Federal Reserve Bank of Minneapolis
Research Department Staff Report 391

Revised January 2008
(First version May 2007)

Are Shocks to the Terms of Trade Shocks to Productivity?

Timothy J. Kehoe

University of Minnesota,
Federal Reserve Bank of Minneapolis,
and National Bureau of Economic Research

Kim J. Ruhl*

University of Texas at Austin

ABSTRACT

International trade is frequently thought of as a production technology in which the inputs are exports and the outputs are imports. Exports are transformed into imports at the rate of the price of exports relative to the price of imports: the reciprocal of the terms of trade. Cast this way, a change in the terms of trade acts as a productivity shock. Or does it? In this paper, we show that this line of reasoning cannot work in standard models. Starting with a simple model and then generalizing, we show that changes in the terms of trade have no first-order effect on productivity when output is measured as chain-weighted real GDP. The terms of trade do affect real income and consumption in a country, and we show how measures of real income change with the terms of trade at business cycle frequencies and during financial crises.

*We would like to thank Narayana Kocherlakota, Ellen McGrattan, Edward Prescott, Katheryn Russ, and an anonymous referee for helpful advice and comments. This work was undertaken with the support of the National Science Foundation under grant SES-0536970. The data used in this paper are available at www.econ.umn.edu/~tkehoe and at www.eco.utexas.edu/~kjr296. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1. Introduction

The terms of trade — the price of imports relative to the price of exports — vary greatly over time and country. This variation makes the terms of trade a natural candidate for explaining country performance. Intuitively, we can think about foreign trade as a production technology: a country's exports are the inputs to the technology, and these inputs are turned into outputs that are recorded as a country's imports. Exports are transformed into imports at the rate that is the ratio of the price of exports to the price of imports, which is just the reciprocal of the terms of trade. Viewed in this way, an increase in the terms of trade acts much like a technology shock: the same amount of exports now produces a smaller amount of imports.

In this paper, we show that standard models do not support this line of reasoning when we measure technology shocks using total factor productivity (TFP) where output is measured using real gross domestic product (GDP). The problem lies in the construction of real GDP. The effect of a shock to the terms of trade on real GDP is not the same as the effect of a productivity shock and is dependent upon the method used to construct real GDP. When real GDP is constructed using the chain-weighting method specified in the United Nations System of National Accounts, terms of trade shocks have no first-order effects if inputs of factors are constant. When real GDP is constructed using fixed base year prices, the effect of a terms of trade shock is ambiguous: in some cases a deterioration of the terms of trade can even increase real GDP! In this paper, we bring this accounting to bear on the relation between the terms of trade and productivity. As productivity is computed using real GDP as the measure of output, the terms of trade cannot have a direct effect on a country's TFP. If factors of production can vary, changes in the terms of trade can cause real GDP to vary, but, even so, there is no first-order effect on TFP. An increase in the terms of trade lowers the purchasing power of the country, which can be painful in terms of consumption and welfare, but does not impact TFP.

The relation between changes in the terms of trade and changes in real GDP is well understood by economists interested in index numbers and national income accounting. Diewert and Morrison (1986) and Kohli (1983, 2004), for example, explain that changes in the terms of trade are considered a price phenomenon, not a real phenomenon, in the construction of real GDP. These researchers also propose other measures of real income that treat terms of trade shocks as if they were technology shocks. We discuss these measures later in the paper.

This paper explores the relation between TFP shocks and terms of trade shocks in models and contrasts this relation with the relation in the data. In particular, we focus on TFP measured using real GDP, rather than on real GDP itself. When we examine data for the United States and Mexico over the past several decades, we see that sharp deteriorations in the terms of trade are accompanied by drops in real GDP and that most of these drops in real GDP are driven by drops in TFP, not drops in factor inputs. Since standard models cannot account for these drops in TFP, our paper identifies a puzzle and poses a challenge for researchers working with open economy models: If we think that terms of trade shocks cause TFP fluctuations, we need to develop a new mechanism for generating this causal relation and build it into our models. Otherwise, we need to come up with some other reasoning to explain why sharp deteriorations in the terms of trade and drops in TFP are highly correlated in the data, especially during financial crises in developing countries like Mexico.

The empirical literature on growth is replete with examples of the association of the terms of trade with output growth and with productivity growth. Easterly, Kremer, Pritchett, and Summers (1993) study a large panel of countries to uncover the sources of long-run growth and aggregate volatility. They conclude that "shocks, especially to the terms of trade, play a large role in explaining variance in growth." In setting out a framework for studying developing country growth, Easterly, Islam, and Stiglitz (2001) find that volatility in the terms of trade is more strongly correlated with volatility in output than are the standard deviations of many of the usual suspects: money growth, fiscal balance, and capital flows, to name a few. Becker and Mauro (2005) use a large panel of countries to study how output drops are related to various external shocks and, using the likelihood of the shock and the associated output drop, compute the cost of the different shocks. They find that the costliest shocks, particularly for developing countries, are terms of trade shocks. The idea underlying many of these conclusions is succinctly summarized by Easterly, Islam, and Stiglitz (2001), who write, "For small *open* economies, adverse terms of trade shocks can have much the same effect as negative technology shocks, and this is one of the important differences between macroeconomics in these economies and that which underlies some of the traditional closed economy models."

In line with the above reasoning, we show that, in standard models, a shock to the terms of trade has an effect on consumption and welfare that is similar to a TFP shock. The analogy between the terms of trade and productivity breaks down when we calculate their effects on real

GDP and on TFP. When real GDP is measured at base period prices and domestic factors of production are held fixed, the effect of a terms of trade shock on real GDP is determined by the current terms of trade relative to the base period terms of trade. If the current import price is the same as the base period price, then the shock has no effect. If the current price is higher than the base period price, the effect is negative, and, conversely, if the current price is lower, the effect is positive. With base period price weighting, a change in the terms of trade can have a first-order effect on GDP, but this result follows from an artifact of the deflation method and not from an underlying structural relationship. When we consider real GDP calculated as a chain-weighted index — as is now the standard for many countries — these artifacts disappear. Changes in the terms of trade do not have a first-order impact on real GDP, and TFP remains unchanged.

We expand the simple examples to show that our results easily generalize to richer environments. We show that a shock to the terms of trade can affect the supply of productive factors like labor and that the effects of these shocks, as in the simple examples, also have an ambiguous impact on real GDP at base period prices. With chain-weighted GDP the ambiguity disappears and the effect on GDP comes only through factor supplies, implying that TFP is not affected by the terms of trade. A third set of results shows how the effect of a terms of trade shock on real GDP and consumption varies with the elasticity of substitution between the domestic factors and the imported input. As the elasticity of substitution decreases, changes in the terms of trade have larger impacts on consumption but smaller impacts on real GDP. When the production function uses domestic and imported inputs in fixed proportions, changes in the terms of trade have a large impact on consumption, but no impact on real GDP.

If the terms of trade do not have a clear effect on measures of real GDP and TFP, where are their effects visible? In national accounting measures, the terms of trade affect gross domestic income (GDI). In a closed economy, real GDI and real GDP are the same, but in an open economy they are not. In section 7, we discuss alternative measures of real income, including the concept of command basis GDP used by the U.S. Bureau of Economic Analysis. These measures do respond to changes in the terms of trade and reflect how the purchasing power of an economy changes as foreign prices change.

The problems highlighted in this paper are part of a much larger issue faced by quantitative researchers. Developing good intuition is paramount in understanding how models work, and constructing analogies, such as the one between the terms of trade and productivity, can be very helpful in developing intuition. When evaluating the quantitative properties of a model, however, the statistics taken from the model must be constructed in the same way as they are in the data. As we show below, it is exactly in this dimension that the analogy between the terms of trade and productivity breaks down. In comparing models to data, the researcher is faced with two choices. Either the statistics can be collected from the model as they are by the economists at the statistical agencies, or the data can be reconfigured to mimic the constructs in the model. We take the first approach in sections 3 through 6 and show how the model's GDP — as it would be constructed by a national income accountant — behaves in unexpected ways. In section 7, we take the second approach and use the data that underlie GDP to construct a measure of national income that corresponds to the analogy between terms of trade shocks and productivity shocks.

This paper identifies a puzzle. In the next section, we show that deteriorations in the terms of trade are frequently accompanied by declines in productivity. We then show that standard models do not generate this relation. If there is a causal mechanism that links shocks to the terms of trade to movements in productivity, researchers need to identify it.

2. Terms of trade and productivity in Mexico and the United States

The data plotted in figure 1 show that contractions in real GDP in the United States accompanied the sharp increases in the terms of trade caused mostly by the rise in petroleum prices that followed the OPEC oil embargo in 1973 and the Iranian Revolution in 1979. (All of the data used in this paper are available at www.econ.umn.edu/~tkehoe and at www.eco.utexas.edu/~kjr296.) In this figure, the terms of trade are calculated as the ratio of the import price deflator and the export price deflator. Consequently, the upward spikes in the terms of trade in 1974 and 1980 are deteriorations. The correlation between changes in the terms of trade and changes in real GDP over the entire period 1970–1990 is only −0.30, but we want to focus on the comovement during the two periods with the large deteriorations in the terms of trade, where the two series move sharply in opposite directions. Writing in the early 1980s, Hamilton (1983) has pointed out that all but one of the post–World War II economic downturns in the United States up until that time were accompanied by an upward spike in the price of imported oil.

Figure 2 presents the analogous data for Mexico, which in 1982 and 1994 entered severe debt financial crises that brought with them sharp increases in the price of imports. The large deteriorations in the terms of trade in 1983, 1986, and 1995 were accompanied by large contractions in real GDP. The correlation coefficient for changes in real GDP and changes in the terms of trade is −0.75 for Mexico.

To examine the relation between changes in the terms of trade and changes in productivity, we calculate TFP as

$$A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}} . \tag{1}$$

Here Y_t is real GDP, K_t is a measure of the real capital stock constructed using the perpetual inventory method, and L_t is a measure of aggregate hours worked. The data for the United States are taken from Kehoe and Ruhl (2005), and the data for Mexico are from Kehoe and Ruhl (2006). (See Conesa, Kehoe, and Ruhl (2007) for a discussion of the issues involved in the construction of this sort of data.) There are two things worth noticing about the data for TFP in figures 1 and 2: First, changes in the terms of trade have the same sort of relation with changes in TFP as they do with changes in real GDP. In particular, the correlation coefficient is −0.46 for the United States and −0.73 for Mexico. Furthermore, the deteriorations in the terms of trade in 1974 and 1980 in the United States and in 1983, 1986, and 1995 in Mexico were accompanied by sharp drops in TFP. Second, the drops in TFP in both countries that occur during the years associated with large deteriorations in the terms of trade were even larger than the drops in real GDP, leaving no room for drops in factor inputs to play a role.

3. A simple model

We begin by considering a simple model in which the single factor of production, labor, is supplied inelastically, and in which there are no distortions or rigidities. We subsequently show how our results extend to models with variable labor supply and to models with distortions. We begin with the case where real GDP is measured in terms of base year prices because the calculations are simpler. We then show how the results can be extended to the case where real GDP is calculated with chain-weighted prices.

3.1. Closed economy

We first consider a closed economy and show that a fall in productivity in the intermediate goods sector produces a fall in GDP and in TFP, a result that does not carry over when we reinterpret the model as that of an open economy in which intermediate goods are imported.

There are two goods produced in this economy at each date t. The first good, the y good, is consumed and is used in the production of the second good, the m good. The y good is produced using labor and intermediate inputs of the m good according to the production function

$$y_t = f(\ell_t, m_t). \tag{2}$$

We assume the production function, f, has constant returns to scale, is concave, and is continuously differentiable. We later analyze the case where f is a fixed proportions production function. The m good is produced using only intermediate inputs of the y good. The production function is

$$m_t = \frac{x_t}{a_t}, \tag{3}$$

where a_t is a unit output requirement that can vary. We assume that the m good is sold in competitive markets at price p_t. The m good producer chooses m_t and x_t to minimize costs and to earn zero profits. The condition that equilibrium profits be zero is

$$p_t = a_t. \tag{4}$$

The feasibility condition is

$$c_t + x_t = y_t = f(\ell_t, m_t). \tag{5}$$

We normalize the price of the y good to be 1. Expenditure on final goods in the closed economy is only consumption, so on the expenditure side, real GDP, Y_t, is

$$Y_t = c_t = y_t - x_t. \tag{6}$$

On the output side, real GDP is calculated as the base period value of gross output minus the base period value of intermediate inputs:

$$Y_t = \left(y_t + p_0 m_t\right) - \left(p_0 m_t + x_t\right) = y_t - x_t, \tag{7}$$

where $p_0 = a_0$ is the base period price of the m good.

To calculate the impact of an increase in a, a decline in productivity in the m good sector, we note that a competitive economy chooses m_t to solve

$$\max f(\overline{\ell}, m_t) - a_t m_t. \tag{8}$$

The first-order condition for this problem is

$$f_m(\overline{\ell}, m_t) = a_t. \tag{9}$$

Using the implicit function theorem, we obtain

$$m'(a_t) = \frac{1}{f_{mm}(\overline{\ell}, m(a_t))} < 0. \tag{10}$$

Suppose that $a_{t+1} > a_t$ increases, that is, that productivity in the intermediate goods sector falls. How does real GDP change? The first-order change is

$$Y(a_{t+1}) - Y(a_t) \approx \frac{dY(a_t)}{da_{t+1}}(a_{t+1} - a_t), \tag{11}$$

where

$$Y(a_{t+1}) = f(\overline{\ell}, m(a_{t+1})) - a_{t+1} m(a_{t+1}) \tag{12}$$

and where $dY(a_t)/da_{t+1}$ in equation (11) denotes the derivative of $Y(a_{t+1})$ in equation (12) evaluated at $a_{t+1} = a_t$. Differentiating $Y(a_{t+1})$, we use the first-order condition (9) to obtain

$$\frac{dY(a_t)}{da_{t+1}} = f_m(\overline{\ell}, m(a_t))m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0. \tag{13}$$

Real GDP and productivity decline.

Equation (11) provides an expression for first-order changes in real GDP when the production function f is continuously differentiable. When f is a fixed proportions function, that is, where

$$y_t = \min\left[\ell_t, m_t / b\right], \tag{14}$$

we can obtain exact expressions. In this case, $m(a_t) = b\overline{\ell}$ and real GDP is

$$Y(a_t) = \overline{\ell} - a_t b\overline{\ell}, \tag{15}$$

which implies that the first-order expression in (11) is exact.

3.2. Open economy

Now consider an open economy with the same structure as that of the closed economy, but where m is now an imported intermediate input, x are exports of the y good, and p is the terms of trade. To make the analysis identical to that in the closed economy, we assume for the moment that there is balanced trade,

$$p_t m_t = x_t . \tag{16}$$

By comparing (16) to (3), we see how the terms of trade in the open economy, p, and the productivity parameter in the closed economy, a, are similar. Notice that we are also assuming that all imports are being used as intermediate goods. We do this for simplicity. It is easy to extend all of the results that follow to a model where some imports are used directly in consumption and, in a model with capital, in investment. Nothing of substance changes.

Real GDP is now

$$Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\overline{\ell}, m_t) - p_0 m_t , \tag{17}$$

where p_0 is price of imports (relative to exports) in the base year. A competitive economy continues to choose m_t to solve

$$\max f(\overline{\ell}, m_t) - p_t m_t \tag{18}$$

with the corresponding first-order condition defining an implicit function $m(p)$:

$$f_m(\overline{\ell}, m_t) = p_t \tag{19}$$

$$m'(p_t) = \frac{1}{f_{mm}(\overline{\ell}, m(p_t))} < 0 . \tag{20}$$

An increase in p — a deterioration in the terms of trade — has the identical impact on consumption and welfare as the decline in productivity in the closed economy. But what happens to real GDP and productivity?

$$Y(p_{t+1}) = f(\bar{\ell}, m(p_{t+1})) - p_0 m(p_{t+1}) \tag{21}$$

$$\frac{dY(p_t)}{dp_{t+1}} = f_m(\bar{\ell}, m(p_t)) m'(p_t) - p_0 m'(p_t) = (p_t - p_0) m'(p_t). \tag{22}$$

To the extent that the terms of trade in the period before the deterioration takes place, p_t, are close to the terms of trade in the base period, p_0, there is no first-order change in measured real GDP or in productivity. Notice that, if $p_t < p_0$, real GDP may even increase in response to a negative terms of trade shock.

Our calculations in equation (22) imply that a change in the terms of trade can have either a positive or negative effect on GDP, depending on the value of the terms of trade in the prior period. As an example, consider the crises in Mexico in 1983 and 1995, episodes that we will explore further in section 7.2. For the crisis in 1983, we use real data in which the base year is 1980. Using the analogue to equation (11), we can calculate the first-order approximation to the change in real GDP:

$$Y(p_{1983}) - Y(p_{1982}) \approx \frac{dY(p_{1982})}{dp_{t+1}} (p_{1983} - p_{1982}) \tag{23}$$

$$Y(p_{1983}) - Y(p_{1982}) \approx (p_{1982} - p_{1980}) m'(p_{1982})(p_{1983} - p_{1982}). \tag{24}$$

Approximating $m'(p_{1982})$ by the ratio of the change in real imports to the change in the terms of trade, we obtain

$$Y(p_{1983}) - Y(p_{1982}) \approx (p_{1982} - p_{1980}) \frac{m_{1983} - m_{1982}}{p_{1983} - p_{1982}} (p_{1983} - p_{1982}) \tag{25}$$

$$Y(p_{1983}) - Y(p_{1982}) \approx (p_{1982} - p_{1980})(m_{1983} - m_{1982}). \tag{26}$$

In 1982, the terms of trade were 3.2 percent higher than in the base year 1980. From 1982 to 1983, real imports fell by 143 billion 1980 pesos. Using equation (26), we can calculate the base period price effect

$$Y(p_{1983}) - Y(p_{1982}) \approx 0.032(-143) = -4.6\,, \tag{27}$$

which implies that the change in real GDP due to the change in the terms of trade is −4.6 billion 1980 pesos, which is −0.09 percent of GDP in 1982. From 1982 to 1983, real GDP fell by 4.3 percent; so the base period price effect contributes to the fall in real GDP, but is very small.

An analogous calculation for the crisis in 1995 reveals the reverse. In 1994 the terms of trade were lower than in the base period, which was 1993 in this case, and the change in real GDP from the base period price effect that we calculate using the analogue to equation (25) is 0.02 percent, that is, the deterioration in the terms of trade actually caused measured real GDP to increase. Real GDP fell by 6.2 percent from 1994 to 1995; so the base period price effect moves in the opposite direction from the change in real GDP, although, again, it is very small.

There are two cases in which the base period prices do not affect the calculation of real GDP in an open economy. The first, which we discuss in the next section, is the use of chained real GDP indices. The second is when f is the fixed proportions function (14), so that real GDP is

$$Y(p_t) = \overline{\ell} - p_0 b \overline{\ell}\,, \tag{28}$$

which does not change at all as the terms of trade change. Notice that the fixed proportions case is where consumption,

$$c(p_t) = (1 - p_t b)\overline{\ell}\,, \tag{29}$$

and therefore welfare, falls the most in response to a deterioration in the terms of trade.

The intuition for our results is simple. A deterioration in the terms of trade causes domestic output to fall, but it also causes imports valued at base period prices to fall. Real GDP is the difference between the two, (21), and the envelope theorem says that the two effects cancel to first-order. With fixed proportions production, the two effects are exactly equal.

3.3. Unbalanced trade

In the previous section, we have assumed in equation (16) that trade is balanced. In this section, we argue that our results extend to models in which trade is not balanced. This case is of obvious interest because, in the data, changes in the terms of trade are often accompanied by changes in the trade balance and because many dynamic open economy models endogenize the trade balance.

Suppose that we have a dynamic model in which the budget constraint in period t is

$$c_t + b_{t+1} = w_t \overline{\ell} + (1+r_t)b_t , \tag{30}$$

where b_t is net borrowing from abroad and r_t is the interest rate and where

$$w_t \overline{\ell} = f_\ell(\overline{\ell}, m_t)\overline{\ell} = f(\overline{\ell}, m_t) - p_t m_t . \tag{31}$$

We can rewrite the budget constraint as

$$c_t = f(\overline{\ell}, m_t) - p_t m_t + (1+r_t)b_t - b_{t+1} . \tag{32}$$

The feasibility condition remains the same as in equation (5): $c_t + x_t = f(\ell_t, m_t)$. Subtracting this feasibility condition from the budget constraint (32), we obtain the trade balance condition

$$b_{t+1} - (1+r_t)b_t = x_t - p_t m_t . \tag{33}$$

(Recall that $b_{t+1} - b_t$ is the current account balance, which differs from the trade balance by $r_t b_t$.)

In the dynamic model, the sequence of foreign borrowing and, possibly, the interest rate are determined endogenously. We do not have to specify the details of this model, however, because these details make no difference to our results for real GDP. Real GDP remains equal to the expression in equation (17), and, as long as the first-order condition $p_t = f_m(\overline{\ell}, m_t)$ holds, we can obtain the result in equation (22). The determination of consumption depends on the trade balance as seen in equation (32) and is considerably more complicated than in the static model. In what follows, we want to compare the effects of changes in terms of trade on consumption and welfare with their effects on real GDP and TFP. To keep this comparison simple, we assume that trade is balanced when we calculate consumption.

4. Chain-weighted real GDP

Currently, the U.S. Bureau of Economic Analysis in its National Income and Product Accounts (NIPA) and the U.N. Statistics Division in its System of National Accounts (SNA) recommend the use of chain-weighted price indices to deflate GDP. In this section we show how the results of the previous section carry over to chain-weighted real GDP. Although the calculations are a little more complicated, an advantage of using chain-weighted real GDP is that the annoying terms involving base period prices disappear.

The United States' NIPA accounting uses Fisher chain-weights. So does Statistics Canada. Most countries that follow the U.N. SNA accounting use Laspeyres chain-weighting of quantities, although both Fisher weighting and Paasche weighting are allowed. We start by showing that Fisher chain-weighting eliminates the terms involving $p_0 - p_t$ and then briefly discuss how this result extends to other indices.

Fisher chain-weighted real GDP is

$$Y_{t+1}(p_{t+1}) = \frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})}{P_{t+1}}, \tag{34}$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and the Laspeyres index between the current period and the previous period:

$$P_{t+1} = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})} \right)^{\frac{1}{2}} \left(\frac{f(\overline{\ell}, m(p_t)) - p_{t+1}m(p_t)}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} P_t, \tag{35}$$

where $P_0 = 1$ in the reference period.

We can substitute (35) into (34), which yields the Fisher chain-weighted quantity index

$$Y(p_{t+1}) = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_{t+1}m(p_t)} \right)^{\frac{1}{2}} \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)} \right)^{\frac{1}{2}} Y(p_t). \tag{36}$$

It is easier to work with the logarithm of chain-weighted GDP and calculate the first-order change as

$$\log Y\left(p_{t+1}\right) - \log Y\left(p_t\right) \approx \frac{d \log Y\left(p_t\right)}{dp_{t+1}}\left(p_{t+1} - p_t\right). \tag{37}$$

The natural logarithm of chain-weighted real GDP is

$$\begin{aligned} \log Y(p_{t+1}) = {} & \frac{1}{2}\log\left(f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})\right) - \frac{1}{2}\log\left(f(\overline{\ell}, m(p_t)) - p_{t+1}m(p_t)\right) \\ & + \frac{1}{2}\log\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right) + \frac{1}{2}\log\frac{\left(f(\overline{\ell}, m(p_t)) - p_t m(p_t)\right)}{P_t^2} \end{aligned} \tag{38}$$

Differentiating, we obtain

$$\frac{d\log Y(p_{t+1})}{dp_{t+1}} = \frac{f_m(\overline{\ell}, m(p_{t+1}))m'(p_{t+1}) - p_{t+1}m'(p_{t+1}) - m(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})\right)} + \frac{m(p_t)}{2\left(f(\overline{\ell}, m(p_t)) - p_{t+1}m(p_t)\right)} + \frac{f_m(\overline{\ell}, m(p_{t+1}))m'(p_{t+1}) - p_t m'(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right)}. \quad (39)$$

Since $f_m(\overline{\ell}, m(p_{t+1})) = p_{t+1}$, this simplifies to

$$\frac{d\log Y(p_{t+1})}{dp_{t+1}} = -\frac{m(p_{t+1})}{2\left(f(\overline{\ell}, m(p_t)) - p_t m(p_t)\right)} + \frac{m(p_t)}{2\left(f(\overline{\ell}, m(p_t)) - p_{t+1}m(p_t)\right)} + \frac{(p_{t+1} - p_t)m'(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right)}. \quad (40)$$

Evaluating this expression at $p_{t+1} = p_t$, we obtain

$$\frac{d\log Y(p_t)}{dp_{t+1}} = 0, \quad (41)$$

which implies that any effect of changes in the terms of trade on chain-weighted real GDP is of second order.

Suppose that, instead of Fisher weighting, the national statistics agency uses Laspeyres weighting of quantities. The relation that corresponds to (36) is

$$Y(p_{t+1}) = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)}\right) Y(p_t). \quad (42)$$

Notice that, in this case, the deflator in (35) is

$$P_{t+1} = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1}m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}\right) P_t, \quad (43)$$

that is, a Paasche price index. In the case where the prices in (42) are those of the second period, p_{t+1} — that is, where the quantity index is Paasche — the deflator in (43) uses the quantity weights in the first period, $f(\overline{\ell}, m(p_t))$ and $m(p_t)$ — that is, the price index is Laspeyres. In either case, a simple argument that follows that in equations (38)–(41), but with less algebra, proves that the first-order effect of a change in the terms of trade on chain-weighted real GDP is zero.

We could have used other functional forms in our calculations. The Törnqvist index, for example, whose chain-weighted price index is of the form

$$P_{t+1} = \left(\frac{p_{t+1}}{p_t} \right)^{\frac{1}{2}\left(\frac{-p_t m(p_t)}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)} + \frac{-p_{t+1} m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})} \right)} P_t , \tag{44}$$

has a strong theoretical foundation and has gained some use among both economics researchers and statistical agencies. All of these functional forms — chain-weighted Fisher, Laspeyres, Paasche, and Törnqvist indices — share the property that changes in the terms of trade do not affect GDP to the first-order. The intuition for this result is simple: in a chain-weighted index, the "base period" is always the preceding period. It is this property of chain weighting, and not a particular functional form, that eliminates the dependency on base period prices.

5. Extensions to the simple model

In this section, we add variable labor supply and distortions to the model. To the extent that shocks to the terms of trade change the labor supply, they can change real GDP, but not productivity. Real GDP can even rise in response to a negative terms of trade shock, although welfare falls. The model with distortions is more complicated. We analyze a model with tariff distortions and show that an increase in tariffs acts like a shock to the terms of trade but has no first-order effects on GDP if initial tariffs are zero.

5.1. Variable labor supply

Suppose that there is a representative consumer who values both consumption and leisure $z = \overline{\ell} - \ell$. The utility function of this consumer is $u(c, z)$, and the consumer solves

$$\max u(c_t, \overline{\ell} - \ell_t) \tag{45}$$

$$\text{s.t. } c_t = w_t \overline{\ell},$$

where $w_t = f_\ell(\ell_t, m_t)$. The first-order condition for this problem is

$$w_t u_c(c_t, \overline{\ell} - \ell_t) = u_z(c_t, \overline{\ell} - \ell_t), \tag{46}$$

which implicitly defines the function $\ell(w)$:

$$w_t u_c(w_t \ell(w_t), \overline{\ell} - \ell(w_t)) = u_z(w_t \ell(w_t), \overline{\ell} - \ell(w_t)) \tag{47}$$

$$\ell'(w_t) = -\frac{u_c(c_t, \overline{\ell} - \ell_t) + u_{cc}(c_t, \overline{\ell} - \ell_t) w_t \ell_t - u_{cz}(c_t, \overline{\ell} - \ell_t)\ell_t}{u_{cc}(c_t, \overline{\ell} - \ell v) w_t^2 - 2u_{cz}(c_t, \overline{\ell} - \ell_t) w_t + u_{zz}(c_t, \overline{\ell} - \ell_t)}. \tag{48}$$

Consider the constant elasticity of substitution case, where

$$u(c,z) = \begin{cases} \left(c^\rho + \gamma z^\rho - 1 - \gamma\right)/\rho & \text{for } \rho \le 1,\ \rho \ne 0 \\ \log c + \gamma \log z & \text{for } \rho = 0 \end{cases}. \tag{49}$$

Here

$$\ell'(w) = \frac{\rho c^{\rho-1}}{(1-\rho)\left(w^2 c^{\rho-2} + \gamma(\overline{\ell} - \ell)^{\rho-2}\right)}. \tag{50}$$

Notice that $\ell'(w)$ has the same sign as ρ.

How do w and m vary with p? We can use the first-order conditions for profit maximization to define implicit functions $w(p)$ and $m(p)$

$$f_\ell(\ell(w(p)), m(p)) = w(p) \tag{51}$$

$$f_m(\ell(w(p)), m(p)) = p. \tag{52}$$

Differentiating, we obtain

$$f_{\ell\ell}(\ell,m)\ell'(w)w'(p) + f_{\ell m}(\ell,m)m'(p) = w'(p) \tag{53}$$

$$f_{\ell m}(\ell,m)\ell'(w)w'(p) + f_{mm}(\ell,m)m'(p) = 1. \tag{54}$$

We can solve this system of two equations in the two unknowns $w'(p)$ and $m'(p)$ to obtain

$$w'(p) = \frac{f_{\ell m}(\ell,m)}{f_{mm}(\ell,m) - \left(f_{mm}(\ell,m) f_{\ell\ell}(\ell,m) - f_{\ell m}(\ell,m)^2\right)\ell'(w)} \tag{55}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell,m)\ell'(w)}{f_{mm}(\ell,m) - \left(f_{mm}(\ell,m) f_{\ell\ell}(\ell,m) - f_{\ell m}(\ell,m)^2\right)\ell'(w)}. \tag{56}$$

As long as the denominator of these expressions is negative, real wages fall with an increase in p, a deterioration of the terms of trade. If $1 - f_{\ell\ell}(\ell, m)\ell'(w)$ is positive, imports fall. Notice that we can construct examples where $\rho < 0$ implies that $\ell'(w) < 0$, so that deteriorations to the terms of trade force the consumer to work more.

Letting $c(p) = f(\ell(w(p)), m) - pm(p)$, we can use the envelope theorem to show that the change in consumer welfare induced by a change in the terms of trade is

$$\frac{d}{dp} u(c(p_t), \overline{\ell} - \ell(w(p_t))) = -u_c(c_t, \overline{\ell} - \ell_t) m_t < 0. \tag{57}$$

What happens to real GDP and productivity when the terms of trade change? First consider real GDP:

$$Y(p_{t+1}) = f(\ell(w(p_{t+1})), m(p_{t+1})) - p_0 m(p_{t+1}) \tag{58}$$

$$\frac{dY(p_t)}{dp_{t+1}} = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + f_m(\ell_t, m_t)m'(p_t) - p_0 m'(p_t) \tag{59}$$

$$\frac{dY(p_t)}{dp_{t+1}} = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t). \tag{60}$$

The first term on the right-hand side of equation (60) is the change in real GDP induced by the change in labor supply, and the second term is the same as in the model with inelastic labor supply. Notice that real GDP can either rise or fall with an increase in the terms of trade, but, if $\ell'(w_t) > 0$, which implies that $w'(p_t) < 0$, and if $(p_t - p_0)m'(p_t)$ is small, real GDP falls. When real GDP is measured using a chain-weighted index and $\ell'(w_t) > 0$, real GDP always falls.

Now consider productivity $Y(p_t)/\ell(w(p_t))$:

$$\frac{d}{dp_{t+1}} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{\ell(w_t)Y'(p_t) - Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}. \tag{61}$$

Substituting in the expressions $Y(p_{t-1})$ in (58) and for $Y'(p_{t-1})$ in (60), we obtain

$$\frac{d}{dp_{t+1}} \frac{Y(p_t)}{\ell(w(p_t))} = \frac{(p_t - p_0)\left(\ell_t m'(p_t) - m_{t-1}\ell'(w_t)w'(p_t)\right)}{\ell_t^2}. \tag{62}$$

Once again, this term is close to zero if $p_t - p_0$ is close to zero. When real GDP is measured with a chain-weighted index, all of the change in GDP comes from a change in labor effort, so there is no effect on TFP.

In the fixed coefficients case, real GDP is

$$Y(p_t) = (1 - p_0 b)\ell(w(p_t)), \qquad (63)$$

and productivity does not change as the terms of trade change.

As our C.E.S. example shows, the impact of a change in the terms of trade on the labor supply is very sensitive to the specification of the consumer's utility function. In a model in which the trade balance varies, a change in the trade balance generated by a change in the terms of trade can induce an additional change in the labor supply. The response of the labor supply to a change in the trade balance is also sensitive to the specification of the consumer's utility function, as pointed out by Chakraborty (2006). Unfortunately, there is little agreement among economists on how to specify consumers' utility for leisure. See Blundell and MaCurdy (1999) for a classic reference and Rogerson and Wallenius (2007) for a recent attempt to reconcile estimates of labor supply elasticities obtained by microeconomists with those obtained by macroeconomists. Fortunately, in terms of the issue addressed in this paper, this uncertainty about the specification of labor supply is not very relevant. If we want to study events like the sharp deteriorations in the terms of trade that have occurred in Mexico and the United States over the past few decades, the growth accounting in section 2 has shown us that the corresponding drops in real GDP were generated mostly by drops in TFP, not by drops in the labor supply.

5.2. Tariffs

In this section, we consider a model with tariff distortions. We model tariff revenues as a lump-sum rebate to the representative consumer. In the presence of tariff distortions, changes in the terms of trade can have first-order effects on real GDP and productivity because the first-order condition (19) is distorted, although these effects are small to the extent that tariffs are small or the production function f is close to fixed coefficients. We also extend our results to tariff changes and show that they are much like terms of trade shocks, except that tariff revenues are spent domestically.

Once again, a useful benchmark is provided by the closed economy model. We assume that the government imposes an *ad valorem* tax τ on intermediate inputs. To keep the discussion simple, assume again that the labor supply is fixed. A competitive economy chooses m_t to solve

$$\max\ f(\bar{\ell}, m_t) - (1+\tau_t)a_t m_t \ . \tag{64}$$

In the case where f is continuously differentiable, the first-order condition is

$$f_m(\bar{\ell}, m_t) = (1+\tau_t)a_t \ . \tag{65}$$

We first consider the case where τ_t is fixed and a_t fluctuates. The implicit function theorem implies that

$$m'(a_t) = \frac{1+\tau_t}{f_{mm}(\bar{\ell}, m((1+\tau_t)a_t))} < 0 \ . \tag{66}$$

How do real GDP change and consumption change?

$$Y(a_{t+1}) = c(a_{t+1}) = f(\bar{\ell}, m((1+\tau_{t+1})a_{t+1})) - a_{t+1}m((1+\tau_{t+1})a_{t+1}) \tag{67}$$

$$\frac{dY(a_t)}{da_{t+1}} = \frac{dc(a_t)}{da_{t+1}} = \tau_t a_t m'((1+\tau_t)a_t) - m((1+\tau_t)a_t) \tag{68}$$

Notice that the tax distortion introduces an additional term into (13).

We now consider the case where a_t is fixed and τ_t fluctuates:

$$m'(\tau_t) = \frac{a_t}{f_{mm}(\bar{\ell}, m((1+\tau_t)a_t))} < 0 \tag{69}$$

$$Y(\tau_{t+1}) = c(\tau_{t+1}) = f(\bar{\ell}, m((1+\tau_{t+1})a_{t+1})) - a_{t+1}m((1+\tau_{t+1})a_{t+1}) \tag{70}$$

$$\frac{dY(\tau_t)}{d\tau_{t+1}} = \frac{dc(\tau_t)}{d\tau_{t+1}} = \tau_t a_t m'(\tau_t) \tag{71}$$

To the extent that the tax before the increase, τ_t, is close to zero, the first-order impact of increasing τ_t is small. In the fixed coefficients case, where $m'(\tau_t) = 0$, real GDP does not change.

In the calculations in the open economy case, where τ_t is an *ad valorem* tariff on imports, fluctuations in tariffs have the same impact on real GDP as do fluctuations in the terms of trade:

$$Y(p_{t+1}) = Y(\tau_{t+1}) = f(\overline{\ell}, m((1+\tau_{t+1})p_{t+1})) - p_0 m((1+\tau_{t+1})p_{t+1}) \quad (72)$$

$$\frac{dY(p_t)}{dp_{t+1}} = ((1+\tau_t)p_t - p_0)m'(p_t) \quad (73)$$

$$\frac{dY(\tau_t)}{d\tau_{t+1}} = ((1+\tau_t)p_t - p_0)m'(\tau_t) \quad (74)$$

Notice that the effect on real GDP of an increase in the terms of trade, or of an increase in the tariff, is close to zero to the extent that either $(1+\tau_t)p_t - p_0$ is close to zero or to the extent that f is close to a fixed proportions function. In terms of the impact on consumption and welfare, the two cases are very different, however:

$$c(p_{t+1}) = c(\tau_{t+1}) = f(\overline{\ell}, m((1+\tau_{t+1})p_{t+1})) - p_{t+1} m((1+\tau_{t+1})p_{t+1}) \quad (75)$$

$$\frac{dc(p_t)}{dp_{t+1}} = \tau_t p_t m'((1+\tau_t)p_t) - m((1+\tau_t)p_t) \quad (76)$$

$$\frac{dc(\tau_t)}{d\tau_{t+1}} = \tau_t p_t m'((1+\tau_t)p_t) \quad (77)$$

Consumption falls much more in the case of a deterioration of the terms of trade than it does when tariffs are increased because the revenue generated is rebated to the representative consumer.

6. Elasticity of substitution

Except for the case where the production function f combines domestic inputs and imported inputs in fixed proportions — where there are analytical formulas for real GDP — we have relied on the implicit function theorem and first-order approximations to determine the impact of terms of trade shocks on real GDP. In this section, we investigate the impacts of large shocks for the case where f is constant elasticity of substitution:

$$f(\ell_t, m_t) = \left((1-\beta)\ell_t^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}, \quad (78)$$

where the parameter β determines the share of imports in production. Again, for simplicity, we consider the case in which labor is fixed, $\ell_t = \overline{\ell}$. The elasticity of substitution between imported intermediates and labor — which corresponds to all domestic inputs in a more general model — is $\sigma = 1/(1-\rho)$. This elasticity is frequently referred to as the Armington elasticity. Like the elasticity of labor supply, the Armington elasticity is the subject of considerable disagreement among reseaerchers. Trade economists estimate this elasticity to be large, on the order of 5−12, while open economy macroeconomists estimate it to be small, on the order of 0−2. See Ruhl (2005) for a recent attempt to reconcile these estimates.

Producers choose m_t to minimize costs,

$$\min p_t m_t \tag{79}$$

$$\text{s.t.} \ \left((1-\beta)\overline{\ell}^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}} \geq y .$$

We can use the first-order condition to obtain the demand for imports

$$m(p_t) = (1-\beta)^{\frac{1}{\rho}} \overline{\ell} \left(p_t^{\frac{\rho}{1-\rho}} \beta^{\frac{-\rho}{1-\rho}} - \beta \right)^{\frac{-1}{\rho}} . \tag{80}$$

This allows us to express real GDP in base period prices as

$$Y(p_t) = \left((1-\beta)\overline{\ell}^{\rho} + \beta m(p_t)^{\rho}\right)^{\frac{1}{\rho}} - p_0 m(p_t). \tag{81}$$

Before studying how real GDP changes are related to the elasticity of substitution, we must choose a value for β. The first-order condition for the producers' problem (79) implies that

$$\frac{m_t}{\left((1-\beta)\overline{\ell}^{\rho} + \beta m_t^{\rho}\right)^{\frac{1}{\rho}}} = \left(\frac{p_t}{\beta}\right)^{\frac{1}{\rho-1}} . \tag{82}$$

The left-hand side of the equation is the share of imports in gross output. For each value of ρ we choose the parameter β so that imports make up 8 percent of gross output when the import price is one. This value is consistent with U.S. data, where imports average 7.8 percent of gross output in the NAICS classified data over the period 1998–2005.

In figure 3, we plot the changes in real GDP that result from changes in the terms of trade for several values of the elasticity of substitution. In this example, we have assumed that the terms of trade in the period prior are the same as those in the base year, so the first-order effect is zero in equation (22). The absence of a first-order effect can be seen in the figure, where the change in real GDP from a small change in the terms of trade is negligible. The figure also shows the impact of larger changes in the terms of trade. The elasticity of substitution between domestic and foreign inputs is commonly specified at or around 2.0 in international real business cycle models. The average annual change in absolute value of the terms of trade for OECD countries is about 3.5 percent. A 3.5 percent increase in the relative price of imports leads to a 0.0058 percent decrease in real GDP when the elasticity of substitution between domestic and foreign factors is 2.0. When the elasticity of substitution is 6.67, the same deterioration of the terms of trade causes a 0.032 percent decrease in real GDP, and, when the elasticity is 0.33, a 3.5 percent increase in the terms of trade decreases real GDP by 0.0019 percent.

Although an increase in the terms of trade has little effect on real GDP, its effect on consumption can be significant. In figure 4, we plot the change in consumption that results from changing the terms of trade. The less substitutable imports are in production, the more painful are increases in the price of imports. In the fixed proportions case, in which real GDP does not change at all with the terms of trade, the consumption and welfare effects of a change in the terms of trade are the largest.

7. Alternative income measures

If real GDP does not accurately reflect the real purchasing power of an open economy, are there measures that do? In this section we discuss measures of real domestic income that incorporate the terms of trade.

GDP in current prices represents the current value of both production and income in both open and closed economies. (Although, in some circumstances, we may want to use gross national product [GNP] as a measure of income because it adjusts for income earned abroad.) Real GDP and real income, though equivalent in a closed economy, are not necessarily equivalent in an open economy. The difference between real GDP and real gross domestic income in the open economy arises from the deflation of the trade balance. Real GDP is computed by deflating the current value of the components of GDP by their respective implicit price deflators, P,

$$GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}, \tag{83}$$

while one method of computing real income is

$$GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}. \tag{84}$$

Notice that real GDP, as a measure of production, values exports as an output and imports as an input, while real GDI values the nominal trade balance in terms of the amount of imports that it purchases. The U.S. Bureau of Economic Analysis (2006) refers to the income measure in equation (84) as *command-basis* GDP, rather than as real gross domestic income as it is defined in the United Nations' 1993 System of National Accounts (2001). (It is actually command-basis GNP, rather than GDP, that is typically reported.) The United Nations (2001) also allows for several definitions of real GDI that differ by the index used to deflate $X_t - M_t$, including the export price index and the domestic absorption price index.

The United Nations (2001) allows for various definitions of the GDI because there is no natural way to deflate the proceeds from foreign trade. The debate over the real trade balance in the national income and product accounts is a long one, going back to the early era of national income accounting. Working within the NIPA framework, Nicholson (1960), Bjerke (1968), and others proposed different methods of deflating the income from trade, with most of them arguing for either the import or the export price deflator, though some (Burge and Geary 1957) propose using one deflator when the trade balance is positive and another when the trade balance is negative. As index number theory progressed, prominent researchers in the field developed alternative indices of welfare and productivity (Diewert and Morrison 1986) and real domestic income (Kohli 2004) that accounted for the terms of trade.

Which method should we use? Mahdavy and Silver (1989) compare these methods and find that, for most industrial countries, the choice of deflator is not important. They find the choice of deflator can be important for non-industrial countries. For simplicity, and to be consistent with the methods used by the BEA, we will use the command-basis GDP measure (84) in what follows.

Command GDP offers an alternative way of viewing a country's performance. For countries in which either the terms of trade have been stable or trade is not an important factor in output, real GDP and command GDP are similar. The United States is a good example of this case. In figure 5 we plot real GDP and command GDP for the United States, as well as the terms of trade. The terms of trade have stayed fairly steady over the last 20 years, and command GDP and real GDP are almost indistinguishable. In contrast, Switzerland's terms of trade have steadily improved, falling 21.4 percent since 1981, as can be seen in figure 6. The figure also shows how command GDP has grown significantly faster than real GDP in Switzerland; from 1981–2005 command GDP grew 10.2 percent more than real GDP. Command GDP grew at 1.8 percent per year over this period, compared to the 1.5 percent per year growth in real GDP. Some Swiss economists, notably Kohli (2004), have used measures similar to command GDP to help explain why they do not believe the Swiss economy is doing poorly, despite the lack of growth in real GDP since 1973. For further discussion of Switzerland's economic performance, including the impact of the terms of trade, see Kehoe and Ruhl (2003, 2005).

Recently, Feenstra, Heston, Timmer, and Deng (2004) have proposed adding the purchasing power parity equivalent of the SNA concept of GDI to the Penn World Table, where the domestic absorption price index is used to deflate the trade balance. They argue that it is this concept of national income that should be used when researchers are interested in studying welfare, while the traditional concept of real GDP should be used when researchers are interested in studying production. The Penn World Table now provides data on both real GDP and real GDI. What concept of real GDP was provided in the Penn World Table previously? Feenstra, Heston, Timmer, and Deng (2004) argue that inconsistencies in the GDP calculations make it neither one nor the other and that these inconsistencies need to be eliminated and both variables need to be reported.

7.1. Business cycle frequencies

It is appropriate to model some countries as small open economies. These countries are small in the sense that they do not influence world prices, and thus the country's terms of trade are exogenously given. It is easy to imagine one of these small open economies being buffeted by shocks to its terms of trade and this in turn affecting the county's GDP. Although terms of trade shocks cannot have much of an effect on real GDP, particularly given the magnitude of these shocks and the low level of substitutability usually assumed in these models, we can use the command GDP measure to calculate how real income changes over the business cycle.

Figures 7 and 8 plot Hodrick-Prescott filtered log real GDP and log command GDP for the United States and Switzerland. We use the standard quarterly smoothing parameter of 1600 in constructing these data. For the United States, the two measures are nearly identical, but this is not true for Switzerland. Command GDP in Switzerland is 16.8 percent less volatile than real GDP. The series do not move as closely either; the correlation coefficient is 0.72.

7.2. Crises

A crisis in a developing economy may be accompanied by deteriorations of the country's terms of trade. Mexico, for example, has weathered two crises in the last 20 years, the first in 1982–83, and the second in 1994–95. As reported in table 1, the terms of trade increased by 26.4 percent from 1982 to 1983 and by 8.7 percent from 1994 to 1995. These periods were also periods of significant declines in output: from 1982 to 1983, real GDP fell by 4.3 percent, and from 1994 to 1995, real GDP fell by 6.2 percent. We have seen in the previous sections that the change in the terms of trade cannot be the cause of the declines in real GDP. How does the situation change when the rising terms of trade are also taken into account?

During the first crisis, real GDP fell by 4.3 percent, but command GDP — real domestic income — fell by 6.5 percent. Command GDP fell by more during the second crisis as well, declining 7.8 percent from 1994 to 1995 compared to the 6.2 percent decline in real GDP over the same period. The output drops associated with financial crises like the ones in Mexico are frequently used as evidence of the painful nature of the withdrawal of credit to a country. The evidence on real domestic income suggests that these "sudden stop" episodes are even more painful than the GDP evidence suggests!

8. Concluding remarks

In standard models, an adverse shock to the terms of trade acts like an adverse shock to productivity along many dimensions: income and consumption fall. In one crucial dimension, however, a terms of trade shock acts nothing like a productivity shock: real GDP, the most common measure of a country's output, is often unchanged in standard models. In returning to our original question, if we are to use real GDP as a measure of production, then total factor productivity also remains unchanged. Although the terms of trade are shocks to a country's income, they are not shocks to a country's productivity.

So how can we account for the relations in figures 1 and 2? This paper shows that we cannot expect standard models to do so. One line of promising research argues that there are other responses to terms of trade shocks. The change in relative prices may induce reallocations across goods and sectors that involve nonproductive activities like retraining, or capital may go idle, both contributing to lower output and measured TFP. The literature on developing country crises is one area in which progress is being made in modeling the frictions that may help account for the relation between the terms of trade, real GDP, and productivity. Beginning with standard models, Meza and Quintin (2007) introduce labor hoarding and variable capital utilization, Mendoza (2006) introduces financial market frictions, and Kehoe and Ruhl (2006) introduce frictions in reallocating labor across sectors. These papers have had some success in replicating the relations discussed above, but the exact specification and the quantitative importance of these frictions remains a question for future research.

Our interest in large deteriorations in the terms of trade has led us to focus on the negative correlation between the terms of trade and TFP that is evident in the data: In figures 1 and 2 when the terms of trade deteriorate sharply, so does productivity. Figure 9, however, presents some data that should serve as a caution to researchers. In contrast to the patterns in figures 1 and 2, in Switzerland over the period 1970–2000, improvements in the terms of trade have been associated with declines in real GDP and in TFP, with correlation coefficients of 0.54 and 0.59, respectively.

References

Becker T. and P. Mauro (2005), "Output Drops and the Shocks that Matter," IMF Working Paper 06172.

Bjerke, K. (1968), "Some Reflections on the Terms of Trade," *Review of Income and Wealth*, 14, 183–98.

Blundell, R. and T. MaCurdy (1999), "Labor Supply: A Review of Alternative Approaches," in O. Ashenfelter and D. Card, editors, *Handbook of Labor Economics,* Volume 3. Elsevier Science, 1559–695.

Bureau of Economic Analysis (2006), "A Guide to the National Income and Product Accounts of the United States," http://www.bea.gov/bea/an/nipaguid.pdf.

Burge, R. W. and R. C. Geary (1957), "Balancing of a System of Accounts in Real Terms," *A System of Price and Quantity Indexes for National Accounts*. United Nations.

Chakraborty, S. (2006), "Modeling Sudden Stops: the Role of Preferences," Baruch College, City University of New York.

Conesa, J. C., T. J. Kehoe, and K. J. Ruhl (2007), "Modeling Great Depressions: The Depression in Finland in the 1990s," *Federal Reserve Bank of Minneapolis Quarterly Review*, 31:1, 16–44.

Diewert, W. E. and C. J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade," *Economic Journal*, 96, 659–79.

Easterly, W., R. Islam, and J. E. Stiglitz (2001), "Shaken and Stirred: Explaining Growth Volatility," in B. Pleskovic and N. Stern, editors, *Annual World Bank Conference on Development Economics 2000*. The World Bank, 191–211.

Easterly, W., M. Kremer, L. Pritchett, and L. H. Summers (1993), "Good Policy or Good Luck? Country Growth Performance and Temporary Shocks," *Journal of Monetary Economics*, 32, 459–83.

Feenstra, R. C., A. Heston, M. P. Timmer, and H. Deng (2004), "Estimating Real Production and Expenditures across Nations: A Proposal for Improving the Penn World Tables," NBER Working Paper 10866.

Hamilton, J. D. (1983), "Oil and the Macroeconomy since World War II," *Journal of Political Economy*, 91, 228–48.

Kehoe, T. J. and K. J. Ruhl (2003), "Recent Great Depressions: Aggregate Growth in New Zealand and Switzerland, 1973–2000," *New Zealand Economic Papers*, 37, 5–40.

Kehoe, T. J. and K. J. Ruhl (2005), "Is Switzerland in a Great Depression?" *Review of Economic Dynamics*, 8, 759–75.

Kehoe, T. J. and K. J. Ruhl (2006), "Sudden Stops, Sectoral Reallocations, and Real Exchange Rates," University of Minnesota and University of Texas at Austin.

Kohli, U. (1983), "Technology and the Demand for Imports," *Southern Economic Journal*, 50, 137–150.

Kohli, U. (2004), "Real GDP, Real Domestic Income, and Terms of Trade Changes," *Journal of International Economics*, 62, 83–106.

Mahdavy, K. and M. Silver (1989), The Measurement of a Nation's Terms of Trade Effect and Real National Disposable Income within a National Accounting Framework," *Journal of the Royal Statistical Society*, 152, 87–107.

Mendoza, E. G. (2006), "Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints: a Fisherian Deflation of Tobin's Q," University of Maryland.

Meza, F and E. Quintin (2007), "Financial Crises and Total Factor Productivity," *The B.E. Journal of Macroeconomics*, 7:1 (Advances), article 33.

Nicholson, J. L. (1960), "The Effects of International Trade on the Measurement of Real National Income," *Economic Journal*, 70, 608–12.

Rogerson, R. and J. Wallenius (2007), "Micro and Macro Elasticities in a Life Cycle Model with Taxes," NBER Working Paper 13017.

Ruhl, K. J. (2005), "The Elasticity Puzzle in International Economics," University of Texas at Austin.

United Nations (2001), *System of National Accounts 1993*, Internet Access System, http://unstats.un.org/unsd/sna1993/introduction.asp.

Table 1

Comparison of real GDP and Command GDP during the crises in Mexico

	Real GDP (percent change)	Command GDP (percent change)	Terms of Trade (percent change)
1982-1983	-4.30	-6.54	26.41
1994-1995	-6.17	-7.75	8.66

Figure 1

United States

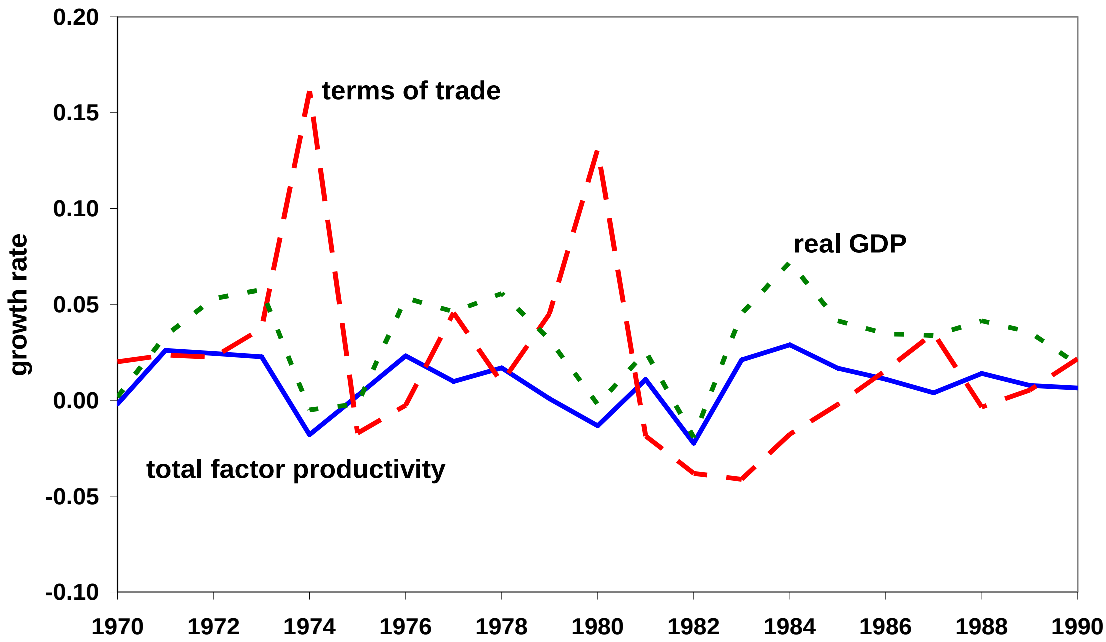

Figure 2

Mexico

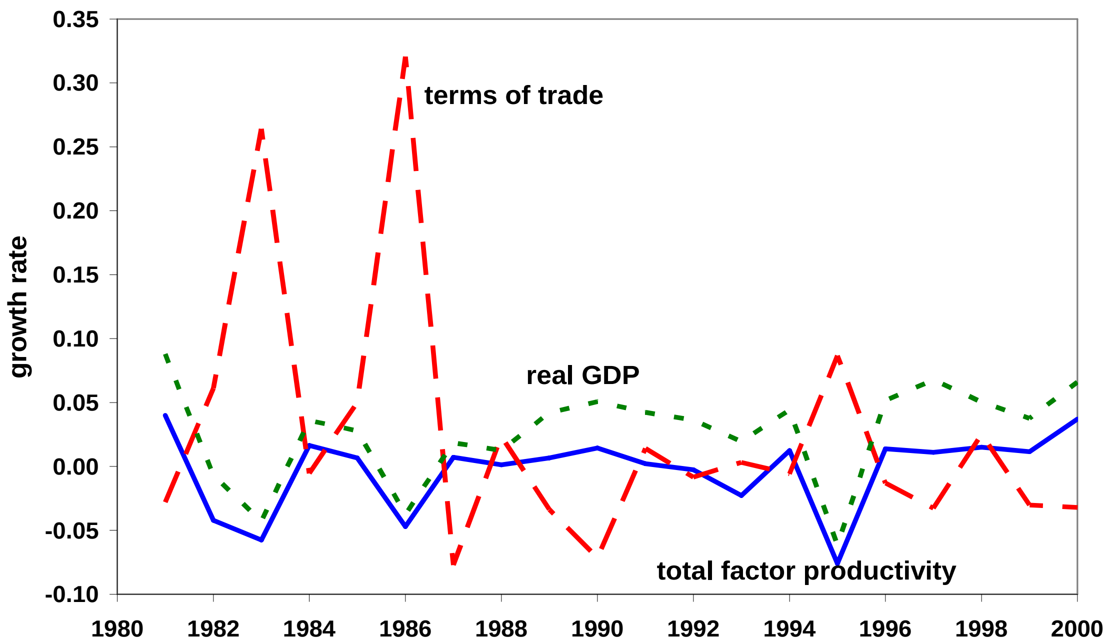

Figure 3

Real GDP and the elasticity of substitution

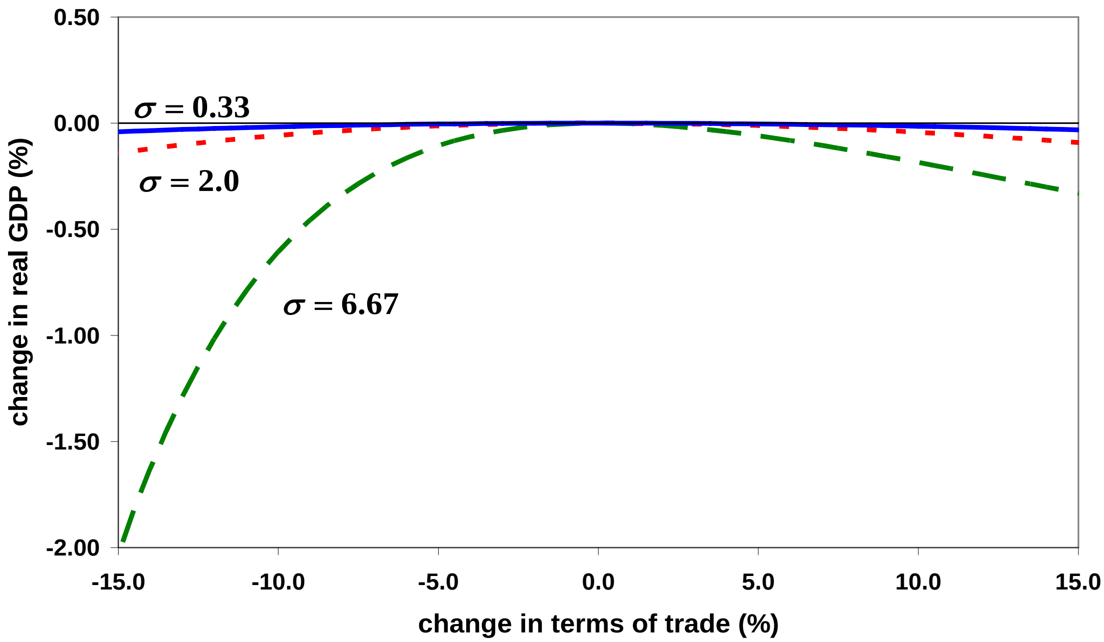

Figure 4

Consumption and the elasticity of substitution

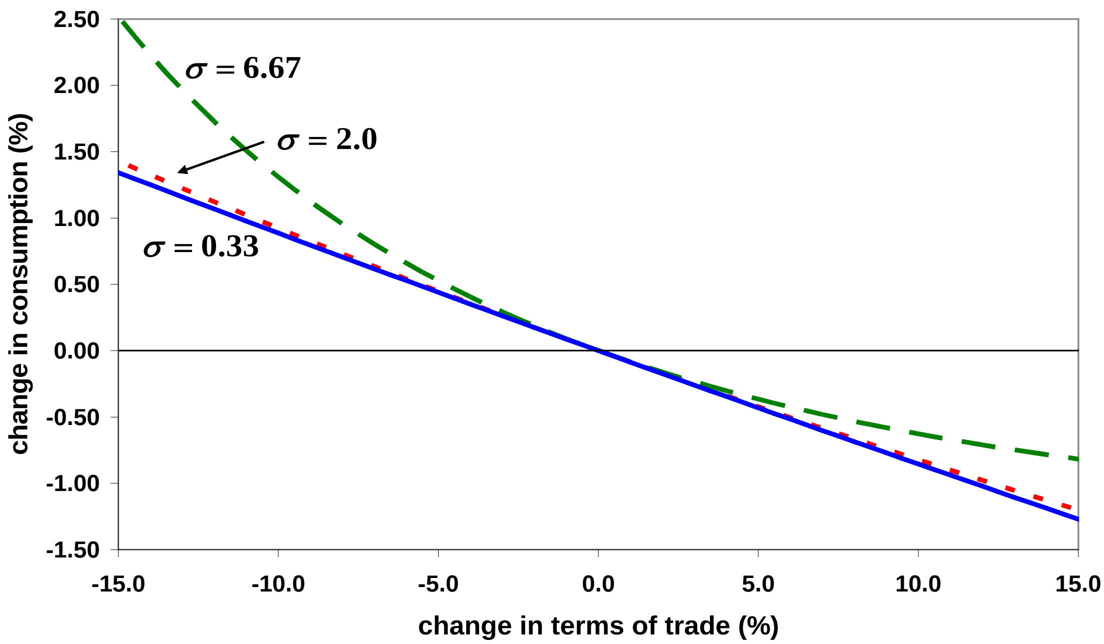

Figure 5

United States

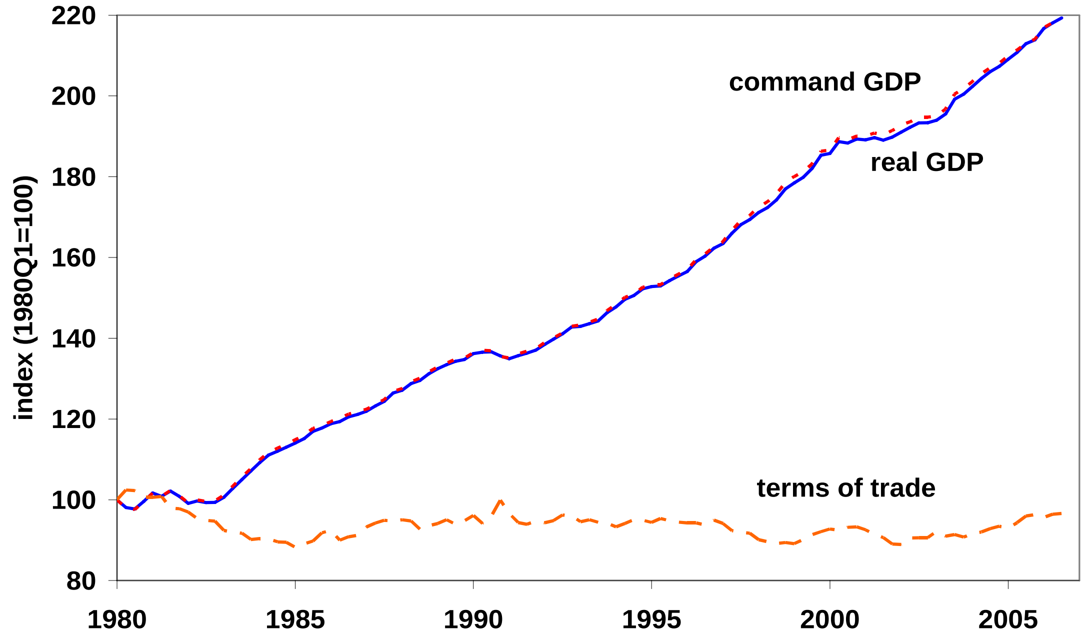

Figure 6

Switzerland

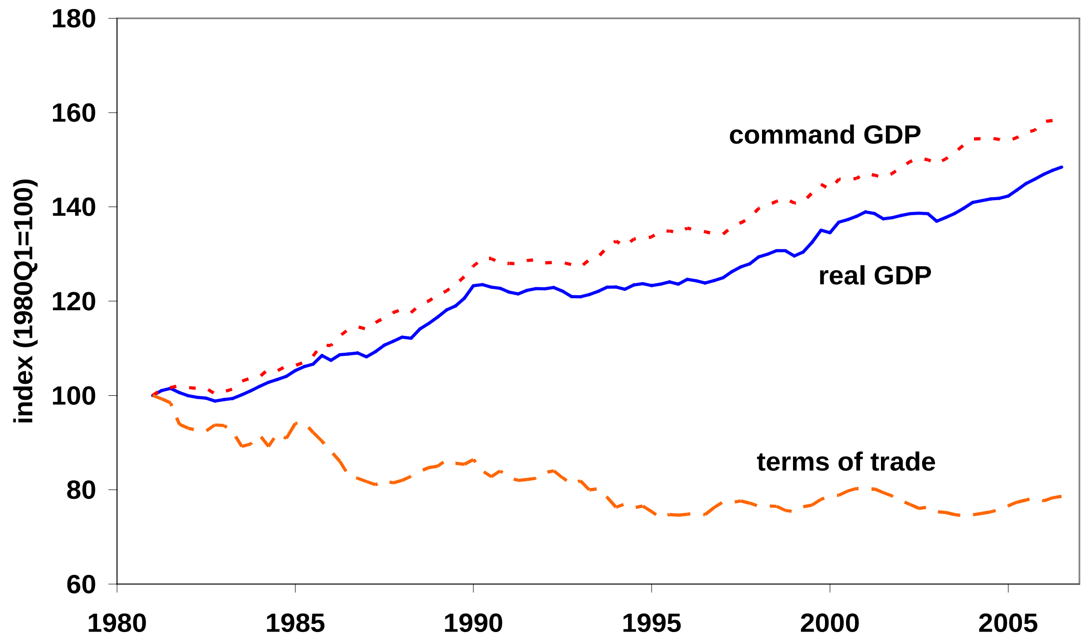

Figure 7

United States

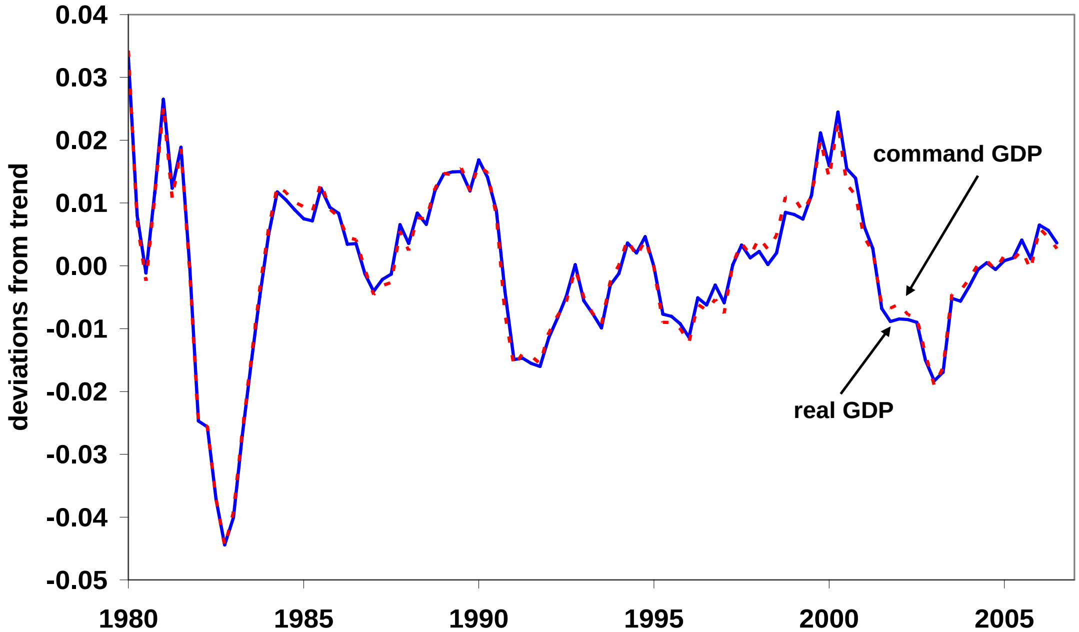

Figure 8

Switzerland

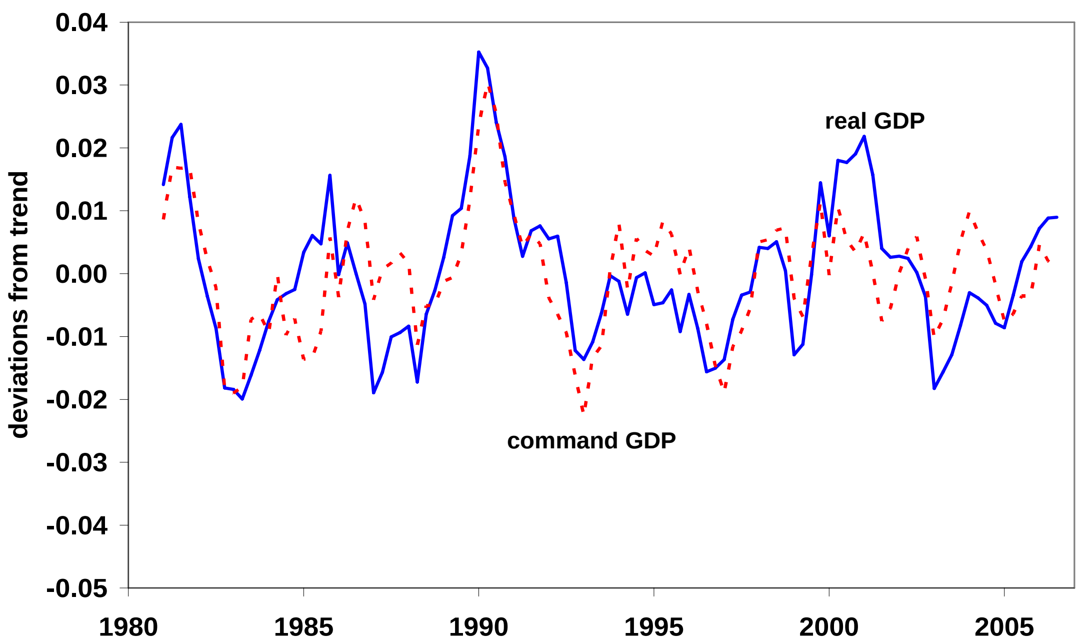

Figure 9

Switzerland