

1st Annual Internal UNB Math Competition

UNB Mathematics Society

March 17, 2016

Rules

1. 40 minutes
2. 15 questions
 - (a) Part A - 6 questions
 - (b) Part B - 6 questions
 - (c) Part C - 3 questions
3. No calculators
4. No collaboration
5. Grading:
 - (a) Part A - 4 points per question
 - (b) Part B - 5 points per question
 - (c) Part C - 7 points per question
6. Penalty for incorrect answer: $1/4$ th of question value
7. Total Points: 75
8. Please detach and write your answers on the last page provided.
9. Good Luck!

Name: _____

Student Number: _____

Part A - 6 Problems (4 Points Each)

1. In 2016, Alvin's birthday was on a hot Wednesday in June. In 2019, what day will his birthday occur?

- A. Sunday B. Tuesday C. Wednesday D. Thursday E. Saturday
-

2. The maximum number of points of intersection of 10 lines is:

- A. 10 B. 25 C. 45 D. 50 E. 100
-

3. The last digit of $3^{1415926}$ is:

- A. 1 B. 3 C. 7 D. 9 E. 0
-

4. The positive integers, starting with 1, are written in ascending order like so: 123456789101112131415....
The digit appearing in the 100th place is:

- A. 0 B. 4 C. 5 D. 6 E. 7
-

5. Richard and Daniel have a bag of marbles. When they split these marbles into two equal piles, one marble remains. When they split these marbles into three equal piles, once again one marble remains. Among the following numbers, which one cannot represent the number of marbles that Richard and Daniel have?

- A. 7 B. 13 C. 25 D. 31 E. 41
-

6. Considering the following statements. Which one of the choices must be true?

- All birds fly.
Some birds are cardinals.
All robins sing.
Some cardinals sing.
All cardinals and robins are birds.

- A. All flying birds are robins. B. All flying cardinals sing. C. Some cardinals do not sing.
D. Birds that do not sing are not robins. E. All cardinals do not fly.

Part B - 6 Problems (5 Points Each)

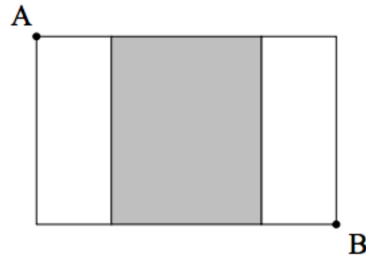
1. In a country of Mathematica, the population consists entirely of mathematicians and politicians. Mathematicians always tell the truth and politicians always lie. One day, 3 persons A, B and C meet. A whispers to B one of these two statements: "I am a mathematician" or "I am a politician" B turns to C and tells him that A claims to be a mathematician. Outraged, C replies that A is not a mathematician, but a politician. How many of these three people are mathematicians.

A. 0 B. 1 C. 2 D. 3 E. Not Enough Information

2. Find the sum of all the integers x for which $2000 < 5^{x-1} < 20000$:

A. 3 B. 1 C. 4 D. 15 E. None of these

3. Two squares, each of area 36 overlap as shown in the diagram to the right. If the overlapping area is $\frac{2}{3}$ of one of the squares, what is the distance between the points A and B?



A. $6\sqrt{2}$ B. 10 C. $6\sqrt{3}$ D. 12 E. None of these

4. For how many integers between 100 and 1000 will the sum of the digits be 7?

A. 8 B. 28 C. 34 D. 64 E. None of these

5. The great mathematician Pouria Poursaied lived in the 19th century. He once made the following statement: "I was x years old in the year x^2 ." What year was he born?

A. 1801 B. 1806 C. 1849 D. 1860 E. 1875

6. Charlotte and Danielle alternate turns tossing a fair coin. Charlotte goes first and each player takes three turns. The first player to toss a tail wins. If neither Charlotte nor Danielle tosses a tail, then neither wins. What is the probability that Charlotte wins?

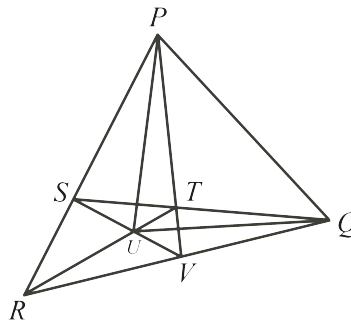
A. $\frac{21}{32}$ B. $\frac{5}{8}$ C. $\frac{3}{7}$ D. $\frac{11}{16}$ E. $\frac{9}{16}$

Part C - 3 Problems (7 Points Each)

1. Rajesh drives along a straight road that goes directly from his house (R) to his Grandfather's house (G). Some of this road is on flat ground and some is downhill or uphill. His car travels downhill at 99 km/h, on flat ground at 77 km/h, and uphill at 63 km/h. It takes Rajesh 3 hours and 40 minutes to drive from R to G. It takes her 4 hours and 20 minutes to drive from R to J. The distance between R and G, in km, is:

A. 205 B. 308 C. $318\frac{2}{3}$ D. $292\frac{3}{5}$ E. $292\frac{1}{9}$

2. In the diagram, $\triangle PQR$ has S on PR and V on RQ . Segments QS and PV intersect at T . Segments RT and SV intersect at U . If the area of $\triangle RTV$ is 66, and the area of $\triangle RSV$ is 77, then the area of $\triangle PQU$ is:



A. 869 B. 836 C. 840 D. 864 E. 847

3. A multiplicative partition of a positive integer $n \geq 2$ is a way of writing n as a product of one or more integers, each greater than 1. Note that we consider a positive integer to be a multiplicative partition of itself. Also, the order of the factors in a partition does not matter; for example, 235 and 253 are considered to be the same partition of 30 . For each positive integer $n \geq 2$, define $P(n)$ to be the number of multiplicative partitions of n . We also define $P(1) = 1$. Note that $P(40) = 7$, since the multiplicative partitions of 40 are 40 , 2×20 , 4×10 , 5×8 , $2 \times 2 \times 10$, $2 \times 4 \times 5$, and $2 \times 2 \times 2 \times 5$.

Determine the value of $P(1000)$.

A. 29 B. 31 C. 33 D. 35 E. 41

Answers

Part A

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

Part B

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.

Part C

- 1.
- 2.
- 3.

Please detach and hand in only this sheet.

Name: _____

Student Number: _____