Abstract
Various sources for the RF-dc differences of micropotentiometers (pots) are analyzed and calculated. The results are shown to agree well with experimental values. A new design is introduced that reduces the RF-dc differences of the pots significantly. Observations show good stability over a long period. This makes the pot suitable as a primary RF and audio standard in the microvolt and millivolt ranges.

Introduction
Significant progress has been made in the last decade to extend the frequency range of the RF pot down to 10 Hz [1-3], and up to 1.2 GHz [4]. The accuracy’s of RF millivolt measurements have also been improved. Since pots are used as primary standards for low voltages, it is necessary to investigate all possible sources of error when determining the RF-dc differences. Equations for calculating the various error sources of the RF-dc differences of pots will be described. A new design is then developed and the RF-dc differences of the pots are reduced significantly. Further improvements are under consideration. Determinations of measurement errors, and necessary precautions to take when performing the measurements in order to achieve the highest accuracy’s will be given.

Figure 1 shows a diagram of the RF pot assembly. When excited with an external RF source, it is designed to provide a precisely determined voltage VRF at its output terminal. The input current Ih flows through the heater of a UHF-type thermoelement (TE) to a disk resistor Ro and the voltage drop across Ro is a low-impedance source of RF voltage. The output voltage is nominally the product of the heater current Ih and the resistance of the disk resistor Ro.

RF-DC Differences of Micropotentiometers
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RF-DC Differences of a Micropotentiometer
The µpot is usually characterized by its RF-dc difference d which is defined by:

\[ d = \frac{V_{RF}}{V_{dc}} - 1 \]

where VRF is the RF output voltage of the pot and Vdc is the average of the positive and negative dc voltage outputs for the same emf output of the thermoelement. The design considerations and characteristics of the pot which affect its RF-dc differences are discussed below.

1. Current errors caused by transmission line effect
A vacuum TE is usually used to measure the RF current through the pot. Any standing wave which exists will cause errors in measuring the RF currents. The current I_{hRF} at the center of TE is less than the current I_{RF} at the disk resistor.

According to the transmission line equation:

\[ I_{hRF} = V_{ph} \sinh(ZY)^{0.5}/(Z/Y)^{0.5} + I_{dc} \cosh(ZY)^{0.5} \]

Where:

\[ V_{ph} = \text{phasor output voltage of pot,} \]
\[ I_{RF} = \text{phasor current at the center of disk resistor,} \]
\[ I_{RF} = \text{phasor current at the center of heater of TE,} \]
\[ I_{dc} = \text{dc current through the heater of TE,} \]
\[ l = \text{distance from the center of heater to the center of disk resistor in cm, see Fig. 1,} \]
\[ L = \text{distributed inductance of heater and lead with length l, in henries,} \]
\[ R = \text{distributed resistance of heater and lead with length l in ohms,} \]
\[ C = \text{distributed capacitance of heater and lead with length l to the shield in F,} \]
\[ Z = R + j\omega L \]
\[ Y = j\omega C \]
If the annular disk resistor $R_o$ is a thin-film resistor constructed using state-of-the-art techniques, the distributed parameters can be neglected. Thus $V_{RF} = I_{RF}R_o$,

$$I_{RF} = I_{RF} \left[ R_o (Y/Z)^{0.5} \sinh(ZY)^{0.5} + \cosh(ZY)^{0.5} \right]$$

and the RF-dc difference caused by the current measurement, $d_c$, is simply:

$$d_c = (V_{RF} - V_{dc})/V_{dc} = (I_{RF} - I_{dc})/I_{dc}$$

Then combining equations (3) and (4):

$$d_c = \{1/[R_o(Y/Z)^{0.5}\sinh(ZY)^{0.5} + \cosh(ZY)^{0.5}\}] - 1$$

Generally, $R_o$ ranges from 1 mΩ to 22 Ω, and the characteristic impedance $(Z/Y)^{0.5}$ is about 200 Ω, thus

$$R_o(Y/Z)^{0.5} \ll 1$$

$$d_c = [1/\cosh(ZY)^{0.5}] - 1$$

Since $\omega CR \ll 1$, and if $\omega L/R \ll 1$:

$$d_c = (\omega^2 LC - CR^2)/2$$

= $\omega^2 LC/2$

For a coaxial line, $L/l = 2 \times 10^{-7} \ln(b/a)$ in H/m, $C/l = 1.8 \times 10^{-10} \ln(b/a)$ in F/m, $a$ is the outer diameter of the inner conductor, $b$ is the inner diameter of the outer conductor. Then

$$d_c = 2.19 \times 10^{-20} f^2$$

where $f$ is the frequency in Hz. In order to reduce the inductance of the thermoelement, it is necessary to reduce both the size of the housing, and the length $l$ as much as practical. An experiment has shown that reducing the length about 2 mm in a commercial unit reduces the RF-dc difference by 1.5% at 900 MHz.

The thermoelement in the pot might sometimes be replaced due to failure. The RF-dc differences are usually not appreciably affected if the replacement is the same type of TE and inserted at the same location.

2. Ground impedance

Ground impedance such as RF resistance and equivalent inductance in the housing, which are present at high frequencies, increases the equivalent length $l$ resulting in differences between the calculated RF-dc differences according to equation (6) and measurement results. Furthermore, the RF-dc differences will be different for various pot housing materials. The calculated RF-dc differences for a typical pot caused by the current transmission line effect and ground impedance according to equation (6) are shown in Table 1.

<table>
<thead>
<tr>
<th>Estimated Equivalent Length (CM)</th>
<th>dc</th>
<th>RF-DC Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 MHz</td>
<td>50 MHz</td>
</tr>
<tr>
<td>2.3</td>
<td>0.0012</td>
<td>0.0029</td>
</tr>
<tr>
<td>3.0</td>
<td>0.0020</td>
<td>0.049</td>
</tr>
</tbody>
</table>

3. Skin effect of the heater and its inner leads

Due to skin effect at high frequencies, the RF resistances of the heater and its inner leads are larger than their dc resistances. Many commercial vacuum TEs are made with copper-coated magnetic leads, so the skin effect is larger than for nonmagnetic leads. There is also a skin effect in platinum leads. Therefore, the RF current is less than the dc value for the same emf output causing negative RF-dc differences. The larger the current range, the larger the diameter of the heater, the smaller the heater resistance, and the larger the skin effect. The RF-dc differences of pots caused only by the skin effect in the TE, $d_s$, is derived as follows:

For the same emf output, $I_{dc}^2R_{dc} = I_{RF}^2R_{RF}$

Then $d_s = I_{RF}/I_{dc} - 1$

= $I_{RF}/I_{dc} - 1 = (R_{dc}/R_{RF})^{0.5} - 1$

Let $\Delta = R_{RF}/R_{dc} - 1$

At frequencies greater than 1 MHz, is proportional to the square root of the frequency. Using Taylor series expansion:

$$d_s = -D/2 = Af^{0.5}$$

where

$R_{dc}$ dc resistance of heater and its internal leads,

$R_{RF}$ RF resistance of heater and its internal leads,

A coefficient determined by individual TE.

The RF-dc differences caused by the skin effect of the TE are proportional to the square root of frequency at frequencies greater than 1 MHz. They are larger when the current rating of TE is larger and may become the major error source around 10 MHz. This error can compensate for the current error caused by the transmission line effect.
4. Errors of annular resistor
The RF-dc difference caused by inductance of the output resistor $R_o$ is

$$d_L = \frac{Z}{R_o} - 1 = \frac{(\omega L)^2}{Z R_o^2}$$

The inductance of a well made annular resistor is extremely small, thus the RF-dc difference is very small.

When the thickness of the resistance film is much smaller than its skin depth, the skin effect of a well made annular resistor can generally be neglected. However, the RF-dc differences may be very large if the annulus is damaged (e.g. poor electrical contact, ceramic substrate broken) when the pot is connected to a nonstandard N male connector. When the resistance value is very small, say, several milliohms for the V ranges, the skin annulus can be considered as a section of coaxial line with a solid conductor as the propagation medium. By using the same method which Selby [5] used, the transfer impedance $Z_m$ which is defined as the output voltage of an annular resistor versus the input current of the same resistor, is:

$$Z_m = \frac{R_o (1 + j)t / \delta}{\text{Sinh}[(1 + j)t / \delta]}$$

where $R_o$ is the dc resistance of annular resistor,

$t$ is the thickness of the film,

$\delta$ is the skin depth.

The RF-dc difference $dR$ caused by the annulus is

$$dR = (Z_m/R_o) - 1 \quad (8)$$

Then, using Taylor series expansion:

$$dR = 1/\left[1 + jt^3/3\delta^3 - t^4/30\delta^4 - jt^5/630\delta^5 + \ldots \right] - 1 \quad (9)$$

The calculated RF-dc differences caused by the annulus are negative and large. This is why the RF-dc differences of microvolt range pots are usually negative. For a 1 milliohm gold film resistor, $t$ is 3.2 m, the RF-dc difference from Eq. (9) at 1 GHz is -5.5%. This is smaller than the measured value because there is a similar effect in the caps of the resistor.

5. Transmission line effect of output connector
At frequencies above a few MHz the distance between reference planes of standard and test device is not negligible compared to the wavelength. When the impedance of the interconnections and the input impedance of the test device are not matched, the RF voltage will usually rise from the center of the annular film resistor to the reference plane of the test device and continue to rise to the input load of the test device. Although the transmission lines effect can be calculated [4-6], the measurements of the load impedance and the total length of the actual transmission line are difficult to obtain practically. When the input impedance of the test device is much higher than the characteristic impedance of the interconnection line, the calculation formula is simply the following [6]

$$d_T = 2.19 \times 10^{-20} |t|^2 f^2 \quad (10)$$

where $|t|$ is the distance between the annular film resistor and the front face of the input load of the test device (not the reference plane of test device). When performing high-frequency voltage calibrations, all interconnecting devices, adapters, and leads between the pot and instrument under test should be as short as possible in order to reduce transmission line errors.

6. Empirical equation for RF-dc difference
From the above analysis, we can obtain an empirical equation for the RF-dc difference of a pot.

$$d = d_0 + d_c + d_s + d_R + d_T$$

$$= d_0 + Af^{0.5} + Bf + Cf^2 + Df^3 \quad (11)$$

where $d_0$ is the ac-dc difference at audio frequency, which can be measured directly, $A$ is related to the skin effect of the thermoelement, $B$, $C$ and $D$ are related to the skin effect of the annular resistor, $C$ is also related to the distributed inductance and capacitance of the thermoelement and annular resistor. $A$, $B$, $C$ and $D$ can be derived by a nonlinear fit from the RF-dc differences of the pot to be measured. The calculated RF-dc differences from equation (11) agree with the measurement data.

7. The improvements in the new design
Based on the theory described above, a new commercial pot housing has been designed and constructed to reduce the error of RF current measurements. The RF-dc differences of pots with new housings are reduced significantly as shown in Fig. 2.

![Fig. 2 The RF-dc differences of Model 440 pot and Model 1351* pot with new design](AL: Aluminum, BR: Brass)
Measurement Method and Results

1. Calibration method and uncertainties
The primary standard pots are calibrated against a bolometer at NIST, Boulder. The bolometer is connected at the output of the pot. The RF output voltage of the pot is derived from the power and impedance measurements for the bolometer.

The calibration of a pot may also be performed by comparison with a calibrated pot [7] or an RF thermal voltage converter. In both the later cases, a transfer or comparison device such as an RF receiver is used to transfer the RF output voltage of the standard to the pot under test. The block diagram of a typical measurement system is shown in Fig. 3. Table 2 lists the uncertainty ranges for special pot calibrations at NIST, Boulder.

<table>
<thead>
<tr>
<th>F (MHz)</th>
<th>Voltage Level (mV)</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>200</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.43</td>
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<tr>
<td></td>
<td>0.2</td>
<td>0.46</td>
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<tr>
<td>100 - 400</td>
<td>200 – 0.2</td>
<td>0.85</td>
</tr>
<tr>
<td>500 - 700</td>
<td>200 – 0.2</td>
<td>1.1</td>
</tr>
<tr>
<td>800 - 1000</td>
<td>200 – 0.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Experimental results show that pots are stable RF voltage standards over a very long period of time. The data in Table 3 indicate that changes in the RF-dc difference of a Ballantine pot* (10mA, 22) over a 20 year period are generally smaller than the measurement uncertainties. This pot was calibrated relative to the standards at NIST, Boulder.

2. Lifetime stability
Table 3. Experimental data showing the stability of a RF pot

<table>
<thead>
<tr>
<th>F (MHz)</th>
<th>Year</th>
<th>RF-DC Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100.0</td>
<td>64.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>500.0</td>
<td>65.00</td>
<td>3.00</td>
</tr>
<tr>
<td>900.0</td>
<td>66.00</td>
<td>13.00</td>
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<td></td>
<td>67.00</td>
<td>14.00</td>
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<td></td>
<td>68.00</td>
<td>15.00</td>
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<td>69.00</td>
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<td></td>
<td>70.00</td>
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<td>77.00</td>
<td>24.00</td>
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<tr>
<td></td>
<td>78.00</td>
<td>25.00</td>
</tr>
</tbody>
</table>


3. Precautions
Since RF pots have frequency responses which extend to 1.2 GHz, they should not be used in areas where the interference from high field strength may be a problem; otherwise, additional shielding will be necessary. The loading effect should also be considered [2], [8].

4) Equation for Calculating Larger RF-DC Differences
The RF-dc differences of pots and thermal voltage converters at several hundred MHz are generally several percent or more, test instruments may also have errors of several percent or more. In order to achieve the best accuracy’s, it is necessary to use the equation:

\[ d_t = d_s + \Delta + d_s \Delta \] (12)

where \( d_s \) and \( d_t \) are the ac-dc differences of reference standard and test devices, and is the difference measured between two devices. The second-order error \( d_s \), which is generally neglected at low frequencies may reach several tenths of one percent to several percent at high frequencies.
Conclusion
Formulas for calculating the RF-dc differences of pots for five major error sources are presented and analyzed. The RF-dc differences of pots are determined by their geometry and materials. They can be reduced by proper designs of the housing and annular resistor. Pots maintain their stability over several decades and are very suitable as primary standards in the microvolt and millivolt ranges from 10 Hz to 1.2 GHz.

References


*Certain commercial equipment, instruments, or materials are identified in this paper in order to adequately specify the experimental procedure. In no case does such identification imply recommendation or endorsement by National Institute of Standards and Technology, nor does it imply that the material or equipment identified is necessarily the best available for the purpose.