BEAM
Bridge to Enter Advanced Mathematics

55 Exchange Place, Suite 603, New York, NY 10005
(888) 264-2793
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Courses Offered at the Bard College Site

Topics Courses

Students take one topics course each week.

Week 1 Course Descriptions

Logic and Proof (Sarah)

When a scientist or lawyer proves something, it means that they've found enough evidence supporting their idea (for how the world works or what really happened during a crime) that they're pretty sure it's correct. However, a mathematician means something different when they talk about proof. In this class, we'll explore what it means to prove something mathematically and practice the kind of thinking that's involved. If you like solving logic puzzles, this is the course for you, as we'll do many of them throughout the week.

Count without Counting (Javier)

In this class we will learn rules or shortcuts that will allow us to count how many objects are in a collection without having to count each of those objects one by one. For example, how can we count without listing one by one the number of different initial lineups for a basketball team that has 10 players in total?

However, when we take a shortcut we need to make sure that it is the right one and that it will not take us in the wrong direction. For example, is it safe to say that a building has 20 floors based on the fact that the largest number in the elevator's buttons of that building is 20? Could I be miscounting or overcounting the number of floors? What if they skipped a number? What if there are other floors whose buttons have a letter instead of a number? In this class we will make sure that we know how to use the shortcuts that we learn so that we are not miscounting or overcounting.

Maps, Puzzles, and Graph Theory (Malcolm)

Have you ever done a puzzle where you had to trace something without lifting your pencil or going over any line twice? Or did you ever have to color a map where you weren't allowed to use the same color on two countries that touched each other? These two activities seem completely unrelated, but they're both examples of questions in a type of math called graph theory. Graph theory has *nothing* to do with the graphs you've probably seen in school! This is NOT about y = mx + b or plotting points on a coordinate plane - it is actually a type of math that most people don't even hear about until college! Graph theory can help us save money while we travel, understand how diseases spread, and find the best way to build roads in a developing country. This class will also teach you how mathematicians think when they're discovering new math that no one has thought of before, such as how to tell when some puzzles are actually impossible to solve, and how to find a "good enough" answer when even a computer would take years to find the perfect solution! In graph theory, you'll learn something totally new, start thinking like a mathematician, and see mathematical structure in places you never would have imagined it.

Squares and the Sum of Two Squares (Karen)

Problem 1: Can you find the dimensions of a square rug so that the area is exactly 5 ft^2?
Problem 2: Can you find the dimensions of two square rugs so that the sum of their areas is 5 ft^2?

In this course we use multiplication, division, patterns, conjecture, logic and proof to determine for what numbers our two rug problems can be solved.

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Week 2 Course Descriptions

Programming in JavaScript (Malcolm)

Every app, video game, website, or program you've ever used was written by a programmer using a programming language. Programming languages are used to tell computers what to do, and they are much more exact than the languages we speak. In this class, you'll join a website that will let you program in a language called JavaScript, and save your programs online. JavaScript is used by programmers to program online games, apps, and websites. In this class, we'll learn the basics together, and then you'll choose a final project that builds off of what you learned in your previous classes, or you can create a new project yourself. Finally, you will learn how to expand your programming knowledge on your own after this summer, so you can keep programming for fun for as long as you want!

Mathematical Modeling (Megan)

In this course, students will develop the basic idea behind mathematical modeling by creating models by hand and using the free software program Vensim to help analyze the models. We will model interactions found in the real world such as rumors spreading in a school, predators and prey, and a cold virus spreading through a summer camp program. By the end of the week, students will create a model to represent a situation they find interesting.

Number Theory: If 4×4=1, does 0×0=1 too? (Tanya)

We will play with numbers, explore and attempt to write a Constitution of a Number Kingdom. You'll have a chance to experience a job of a Supreme Court judge i.e. to decide important cases based on our Constitution. We might also write a fairy tale which is pretty enlightening as fairy tales usually are, collect some cute little number systems, and look at the big picture through a magnifying glass.

Mosaics, Symmetries and ¿Dancing? (Javier)

Have you noticed that the New Year's Eve ball that comes down every year in Times Square is not perfectly rounded? It is really made out of a bunch of smaller triangles!! Could we make a ball like this using just stop signs? Have you noticed that soccer balls are hybrids of pentagons and hexagons? Could we do a hybrid ball with stop signs and triangles?

All these questions have to do with arranging together regular polygons (these arranges need not to form a ball, for example the polygons could also be sitting on a wall). We will call one of these kind of arranges a ***mosaic***. Along history mosaics have been some of the most beautiful pieces of art and mathematics is used to create the harmony and beauty that they possess.

In this class we will explore the questions listed above and many more!

We will also explore what is the relationship between dancing and the way we can move mosaics without changing their shape and we will move and dance in order to do it!

Week 3 Course Descriptions

Information Theory (Sarah)

Can you count past a thousand on your fingers? How did the Voyager spacecraft send images of Jupiter and Saturn back to Earth? And what (on earth) do these questions have to do with each other? This course will dive into information theory, a field at the intersection of mathematics, statistics, physics, computer science, electrical engineering, and neuroscience. We will focus on two very important questions: What is information, and how can we efficiently store and transmit it?

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Voting Theory: Is there really a fair way to make decisions in a group? (Megan)

In many decision making situations, it is necessary to gather the group consensus. This happens when a friends decide which movie to watch, when a company decides which product design to manufacture, and when a democratic country elects its leaders. While the basic idea of voting is fairly universal, the method by which those votes are used to determine a winner can vary. In a group of friends, you may decide upon a movie by voting for all the movies you're willing to watch, with the winner being the one with the greatest approval. A company might eliminate unpopular designs then revote on the remaining. A country might look for the candidate with the most votes. In deciding upon a winner, there is always one main goal: to reflect the preferences of the people in the fairest way possible. In this class, we will explore different ways to pick a winner of a vote and discover the potential flaws in the way decisions are made. We will also consider how voting systems change if some people's votes have more influence or weight than other people's votes and how that changes the group dynamics.

Amazing Discovery, which after 20 years of simple calculations changed the world (Tanya)

Can you imagine a world with no calculators - computers - any electronic devices? Can a piece of wood become a calculator in clever hands? I am not talking about abacus! People build roads, bridges, Egyptian pyramids (they are still standing!) in the world where all calculations had to be done by hand. And numbers were kind of big and not "nice"! We will play lots of games, tear some paper, learn to add infinitely many small things in the process and with any luck, try to wrap our heads around the ideas which allow a piece of wood to become so very-very-very magical!

Sequences: Triangles, Squares, Pentagons, Hexagons, and Euler's Partition Formula (Karen)

In this course we look at examples of famous sequences. We will focus on three sequences; the sequence of triangular numbers, the sequence of square numbers, and the sequence of pentagonal numbers. We know which numbers are squares. But what are triangular numbers? What are pentagonal numbers? We will also explore partitions (and if we have time I will tell you what Ramanujan proved about partitions).

Problem Solving Course Descriptions

Math Team Strategies (Nate)

If you are competitive and love the challenge of quickly solving problems that most other kids can't solve at all this is the course for you! Math Team Strategies is an introduction to the problem solving strategies that are helpful for math contests. Once you are able to master the problem solving techniques presented in this course, AND you learn when and how to use them, you may have the chance to represent your school in local math competitions, and if you do well, you may even have a chance to compete against kids from all over the country!

Solving Big Problems (Malcolm)

You may find that it only takes you a few minutes to solve the math problems that you see in school. But real math---the kind we use to build bridges or to enable computers to send data to each other, the kind that makes us go "wow!" and which people think is so great that they spend all their lives doing it---those problems take creativity, and often take much longer to solve. You have to think deeply and really piece things together to figure them out. In this class, we will work together to solve problems that take hours or even days to solve. We will build stamina for solving this type of problem, and learn strategies for making progress on problems, even when you feel stuck. You'll learn how to solve problems that are deep, interesting, fun, and hard- in other words, Big Problems.

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Topics Courses

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Week 1 Course Descriptions

Exploring Infinity (Aaron)

Do all infinite sets have the same size or are there different sizes of infinity? We'll spend the week exploring this fascinating question. Along the way we'll learn about methods of mathematical proof and deepen our understanding of various types of numbers: integers, rational numbers, irrational numbers, and more.

SET and Combinatorics (Dan)

You probably think you know everything there is to know about counting. "One, two, three, ..." – this is pretty boring! But did you know that there is an entire branch of mathematics, called combinatorics, which is concerned with counting stuff? In this class we will explore many fun problems in combinatorics, including chess, license plate numbers, and athletic competitions like racing. Our biggest source of combinatorics problems will be the card game SET, and we will consider many of the mathematical properties of this game. Through the game of SET, we will also encounter probability, modular arithmetic, vectors and perhaps even a little finite geometry. We'll play a lot of games in this class, and you will be out of your seat and moving around in almost every class period. Come prepared to move, have fun, and count!

Logic and Proof (Marcus)

This course will introduce the foundations of mathematical logic and explore how they are involved in proving (and disproving) mathematical ideas. We will discuss why proof is an important component of mathematics, explore some of the most common mistakes made in mathematical proofs, and see some beautiful tactics used to prove mathematical statements.

Numbers, Patterns, and Proofs (Reva)

Just underneath the surface of your regular, everyday numbers like 0, 1, 2, 3, etc. is a whole world of fascinating mathematics. These numbers make up all sorts of amazing patterns, and some numbers act in particularly special ways. Maybe you've already wondered whether the number 0 was even, odd, both, or neither. Maybe you've noticed a pattern with some kinds of numbers (like the perfect squares), but you aren't sure if it's just a coincidence. Or you might have learned a trick for telling if a number was divisible by 9, but no one showed you why the same trick works for 3 but doesn't work for 7. In this class we'll explore this world of Number Theory, and along the way we'll learn ways to give a proof that demonstrates why something has to be true, or to convince someone else that their idea is false!

Week 2 Course Descriptions

Winning Games and Writing Code (Aaron)

We will play a variety of two player games (with no chance) and try to find winning strategies to those games. A "winning strategy" is a strategy for one of the players that, if followed, will guarantee a win for that player, no matter what the other player does. We'll also learn about the programing language Python and we'll teach computers to play those games.

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Sequences and Series: The Language of Pattern (Cory)

A sequence is a list of objects. Those objects can be numbers, shapes, or even trees! Although the definition of a sequence is simple, it is one of the most useful tools for looking for patterns in our universe. In this course, we will focus on special sequences as they arise in nature and science, and we will see how they can be used to understand the world around us.

Projective Planes (Dan)

What is a line? Does it have to be straight? You probably already know a lot about geometry without even realizing it. In this course, we will investigate a different type of geometry where a lot of what seems obvious about geometry is no longer true. We will study objects called incidence structures, and introduce rules which will give us more specific types of structures. This course will be abstract, but it is also very visual – almost everything we study will be sketched out on paper, and many of our examples are quite beautiful. We will also connect some of the concepts of this course to the game of Spot It!

Cryptography: The Science of Secrets (Siddhi)

Throughout history, military and political leaders have developed methods for exchanging top-secret information. We will learn about these different methods of “encryption” and “decryption” of messages, and discover how math is used to make it harder for codes to be cracked. We'll end the class by discussing modern techniques and technologies used by computers to secretly send messages.

Week 3 Course Descriptions

Aviation and the Wonder of Flight (Cory)

From tiny birds to massive four-engine, double-decker airliners, the concept of flight seems like magic to most. How can anything with mass seemingly "float" in thin, invisible air? This course aims to explain the science behind the wonder of flight through lecture, group work, experimentation, and even some flight simulation.

It Takes Two! (Marcus)

This course will introduce a new system of mathematics in which there only exist 2 symbols to represent numbers instead of our usual 10 symbols. We will explore the pros and cons of this beautiful mathematical system, and see some surprising applications.

Maps, Puzzles, and Graph Theory (Reva)

Have you ever done one of those puzzles where you had to trace something (like a house) without lifting your pencil or going over any line twice? Or did you ever have to color a map where you weren’t allowed to use the same color on two countries that touched each other? Amazingly, even though these two activities seem completely unrelated, they’re both examples of challenges that we could solve with a type of math called graph theory. (And this graph theory has *nothing* to do with the graphs you’ve probably seen in school!) This kind of math is great for exploring all sorts of problems, like scheduling courses so that no student ends up in two classes at the same time, or finding a way to build roads in a developing country so that all the major communities are connected. We’ll also learn how to tell when some puzzles are actually impossible to solve, and how to find a “good enough” answer when even a computer would take years to find the perfect solution!

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Knot Theory (Siddhi)

What do you think of when you hear the word “knotted”? While your shoe laces might come to mind, a mathematician means something different when they talk about knots. We wouldn't call them knots at all! In this course, we'll talk about the mathematical theory of knots, and discuss questions like “what is a knot?” and “when are two knots the same?”. We'll tackle these questions -- and more! -- by drawing pictures and building models.

Problem Solving Course Descriptions

Math Team Strategies (Matt)

What can you do when you’re not even sure how to get started on a tough math problem? In this class, you will learn and discover a variety of strategies for getting moving whenever that happens. These techniques are especially helpful on math contests as well as standardized tests, but are useful all across mathematics.

Another major theme of the class, because contests and exams are time-limited, is finding ways to avoid doing long, tedious, repetitive calculations. Such methods are often extremely satisfying and elegant as well as efficient. The example problem demonstrates this type of situation, where trying to answer the question in the obvious way is not practical.

Of course, another major piece of being on a team is teamwork. Although all classes at BEAM are highly interactive and collaborative, Math Team Strategies places a special emphasis on the “how” of working together. You will discuss how best to communicate your ideas to your classmates and present at the board—skills that will serve you well in mathematics and beyond!

Solving Big Problems (Sarah)

You may find that it only takes you a few minutes to solve the math problems that you see in school. But real math---the kind we use to build bridges or to enable computers to send data to each other, the kind that makes us go “wow!” and which people think is so great that they spend all their lives doing it---those problems take creativity, and often take much longer to solve. You have to think deeply and really piece things together to figure them out. In this class, we will work together to solve problems that take hours or even days to solve. We will build stamina for solving this type of problem, and learn strategies for making progress on problems, even when you feel stuck. You'll learn how to solve problems that are deep, interesting, fun, and hard- in other words, Big Problems.