, as the discussion in text would suggest. We denote the share of the population taking $a = 1$ *ex post* by Γ.

Given a distribution of valuations, F, Γ uniquely defines the size of the policy intervention, which we refer to by X. We may define $X = P + R$, although recall that Lemma 1 establishes that at most one of P or R will be strictly greater than zero. The relationship is given by $X(\Gamma) = -F^{-1}(1 - \Gamma)$. The choice forgo a policy intervention is given by $(\Gamma_{\emptyset}, \phi)$.

Lemma 5 establishes that the (Γ, θ) choice problem yields identical solutions to the original formulation in which the policymaker chose an optimal (P, R). Were $(P^*, 0)$ the optimal policy under the original formulation, then (Γ^*, θ^*) is the optimal policy under the reframing, where $\theta^* = p$ and $\Gamma^* = 1 - F(-P^*)$. The analogous statement holds for R. Lemma 1 plays a crucial role in the proof.

Lemma 5. $(\Gamma^*, \theta^*) := \max_{\Gamma \geq \Gamma_{\emptyset}, \theta \in \{\phi, r, p\}} W(\Gamma) - \int_0^{\Gamma} X(\tilde{\Gamma}) d\tilde{\Gamma} - C_{\theta}(\Gamma)$ *iff* $(\mathbb{I}_{\theta^* = p} \cdot X(\Gamma^*), \mathbb{I}_{\theta^* = r} \cdot X(\Gamma^*)) = (P^*, R^*) := \max_{P \geq 0, R \geq 0} W(1 - F(-P - R)) + \int_{-P-R}^{\overline{v}} v f(v) dv - \mathbb{I}_{P > 0} C_p(\Gamma) - \mathbb{I}_{R > 0} C_r(\Gamma)$.

NB: $C_p'(\Gamma) < 0$**, while** $C_r'(\Gamma) > 0$**, where each derivative is taken with respect to** Γ**.**

Proof of Lemma 5. By Lemma 1, we know that the optimal policy, (P^*, R^*) lies in the set $\{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$.

By the independence of irrelevant alternatives axiom, we know that eliminating the set of policies $\{(P, R) | P > 0, R > 0\}$ and maximizing over $(P, R) \in \{(P, 0) | P \geq 0\} \cup \{(0, R) | R \geq 0\}$ instead of $(P, R) \in \mathbb{R}_+^2$ will yield the same solutions.

Then consider (P^*, R^*) and (Γ^*, θ^*). The one-to-one and onto transformation between the two pairs is sufficient. If $(\mathbb{I}_{\theta^* = p} \cdot X(\Gamma^*), \mathbb{I}_{\theta^* = r} \cdot X(\Gamma^*)) \neq (P^*, R^*)$, then it would suggest some other $(\tilde{P}, \tilde{R})$ were the solution to maximizing social welfare using the (P, R) formulation. This would contradict the supposition that (P^*, R^*) were the optimal choices of punishment and reward. The same argument in reverse (from (P^*, R^*) to (Γ^*, θ^*)) completes the proof. ■

We rewrite the social planner's utility function in accordance with the above transformation:

$$U^{SP}(\Gamma, \theta) = \begin{cases} U_p^{SP}(\Gamma) = W(\Gamma) - \int_0^{\Gamma} X(\tilde{\Gamma}) d\tilde{\Gamma} - C_p(\Gamma) & \text{if } \theta = p, \Gamma > \Gamma_{\emptyset} \\ U_r^{SP}(\Gamma) = W(\Gamma) - \int_0^{\Gamma} X(\tilde{\Gamma}) d\tilde{\Gamma} - C_r(\Gamma) & \text{if } \theta = r, \Gamma > \Gamma_{\emptyset} \\ U_{\phi}^{SP}(\Gamma) = W(\Gamma) - \int_0^{\Gamma} X(\tilde{\Gamma}) d\tilde{\Gamma} & \text{if } \theta = \phi, \Gamma = \Gamma_{\emptyset} \end{cases} . \tag{5}$$

The restriction to $(P, R) \geq 0$ becomes $\Gamma \geq \Gamma_{\emptyset} := 1 - F(0)$, which we impose on the social planner's transformed maximization problem.

Ashworth & Bueno de Mesquita (2006) provide conditions of complementarity, formally supermodularity, under which such results are possible without further parameterization. Specifically, we seek to show each pair of arguments of the utility function has increasing differences. This amounts to demonstrating that the incremental return of each pair of arguments is increasing.

We derive these results in reference to U^{SP}, where $\theta \in \{p, r\}$. In effect, we treat the comparison to status quo utility under $(\Gamma_\emptyset, \phi)$ as a second step, after first choosing between p and r. In the context of Equation 5, we say U^{SP} has increasing differences in the level of compliance achieved, Γ, and the choice of policy type, $\theta \in \{p, r\}$, if for all $\hat{\Gamma} > \Gamma$ and $\hat{\theta} > \theta$ (i.e., $p > r$),

$$U^{SP}(\hat{\Gamma}, \hat{\theta}) - U^{SP}(\Gamma, \hat{\theta}) \geq U^{SP}(\hat{\Gamma}, \theta) - U^{SP}(\Gamma, \theta). \tag{6}$$

Indeed, this does hold for the social planner's objective function, reducing to

$$-C_p(\hat{\Gamma}) - C_p(\Gamma) \geq -C_r(\hat{\Gamma}) + C_r(\Gamma),\text{[23]} \tag{7}$$

where the left-hand side is positive and the right-hand side is negative.

An increase in the level of compliance (Γ's incremental return) is more attractive under the use of disincentives than under to the use of incentives (an increase in θ). Equivalently, the use of punishments is more attractive relative to the use of rewards (θ's incremental return) as compliance (Γ) increases. This is a direct result of the asymmetric way in which costs accrue under each incentive and disincentive policies.

If it can then be shown that U^{SP} has increasing differences with respect to an exogenous parameter and each choice variable, then we may conclude that an increase in that parameter leads to an increase in the optimal choice of (Γ, θ), $\Gamma > 0, \theta \neq \phi$. We need not worry about indirect effects. The pairwise complementarity of parameters and choice variables (i.e., supermodularity) ensures that any indirect effects only enhance the direct effects. We then ask how the utility given by the optimal choice of a "non-zero" policy intervention changes relative to the status quo utility under $(\Gamma_\emptyset, \phi)$ in response to parameter changes.

A.2 Comparative statics of social planner's optimal policy

Proposition 1' (a)**.** *Let the optimal level of compliance, $\Gamma \in [\Gamma_\emptyset, 1]$, and type of policy, $\theta \in \{p, r\}$, under the function W be given by (Γ^*, θ^*). Let $(\hat{\Gamma}^*, \hat{\theta}^*)$ be the optimal compliance and policy under $\hat{W}$.*

If $\hat{W}' \geq W'$, such that the marginal benefit of compliance under $\hat{W}$ is weakly higher than under W for all levels of compliance, then $\hat{\Gamma}^* \geq \Gamma^*$. If $\Gamma^* > \Gamma_\emptyset$, then $\hat{\theta}^* \geq \theta^*$.

[23]This constitutes a proof of Lemma 2.

Proposition 1' (b). *Let (Γ^*, θ^*) be the optimal level of compliance and type of policy under the distribution of valuations F. Let $(\hat{\Gamma}^*, \hat{\theta}^*)$ be the optimal compliance and policy under $\hat{F}$.*

If $\hat{F} \leq F$, such that $\hat{F}$ first-order stochastically dominates F, and if $\Gamma^*, \hat{\Gamma}^* > \Gamma_\emptyset$, then $\hat{\Gamma}^* \geq \Gamma^*, \hat{\theta}^* \geq \theta^*$.

Proposition 1' (c). *Let (Γ^*, θ^*) be the optimal level of compliance and type of policy under the function C_p. Let $(\hat{\Gamma}^*, \hat{\theta}^*)$ be the optimal compliance and policy under $\hat{C}_p$. Further, suppose $\hat{C}_p' \leq C_p'$, such that the marginal cost of punishing non-compliers is weakly less under $\hat{C}_p$ than under C_p for all levels of compliance.*

A change from C_p to $\hat{C}_p$ increases the incremental return of U^{SP} from an increase in Γ but decreases the incremental return of U^{SP} with respect to a change from $\theta = r$ to $\theta = p$. As such, the relationships of $\hat{\Gamma}^*$ to Γ^* and $\hat{\theta}^*$ to θ^* are ambiguous, even if $\Gamma^*, \hat{\Gamma}^* > 0$.

The same is true for C_r and $\hat{C}_r$, where $\hat{C}_r' \leq C_r'$ such that the marginal cost of rewarding compliers is weakly greater under $\hat{C}_r$ than under C_r for all levels of compliance.

Proof of Proposition 1. We seek to apply Theorem 5 from Milgrom & Shannon (1994). We have already shown in text that U^{SP} is supermodular in (Γ, θ). Additionally, $\{[\Gamma_\emptyset, 1] \times \{p, r\}\}$ is a lattice satisfying the necessary condition on the set from which the choice variables (Γ, θ) are drawn. It remains to be shown whether U^{SP} has increasing differences in $(\Gamma, \theta; W(\cdot), F(\cdot), C_p(\cdot), C_r(\cdot))$, with partial orderings for the latter four arguments supplied in the Proposition and further clarified below. To do so, we must demonstrate increasing differences in each choice variable-parameter pair.

For Propositions 1(a)-(b), our aim is to show U^{SP} does have increasing differences in $(\Gamma, \theta; W(\cdot), F(\cdot))$. As such, $(\Gamma^*, \theta^*) = \underset{(\Gamma,\theta)\in\{[\Gamma_\emptyset,1]\times\{p,r\}\}}{\arg\max} U^{SP}(\Gamma, \theta; W(\cdot), F(\cdot))$ is monotone nondecreasing in $(W(\cdot), F(\cdot), \Gamma_\emptyset)$.[24] For Proposition 1(c), we wish to demonstrate that $U^{SP}(\Gamma, \theta; C_p, C_r)$ has increasing differences in $(\Gamma; C_p)$ and $(\Gamma; C_r)$ but decreasing differences in $(\theta; C_p)$ and $(\theta; C_r)$. Thus, we cannot infer that (Γ^*, θ^*) is monotone nondecreasing in (C_p, C_r).

For each parameter, we adopt a mix of techniques. To show increasing differences in θ and the parameter, we compare the incremental return of a discrete increase in the parameter at $\theta = r$ and $\theta = p$. This follows the approach taken to show the increasing differences of (Γ, θ) in text.

We proceed differently to show increasing differences in Γ and the parameter. For example, for $W(\cdot)$, we examine $\frac{\partial}{\partial\Gamma}(U^{SP}(\Gamma, \theta; \hat{W}) - U^{SP}(\Gamma, \theta; W))$. If that quantity is weakly positive, increasing differences may be inferred. Note that

$$\frac{\partial}{\partial\Gamma} U^{SP}(\Gamma, \theta; W(\cdot), F(\cdot), C_p(\cdot), C_r(\cdot)) = W'(\Gamma) + F^{-1}(1 - \Gamma) - C_\theta(\Gamma).$$

[24] We address the matter of $\Gamma_\emptyset$ after item 2 below.

We clarify partial orderings using $\succ$ to avoid ambiguity with numerical statements about the parameters, although nothing regarding preferences should be inferred.

1. Partially order the set of functions $\{W(\cdot)|W' \geq 0\}$ with the rule:

$$\hat{W} \succ W \Leftrightarrow \hat{W}' > W', \forall \Gamma.$$

$$(\Gamma, W): \ \tfrac{\partial}{\partial \Gamma}(U^{SP}(\Gamma, \theta; \hat{W}) - U^{SP}(\Gamma, \theta; W)) = \hat{W}'(\Gamma) - W'(\Gamma) \geq 0, \ (> \text{ for } \Gamma < 1)$$

$$(\theta, W): \ U^{SP}(\Gamma, \hat{\theta}; \hat{W}) - U^{SP}(\Gamma, \theta; \hat{W}) - [U^{SP}(\Gamma, \hat{\theta}; W) - U^{SP}(\Gamma, \theta; W)] = 0$$

2. Employ the partial ordering given by first-order stochastic dominance to order the set of distributions over valuations, $F(v)$, such that:

$$\hat{F} \succ F \Leftrightarrow \hat{F} < F, \forall v \in \text{int}(\text{supp}(F)).$$

$$(\Gamma, F): \ \tfrac{\partial}{\partial \Gamma}(U^{SP}(\Gamma, \theta; \hat{F}) - U^{SP}(\Gamma, \theta; F)) = \hat{F}^{-1}(1-\Gamma) - F^{-1}(1-\Gamma) \geq 0, \ (> \text{ for } \Gamma < 1)$$

$$(\theta, F): \ U^{SP}(\Gamma, \hat{\theta}; \hat{F}) - U^{SP}(\Gamma, \theta; \hat{F}) - [U^{SP}(\Gamma, \hat{\theta}; F) - U^{SP}(\Gamma, \theta; F)] = 0$$

   Should a change in a parameter affect the constraint set for the the maximization problem, Theorem 4 from Milgrom & Shannon (1994) provides the condition under which we may still infer monotone comparative statics. Specifically, as long as the constraint set is increasing (in the strong set order) and the strict single crossing property is satisfied in the parameter, we may proceed as before. In this case, the constraint set is $[\Gamma_\emptyset, 1] = (1 - F(0), 1)$, which is strictly smaller than $[1 - \hat{F}(0), 1]$ in the strong set ordering.

3. Partially order the set of functions $\{C_p(\cdot)|C_p' \leq 0, C_p(1) = \underline{C}_p \geq 0\}$ with the rule:

$$\hat{C}_p \succ C_p \Leftrightarrow \hat{C}_p' < C_p', \forall \Gamma < 1.$$

   Note this implies $\hat{C}_p(\Gamma) > \hat{C}_p(\Gamma), \forall \Gamma < 1$.

$$(\Gamma, C_p): \ \tfrac{\partial}{\partial \Gamma}(U^{SP}(\Gamma, \theta; \hat{C}_p) - U^{SP}(\Gamma, \theta; C_p)) = C_p'(\Gamma) - \hat{C}_p'(\Gamma) \geq 0$$

$$(\theta, C_p): \ U^{SP}(\Gamma, \hat{\theta}; \hat{C}_p) - U^{SP}(\Gamma, \theta; \hat{C}_p) - [U^{SP}(\Gamma, \hat{\theta}; C_p) - U^{SP}(\Gamma, \theta; C_p)] = -\hat{C}_p(\Gamma) + C_p(\Gamma) \leq 0$$

4. Partially order the set of functions $\{C_r(\cdot)|C_r' \geq 0, C_r(0) = \underline{C}_r \geq 0\}$ with the rule:

$$\hat{C}_r \succ C_r \Leftrightarrow \hat{C}_r' < C_r', \forall \Gamma > 0.$$

Note this implies $\hat{C}_r(\Gamma) < C_r(\Gamma), \forall \Gamma > 0$.

$$(\Gamma, C_r): \ \frac{\partial}{\partial \Gamma}(U^{SP}(\Gamma, \theta; \hat{C}_r) - U^{SP}(\Gamma, \theta; C_r)) = -C_r'(\Gamma) - \hat{C}_r'(\Gamma) \geq 0$$

$$(\theta, C_r): \ U^{SP}(\Gamma, \hat{\theta}; \hat{C}_r) - U^{SP}(\Gamma, \theta; \hat{C}_r) - [U^{SP}(\Gamma, \hat{\theta}; C_r) - U^{SP}(\Gamma, \theta; C_r)] = -\hat{C}_r(\Gamma) + C_r(\Gamma) \leq 0$$

This concludes the proof. ■

B Proofs from Sections 5-6: Majority Preference

B.1 Understanding Preferences across the Population

Lemma 3. *Let $(P^{**}, R^{**}) := \arg\max_{(P,R)} U^{MV}(P, R)$. There does not exist a policy more preferred by a majority of individuals than (P^{**}, R^{**}).*

Proof of Lemma 3. $U^i(P_1^{**}, R_1^{**}) - U^i(P_0^{**}, R_0^{**})$ is strictly increasing in v_i. Set $v^{\tilde{i}}$ s.t. $U^{\tilde{i}}(P_1^{**}, R_1^{**}) = U^{\tilde{i}}(P_0^{**}, R_0^{**})$. It must be that $\forall i$ s.t. $v^i > v^{\tilde{i}}$, $U^{\tilde{i}}(P_1^{**}, R_1^{**}) > U^{\tilde{i}}(P_0^{**}, R_0^{**})$.

From the monotonicity of the preference relation with respect to v^i and an application of the median voter theorem, we conclude that the majority preference relation is equivalent to the preference relation of the individual with the median valuation. It is also the case that if the median voter prefers the lack of a policy intervention to the better of (P_1^{**}, R_1^{**}) and (P_0^{**}, R_0^{**}), so will all those taking the same action under $(P, R) = (0, 0)$. ■

Lemma 4. *All members of the population prefer the use of either an incentive or a disincentive policy – but not both – to achieve any given level of compliance.*

Proof of Lemma 4. Let $P = \varphi X$ and $R = (1 - \varphi)X$, with $\varphi \in [0, 1]$. Then consider:

$$U^i(X, \varphi) = \begin{cases} W(1 - F(-X)) + F(-X)X - \mathbb{I}_{\varphi>0}C_p(F(-X)) + v^i - \mathbb{I}_{\varphi<1}C_r(1 - F(-X)) + v^i \\ \text{ if } v^i \geq -X \\ W(1 - F(-X)) - (1 - F(-X))X - \mathbb{I}_{\varphi>0}C_p(F(-X)) - \mathbb{I}_{\varphi<1}C_r(1 - F(-X)) \\ \text{ if } v^i < -X \end{cases} \tag{8}$$

In either case, the only terms that depend on φ are the cost terms. Any share choosing $1 - F(-X)$ could be induced with only one policy, and this would reduce the cost. As above, this result holds (albeit less starkly) even if the cost of each type of policy rises in the size of punishment or reward, not just in the measure of the population to which it is applied. ■

B.2 Comparative statics of majority-preferred policy

The office-motivated politician, seeks to maximize the decisive voter's utility by choosing a level of compliance among the population (Γ) and a type of policy (θ). The politician must, however, take into account whether the decisive voter will herself choose $a = 1$ *ex post*. As such, it is useful to rewrite the utility function of the decisive voter in terms of the (Γ, θ) formulation, recalling that $-X = F^{-1}(1-\Gamma)$, where $X = P + R$, and that the decisive voter will choose $a = 1$ when $v^{MV} \geq -X \Leftrightarrow \Gamma > \frac{1}{2}$.

$$U^{MV}(\Gamma, \theta) = \begin{cases} U^{MV}_{1,\theta}(\Gamma) = W(\Gamma) - (1-\Gamma)F^{-1}(1-\Gamma) - C_\theta(\Gamma) + v^{MV} & \text{if } \Gamma \geq \frac{1}{2} \\ U^{MV}_{0,\theta}(\Gamma) = W(\Gamma) + \Gamma F^{-1}(1-\Gamma) - C_\theta(\Gamma) & \text{if } \Gamma < \frac{1}{2} \end{cases} \tag{9}$$

As above, we wish to demonstrate that U^{MV} has increasing differences in (Γ, θ). If this can be shown, then if U^{MV} has increasing differences in an exogenous variable and each choice variable, we may draw conclusions to the effect that an increase in the exogenous parameter makes higher compliance and the use of punishments more favorable. From there, we may derive results about the compliance of the decisive voter and the circumstances in which $a = 0$ might be encouraged. Because the pairwise complementarity ensures that indirect effects only enhance direct effects, we may be satisfied that the conclusions drawn from such analysis are particularly robust.

Demonstrating increasing differences in $(\Gamma, \theta), \theta \in \{p, r\}$[25] proceeds exactly as did the inequality in 6, although showing increasing differences in the choice variables and the parameters of interest presents a small hurdle. As above, we seek increasing differences with respect to pointwise increases in the marginal benefit, W, and first-order stochastic increases of the distribution of valuations for compliance, F. The latter, however, is no longer straightforward without additional structure. As such, we limit consideration to "shifts" in a distribution, a particular type of first-order stochastic increases. Specifically, we say $\hat{F}$ represents a shift of F if for $v \sim F, \hat{v} \sim \hat{F}$, $\hat{v} = \mu + v, \mu \geq 0$. Ultimately, the results of Proposition 1 apply to the Condorcet winning policy as they did to the social-welfare maximizing policy, with an additional implication regarding the majority's *ex post* decision to comply.

Proposition 3. *Let the Condorcet winning policy, consisting of a level of compliance, $\Gamma \in [0, 1]$, and type of policy, $\theta \in \{p, r\}$, under the functions W, F, C_p, and C_r be given by $(\Gamma^{**}, \theta^{**})$. Further, let the decisive voter's decision to comply or not under $(\Gamma^{**}, \theta^{**})$, $a^{MV}(\Gamma^{**}, \theta^{**}) \in \{0, 1\}$, be given by α^{**}.*

(a) Let $(\hat{\Gamma}^{**}, \hat{\theta}^{**})$ be the Condorcet winning policy under $\hat{W}$ (holding the other functions at the values above) and $\hat{\alpha}^{**}$ be the action choice of a majority of the population.

[25] Recall that we adopt the ordering $p > r$.

If $\hat{W} \geq W'$, such that the marginal benefit of compliance under $\hat{W}$ is weakly higher than under W for all levels of compliance, then $\hat{\Gamma}^{**} \geq \Gamma^{**}$ and $\hat{\alpha}^{**} \geq \alpha^{**}$. If $\Gamma^{**} > \Gamma_{\emptyset}$, then $\hat{\theta}^{**} \geq \theta^{**}$.

(b) Let $(\hat{\Gamma}^{**}, \hat{\theta}^{**})$ be the Condorcet winning policy under $\hat{F}$ and $\hat{\alpha}^{**}$ be the action choice of a majority of the population.

If $\hat{F} \leq F$, such that the distribution of valuations under $\hat{F}$ is a shift of the distribution under F, then if $\theta^{**}, \hat{\theta}^{**} \neq \phi$, then $\hat{\Gamma}^{**} \geq \Gamma^{**}$, $\hat{\theta}^{**} \geq \theta^{**}$, and $\hat{\alpha}^{**} \geq \alpha^{**}$.

(c) Let $(\hat{\Gamma}^{**}, \hat{\theta}^{**})$ be the Condorcet winning policy under $\hat{C}_p$ and/or $\hat{C}_r$, where $\hat{C}_p' \leq C_p'$ and $\hat{C}_r' \leq C_r'$.

The relationships of $\hat{\Gamma}^{**}$ to Γ^{**}, $\hat{\theta}^{**}$ to θ^{**}, and $\hat{\alpha}^{**}$ to α^{**} are ambiguous.

Proof of Proposition 3. We need only show increasing differences or the single-crossing property in $\{\Gamma, \theta\} \times \{W, F\}$, with W ordered by pointwise larger first derivatives and F ordered by first-order stochastic dominance. Recall that α denotes the *ex post* compliance of the decisive voter, and thus a majority of the population.

1. $\hat{W} \succ W \Rightarrow \hat{W}' > W', \forall \Gamma \in [0,1]$

$(\Gamma, W):\ \frac{\partial}{\partial \Gamma}[U^{MV}(\Gamma, \theta; \hat{W}) - U^{MV}(\Gamma, \theta; W)] = \hat{W}' - W' \geq 0$

$(\theta, W):\ U^{MV}(\Gamma, \hat{\theta}; \hat{W}) - U^{MV}(\Gamma, \theta; \hat{W}) - [U^{MV}(\Gamma, \hat{\theta}; W) - U^{MV}(\Gamma, \theta; W)] = 0$

2. $\hat{F}(\hat{v}) \succ F(v) \Rightarrow \hat{v} = \mu + v,\ \mu \geq 0.$

$(\Gamma, F):\ \frac{\partial^2}{\partial \Gamma \partial \mu} = \frac{\partial}{\partial \mu} F^{-1}(\Gamma, \mu) > 0, \forall \Gamma \in [0,1]$

$(\theta, F):\ U^{MV}(\Gamma, \hat{\theta}; \hat{F}) - U^{MV}(\Gamma, \theta; \hat{F}) - [U^{MV}(\Gamma, \hat{\theta}; F) - U^{MV}(\Gamma, \theta; F)] = 0$

3. We cannot definitively sign the change in the Condorcet winning policy given the partial ordering on the costs for the same reason as in Proposition 1.

Finally, α is an increasing function of Γ^{**}, and the effects of compliance/non-compliance were accounted for in the analysis of U^{MV}. ■

Proposition 2'. *Let (Γ^*, θ^*) be the social-welfare maximizing level of compliance and type of policy, and let $(\Gamma^{**}, \theta^{**})$ be the policy (inducing $a = 1$) that maximizes the decisive voter's utility, i.e., the Condorcet winning policy if limited to $\Gamma \in [\Gamma_{\emptyset}, 1] \Leftrightarrow P, R > 0$. Suppose both policies entail active interventions, such that $\theta^*, \theta^{**} \neq \phi$.*

A majority neither punishes itself for non-compliance nor rewards a minority of compliers sufficiently to achieve the optimal level of compliance. Formally, if $\Gamma < 1/2 \Leftrightarrow \alpha = 0$, such that a majority of the population chooses $a = 0$, and $\theta \neq \phi$, then $\Gamma^{**} < \Gamma^*$.

A majority either rewards itself or punishes the minority excessively, so as to achieve greater than the optimal level of compliance. Formally, if $\Gamma > 1/2 \Leftrightarrow \alpha = 0$*, such that a majority of the population chooses* $a = 0$*, and* $\theta \neq \phi$*, then* $\Gamma^{**} > \Gamma^*$*.*

Increases in the marginal benefit of compliance or an upward shift in the distribution of valuations favor the outcomes in which a majority of compliers imposes a larger-than-efficient disincentive on the minority of non-compliers, or in which a majority of non-compliers institutes a smaller-than-efficient incentive for the minority of compliers.

Proof of Proposition 2'. Consider the following series of inequalities:

$$\begin{array}{rcl} \frac{\partial U_{1,\theta}^{MV}}{\partial \Gamma} & = & W' + \frac{1-\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta \\ & \geq & \\ \frac{\partial U_{\theta}^{SP}}{\partial \Gamma} & = & W' + F^{-1}(1-\Gamma) - C'_\theta \qquad\qquad\qquad , \forall \Gamma \in [\Gamma_\emptyset, 1]. \\ & \geq & \\ \frac{\partial U_{0,\theta}^{MV}}{\partial \Gamma} & = & W' - \frac{\Gamma}{f(F^{-1}(1-\Gamma))} + F^{-1}(1-\Gamma) - C'_\theta \end{array}$$

Viewed in the context of Proposition 1(a), were we to define $\hat{W}' = W' + \frac{1-\Gamma}{f(1-\Gamma)}$ and $\hat{\hat{W}}' = W' - \frac{\Gamma}{f(1-\Gamma)}$, we would find $\hat{W}' \geq W'$ and $W' \geq \hat{\hat{W}}'$ pointwise and could draw the same conclusion regarding increasing differences in (Γ, α).

Applying Proposition 1 to $U_{0,\theta}^{MV}$ and $U_{1,\theta}^{MV}$, the results regarding $\Gamma^{**} \gtrless \Gamma^*$ follow immediately. The indirect effect of increasing Γ is that $\theta^{**} \geq \theta^*$.

The comparative statics stem from an application of Proposition 3. ■

As noted in text, further illustration of the preceding result may be found in Figures 5-6.

Corollary 1'. *Let the Condorcet winning policy, consisting of a level of compliance,* $\Gamma \in [0, 1]$*, and type of policy,* $\theta \in \{\phi, p, r\}$*, under the functions* W*,* F*,* C_p*, and* C_r *be given by* $(\Gamma^{**}, \theta^{**})$*.*

(a) *Let* $(\hat{\Gamma}^{**}, \hat{\theta}^{**})$ *be the Condorcet winning policy under* $\hat{W}$ *(holding the other functions at the values above).*

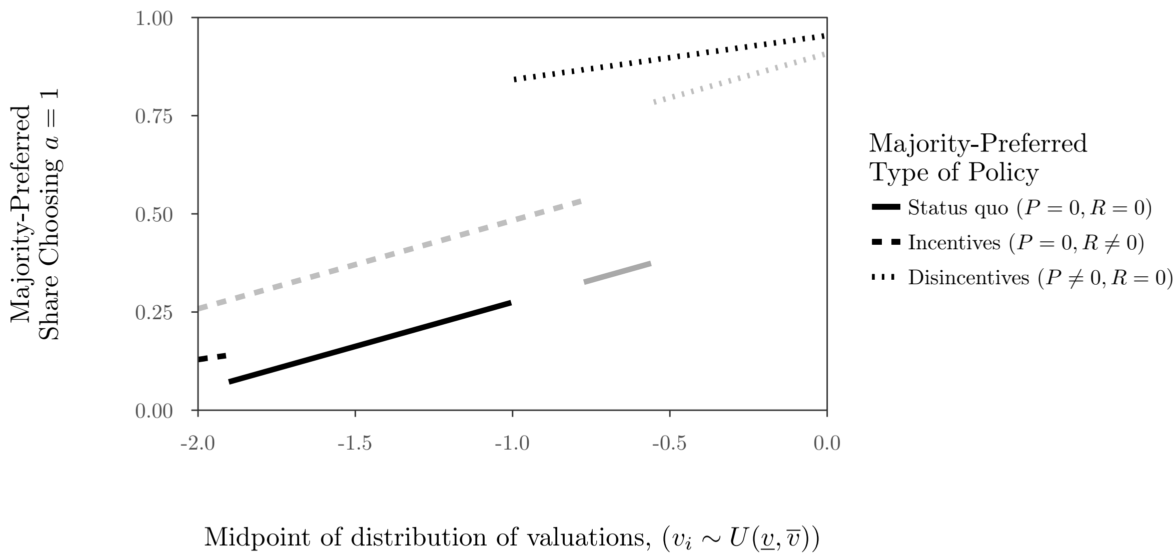
   If $\hat{W} \geq W'$*, such that the marginal benefit of compliance under* $\hat{W}$ *is weakly higher than under* W *for all levels of compliance, and if* $\hat{\Gamma}^{**} < \Gamma_\emptyset$*, then* $\Gamma^{**} < \hat{\Gamma}^{**} < \Gamma_\emptyset$*, and* $\theta^{**} \leq \hat{\theta}^{**}$*.*

(b) *Let* $(\hat{\Gamma}^{**}, \hat{\theta}^{**})$ *be the Condorcet winning policy under* $\hat{F}$*.*

   If $\hat{F} \leq F$*, such that the distribution of valuations under* $\hat{F}$ *is an upwards shift of the distribution under* F*, then if* $\Gamma^{**}, \hat{\Gamma}^{**} < \Gamma_\emptyset$*, then* $\hat{\Gamma}^{**} \geq \Gamma^{**}$ *and* $\hat{\theta}^{**} \geq \theta^{**}$*.*

Proof of Corollary 1'. Follows directly from Propositions 1-2. ■

Figure 5: The effect of shifting upwards the distribution of valuations for choosing $a = 1$ rather than $a = 0$ on the majority-preferred share of the population choosing $a = 1$ and type of policy



Notes: The dotted line represents the status quo share of the population choosing $a = 1$, $1 - F(0)$. The gray lines correspond to the social planner's optimal share choosing $a = 1$ and type of policy. The horizontal axis measures $(\overline{v} + \underline{v})/2$, which corresponds to the decisive voter's valuation for choosing $a = 1$. An increase in this quantity also signifies an upward shift in the distribution of valuations, $F(\cdot)$.

C Proofs from Section 7: Introducing Unaffected Subpopulations

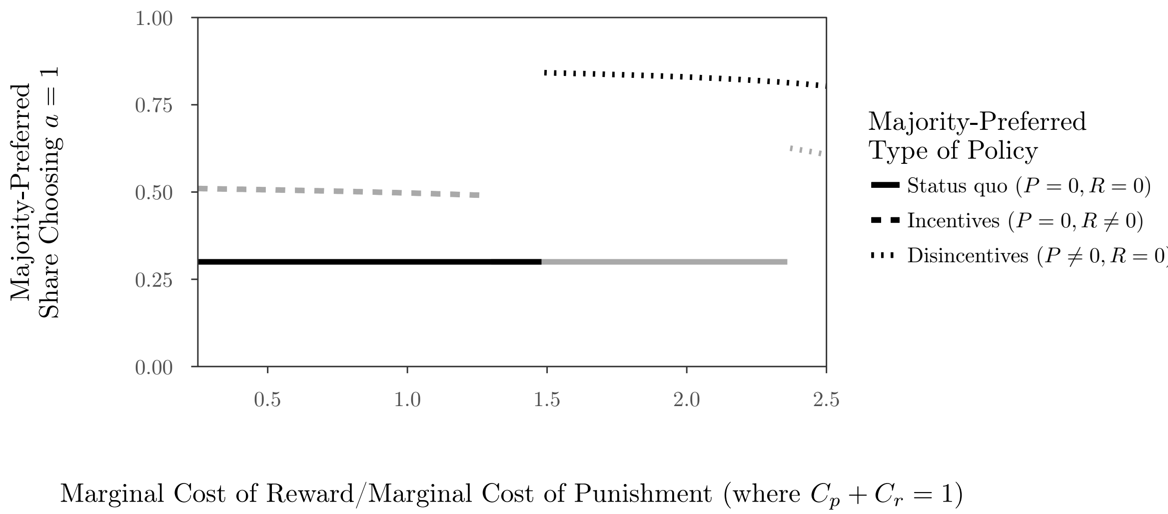
Lemma 6. *A member of the unaffected subpopulation's most-preferred policy does not include both types of policy, i.e., it is never the case that $P \neq 0$ and $R \neq 0$.*

Proof of Lemma 6. Let $P = \varphi X$ and $R = (1 - \varphi)X$, with $\varphi \in [0, 1]$. The size of the unaffected subpopulation is $\lambda \in (0, 1)$. Then consider:

$$\begin{aligned} U^i(X, \varphi; \lambda) &= W((1-\lambda)(1 - F(-X))) \\ &\quad -(1-\lambda)(1-\varphi)X(1 - F(-X)) - \mathbb{I}_{\varphi<1} C_r((1-\lambda)(1 - F(-X))) \\ &\quad +(1-\lambda)\varphi X F(-X) - \mathbb{I}_{\varphi>0} C_p((1-\lambda)F(-X)) \\ &= W((1-\lambda)(1 - F(-X))) \\ &\quad -(1-\lambda)X(\varphi - (1 - F)) \\ &\quad -\mathbb{I}_{\varphi<1} C_r((1-\lambda)(1 - F(-X))) - \mathbb{I}_{\varphi>0} C_p((1-\lambda)F(-X)) \end{aligned}$$

Given some share of the population choosing $a = 1$, what is the best value of φ? If $X - C_p(F(-X)) >$

Figure 6: The effect of decreasing the marginal cost of disincentives relative to the marginal cost of incentives on the majority-preferred share of the population choosing $a = 1$ and type of policy



Notes: The dotted line represents the status quo share of the population choosing $a = 1$, $1 - F(0)$. The gray lines correspond to the social planner's optimal share choosing $a = 1$ and type of policy. The ratio of marginal cost coefficients, C_r/C_p, is increasing along the horizontal axis. The restriction that $C_p + C_r = 1$ facilitates visual comparison but is not a restriction that applies more broadly.

$-C_r(1 - F(-X))$, then φ should be set to 1, i.e., only P used, and vice versa. If equal, one would not want to double up on the cost, but only use one or the other. Indeed, the left-hand side is increasing in X, while the right-hand side is decreasing in X. As such, at high shares of the population choosing $a = 1$, P is the better policy instrument, and again vice versa. ■

Corollary 2' *Assume that $\lambda \geq \frac{1}{2}$, such that a majority of the population will receive neither incentive nor disincentive under a policy intervention.*

Let $(\Gamma^{\ell *}_{1,p}, p)$ be the policy that maximizes $W((1-\lambda)\Gamma) - (1-\lambda)(1-\Gamma)F^{-1}(1-\Gamma) - C_p((1-\lambda)\Gamma)$ and $(\Gamma^{\ell *}_{0,r}, r)$ be the policy that maximizes $W((1-\lambda)\Gamma) + (1-\lambda)\Gamma F^{-1}(1-\Gamma) - C_r((1-\lambda)\Gamma)$. It is the case that $\Gamma^{\ell *}_{1,p} > \Gamma^{\ell *}_{0,r}$.

Let the Condorcet winning policy under the functions W, F, C_p, and C_r be given by $(\Gamma^{\ell *}, \theta^{\ell *})$. It is the case that $(\Gamma^{\ell *}, \theta^{\ell *}) \in \{(\Gamma^{\ell *}_{1,p}, p), (\Gamma_{\emptyset}, \phi), (\Gamma^{\ell *}_{0,r}, r)\}$.

(a) *Let $(\hat{\Gamma}^{\ell *}, \hat{\theta}^{\ell *})$ be the Condorcet winning policy under $\hat{W}$.*

It must be that $(\hat{\Gamma}^{\ell *}, \hat{\theta}^{\ell *}) \in \{(\Gamma^{\ell *}_{1,p}, p), (\Gamma_{\emptyset}, \phi), (\Gamma^{\ell *}_{0,r}, r)\}$.

If $\hat{W} \geq W'$, such that the marginal benefit of compliance under $\hat{W}$ is weakly higher than under W for all levels of compliance, and if $\hat{\theta}^{\ell *}, \theta^{\ell *} \neq \phi$, then $\hat{\Gamma}^{\ell *} \geq \Gamma^{\ell *}$.

(b) *Let* $(\hat{\Gamma}^{\ell *}, \hat{\theta}^{\ell *})$ *be the Condorcet winning policy under* $\hat{F}$*.*

It must be that $(\Gamma^{\ell *}, \theta^{\ell *}) \in \{(\Gamma^{\ell *}_{1,p}, p), (\Gamma_{\emptyset}, \phi), (\Gamma^{\ell *}_{0,r}, r)\}$*.*

If $\hat{F} \leq F$*, such that the distribution of valuations under* $\hat{F}$ *is an upwards shift of the distribution under* F*, and if* $\hat{\theta}^{\ell *}, \theta^{\ell *} \neq \phi$*, then* $\hat{\Gamma}^{\ell *} \geq \Gamma^{\ell *}$*.*

Proof of Corollary 2'. This follows directly from Proposition 3 and the discussion in Section 7. ■