BEAM Solvers

Congratulations to the successful solvers from Challenge Set 2!

Top Solvers (4+ Solutions, win a prize!)

- Aamirah Spencer
- Alex Torres
- BJ Kemp
- Brent Anderson
- Brianna Graham
- Camia King Cupid
- Cindy Truong
- Daniel Myers
- Eils Thornton
- Fatima Soussi
- Gittan Johnson
- Ilana Ocampo
- Ivory Wang
- Jacob Tom-Wong
- Jason Lin

- Jeremiah Beaufils
- Justin Perez
- Kali Sears
- Karen Zhao
- Karen Ramirez
- Liberty Sumners
- Melvin Chino-Hernandez
- Minhaj Uddin
- Mohammad Alhusaini
- Nicole Belisario
- Otto Recondo
- Rawin Hidalgo
- Raynelis Villa
- Yilin Li
- Youmna Nasr

Solvers (2+ Solutions)

- Camren Hall
- Conaré Lucas
- Faiyaz Hasan

- Hawa Drame
- José López
- Maryam Yaqoob
Challenge Set 2 Solutions

Problem 1
Place each of the numbers 1, 2, 3, 4, 5, 6, 7, and 8 in the circles at right so that no two numbers that are one apart are in circles connected by a line. For example, 2 and 3 cannot be in circles that are connected by a line because they are one apart.

Problem 1 Solution
Trying out every possible solution would take much too long, but some thinking lets you see that 1 and 8 have to be in the middle circles. Why? Each of the middle circles is connected to 6 other circles, which means there is only one circle each middle circle is not connected to:

But that means 2 can’t go in a middle circle, because 2 is one apart from 1 and 3, which means they’d both have to go into that one circle not connected to a middle circle. Something like that happens for every number except for 1 and 8 – they’re only one apart from one other number. So the center circles have to be 1 and 8 or they could be 8 and 1.
If we place them 1 and 8 it looks like this

![Graph with nodes 1 and 8 placed]

There is only one circle not connected to the circle with the 1, so that circle has to have the 2. In the same way, there's only one circle not connected to the circle with the 8, so that circle has to have the 7.

![Graph with nodes 7, 1, 8, and 2 placed]

The two circles left on the top are connected to each other, and so are the two circles on the bottom. The numbers left are 3, 4, 5, and 6. If we consider 4, it can only be next to 6 but not next to 3 or 5. So if 4 is on top, then the other number on top has to be 6. But 6 can't be next to the 7, so that means it has to be on the right side.
That would look like this:

Note that we could just as easily have put the 4 and 6 on the bottom as well and it would be the same, since they are connected to the other in the same way.

Now we only need to put the 3 and 5 in the last two circles. The 3 can't be next to the 2, so it has to be:

No guessing required!
Problem 2

If $\frac{22}{111}$ is written in decimal form, what is the 92nd digit after the decimal point?

**Problem 2 Solution**

This problem can be done a lot like Problem 2 in Challenge Set 1! In fact, you could start by using the trick you learned from the solutions:

If you have a 3-digit number over 999, then its decimal form is just its 3 digits repeating to the right of the decimal point.

Now $\frac{22}{111}$ doesn’t fit this form, but we can multiply the top and bottom by 9 to get $\frac{198}{999}$. This tells us that the decimal form of $\frac{22}{111}$ is just 198 repeating to the right of the decimal place.

We could also check this using long division:

\[
\begin{array}{c|c}
111 & 2 \hspace{0.5cm} 2 \hspace{0.5cm} 0 \\
\hline
& 1 \hspace{0.5cm} 1 \hspace{0.5cm} 1 \\
& 1 \hspace{0.5cm} 0 \hspace{0.5cm} 9 \hspace{0.5cm} 0 \\
& 9 \hspace{0.5cm} 9 \hspace{0.5cm} 9 \\
\hline
& 9 \hspace{0.5cm} 1 \hspace{0.5cm} 0 \\
& 8 \hspace{0.5cm} 8 \hspace{0.5cm} 8 \\
\hline
& 2 \hspace{0.5cm} 2 \\
\end{array}
\]

The remainder you end up with here is 22, just like what you started with. So if you kept doing long division, you would just repeat the 1, 9, and 8. So the result is what the trick predicted:

0.198198...

The pattern repeats every 3 digits! So digits 1-3 are 198, and digits 4-6 are 198, and so on. You can break up 92 = 90 + 2. Because 90 is 3×30, in the 90th digit the number has just finished repeating 198 (the 30th time the pattern repeats). Then digit 91 is 1, and digit 92 is 9. The 92nd place must be a 9.
Problem 3

In the diagram on the right, the number in each square is the product of the two squares just below it. For example, since $2 \times 5 = 10$, 10 is in the square above 2 and 5. Fill in the rest of the squares.

Problem 3 Solution

Like many math problems, if you go step-by-step you can solve it. We will show one order of filling in the boxes, but you might have done it a different way and that’s okay!

There is a blank box over a 2 and 10, and $2 \times 10 = 20$, so that box has to be 20. Let’s fill that in:

Next, there’s a blank box above that 20 and the 6 next to it. Since $6 \times 20 = 120$, we can put a 120 in that blank box.

Next, the 6 is above a 2 and a blank box. Since $2 \times 3 = 6$, that blank box must have a 3 in it. Remember how the number pyramid last month used addition, and so you had to subtract to go down the pyramid? Here, the number pyramid is based on multiplication, so you have to divide to go down the pyramid!
Now, there’s a blank box with a 2 above it and a 2 next to it.
So that box must have a 1, since $2 \times 1 = 2$ (or, if you want to think of it as division, because $2 \div 2 = 1$).

The last blank box is next to a 1 and below a 3.
So it has to be a 3 too, since $1 \times 3 = 3$, or using division, because $3 \div 1 = 3$.

**Problem 4**

Go to this website and watch the video on how to sum every number between 1 and 100:

https://aops.com/videos/prealgebra/chapter1/19

Think about how the video does this, and use it to add every number between 1 and 200:

$$1 + 2 + 3 + \cdots + 198 + 199 + 200 =$$

**Problem 4 Solution**

In the video, we pair numbers so that every pair adds to 101 and there are 50 pairs, so we get

$$101 \times 50 = 5050.$$ 

We do the same thing with this sum. When we pair 1 and 200, the outermost numbers, we get 201:

$$1 + 2 + 3 + \cdots + 198 + 199 + 200 = 201.$$
Similarly, when we pair the next inner-most numbers, the 2 and the 199, they also sum to 201.

\[ 1 + 2 + 3 + \cdots + 198 + 199 + 200 = 201 \]

This continues if we pair the numbers like this:

\[ 3 + 198 = 201 \]
\[ 4 + 197 = 201 \]
\[ 5 + 196 = 201 \]
\[ 6 + 195 = 201 \]

… and continuing on like that!

How many pairs are there? In the middle, the last numbers added are 100+101=201, so each number between 1 and 100 pairs with a number between 101 and 200. In total, each number 1–100 is part of one pair, so there are exactly 100 pairs. This becomes 100 201’s, so the final sum (and answer to the problem) is

\[ 100 \times 201 = 20100. \]

**Problem 5**

Otto starts doodling in his notebook. First, he draws a hexagon, which has 6 sides. Then he draws two hexagons next to each other, and he counts 11 sides. Then he draws three hexagons next to each other, and he counts 16 sides. If he keeps going, how many sides will there be when he draws 50 hexagons next to each other?

**Problem 5 Solution**

The first hexagon Otto draws has 6 sides. Every time he draws another hexagon connected to the end, he has to add 5 more sides. To get to 50 hexagons total, he adds 49 hexagons to her first hexagon, each of which will add 5 sides to the chain. This means the total number of sides are

\[ 6 + 5 \times 49 = 6 + 245 = 251 \]

Where the 6 comes from the first hexagon, and then we add a 5 for each hexagon added after that. Since this adds to 251, this is the answer.
Problem 6

A BEAM student always tells the truth on Thursdays and Fridays, always tells lies on Tuesdays, and randomly tells the truth or lies on other days of the week. On seven consecutive days, the student was asked her name, and on the first six days she gave these answers in order:

Cindy, Ivory, Cindy, Ivory, Karen, Ivory

What answer(s) could she have given on the seventh day? A full answer to this question must explain why no other answers are possible.

Problem 6 Solution

We know that the BEAM student always tells the truth on Thursdays and Fridays, so there are two days in a row where the student will say the same thing (their true name). Since the answers she gave are in order and none of them have the same name twice in a row, this means the 7th day must repeat her true name, either Cindy or Ivory. In other words, either the last day is Thursday (and then the missing day is Friday, and she says Ivory), or the first day is Friday (and the missing day was a Thursday, and she says Cindy). So her name must either be Cindy or Ivory.

Case 1: The last day is Thursday. This means that she said her name was Ivory on Thursday, and since she tells the truth on Thursdays, her name really is Ivory.

Does this make sense with the rest of the days? If the last day is Thursday, the six days listed are:

Saturday  Sunday  Monday  Tuesday  Wednesday  Thursday
Cindy  Ivory  Cindy  Ivory  Karen  Ivory

But we also know that the BEAM student always lies on Tuesday, so we should check that she is lying on Tuesday. But by the above, she says her name is Ivory on Tuesday. We already determined that her name must be Ivory, so this isn’t a lie! This is a contradiction; it cannot happen. So the last day cannot be Thursday.

Case 2: The first day is Friday. Since the name she gives on the first day is Cindy, and the problem tells us she always tells the truth on Friday, her name must be Cindy.

Does this make sense with the rest of the days? If Friday is the first day, then the days look like this:

Friday  Saturday  Sunday  Monday  Tuesday  Wednesday
Cindy  Ivory  Cindy  Ivory  Karen  Ivory
We know she always tells the truth on Thursdays and Fridays, so if this case works she must be Cindy. Does it match the other parts of the problem? The problem says she lies on Tuesdays. For this case, on Tuesday she says she's Karen, which would be a lie. Since she can either lie or tell the truth on the other days, this is consistent and so her name really is Cindy.

Since the next day is Thursday, she tells the truth the next day, and the answer to the problem is Cindy.

**Problem 7**

A book has 30 stories. Each story takes up a different number of pages: one takes up 1 page, another takes up 2 pages, all the way up to the thirtieth story which takes up 30 pages. The first story starts on page 1. If the stories can go in any order, what is the greatest number of stories that can start on an odd page?

**Problem 7 Solution**

Since an even number plus an odd number is an odd number, if a chapter with an even number of pages starts on an odd page, then the next chapter will also start on an odd page. So if all the chapters that are an even number of pages come first, then since the first one starts on page 1 (an odd number), they will all start on an odd page. That's 15 chapters that all start on an odd page already!

Now we have to do the chapters that have an odd number of pages. When a chapter with an odd number of pages starts on an odd numbered page, the following chapter starts on an even page, because an odd plus an odd is an even. On the other hand, when a chapter with an odd number of pages starts on an even numbered page, the following chapter will start on an even page, because an even plus an odd is an odd. So when we’re down to chapters with only odd numbers of pages, they alternate which chapters start on an odd page.

The first chapter with an odd number of pages starts on an odd page. The second starts on an even page. The third starts on an odd page. This pattern continues until the fifteenth, which starts on an odd page. In total, that’s 8 that start on an odd page number.

Thus, the total number of chapters that start on an odd page will be $15 + 8 = 23$.

*Problems 6 and 7 are from the Math Kangaroo contest.*