Federal Reserve Bank of Minneapolis
Research Department Staff Report

May 2016

The Opportunity Costs of Entrepreneurs in International Trade*

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ABSTRACT

We show that a heterogeneous-firm trade model with fixed operating costs has the same aggregate outcomes as a span-of-control model (Lucas, 1978). The fixed operating cost in the heterogeneous-firm model is the entrepreneur's forgone wage in the span-of-control model.

JEL codes: F10, F11.
Keywords: Chaney model, span-of-control model, firm heterogeneity, international trade.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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1. Introduction

Models in which heterogeneous firms face fixed costs of entering markets are a standard part of the economist's toolkit. A seminal version of this model is found in Melitz (2003), but a popular variation is found in Chaney (2008). Chaney (2008) makes three significant simplifying assumptions: the underlying distribution of productivity follows a Pareto distribution; a global equity fund allows agents to receive a share of the profits of firms in foreign countries; and the measure of potential firms is exogenous. The first two assumptions greatly simply the model, making it analytically tractable. In this paper, we provide a reinterpretation of the third assumption.

We show that the heterogeneous-firm model with a fixed measure of potential firms is a multi-country extension of the span-of-control model in Lucas (1978). In the span-of-control model, each agent is endowed with an ability that is used if the agent decides to create a firm. In this way, the number of potential firms is bounded by the number of agents in the model. This extended span-of-control model and the model developed by Chaney (2008) have identical aggregate variables, for example, the distribution of firms in operation, the distribution of exports, and aggregate consumption. The two models differ, in that, in the span-of-control model, firms are owned by individual agents, so individual income and consumption vary.

To align the two models, we extend the closed economy span-of-control model by introducing several countries, each requiring a fixed cost to service and each populated by agents with love-for-variety preferences as in Dixit and Stiglitz (1977). Each agent is endowed with a managerial talent and one unit of labor. If the agent chooses to operate a firm, he forgoes the wage he would have earned by supplying his unit of labor. This is equivalent to operating a firm in the heterogeneous-firm model when the fixed cost of doing so is one unit of labor: In both models, a technology is used only when it is worth using one unit of labor to do so.

An interesting implication of the span-of-control interpretation of Chaney (2008) is the change in the income distribution that results from trade liberalization. When firms are owned by individual agents, the change in firm profits induced by liberalization is passed through to the owner’s income. Melitz (2003) focuses on the case in which trade liberalization causes more productive firms to expand and their profits to increase, but other, less productive, firms to exit the market in response to increasing domestic wages. In this case, in our span-of-control model, trade liberalization causes income distribution to become more unequal.

There is limited work that uses the span-of-control model to analyze international trade. Ma (2015) develops a Lucas span-of-control model in which trade is generated by Dixit-Stiglitz preferences, similar to Melitz (2003). The Garicano (2000) model can be interpreted as a generalization of the Lucas span-of-control model emphasizing the structure of the firm. Antras, Garicano, and Rossi-Hansberg (2006) develop a version of the Garicano model in an international framework to study offshoring.

2. A trade model with fixed costs of entry and a fixed measure of potential firms

We first present a version of Chaney's (2008) model. Our model differs from Chaney's in that he assumes that all agents own shares in a global fund that redistributes the profits earned by firms in all the countries. We assume, instead, that agents are the claimants only to the profits earned by firms based in their country. To show that the model has an equilibrium in which the aggregate variables are the same as in the span-of-control model, we initially impose three assumptions on our version of the Chaney model: First, the fixed cost of setting up a firm to produce for domestic consumption is greater than or equal to one unit of labor. Second, parameter values are such that any firm that finds it profitable to export also finds it profitable to produce for domestic consumption. Third, the measure of potential firms is equal to the measure of workers. We later explain how to weaken these assumptions.

2.1. Agents

The world economy consists of n countries. Country i is populated by a continuum of agents of measure $\bar{\ell}_i$, each endowed with one unit of labor. There are two types of firms: homogeneous-good producers and differentiated-good producers. Each agent in country i owns an equal share in all firms that operate in that country.

Each agent in country i has income $w_i + \pi_i / \bar{\ell}_i$. Since the utility function is homothetic, we model a representative agent who supplies $\bar{\ell}_i$ units of labor, receives profits, π_i, and chooses consumption of the homogeneous and differentiated goods to solve

$$\max_{c_{0i}, c_i(\omega)} (1-\alpha)\log(c_{0i}) + (\alpha / \rho)\log \int_0^{m_i} c_i(\omega)^\rho \, d\omega \tag{1}$$

$$\text{s.t. } p_{0i}c_{0i} + \int_0^{m_i} p_i(\omega)c_i(\omega)d\omega = w_i\overline{\ell}_i + \pi_i ,$$

where $c_{0i} > 0$ is the consumption of the homogeneous good, and $p_{0i} > 0$ is its price; $m_i > 0$ is the measure of differentiated goods consumed in country i; $c_i(\omega)$ is the consumption of variety ω consumed in country i, and $p_i(\omega) > 0$ is its price. The parameter α, $1 > \alpha > 0$, governs the importance of differentiated goods relative to homogeneous goods and ρ, $1 > \rho > 0$, governs the elasticity of substitution between differentiated varieties. The wage is w_i.

The agent's demand function for differentiated good ω is

$$c(p_i(\omega), P_i, w_i\overline{\ell}_i + \pi_i) = \frac{\alpha(w_i\overline{\ell}_i + \pi_i)}{p_i(\omega)^{\frac{1}{1-\rho}} P_i^{\frac{-\rho}{1-\rho}}} , \tag{2}$$

where

$$P_i = \left[\int_0^{m_i} p_i(\omega)^{\frac{-\rho}{1-\rho}} d\omega \right]^{-\frac{1-\rho}{\rho}} . \tag{3}$$

2.2. Homogeneous-good firms

The homogeneous good, good 0, is produced using a constant returns to scale production function, $y_{0i} = a_i\ell_{0i}$, and sold in competitive markets. Good 0 is freely traded, so the solution to the firm's profit-maximization problem is

$$p_0 = p_{0i} = \frac{w_i}{a_i} \tag{4}$$

if country i produces good 0. We choose good 0 as the numeraire and set $p_0 = 1$.

2.3. Differentiated-good firms

Country i is endowed with a measure of potential differentiated-good firms, μ_i. We assume that $\mu_i = \overline{\ell}_i$. Each potential firm can produce a unique good, ω, with marginal productivity $x(\omega) \geq 0$ drawn from the cumulative probability distribution $G_i(x)$. Chaney assumes that this distribution is Pareto, $G_i(x) = 1 - \underline{x}_i^{\gamma_i} x^{-\gamma_i}$ for $x \geq \underline{x}_i$. An attractive feature of the model is that, if $G_i(x)$ is Pareto and the curvature parameter $\gamma_i = \gamma$ is the same for all countries, then the model equilibrium can be calculated analytically.

Differentiated-good firms are monopolistic competitors. The firm in country i that produces good ω for sale in country j has the production function

$$y_{ij}(\omega) = \frac{x_i(\omega)}{\tau_{ij}} \max[\ell_{ij} - \kappa_{ij}, 0]. \tag{5}$$

Here κ_{ij}, is the fixed cost of exporting from country i to country j, and $\tau_{ij} / x_i(\omega)$ is the variable cost. The firm must ship $\tau_{ij} \geq 1$ units of the good in order for one unit to arrive at the destination; we set $\tau_{ii} = 1$. We assume that $\kappa_{ii} \geq 1$.

Differentiated-good firms produce with a constant marginal cost, so the firm's maximization problem can be separated across destination markets. The firm takes as given the demand for its good, $c(p_{ij}(\omega), P_j, w_j \bar{\ell}_j + \pi_j)$, and chooses its price $p_{ij}(\omega)$ to maximize profits

$$\pi_{ij}(\omega) = \max_p \left\{ \left(p - \frac{w_i \tau_{ij}}{x_i(\omega)} \right) c(p, P_j, w_j \bar{\ell}_j + \pi_j) - w_i \kappa_{ij}, 0 \right\}. \tag{6}$$

The profit-maximizing price is

$$p_{ij}(\omega) = \frac{w_i \tau_{ij}}{\rho x_i(\omega)} \quad . \tag{7}$$

2.4. Market clearing and equilibrium

Labor market clearing implies that, for $i = 1, \ldots, n$,

$$\ell_{0i} + \sum_{j=1}^{n} \int_0^{m_j} \left(\ell_{ij}(\omega) + \kappa_{ij}(\omega) \right) d\omega = \bar{\ell}_i. \tag{8}$$

The pricing rule (7) implies that more productive firms charge lower prices and earn larger profits. The technologies operated in equilibrium are characterized by a cutoff value, $\hat{x}_{ij}$, which is the largest productivity level x such that

$$\pi_{ij}(\hat{x}_{ij}) = 0 . \tag{9}$$

Firms in country i with productivity greater than $\hat{x}_{ij}$ sell to country j, and firms with productivity less than $\hat{x}_{ij}$ do not. We assume that parameters are such that $\hat{x}_{ij} \geq \hat{x}_{ii}$ for all country pairs i, j. Typically, following Melitz (2003), researchers assume that $\kappa_{ij} > \kappa_{ii}$ for $j \neq i$, to ensure that this condition holds. Since we allow for considerable asymmetries across countries, we assume the condition directly.

3. A span-of-control model with international trade

In this section, we generalize the Lucas (1978) span-of-control model to incorporate international trade and imperfect competition as in Dixit and Stiglitz (1977). In Lucas's model, agent ω is characterized by a talent for operating a firm, $x(\omega)$. Each agent chooses to operate a firm of his own or to work for a wage in another agent's firm. In equilibrium, more talented agents choose to operate their own firms while less talented agents supply labor to other firms.

3.1. Agents

Once again, the world economy consists of n countries, each populated by a continuum of agents of measure $\bar{\ell}_i$, each endowed with one unit of labor. Again, there are two types of firms: homogeneous-good producers and differentiated-good producers. Agent ω' has income $I(\omega')$ and chooses consumption to solve

$$\max_{c_{0i}(\omega'), c_i(\omega,\omega')} (1-\alpha)\log(c_{0i}(\omega')) + (\alpha/\rho)\log\int_0^{m_i} c_i(\omega,\omega')^{\rho}\, d\omega \qquad (10)$$

$$\text{s.t. } p_{0i}c_{0i}(\omega') + \int_0^{m_i} p_i(\omega)c_i(\omega,\omega')d\omega = I_i(\omega').$$

Problem (10) is similar to problem (1), except that agents' incomes are heterogeneous, which implies that households' consumptions are heterogeneous. Since the utility function is homothetic, the demand function of agent ω' for differentiated good ω is

$$c(p_i(\omega), P_i, I_i(\omega')) = c(p_i(\omega), P_i, 1)I_i(\omega'), \qquad (11)$$

and the aggregate consumption of any differentiated good depends on the aggregate income of individuals, but not on the distribution of this income across agents,

$$\int_0^{\bar{\ell}_i} c(p_i(\omega), P_i, I_i(\omega'))d\omega' = c\left(p_i(\omega), P_i, \int_0^{\bar{\ell}_i} I_i(\omega')d\omega'\right). \quad (12)$$

3.2. Firms

In this model, homogeneous-goods firms are the same as in the model of Section 2, but differentiated-good producers are different. We change the interpretation of the production technology. Rather than the disembodied technologies in the previous model, we assume that each agent is endowed with a technology that only he can operate by supplying his one unit of labor as part of the management of the firm. This is why we assume that $\kappa_{ii} \geq 1$. Suppose, for example, that an agent in country 1 chooses to be an entrepreneur and to operate a firm that sells only to the domestic market. Then he himself works in managing the firm, which is part of the fixed costs, and hires $\kappa_{11} - 1$ units of labor to cover the rest of the fixed costs. Similarly, if he chooses to be an entrepreneur and to operate a firm that sells to both the domestic market and one foreign market, say country 2, then he himself works in managing the firm and hires $\kappa_{11} + \kappa_{12} - 1$ to cover the rest of the fixed costs.

The firm operated by agent ω' produces differentiated good ω' with marginal productivity $x(\omega')$, where the distribution of technologies across households is again described by $G_i(x)$.[1] If an agent operates his technology, he forgoes the wage he would have earned by working for another firm. Since each agent is endowed with one unit of labor, this opportunity cost is w_i. Profits from selling in the domestic market are

$$\pi_{ij}(\omega) = \max_p \left\{ \left(p - \frac{w_i}{x_i(\omega)} \right) c\left(p, P_j, \int_0^{\bar{\ell}_i} I_j(\omega')d\omega' \right) - w_i, 0 \right\}. \quad (13)$$

Once a household has opened a firm, he can sell to the domestic market. For each additional country the household chooses to serve, he hires κ_{ij} units of labor and faces the iceberg trade cost τ_{ij}. The entrepreneur's problem in this case is the same as in (6).

[1] In Lucas's (1978) model, agents differ in their managerial talent, which scales a production function with decreasing returns to scale in the other factors. In our model, the agent's managerial talent scales a production function with constant or increasing returns to scale. The concavity of profits with respect to labor follows from demand.

3.3. Market clearing and equilibrium

Labor market clearing implies that, for $i = 1, \ldots, n$,

$$\ell_{0i} + \int_0^{m_i} \ell_{ii}(\omega) d\omega + \sum_{j \neq i} \int_0^{m_j} \left(\ell_{ij}(\omega) + \kappa_{ij}(\omega) \right) d\omega = \overline{\ell}_i - m_i . \tag{14}$$

The second term in (14) is the labor needed by firms to serve the domestic market: the production labor hired and the entrepreneur's labor. The third term is the labor needed to serve foreign markets, and the first term is the labor needed to produce the homogeneous good.

If an agent chooses to become an entrepreneur and operate his technology, he earns

$$\pi_i(x(\omega)) = \sum_{j=1}^{n} (x(\omega)) . \tag{15}$$

If he chooses to supply labor to another entrepreneur, he earns w_i . An agent's income is

$$I_i(\omega) \ = \max \left\{ w_i, \pi_i(x(\omega)) \right\}. \tag{16}$$

The individual that is indifferent between operating his own firm or providing labor to another firm is characterized by a cutoff value, $\hat{x}_{ii}(\omega)$, which is the largest x such that

$$\pi_{ii}(\hat{x}_{ii}(\omega)) - w_i = 0. \tag{17}$$

Individuals in country i with productivity greater than $\hat{x}_{ii}$ choose to operate their firms, while individuals with productivity below provide labor. To be consistent with the models of international trade, we assume that parameters are such that $\hat{x}_{ii}(\omega) \leq \hat{x}_{ij}(\omega)$ for all i, j , where $\hat{x}_{ij}(\omega)$ is determined as in (9).

4. The heterogeneous-firm trade model as a span-of-control model

In the heterogeneous-firm trade model, there are agents and anonymous technologies, whereas, in the span-of-control model, technologies are embodied in the agents. The first step in making the two models equivalent is to set the mass of technologies in the heterogeneous-firm model to equal the number of agents, $\mu_i = \overline{\ell}_i$.

Second, if κ_{ii}^{s} is a fixed cost of serving the domestic market in the span-of-control model (above and beyond the opportunity cost of the entrepreneur), then the entry cost in the heterogeneous-firm model is $\kappa_{ii} = \kappa_{ii}^s + 1$, where the extra unit paid in the heterogeneous-firm model captures the time of the agent that manages the firm in the span-of-control model. This aligns the cost of serving the domestic market in the heterogeneous-firm model with the cost of serving the domestic market in the span-of-control model.

Given these two assumptions, the firms' problem in the heterogeneous-firm model and the entrepreneurs' problem in the span-of-control model coincide, and the same set of technologies are operated in both models. Since preferences are homothetic, aggregate consumption expenditures and aggregate trade flows in the two models are identical. When we interpret the exogenous set of technologies in the heterogeneous-firm model as technologies endowed upon agents as in the span-of-control model, the aggregate variables in the two models are identical.

While the aggregate variables are identical in the two models, the agent-level consumption and income in the two models are not. The heterogeneous-firm model assumes that agents own equal shares in operating firms, but, in the span-of-control model, the entrepreneur earns the profits of his firm only. To equate the agent-level distribution of income (and consumption) across the two models, we need only to randomly assign one technology to each agent in the trade model.

We can generalize the assumption that parameters are such that $\hat{x}_{ii} \le \hat{x}_{ij}$ for all country pairs i, j in the heterogeneous-firm model in one of two ways. First, we could require that, in addition to the fixed costs κ_{ii} of producing for domestic consumption and κ_{ij} of exporting, there is a fixed cost of one unit of labor to set up the firm to engage in any sort of activity. With this specification, we would not require that $\kappa_{ii} > 1$, and the fixed costs would be identical in the heterogeneous-firm model and the span-of-control model. Second, and alternatively, we could require firms to pay the fixed costs of producing for domestic production whenever they choose to export. In this case, losses in producing for domestic production would be covered by profits in exporting, and the cutoff productivity for domestic production, (9), would become

$$\pi_{ii}(\hat{x}_{ii}) + \sum_{j \ne i} \max[\pi_{ij}(\hat{x}_{ii}), 0] = 0. \qquad (18)$$

Now, the cutoff for exporting from country i to country j would be the maximum of the zero-profit cutoff $\hat{x}_{ij}$ defined by condition (9) and the cutoff $\hat{x}_{ii}$ defined by condition (18).

To generalize the assumptions that the fixed cost of setting up a firm is at least one unit of labor and that the potential measure of firms is equal to the measure of workers, we could simply change the units by which labor is measured. With natural units for firms and labor, however, say millions of firms and millions of workers, or millions of workers per year in a dynamic model, we can easily change the assumption $\mu_i = \overline{\ell}_i$ to the assumption that $\mu_i \leq \overline{\ell}_i$. In this case, $\overline{\ell}_i - \mu_i$ would be the measure of workers with no ability to set up a firm.

Our span-of-control model easily generalizes to a model in which agents have heterogeneous labor abilities as well as heterogeneous entrepreneurial abilities. Such a model is equivalent to a heterogeneous-firm model with heterogeneous fixed costs.

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