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The Interaction and Sequencing of Policy Reforms*

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ABSTRACT

In what order should a developing country adopt policy reforms? Do some policies complement each other? Do others substitute for each other? To address these questions, we develop a two-country dynamic general equilibrium model with entry and exit of firms that are monopolistic competitors. The model includes barriers to entry of new firms, barriers to international trade, and barriers to contract enforcement. We find that the same reform can have very different effects on other economic outcomes, depending on the types of distortions present. In our model, we find that reforms to trade barriers and barriers to the entry of new firms are substitutable, as are reforms to contract enforcement and trade barriers. In contrast, we find that reforms to contract enforcement and the barriers to entry are complementary. Finally, the optimal sequence of reforms requires reforming trade barriers before contract enforcement.

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1. Introduction

Policy makers in developing economies are called upon to make a wide array of structural reforms. The International Monetary Fund, in its “Article IV” consultation with Brazil, for example, recommends that Brazil, among other things, liberalize trade, overhaul the tax system, and reform the banking sector (International Monetary Fund, 2015). Faced with a multitude of reforms, in what order should a country adopt policy reforms? If political will limits the scope for reform, which subsets of reforms are optimal?

To address these questions, we develop a two-country dynamic general equilibrium model with entry and exit of firms, as in Hopenhayn (1992), that are monopolistic competitors, as in Melitz (2003). The model includes barriers to the creation of new firms, barriers to international trade, and barriers to contract enforcement, as in Kehoe and Levine (1993) and Albuquerque and Hopenhayn (2004). Although our model is simple, it produces rich firm dynamics. In countries with imperfect contract enforcement, for example, some firms must borrow to begin production, earn profits in the domestic market to pay down their debt, and then accumulate assets to finance their entry into the export market. In spite of these rich micro dynamics, the model has a balanced growth path that is easy to characterize.

We calibrate the model to match several features of the U.S. economy, focusing on the size distribution of establishments and the fraction of plants that export. We also consider distorted economies that have the same preferences and technologies as the United States, but are on lower balanced growth paths because of one or more policy distortions that increase the barriers to firm entry, increase the barriers to international trade, or weaken contract enforcement. Using these distorted economies, we investigate how the sequencing of three reforms — reducing barriers to firm creation, reducing barriers to international trade, and strengthening contract enforcement — determines the welfare gain from reform.

We focus on a world with two symmetric countries that coordinate, for example, by means of a free trade agreement, to enact identical reforms at the same time. We parameterize the three reforms — to entry barriers, trade barriers, and contract enforcement — so that each reform enacted separately would produce the same increase in balanced growth path output per capita. We consider the six possible sequences of the three reforms. In each sequence, the second reform follows the first reform by four years, and the third reform follows the second by four years. We include these lags to reflect political and administrative constraints.

How important is the sequencing of reforms? To answer this question, we compute the welfare difference between the best and worst reform sequences in terms of both the per-year real income increment and the share of period-zero consumption that the household in our model would need to be indifferent between the two sequences of reforms. The household needs an extra 4.8 percent of its period-zero consumption to be indifferent between the best and the worst sequence of reforms. In terms of the per-year real income equivalent, we find the difference between the best and worst sequence of reforms amounts to a perpetual increase of 0.11 percent of real income per year. To put that into perspective, researchers in the business cycle literature follow Lucas (1987) in finding welfare gains of this order of magnitude when they consider permanently eliminating business cycle variation — we find these gains from merely changing the order in which the same three reforms are carried out.

We find that the best sequence of reforms is to first decrease trade costs, then to improve contract enforcement, and, finally, to decrease the cost of firm creation. This result is driven by the behavior of firm entry in the model. The increase in competition from the lower trade costs leads to a strong selection of firms in the economy in which the expansion of efficient firms — who choose to become exporters — crowds out the entry of less efficient firms. Reforms to contract enforcement or entry costs lead to an increase in firm entry — and this entry is biased toward inefficient firms that would not have been able to enter if trade costs had been lowered first. By liberalizing international trade first, we impose the firm selection early, which dissuades inefficient firms from entering later when contract enforcement and firm entry costs are reformed.

With similar reasoning, we find that first improving contract enforcement, then decreasing trade costs, and finally decreasing firm entry costs is the worst sequence of reforms. Under this sequence, the reform to enforcement generates an increase in firm creation. Many of these new firms would not have been able to enter if trade costs had been reformed first. As monopolistic competitors, firms in our model never choose to exit production, but die exogenously over time. This generates hysteresis, as firms that have entered do not exit, even as they become less profitable. For simplicity, we have abstracted from a fixed production cost, which would yield endogenous firm exit. If we include a fixed production cost, but one that is smaller than the cost of creating a firm, firms would still delay exit, generating the same type of hysteresis we have in our model with only exogenous firm exit. Reform sequences that generate

entry of inefficient firms in the beginning of the reform period saddle the economy with less efficient distributions of firms during the transition, reducing the welfare gain from reform.

We should stress that the welfare differences we find are driven solely by the sequencing of the reforms. Regardless of the order in which the reforms are implemented, all of the equilibria converge to the same balanced growth path.

How do reforms interact with each other? We classify reforms as being *substitutes* if, once a country has enacted one reform, the percentage increase in GDP from enacting the other reform decreases. Similarly, two policies are *complements* if, once a country has enacted one reform, the percentage increase in GDP from enacting the other reform increases. We find that reforming trade costs is a substitute for reforming enforcement or reforming entry costs, but reforming entry costs and enforcement are complements. Again, this result is driven by the incentives for firm entry and exit. Lowering trade barriers increases competition from foreign firms, generating selection that makes it more difficult for less efficient firms to enter the domestic market. Reforming enforcement or entry barriers, however, makes it easier for these less-efficient firms to enter. These two types of reforms work against each other, so implementing one reform will weaken the impact of implementing the second one.

Since both enforcement and entry reform make it easier for firms to enter the domestic market, the two reforms are complementary. The complementarity of enforcement reform and entry cost reform implies that, if policy makers are constrained to only implement two reforms, they should choose these two and skip the reforms to international trade.

There is a large literature on the sequencing and interaction of policy reforms. Some of this research, for example, Aziz and Wescott (1997), Edwards (1990), and Martinelli and Tommasi (1997), focuses on less developed countries, especially those in Latin America. Another part of this literature, for example, Campos and Corcelli (2002), focuses on the transition economies in Eastern Europe. The collection of papers edited by Krueger (2000) combines research on the two areas. This existing literature is mostly concerned with the timing of capital account liberalization and the associated cross-border capital flows, typically in a representative-firm framework.

The capital account does not play a role in our model of two symmetric countries. Rather, our focus is on the firm-level implications of structural reform and the ways that firm entry and exit depend on the sequencing of reforms. Nonetheless, our results are relevant for the

debate in this literature. In comparing economic growth in Mexico and China, for example, Kehoe and Meza (2011) conclude that Mexico would have experienced better economic performance subsequent to its rapid growth during the period 1950–1981 if it had opened to trade and foreign investment early in this period. Our model provides theoretical justification and an intuition for this conclusion: If a country is going to eventually reform foreign trade and investment, it is better to do this early in its industrialization process so that the economy has a composition of firms more suited to competition in international markets.

The structural reform literature typically considers a particular policy distortion in isolation. Closer to our work, however, is a growing literature that analyzes the impact of multiple frictions in heterogeneous firm models. Buera, Kaboski, and Shin (2011) and Bah and Fang (2014) construct models with both financial frictions and firm entry costs. They show that the larger are entry costs, the more sensitive are firms to financial frictions. Their work is related to our analysis of the complementarity of reforming these two distortions, but their focus is on the steady-state differences in aggregate output and productivity and not the sequencing of reforms or the transitional dynamics from reform.

Our model is simple and stylized. The simplicity of our model allows us to focus on a limited set of results for which we can develop intuition. Our framework could be generalized substantially, however, and doing so would be worthwhile. We could, for example, model trade costs as iceberg transportation costs or tariffs rather than as the fixed trade costs in our model, which can be thought of as non-tariff trade barriers. While we model our two countries as symmetric and we study symmetric reforms, we could instead include more than two countries in our model and we could model them as asymmetric. We suspect the most significant departures from our results would be generated by asymmetric reforms. If a country unilaterally lowers the costs of foreign entry into its market, domestic households would benefit from more varieties of foreign goods, for example, but unilateral trade liberalization would generate an unfavorable shift in the terms of trade that could make domestic households worse off. Although generalizations of our model are likely to change many of our results, we suspect that our most important result — that any reform package that includes a trade reform should include it early in the sequence of reforms — is robust. Any sequence of reforms that pushes the trade reform to later in the sequence would induce firms to pay fixed costs to enter early in the reform sequence even though these firms would later be too inefficient to enter.

In Section 2, we develop a dynamic two-country general equilibrium model with three policy distortions. In Section 3, we characterize the balanced growth path of this model. In Section 4, we use a calibrated version of the model to quantitatively investigate the gains from various reform sequences. In Section 5, we conclude.

2. Model

In this section we develop a two-country dynamic general equilibrium model based on Chaney (2008) and Asturias et al. (2015). There is a representative household in each country. The production side of the economy comprises a representative final good producer and a continuum of monopolistically competitive intermediate goods producers. The intermediate good firms face an entry cost to operate domestically, a trade cost, and endogenous borrowing constraints that arise from the limited enforcement of contracts.

2.1. Households

The representative household in country $i = 1, 2$ is endowed with L_i units of labor, which it inelastically supplies to the intermediate goods firms. The problem of the household is

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \log C_{it} \\ & \text{s.t. } P_{it} C_{it} + q_{it+1} B_{it+1} = w_{it} L_i + D_{it} + B_{it} \\ & C_{it} \geq 0, \text{ no Ponzi schemes, } B_{i0} \text{ given,} \end{aligned} \quad (1)$$

where β , $1 > \beta > 0$, is the discount factor, C_{it} is consumption of the final good, P_{it} is its price, q_{it+1} is the price of a one-period bond, B_{it+1} is the face value of one-period debt purchased, w_{it} is the wage rate, and D_{it} is the aggregate dividends paid by domestic firms — those firms that were created in country i . We assume that there is no borrowing and lending across countries, so the price of bonds can differ across countries.

2.2. Final good producers

We model perfectly competitive final good firms that purchase intermediate goods and assemble them to produce the final good. The representative final good firm in country i solves

$$\min \int_{\omega \in \Omega_i^d} p_{it}^d(\omega) y_{it}^d(\omega) d\omega + \int_{\omega \in \Omega_i^e} p_{jt}^e(\omega) y_{jt}^e(\omega) d\omega$$

$$\text{s.t. } \left(\int_{\omega \in \Omega_{it}^d} y_{it}^d(\omega)^\rho d\omega + \int_{\omega \in \Omega_{jt}^e} y_{jt}^e(\omega)^\rho d\omega \right)^{\frac{1}{\rho}} = Y_{it} , \quad (2)$$

where $p_{it}^d(\omega)$ and $y_{it}^d(\omega)$ are the price and quantity of intermediate good ω and Ω_{it}^d is the set of intermediate goods produced for domestic consumption in country i , and $p_{jt}^e(\omega)$ and $y_{jt}^e(\omega)$ are the price and quantity of intermediate good ω and Ω_{jt}^e is the set of intermediate goods produced for export in country $j \neq i$. The elasticity of substitution between intermediate goods is $1/(1-\rho) > 1$, and Y_{it} is real aggregate output.

Solving the final good firm's problem yields the demand function for the domestically produced good ω from country i ,

$$y_{it}^d(\omega) = Y_{it} \left(\frac{p_{it}^d(\omega)}{P_{it}} \right)^{\frac{-1}{1-\rho}} , \quad (3)$$

and the demand function for the imported good ω from country $j \neq i$,

$$y_{jt}^e(\omega) = Y_{it} \left(\frac{p_{jt}^e(\omega)}{P_{it}} \right)^{\frac{-1}{1-\rho}} . \quad (4)$$

The price of the final good is

$$P_{it} = \left(\int_{\omega \in \Omega_{it}^d} p_{it}^d(\omega)^{\frac{-\rho}{1-\rho}} d\omega + \int_{\omega \in \Omega_{jt}^e} p_{jt}^e(\omega)^{\frac{-\rho}{1-\rho}} d\omega \right)^{\frac{1-\rho}{-\rho}} . \quad (5)$$

2.3. Intermediate goods producers

There is a continuum of heterogeneous intermediate good firms. In each period, a measure μ_i of potential entrants arrives with marginal productivities, x , drawn from the distribution $F_{it}(x)$. A potential entrant must hire κ_i^d units of domestic labor if it will produce for the domestic market. The potential entrant who enters at time t begins production in $t+1$. Potential entrants who choose not to enter cannot enter in subsequent periods.

Intermediate good producers may also choose to hire κ_i^e units of foreign labor to produce for the foreign market. We require foreign labor for the trade cost because we think of this fixed

cost as being partially determined by the government in the foreign country. If a firm chooses to enter the export market at time t , it begins to export in $t+1$. Furthermore, we assume that $\kappa_i^e > \kappa_i^d$, although, for asymmetric countries, this does not imply that $w_j \kappa_i^e > w_i \kappa_i^d$.

To keep our analysis simple, we assume that, once a firm has paid the fixed cost to operate domestically or to export, there are no further fixed costs associated with either activity. Thus, a firm does not voluntarily exit domestic production or cease exporting to the other country. Firms die with probability δ every period.

The firm producing good ω uses labor to produce according to

$$y_{it}(\omega) = x_{it}(\omega)l_{it}(\omega) \quad (6)$$

where $x_{it}(\omega)$ is the productivity of firm ω .

Conditional on choosing to sell to the domestic market, firm ω chooses its domestic price to maximize profits,

$$\pi_{it}^d(\omega) = \max p_{it}^d(\omega)y_{it}^d(\omega) - w_{it} \frac{y_{it}^d(\omega)}{x_{it}(\omega)}. \quad (7)$$

The solution to this profit-maximization problem yields the standard constant markup pricing,

$$p_{it}^d(\omega) = \frac{w_{it}}{\rho x_{it}(\omega)}. \quad (8)$$

Conditional on exporting, firm ω solves

$$\pi_{it}^e(\omega) = \max p_{it}^e(\omega)y_{it}^e(\omega) - w_{it} \frac{y_{it}^e(\omega)}{x_{it}(\omega)}. \quad (9)$$

Since we assume that there are no transportation costs or tariffs, the firm charges the same price in the export and domestic markets, $p_{it}^e(\omega) = p_{it}^d(\omega)$. Notice that every firm with productivity x chooses the same price. In what follows, we no longer characterize a good by its name ω but by the productivity x of the firm that produces it.

The fixed costs that firms pay to enter domestic and foreign markets must be paid before production takes place. This implies that firms must finance these costs by issuing debt. The amount of debt the firm can issue, however, is limited by the strength of contract enforcement in the economy. We assume that the manager of the firm can abscond with a fraction $1 - \theta_i$ of the

value of the firm in the case of default. We interpret this possibility as the result of imperfect contract enforcement.

The price of risk-free debt is q_{it+1} . When a firm issues debt, there is a possibility that the firm will exogenously cease operations and not repay its debt. This exogenous firm death occurs with probability δ . In equilibrium, the price of the firm's debt will be $(1-\delta)q_{it+1}$, so that the return on firm debt matches the risk-free rate,

$$\frac{1}{q_{it+1}} = (1-\delta) \times \frac{1}{(1-\delta)q_{it+1}} + \delta \times 0. \quad (10)$$

An existing exporter, with productivity x and existing debt b , chooses new holdings of debt, b' , and dividend payments, d , to solve the dynamic programming problem

$$\begin{aligned} V_{it}^e(b, x) &= \max d + q_{it+1}(1-\delta)V_{it+1}^e(b', x) \\ \text{s.t. } V_{it}^e(b, x) &\geq (1-\theta_i)V_{it}^e(0, x) \\ d &= \pi_{it}^d(x) + \pi_{it}^e(x) + (1-\delta)q_{it+1}b' - b \\ d &\geq 0. \end{aligned} \quad (11)$$

The first constraint is the enforcement constraint, which limits the amount of debt that the firm can issue, where θ_i , $1 \geq \theta_i > 0$, governs the degree of contract enforcement in country i . If $\theta_i = 1$, there is perfect contract enforcement; if $\theta_i = 0$, there is no contract enforcement and borrowing is impossible. We rule out $\theta_i = 0$, since firms in our model require some borrowing for production to take place. The second constraint defines the dividend payment. The firm cannot choose negative dividends, as this would circumvent the enforcement constraint.

Besides choosing its debt level b' , an existing non-exporter chooses either to continue to produce for only the domestic market and pay dividends d^n or to enter the export market and pay dividends d^e . An existing non-exporter solves

$$\begin{aligned} V_{it}^n(b, x) &= \max \left\{ d^n + q_{it+1}(1-\delta)V_{it+1}^n(b', x), d^e + q_{it+1}(1-\delta)V_{it+1}^e(b', x) \right\} \\ V_{it}^n(b, x) &\geq (1-\theta_i)V_{it}^n(0, x) \\ d^n &= \pi_{it}^d(x) + (1-\delta)q_{it+1}b' - b \\ d^e &= \pi_{it}^d(x) + (1-\delta)q_{it+1}b' - b - w_{jt}K_i^e \end{aligned} \quad (12)$$

$$d^n \geq 0, d^e \geq 0.$$

Notice that, if the firm chooses to become an exporter, the time-to-build requirement implies that the firm pays the fixed cost ($w_{jt}\kappa_i^e$) in the current period, but does not export until the next period.

2.4. Entry decisions

In each period, measure μ_i of potential entrants is born. Their productivities are drawn from a Pareto distribution,

$$F_{it}(x) = 1 - \left(\frac{x}{\underline{x}_i g^t} \right)^{-\gamma}, \quad x \geq \underline{x}_i g^t, \quad (13)$$

which has a mean that grows at rate $g - 1$. We impose the standard condition for this sort of model, $\gamma(1 - \rho) - \rho > 0$, which is necessary for the distribution of profits to have a finite mean. The continual improvement of the technologies available to new firms drives the long-run aggregate growth in the model: Older firms exit and are replaced by new entrants who are, on average, more productive.

A potential entrant with productivity x does not produce at age $k = 0$ because of the time-to-build requirement. This potential entrant borrows to pay the fixed cost $w_{it}\kappa_i^d$ only if two conditions are satisfied. First, the value of the firm must be greater than zero if it enters and, second, there must exist a debt path such that the enforcement constraints in all subsequent periods are satisfied. The first condition is satisfied if

$$V_{it+1}^n \left(\frac{w_{it}\kappa_i^d}{(1 - \delta)q_{it+1}}, x \right) \geq 0. \quad (14)$$

Notice that the first term in the value function is the debt of the firm in $t + 1$ if it sells only in the domestic market. We denote \hat{x}_{i0t}^d as a potential entrant's minimum productivity necessary to enter at time t . More generally, \hat{x}_{ikt}^d is the minimum productivity of firms of age k that continue to operate at time t . Since there are no fixed costs to pay each period, $\hat{x}_{ikt}^d = \hat{x}_{i,k-1,t-1}^d$ for all $k \geq 1$. That is, firms only exit the domestic market exogenously.

Similarly, a potential exporter with productivity x and debt b pays the fixed cost $w_{it}\kappa_i^e$ to enter the export market only if the value of the firm if it enters the export market is greater than the value of the firm if it remains only serving the domestic market,

$$V_{it+1}^e \left(\frac{b + w_{jt}\kappa_i^e - \pi_{it}^d(x)}{(1-\delta)q_{it+1}}, x \right) \geq V_{it+1}^n \left(\frac{b - \pi_{it}^d(x)}{(1-\delta)q_{it+1}}, x \right), \quad (15)$$

and if there exists a debt path such that the enforcement constraints in all subsequent periods are satisfied. Notice that the first term in the exporter value function in (15) is the debt in period $t+1$ of the firm after it pays the fixed cost to enter the export market.

Finally, a potential entrant with productivity x borrows and pays both the fixed cost $w_{it}\kappa_i^d$ to enter the domestic market and the fixed cost $w_{jt}\kappa_i^e$ to the export market only if the value of the firm if it enters both markets is greater than the value of only entering the domestic market, which in turn is greater than zero,

$$V_{it+1}^e \left(\frac{w_{it}\kappa_i^d + w_{jt}\kappa_i^e}{(1-\delta)q_{it+1}}, x \right) \geq V_{it+1}^n \left(\frac{w_{it}\kappa_i^d}{(1-\delta)q_{it+1}}, x \right) \geq 0, \quad (16)$$

and if there exists a debt path such that the enforcement constraints in all subsequent periods are satisfied.

We denote \hat{x}_{ikt}^e as the minimum productivity of firms of age $k \geq 0$ who pay the trade cost at age $\ell \geq 0$. Consequently, a potential entrant's minimum productivity necessary to enter both the domestic market and the export market at age zero is \hat{x}_{i00t}^e . In a country with perfect enforcement ($\theta_i = 1$), any firm that will ever export pays the export fixed cost at age $\ell = 0$. As enforcement worsens (θ_i falls), less efficient exporters take longer to export because they must decrease their debts to satisfy the enforcement constraint, and, consequently, $\hat{x}_{i00t}^e > \hat{x}_{i01t}^e > \dots > \hat{x}_{i0\hat{n}_{it}t}^e$, where \hat{n}_{it} denotes the oldest age at which a firm who enters in time t pays the trade cost. Finally, define \hat{x}_{ikt}^e as the minimum productivity of all exporting firms with age $k \geq 1$. For the cohort of firms with age $k \leq \hat{n}_{it} + 1$, we would expect that $\hat{x}_{ikt}^e = \hat{x}_{i,k-1,k-1,t}^e$, since firms in this cohort are still paying the trade cost to become exporters. For older cohorts

with age $k > \hat{n}_i + 1$, the export thresholds are given by $\hat{x}_{ikt}^e = \hat{x}_{i,k-1,t-1}^e$, since firms in this cohort are no longer becoming exporters.

The measure of exporting firms, η_{it}^e , evolves according to

$$\eta_{it+1}^e = (\eta_{it}^e + \lambda_{it}^e)(1 - \delta), \quad (17)$$

where λ_{it}^e is the measure of new exporters,

$$\lambda_{it}^e = \mu_i \left(1 - F_{it}(\hat{x}_{i00t}^e)\right) + \mu_i \sum_{k=1}^{\hat{n}_i} (1 - \delta)^k \left[F_{it-k}(\hat{x}_{ikt}^e) - F_{it-k}(\hat{x}_{ikk}^e) \right]. \quad (18)$$

The first term in the right-hand side of (18) is the measure of new entrants who immediately pay the trade cost to access the export market, and the second term is the measure of existing age- k firms who pay the trade cost in period t .

The measure of all domestic firms, η_{it}^d , evolves according to

$$\eta_{it+1}^d = (\eta_{it}^d + \lambda_{it}^d)(1 - \delta), \quad (19)$$

where λ_{it}^d is the mass of new firms,

$$\lambda_{it}^d = \mu_i \left(1 - F_{it}(\hat{x}_{i0t}^d)\right). \quad (20)$$

2.5. Equilibrium

We focus on balanced growth paths and the transitions between them, but before defining a balanced growth path, we first define an equilibrium. To define an equilibrium, we need to provide, as initial conditions, the measures of domestic and exporting firms of all ages operating in period zero. To define these measures, we need the minimum productivities of operating firms, $\hat{x}_{1k0}^d, \hat{x}_{2k0}^d$, and $\{\hat{x}_{1k\ell 0}^e, \hat{x}_{2k\ell 0}^e\}_{\ell \geq 0}$. Using the minimum productivities to specify the measures of firms operating in period zero requires us to specify the distributions of productivities from which these existing firms were drawn. We do this using Pareto distributions analogous to those for firms born in period zero and later,

$$F_{i,-k}(x) = 1 - \left(\frac{x}{\underline{x}_i g^{-k}} \right)^{-\gamma}, \quad x \geq \underline{x}_i g^{-k}, \quad (21)$$

for $i=1,2$ and $k \geq 1$. Additionally, we need to specify the bond holdings of households, B_{i0} and B_{20} , and the bond holdings of firms, $b_{1k0}(x)$ for $x \geq \hat{x}_{1k0}^d$, $k \geq 1$ and $b_{2k0}(x)$ for $x \geq \hat{x}_{2k0}^d$, $k \geq 1$. We require that these initial conditions for bond holdings by households and bond holdings by firms are consistent,

$$B_{i0} = \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ik0}^d}^{\infty} b_{ik0}(x) dF_{i,-k}(x), \quad i=1,2. \quad (22)$$

Definition: Given the initial conditions, an *equilibrium* is, for $i=1,2$, sequences of prices $\{w_{it}, P_{it}, q_{it+1}\}_{t=0}^{\infty}$, aggregate output, consumption, dividends and bond holdings, $\{Y_{it}, C_{it}, D_{it}, B_{it+1}\}_{t=0}^{\infty}$, entry threshold values $\{\hat{x}_{ikt}^d, \hat{x}_{ikt}^e\}_{k,\ell,t=0}^{\infty}$, $x > 0$, measures of new entrants $\{\lambda_{it}^d, \lambda_{it}^e\}_{t=0}^{\infty}$, prices and allocations for intermediate good firms that produce for the domestic market $\{p_{it}^d(x), y_{it}^d(x), l_{it}^d(x)\}_{t=0}^{\infty}$, $x > 0$, prices and allocations for intermediate firms that produce for the export market $\{p_{it}^e(x), y_{it}^e(x), l_{it}^e(x)\}_{t=0}^{\infty}$, $x > 0$, and debt levels and dividends for intermediate good firms $\{b_{ikt+1}(x), d_{ikt}(x)\}_{t=0}^{\infty}_{k=0}$ $x \geq 0$ such that

1. Given $\{w_{it}, P_{it}, q_{it+1}, D_{it}\}_{t=0}^{\infty}$ and B_{i0} , the household in country i chooses $\{C_{it}, B_{it+1}\}_{t=0}^{\infty}$ to solve its utility maximization problem (1).
2. Given $\{w_{it}, P_{it}, Y_{it}\}_{t=0}^{\infty}$, the intermediate good firm with productivity $x > 0$ in country i chooses $\{p_{it}^d(x), y_{it}^d(x), l_{it}^d(x)\}_{t=0}^{\infty}$ to solve (7) and, given $\{Y_{jt}, P_{jt}\}_{t=0}^{\infty}$ for $j \neq i$, chooses $\{p_{it}^e(x), y_{it}^e(x), l_{it}^e(x)\}_{t=0}^{\infty}$ to solve (9).
3. Given $\{w_{it}, Y_{it}, q_{it+1}\}_{t=0}^{\infty}$ and $\{w_{jt}, Y_{jt}\}_{t=0}^{\infty}$ for $j \neq i$, the intermediate firm with productivity $x > 0$ in country i chooses $\{b_{ikt+1}(x), d_{ikt}(x)\}_{k,t=0}^{\infty}$ and makes entry and export decisions consistent with $\{\hat{x}_{ikt}^d, \hat{x}_{ikt}^e\}_{k,\ell,t=0}^{\infty}$ to solve the dynamic programming problems of the non-exporting firm in (12) and of the exporting firm in (11).

4. Given $\{Y_{it}, p_{it}^d(x), p_{jt}^e(x)\}_{t=0}^{\infty}$, $j \neq i$, final good firms in country i choose $\{y_{it}^d(x), y_{jt}^e(x)\}_{t=0}^{\infty}$, $j \neq i$, to solve the cost minimization problem (2), and earn zero profits (5).

5. The entry threshold values $\{\hat{x}_{ikt}^d, \hat{x}_{ikt}^e\}_{k,\ell,t=0}^{\infty}$ and the measures of new entrants $\{\lambda_{it}^d, \lambda_{it}^e\}_{t=0}^{\infty}$ satisfy conditions (20) and (18).

6. Labor markets clear in country i for all $t \geq 0$,

$$L_i = \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^d}^{\infty} l_{it}^d(x) dF_{it-k}(x) + \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^e}^{\infty} l_{it}^e(x) dF_{it-k}(x) + \lambda_{it}^d \kappa_i^d + \lambda_{jt}^e \kappa_j^e. \quad (23)$$

7. The bond market clears in country i in all periods $t \geq 0$,

$$B_{it+1} = \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^d}^{\infty} b_{ikt+1}(x) dF_{it-k}(x). \quad (24)$$

8. Aggregate dividends are the sum of firm dividend payments in country i for all $t \geq 0$,

$$D_{it} = \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^d}^{\infty} d_{ikt+1}(x) dF_{it-k}(x).$$

9. Trade is balanced for all $t \geq 0$,

$$\mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^d}^{\infty} p_{it}^e(x) y_{it}^e(x) dF_{it-k}(x) = \mu_j \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{jkt}^e}^{\infty} p_{jt}^e(x) y_{jt}^e(x) dF_{jt-k}(x). \quad (25)$$

3. Balanced growth

In this section, we prove that the model has a balanced growth path, and we characterize the behavior of its key variables. To make our characterization of a balanced growth path simple, we assume that κ_i^d is low enough relative to κ_i^e so that on the balanced growth path the marginal entrant never exports and only produces for the domestic market. We also assume that the elasticity of substitution between intermediate goods is large enough, $1/(1-\rho) > 2$, so that a firm's profits decrease over time.

Definition: A *balanced growth path* is an equilibrium, for the appropriate initial conditions, such that

$$\frac{w_{it+1}}{w_{it}} = \frac{Y_{it+1}}{Y_{it}} = \frac{C_{it+1}}{C_{it}} = \frac{B_{it+1}}{B_{it}} = \frac{D_{it+1}}{D_{it}} = \frac{\hat{x}_{i,k,t+1}^d}{\hat{x}_{ikt}^d} = \frac{\hat{x}_{i,k,\ell,t+1}^e}{\hat{x}_{ik\ell t}^e} = g, \quad (26)$$

$$\frac{p_{i,t+1}^d(gx)}{p_{it}^d(x)} = \frac{p_{i,t+1}^e(gx)}{p_{it}^e(x)} = \frac{l_{i,t+1}^d(gx)}{l_{it}^d(x)} = \frac{l_{i,t+1}^e(gx)}{l_{it}^e(x)} = 1, \quad (27)$$

$$\frac{y_{i,t+1}^d(gx)}{y_{it}^d(x)} = \frac{y_{i,t+1}^e(gx)}{y_{it}^e(x)} = \frac{b_{i,k,t+1}(gx)}{b_{ikt}(x)} = \frac{d_{i,k,t+1}(gx)}{d_{ikt}(x)} = g, \quad (28)$$

$P_{1t} = P_1$, $P_{2t} = P_2$ $q_{1t+1} = q_{2t+1} = \beta / g$, $\lambda_{1t}^d = \lambda_1^d$, $\lambda_{2t}^d = \lambda_2^d$, $\lambda_{1t}^e = \lambda_1^e$, $\lambda_{2t}^e = \lambda_2^e$ for all $t, k, \ell \geq 0$.

We could have specified a balanced growth path only in terms of the values of the variables in period zero rather than sequences for all $t \geq 0$. Our definition makes very clear, however, that a balanced growth path corresponds to a set of sequences where the elements all either grow by the same factor, g , or stay constant.

On the balanced growth path, growth in the economy is driven by the continual entry of new firms that are, on average, more productive than the previous cohorts. The growth rate of output, consumption, and both components of income grow at the rate $g - 1$, which is the rate at which the mean of the productivity distribution of potential entrants grows. Next, we characterize the productivity cutoff for the marginal entrant on the balanced growth path.

Lemma 1. *On any balanced growth path, the enforcement constraint of the marginal entrant at time t holds with equality only when $k = 1$,*

$$V_{it+1}^n \left(b_{i,0,t+1} \left(\hat{x}_{i0t}^d \right), \hat{x}_{i0t}^d \right) = (1 - \theta_i) V_{it+1}^n \left(0, \hat{x}_{i0t}^d \right) \quad (29)$$

$$V_{it+k}^n \left(b_{i,k-1,t+k} (x), x \right) > (1 - \theta_i) V_{it+k}^n \left(0, x \right) \quad (30)$$

for all $t, k > 1$ and $x \geq \hat{x}_{kt}^d$.

Proof: See Appendix B.

Notice that, although the enforcement constraint holds with equality for the marginal entrant when $k = 1$, it does not bind in the sense that the constraint distorts the decision of this marginal entrant. Instead, the enforcement constraint binds in determining the marginal entrant. If we loosen the enforcement constraint by increasing θ_i , firms with lower productivity enter.

This is an attractive feature of the model in terms of characterizing balanced growth paths, and it is the product of the costs of entry being fixed costs. Either firms pay this fixed cost and enter or they do not.

Using (29), we can derive the expression for \hat{x}_{i0t}^d ,

$$\hat{x}_{i0t}^d = \tilde{\kappa}_i^d \frac{1-\rho}{\rho} \left(\frac{1}{1-\rho} \frac{w_{it}}{P_{it} Y_{it}} \right)^{\frac{1-\rho}{\rho}} \frac{1}{\rho} \frac{w_{it}}{P_{it}}, \quad (31)$$

where

$$\tilde{\kappa}_i^d = \frac{\kappa_i^d}{\theta_i \sum_{k=1}^{\infty} \beta^k (1-\delta)^k g^{\frac{k-\rho}{1-\rho}}}. \quad (32)$$

The cutoff in (31) looks similar to that in a static model except that the entry cost in that expression is replaced with $\tilde{\kappa}_i^d$, which we interpret as the effective entry cost. Notice that changes in the enforcement constraint change the effective entry cost that firms face. In the case that θ_i approaches 0, then $\tilde{\kappa}_i^d$ approaches infinity.

We now characterize the productivity cutoff for the marginal exporter.

Lemma 2. *On any balanced growth path, the enforcement constraint of the marginal firm of age k at time t who pays the trade cost at age ℓ*

1. *either holds with equality only at age $\ell + 1$:*

$$V_{i,\ell+1,t-k+\ell+1}^e \left(b_{i,\ell,t-k+\ell+1} \left(\hat{x}_{ik\ell t}^e \right), \hat{x}_{ik\ell t}^e \right) = (1-\theta_i) V_{i,\ell+1,t-k+\ell+1}^e \left(0, \hat{x}_{ik\ell t}^e \right) \quad (33)$$

$$V_{i,h,t-k+h}^e \left(b_{i,h-1,t-k+h} (x), x \right) > (1-\theta_i) V_{i,h,t-k+h}^e \left(0, x \right), \text{ for all } h > \ell + 1, t, x \geq \hat{x}_{ik\ell t}^e \quad (34)$$

2. *or is slack, in which case,*

$$\begin{aligned} & V_{i,\ell+1,t-k+\ell+1}^e \left(\frac{b_{i,\ell-1,t-k+\ell} \left(\hat{x}_{ik\ell t}^e \right) + \kappa_i^e w_{i,t-k+\ell} - \pi_{i,t-k+\ell}^d \left(\hat{x}_{ik\ell t}^e \right)}{(1-\delta) q_{i,t-k+\ell+1}}, \hat{x}_{ik\ell t}^e \right) \\ & = V_{i,\ell+1,t-k+\ell+1}^n \left(\frac{b_{i,\ell-1,t-k+\ell} \left(\hat{x}_{ik\ell t}^e \right) - \pi_{i,t-k+\ell}^d \left(\hat{x}_{ik\ell t}^e \right)}{(1-\delta) q_{i,t-k+\ell+1}}, \hat{x}_{ik\ell t}^e \right) \end{aligned} \quad (35)$$

Proof: See Appendix B.

The lemma says that, at some ages, the marginal exporter is determined by the enforcement constraint, but at other ages, it is determined by the condition that entering the export market is at least as profitable as not entering. The enforcement constraint binds in the sense that it distorts the timing of when a firm begins to export. Conditional on exporting though, it does not distort the decisions of the firm.

Using (33), we can derive an expression for the constrained marginal exporter's productivity, $\hat{x}_{ik\ell t}^{ec}$,

$$\hat{x}_{ik\ell t}^{ec} = \tilde{\kappa}_{i\ell}^{ec} \frac{1-\rho}{\rho} \left(\frac{1}{1-\rho} \frac{w_{i,t-k}}{P_{i,t-k} Y_{i,t-k}} \right)^{\frac{1-\rho}{\rho}} \frac{1}{\rho} \frac{w_{i,t-k}}{P_{i,t-k}} \quad (36)$$

where

$$\tilde{\kappa}_{i\ell}^{ec} = \frac{\Delta_i^w (1-\delta)^\ell \beta^\ell \kappa_i^e + \kappa_i^d}{\theta_i \left[1 + \Delta_i^Y \Delta_i^{P^{1-\rho}} \right] \sum_{m=\ell+1}^{\infty} (1-\delta)^m \beta^m g^{\frac{m-\rho}{1-\rho}} + \sum_{m=1}^{\ell} (1-\delta)^m \beta^m g^{\frac{m-\rho}{1-\rho}}} \quad (37)$$

and $\Delta_i^w = w_{j_0} / w_{i_0}$, $\Delta_i^Y = Y_{j_0} / Y_{i_0}$, and $\Delta_i^P = P_j / P_i$ for $j \neq i$. Like the condition in (31), the cutoff in (36) looks similar to that in a static model for the minimum productivity of an exporter, except that the fixed cost in that expression is replaced by $\tilde{\kappa}_{i\ell}^{ec}$, which we interpret as the effective trade cost. Once again, changes in the enforcement constraint change the effective trade cost that firms face. Changes in the enforcement parameter in (37) affect the effective trade cost differently from the way they do in the effective entry cost in (32). In particular, as θ_i approaches zero, $\tilde{\kappa}_{i\ell}^{ec}$ does not approach infinity although $\tilde{\kappa}_i^d$ does. The reason is that firms can self-finance using profits from the domestic market and then use that to pay the fixed cost to export. Also, $\tilde{\kappa}_{i\ell}^{ec}$ is decreasing in age ℓ , and therefore $\hat{x}_{ik\ell t}^{ec}$ is also decreasing in age. In other words, less efficient firms take longer to export.

Using (35), we can derive the expression for the unconstrained marginal exporter's productivity $\hat{x}_{ik\ell t}^{eu}$,

$$\hat{x}_{ik\ell t}^{eu} = \tilde{K}_{i\ell}^{eu} \frac{1-\rho}{\rho} \left(\frac{1}{1-\rho} \frac{w_{i,t-k}}{P_{i,t-k} Y_{i,t-k}} \right)^{\frac{1-\rho}{\rho}} \frac{1}{\rho} \frac{w_{i,t-k}}{P_{i,t-k}} \quad (38)$$

where

$$\tilde{K}_{i\ell}^{eu} = \frac{\Delta_i^w K_i^e}{\Delta_i^y \Delta_i^{P^{1-\rho}} \sum_{m=1}^{\infty} (1-\delta)^m \beta^m g^{\frac{m-\rho}{1-\rho}}} g^{\frac{\ell-\rho}{1-\rho}}. \quad (39)$$

Notice that the expression for $\hat{x}_{ik\ell t}^{eu}$ is increasing in age ℓ . The reason is that the more a firm waits to pay the trade cost, the more profitable it needs to be. Thus, in an unconstrained environment, any exporter will pay the trade cost at age $\ell = 0$. In general, the marginal exporter productivity is

$$\hat{x}_{ik\ell t}^e = \max \left\{ \hat{x}_{ik\ell t}^{ec}, \hat{x}_{ik\ell t}^{eu} \right\}. \quad (40)$$

Let $n_{it}(x)$ be the age at which a firm of productivity x born at time t pays the trade cost. Furthermore, let \hat{n}_{it} be the oldest age of the latest export entrant, which can be found by

$$\hat{n}_{it} = \min \left\{ \ell = 1, 2, \dots, \infty \mid \hat{x}_{ik\ell t}^{eu} \geq \hat{x}_{ik\ell t}^{ec} \right\}. \quad (41)$$

The intuition for this is simple. If contract enforcement is sufficiently bad, less efficient firms must pay down their existing debt to finance the trade cost. As the firms age, however, their profits decline as the wage increases. Therefore, at age $\hat{n}_{it} + 1$, it is no longer profitable to enter the export market, even though the enforcement constraint is no longer binding. Thus, the minimum productivity for exporting firms that are of age k at time t is given by

$$\hat{x}_{ikt}^e = \begin{cases} \hat{x}_{i,0,k-1,t-k}^{ec} & \text{if } k \leq \hat{n}_{it} \\ \hat{x}_{i,0,\hat{n}_{it},t-k}^{eu} & \text{otherwise} \end{cases}. \quad (42)$$

The first line of (42) refers to the cohort of firms who still have some firms paying the trade costs, and thus the minimum exporter productivity is given by the marginal constrained exporter who paid its trade cost in the previous period. The second line refers to the cohort of firms who no longer have firms paying trade costs; in this case, the minimum exporter productivity is

determined by the marginal unconstrained exporter at the last age any firm pays the trade cost, given by \hat{n}_{it} .

The household's income is the sum of its labor income and net capital income. Net capital income, A_{it} , is the sum of firm profits net of entry and trade costs,

$$A_{it} = \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^d}^{\infty} \pi_{it}^d(x) dF_{t-k}(x) + \mu_i \sum_{k=1}^{\infty} (1-\delta)^k \int_{\hat{x}_{ikt}^e}^{\infty} \pi_{it}^e(x) dF_{t-k}(x) - \lambda_{it}^d w_{it} \mathbf{K}_i^d - \lambda_{it}^e w_{it} \mathbf{K}_i^e. \quad (43)$$

In equilibrium, net capital income is equal to the sum of aggregate dividends and net debt income,

$$A_{it} = D_{it} + B_{it} - q_{it+1} B_{it+1}. \quad (44)$$

Proposition 1. *A balanced growth path exists.*

Proof: On the balanced growth path, the aggregate variables are

$$\frac{w_{it}}{P_i} = g^t \rho \left(\frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} v_i \right)^{\frac{1}{\gamma}} \left[\frac{(1-\rho)L_i}{1-\xi_i} \right]^{\frac{\gamma(1-\rho)-\rho}{\gamma}} \quad (45)$$

$$\frac{A_{it}}{w_{it}} = \frac{\xi_i}{1-\xi_i} L_i \quad (46)$$

$$P_i C_{it} = P_i Y_{it} = w_{it} L_i + A_{it} \quad (47)$$

where v_i and ξ_i , the ratio of net capital income to total output, are positive constants. From the above equations, we see that w_{it} , A_{it} , C_{it} , and Y_{it} grow by g and satisfy the equilibrium conditions. Furthermore, the cutoffs \hat{x}_{i0t}^d and $\hat{x}_{ik,\ell,t+k}^e$, given by (31), (36), (38), and (40), also grow by g . Finally, from the first-order condition of the household and applying the balanced growth path conditions, we obtain $q_{it+1} = \beta/g$. See Appendix B for details. \square

To better understand the model's balanced growth paths, we turn to the evolution of debt, profits, and dividends of firms that service only the domestic market and those that do not immediately become exporters. As Figure 1 shows, a firm that only operates in the domestic market during its life takes on debt at age zero. The firm then uses its profits in the subsequent

periods to pay off that debt. Upon paying off the debt, the firm issues all of its profits as dividends. Notice that the profitability of the firm declines through time. This is because, as more productive firms enter, an existing firm becomes relatively unproductive.

Figure 1: Debt, profits, and dividends of a domestic firm

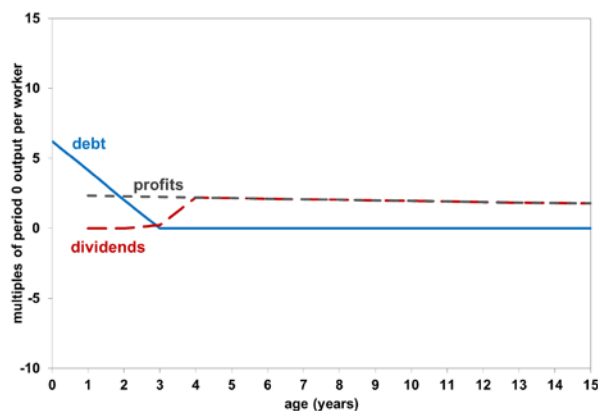
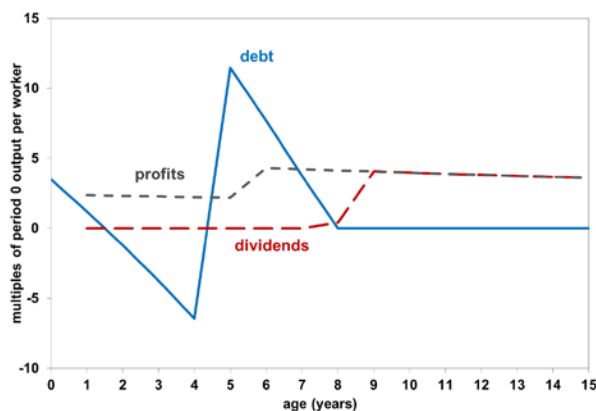


Figure 2: Debt, profits, and dividends of an eventual exporter



The debt, profits, and dividends of a firm that does not immediately export, as seen in Figure 2, looks similar to the firm that only operates in the domestic market except for two differences. First, an eventual exporter is more productive and thus pays down its debt faster. Second, after paying down its debt, the firm does not issue dividends. Instead, the firm saves so that it can pay the fixed cost to enter the export market. After entering the export market, both the firm's profitability and its debt level increase. Next, the firm uses its profits to pay down this additional debt. Upon retiring its debt, the exporting firm issues all of its profits as dividends.

4. Quantitative exercises

In this section, we use the model to perform quantitative exercises to determine how the sequencing of reforms affects the welfare gains from these reforms. For simplicity, we model two symmetric countries in which $L_1 = L_2$, $\mu_1 = \mu_2$, $\kappa_1^d = \kappa_2^d$, $\kappa_1^e = \kappa_2^e$, $\theta_1 = \theta_2$, $\underline{x}_1 = \underline{x}_2$. We begin by calibrating the model to the U.S. economy, which trades with a symmetric economy that represents the rest of the world.

We examine the effects of conducting symmetric reforms. In particular, we investigate the effects of six possible reform sequences, with each reform occurring every four years. The first reform is unexpected by the agents, but the subsequent reforms are foreseen.

We focus on symmetric countries and symmetric reforms only to keep our analysis simple. We could calibrate the model to asymmetric countries, and we could analyze the impact of asymmetric reforms.

4.1. Calibration

We choose parameters so that the model's equilibrium matches several features of the U.S. economy, focusing on the size distribution of establishments and the number of plants that export. We summarize the parameters in Table 1.

We normalize the labor endowment, L , to one. We set the fixed cost to operate domestically, κ^d , so that the model matches the average establishment size in the United States of 16.0 employees (1981–2000, U.S. Census, *Statistics of U.S. Businesses*). We choose the trade cost, κ^e , so that the model matches the observed fraction of manufacturing plants that export, 0.21 (Bernard, et al., 2003). The parameter that governs enforcement, θ , is set so that the model matches the debt-to-revenue ratio of firms aged less than five years, 0.27 (2003, *Survey of Small Business Finances*). The *Survey of Small Business Finances* surveys firms with less than 500 employees, which account for most of the new firms created in the United States. For example, in 2010–2011, 99.98 percent of new firms employed less than 500 workers (U.S. Census, *Statistics of U.S. Businesses*).

The curvature parameter of the Pareto distribution, γ , is set so that the model matches the standard deviation of the U.S. establishment size distribution, which averages 91.2 workers (1981–2000, United States Census, *Statistics of U.S. Businesses*). The probability that the firm dies, δ , is set so that the establishment death rate is 10 percent per year (2010–2011, U.S. Census, *Statistics of U.S. Businesses*). The discount factor, β , is set to generate a real interest rate of four percent per year. Finally, we set the entrant productivity growth factor, g , so that, in the balanced growth path, output per capita grows at two percent per year — the historical U.S. average.

Table 1: Calibrated parameters

parameter		value	target
fixed cost domestic	κ^d	8.6	average U.S. establishment size: 16.0
fixed cost trade	κ^e	40.9	fraction of U.S. exporting establishments: 21 percent
enforcement	θ	0.38	debt/revenue of firms of age less than 5 years: 0.27
Pareto distribution parameter	γ	4.03	std. dev. of U.S. establishment size: 91.2
death rate	δ	0.10	U.S. establishment death rate: 10 percent per year
discount factor	β	0.98	U.S. real interest rate: 4 percent per year
entrant productivity growth	g	1.02	U.S. output growth rate: 2 percent per year

There are three parameters that we do not calibrate. We set the elasticity of substitution across goods, $1/(1-\rho)$, to three. This elasticity of substitution is similar to that of Simonovska and Waugh (2014). These authors find the elasticity of substitution for a Melitz (2003) model to be 3.69. We set the mass of potential entrants, μ , and the minimum productivity level, \underline{x} , to one. Given our assumption that firm productivities are distributed according to a Pareto distribution, these final two parameter choices are without loss of generality as long as the mass of potential entrants is large enough and the minimum productivity is low enough that the entry cutoffs are always strictly greater than $\underline{x}g^t$.

4.2. Creating a benchmark distorted economy

Using the calibrated model, we create a benchmark distorted economy that has all three distortions (high entry costs, high trade costs, and poor enforcement of contracts). We use this benchmark distorted economy to study the optimal sequence of reforms. The spirit of the exercise is that this economy has the same technology and preferences as the United States, but the economies have different levels of distortions.

As a first step, we solve for the balanced growth path of three economies, each of which is distorted by a single policy. In the first economy, we raise entry barriers so that output drops by three percent, which requires increasing κ^d from 8.6 to 10.0. In the second distorted economy, we raise trade costs so that output drops by three percent, which requires raising κ^e

from 40.9 to 91.9. In the third distorted economy, we lower contract enforcement so that output drops by three percent, which requires lowering θ from 0.39 to 0.32.

The distortions we have imputed above are comparable, as each of them results in a decline in income of three percent in the balanced growth path. The *benchmark distorted economy* is the one that has all three distortions. Since we are considering the case of symmetric reforms, the same parameters are used for both domestic and foreign countries.

4.3. Comparing balanced growth paths

In this section, we study the interaction among the three policies using information from the balanced growth path. In the next section, we consider the transitions paths from the reforms and the welfare effects of the reform sequences.

Table 2 reports the policy parameters for each balanced growth path along with their corresponding income levels. The income levels have been normalized to the benchmark distorted economy for easier comparison. Notice that reforming trade costs induces the largest increase in income when starting from the benchmark distorted economy, even though all reforms increase income by the same amount when they are the only distortion present.

Table 2: Balanced growth path output gain from reform

reforms	κ^d	κ^e	θ	Y/L (benchmark = 100)
no reforms (benchmark)	10.0	91.9	0.32	100.00
enforcement	10.0	91.9	0.39	103.17
entry costs	8.6	91.9	0.32	103.44
trade costs	10.0	40.9	0.32	103.71
entry costs and enforcement	8.6	91.9	0.39	106.75
trade costs and enforcement	10.0	40.9	0.39	106.75
entry costs and trade costs	8.6	40.9	0.32	106.75
all reforms (United States)	8.6	40.9	0.39	110.01

We categorize policy pairs as being either complementary or substitutable. Two policies are *substitutes* if, once a country has enacted one reform, the percentage increase in GDP from

enacting the other reform decreases. Similarly, two policies are *complements* if, once a country has enacted one reform, the percentage increase in GDP from enacting the other reform increases.

We can determine whether policies are substitutes or complements using the information in Table 2. For example, suppose that we begin with all three distortions present. We find that reducing trade costs and reducing entry costs are substitutes. To arrive at this conclusion, we observe that reducing trade costs increases output by 3.71 percent (from 100.0 to 103.71). If the economy already had lower entry costs, however, the same reduction in trade costs increases output by only 3.20 percent (from 103.44 to 106.75). We summarize our findings in Table 3: Reforms that reduce trade costs are substitutable with the other reforms, but contract enforcement and entry barriers are complementary.

Table 3: Complementarity and substitutability of reforms

reform #1	reform #2	
trade costs	entry costs	substitutable
trade costs	enforcement	substitutable
enforcement	entry costs	complementary

We can gain intuition into why policies are substitutes or complements by analyzing how the number of varieties available to households changes in each balanced growth path. To do so, we decompose the change in the total number of varieties available to consumers,

$$\frac{\Delta V}{V} = \frac{\Delta D^{NE}}{V} + \frac{\Delta D^E}{V} + \frac{\Delta F^E}{V}, \quad (48)$$

where V is the mass of varieties available to consumers, D^{NE} is the mass of domestic non-exporters, D^E is the mass of domestic exporters, and F^E is the mass of foreign exporters.

The results from this decomposition are reported in Table 4, which has been sorted in descending order of the percentage change in total varieties. We find that reforms can lead to very different outcomes in the composition of firms in the economy. When we reduce trade costs, we find a small increase in the total varieties available to consumers, 0.4 percent. Reducing trade costs leads to fewer inefficient firms that exist to only serve the domestic market, but this reduction in domestic-oriented firms is offset by increases in domestic exporting firms and foreign exporters.

Table 4: Change in varieties from one reform

reform	percent change			
	total varieties	domestic non-exporters	domestic exporters	foreign exporters
enforcement	22.3	23.2	-0.5	-0.5
entry costs	17.9	19.5	-0.8	-0.8
trade costs	0.4	-38.4	19.4	19.4

Notes: Columns report the percent change, from the initial balanced growth path to the new balanced growth path, when a single reform is implemented in the benchmark model.

The model generates larger increases in the number of varieties with improvements in contract enforcement (22.3 percent) and reductions in entry costs (17.9 percent). This increase in the number of varieties comes from firms that only serve the domestic market. Reforming contract enforcement or firm entry costs has a small negative impact on export activity.

Notice that the number of exporting firms declines even though reforms to contract enforcement benefit exporting firms by improving their ability to borrow to pay the trade cost. This outcome is the result of two opposing forces in the model. First, after a reform, firms find it easier to enter the export market, which increases the number of exporters. This can be seen by examining the effective trade cost that firms face in (37): An increase in θ leads to a reduction in the effective trade cost. Second, the general equilibrium effects from the surge of new firms crowds out exporting firms by increasing the real wage. The increase in the real wage dominates, and the number of exporting firms shrinks. In the appendix, we report the decomposition in (48) for an economy implementing a second and third reform (Table 9 and Table 10). We consistently find that reforms to enforcement lead to the largest increase in the number of varieties available to consumers, followed by reforms to entry costs. Furthermore, it is always the case that reforms to enforcement and entry costs, through general equilibrium effects, lead to a crowding out of exporting firms.

The results in Table 4 highlight the forces that drive the complementarity or substitutability of reforms. Reforming enforcement or entry costs makes entry easier for relatively inefficient firms, increasing the number of non-exporting firms. Trade liberalization, however, decreases the number of non-exporting firms and transfers resources to exporting firms. When these two types of policies are implemented together, they work against each other,

reducing the effectiveness of the second reform. Reforming enforcement costs and entry costs together combines two policies that increase the number of non-exporting firms, making the second reform more effective.

The distribution of firm types in the economy is reflected in the aggregate price index. Reforms to enforcement or entry costs lead to more firms, which lowers the price index. Reforms to trade costs lead to an expansion of low-price firms, which also lowers the price index. In Table 5, we report the percentage change in the price index along with the domestic and import price indexes. As expected, we see that all reforms lead to reductions in the overall price index. Reforms to entry costs or enforcement work through the domestic price index, while reforms to trade costs work through the import price index.

Table 5: Change in the price index from one reform

reform	percent change		
	price index	domestic price index	import price index
enforcement	-4.1	-5.4	0.9
entry costs	-3.1	-4.5	2.8
trade costs	-4.9	6.0	-27.3

Notes: Columns report the percent change, from the initial balanced growth path to the new balanced growth path, when a single reform is implemented in the benchmark model.

The behavior of the price index allows us to see why reforming the trade sector leads to the largest gain in balanced growth path output (Table 2) even though the reform generates the smallest change in the number of varieties available for consumption (Table 4). Reform to enforcement or entry costs increases the number of relatively low-productivity firms that can profitably produce for the domestic market. Following these reforms, the domestic price index falls modestly. This decrease in the price index is driven mostly by the increase in varieties — the new entrants charge relatively high prices. Trade reform, in contrast to the other two, generates a large decrease in the import price index, which leads to the largest overall decline in the aggregate price index, and, thus, the greatest increase in real output among the three reforms. The decrease in the import price index comes from two sources. First, new imported varieties from abroad increase the varieties available for consumption. Second, these imported varieties are produced by relatively high-productivity firms in the other country, so the new imported

varieties are sold at low prices. This second effect generates a stronger decrease in the import price index.

4.4. Evaluating the welfare gains from one reform

As a next step, we compute the transition path for the case in which one unexpected reform is implemented. In Sections 4.5 and 4.6, we will measure the welfare gains in the case in which two and three reforms are implemented.

Table 6 reports the welfare gains from conducting each of the three reforms in terms of the change in permanent real income. We calculate permanent real income as

$$\exp\left[(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}\log C_{it}\right]. \quad (49)$$

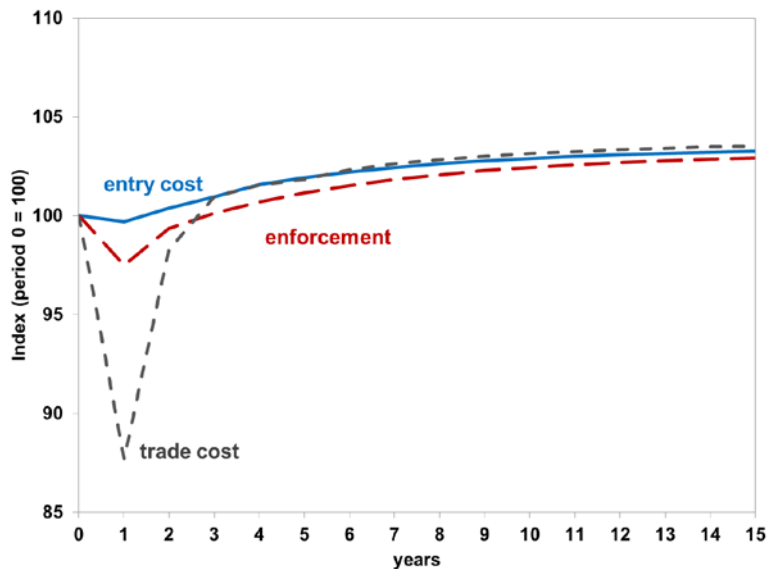
First, we see that the welfare differences across reforms can be large, amounting to 0.40 percent in terms of real income. Second, we find that, although the reform to trade costs results in the highest balanced growth path consumption level (Table 2), it is reforming entry costs that results in the highest welfare gain once we consider the transition dynamics. The intuition behind this result can be seen in Figure 3, where we plot consumption, detrended by g^t , following the reform. Compared with the entry cost reform, we observe a larger decrease in consumption following the trade cost reform, because there is a large increase in the number of firms that pay the fixed trade cost. This initial drop in consumption results in the reform to entry costs being more beneficial, even though the trade cost reform eventually results in higher consumption. Finally, we find that, after reform, the economy takes significant time to reach the new balanced growth path. For example, in the case of trade reforms, it takes the economy 17 years to be 0.1 percent away from the new balanced growth path.

Table 6: Welfare gains from one reform

reform	real income index (100 = no reform)
entry costs	103.09
trade costs	103.02
enforcement	102.69

Notes: Real income is computed according to (49). The calculation includes the transition from the initial balanced growth path to the new balanced growth path when a single reform is implemented in the benchmark model.

Figure 3: Detrended consumption for one reform



4.5. Evaluating the gains from conducting two reforms

Next, we evaluate the gains from enacting two reforms. The second reform takes place four years after the first. We impose the four-year lags to reflect political and administrative constraints in implementing reform. The first reform is unexpected, but, after the first reform, agents foresee the second reform.

Table 7 reports the welfare gains, in descending order, from implementing each possible sequence of two reforms. We find significant differences in welfare outcomes. The difference

between the best and worst reforms is 0.34 percent of real income, which is equivalent to 17.3 percent of first period consumption.

Table 7: Welfare gains from two reforms

reform #1	reform #2	real income index (100 = no reform)
entry costs	enforcement	105.70
enforcement	entry costs	105.62
trade costs	entry costs	105.53
entry costs	trade costs	105.50
trade costs	enforcement	105.47
enforcement	trade costs	105.36

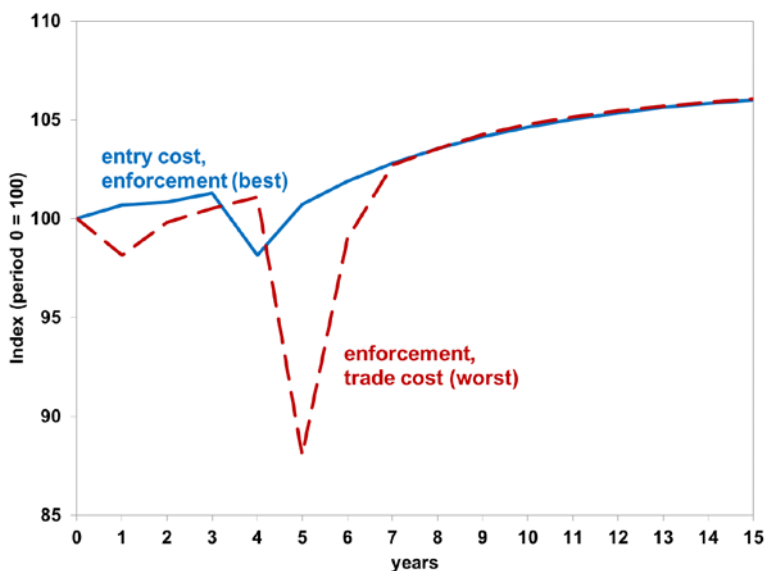
Notes: Real income is computed according to (49). The calculation includes the transition from the initial balanced growth path to the new balanced growth path when two reforms are implemented in the benchmark model.

The best sequence of reforms first lowers entry costs and then improves contract enforcement. The second best reform sequence is the same pair of reforms, but with their order reversed. Notice that, if only two reforms are conducted, then policy makers should avoid reductions in trade costs. This is for two reasons. First, as can be seen in Figure 4, trade cost reforms induce a larger initial drop in consumption compared with other reforms. Second, trade reforms are substitutable to the other reforms, while reforms to entry costs and contract enforcement are complementary.

If trade costs are to be reformed, then sequencing plays an important role: It is preferable to lower trade costs before conducting other reforms. This difference is most stark in the case of reforming trade costs and then enforcement. The gains from this sequence are 0.11 percent higher than from the sequence in which we reform enforcement and then trade costs. This is because, if we reform entry costs or enforcement first, we increase the number of relatively inefficient non-exporting firms in the economy. If we then reform trade costs, the selection induced by the exporting decision will make these new firms obsolete. The hysteresis generated by the entry cost means that the economy will carry these inefficient firms for several periods before they eventually die off. The optimal sequencing of reforms imposes the selection from

reducing trade costs first, so that the reform to enforcement or to entry costs does not generate the entry of soon-to-be unwanted firms.

Figure 4: Detrended consumption for two reforms



4.6. Evaluating the gains from conducting three reforms

To determine the optimal sequence of reform, we compute the equilibrium of the model under six possible sequences of three reforms. As before, the first reform is unexpected by agents. After the first reform takes place, the agents foresee reforms two and three, which take place four years and eight years after the initial reform.

In Table 8, we report, in descending order, the welfare gain from the six possible reform sequences. We find that the difference between the best and worst sequence is 0.11 percent of real income, which is equivalent to 4.8 percent of first period consumption. Since each sequence of reforms generates the same balanced growth path, this increase in welfare is solely the result of differences in the transition paths that arise from the ordering of reforms.

Table 8 indicates that the best reform sequences involve reducing trade costs first and the worst reform sequences involve improving enforcement first. We also find that improving enforcement before lowering trade costs is a poor combination. It is striking to note that in the best and worst sequences, entry cost reforms occur last: The difference is the ordering of reforms to trade costs and enforcement.

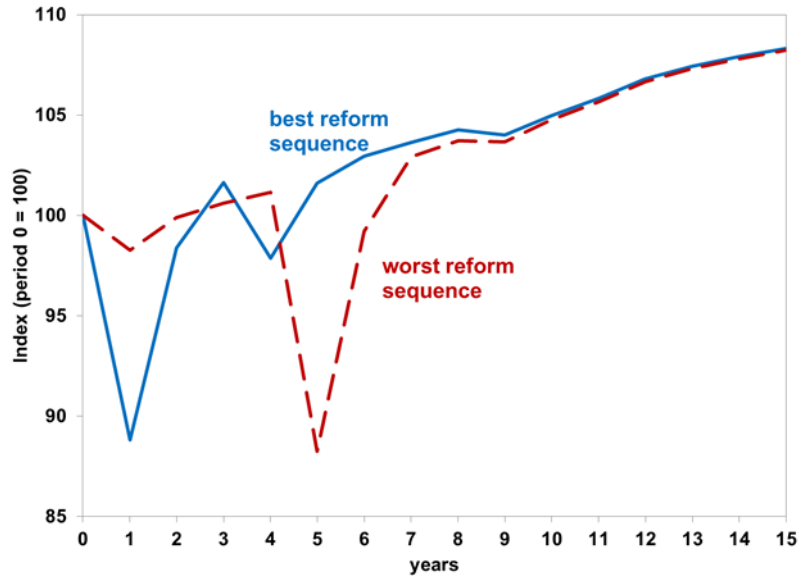
Table 8: Welfare gains from three reforms

reform #1	reform #2	reform #3	real income index (100 = no reform)
trade costs	enforcement	entry costs	107.91
trade costs	entry costs	enforcement	107.90
entry costs	enforcement	trade costs	107.89
entry costs	trade costs	enforcement	107.88
enforcement	entry costs	trade costs	107.81
enforcement	trade costs	entry costs	107.80

Notes: Real income is computed according to (49). The calculation includes the transition from the initial balanced growth path to the new balanced growth path when three reforms are implemented in the benchmark model.

In Figure 5 we plot the evolution of detrended consumption in the best and worst reform sequences. Reforms take place in years one, five, and nine. First, notice the dip in consumption that takes place when trade costs are lowered. This is driven by the entry of new exporters, who divert labor from production to pay the fixed cost to enter the export market. This dip in consumption is smaller in the case of the best reform sequence (-9.4 vs. -11.0 percent). Second, notice that, following the reform in trade costs, consumption in the worst reform sequence is consistently below that of the best until they converge to the same balanced growth path.

Figure 5: Detrended consumption for three reforms



As in the two-reform case — and for the same reasons — we find that trade costs should be reformed before entry costs or enforcement. Decreasing trade barriers leads to a strong selection on productivity that makes it harder for less efficient firms to compete. In Figure 6 and Figure 7, we plot the mass of exporters and the mass of non-exporters in the best and worst reform sequences. Each sequence of reforms will eventually generate the same distribution of firms, but the transition paths can be quite different and lead to large and persistent differences in the composition of firms in operation.

In the best reform sequence, the mass of exporters begins to grow immediately, and the initial decline in non-exporting firms is driven mostly by the conversion of non-exporting firms to exporting firms. Converting non-exporting firms to exporting firms takes time, as constrained firms need to improve their balance sheets before they are able to finance the trade costs. In year five, the mass of exporters sharply increases, as the reform to contract enforcement increases the availability of finance. The decline in the mass of non-exporters levels off in year five as enforcement reform lowers the productivity threshold for entry into the domestic market. When entry costs are reformed in year nine, the mass of non-exporters begins to increase as it converges to its balanced growth path.

In the worst reform sequence, the improvement in enforcement leads to an increase in non-exporting firms. This inflow of new firms increases the real wage, which has a slight negative effect on the mass of exporters. In year five, the trade costs are reformed. This reform generates an increase in exporters and an increase in the productivity level needed to profitably enter the domestic market. Since reforms to enforcement already occurred, the economy had already increased its stock of low productivity non-exporters. These firm types are no longer profitable from the point of view of a new entrant, but the firms that are already in the market will remain until they die exogenously. Carrying these inefficient firms along the transition path decreases the gains from this sequence of reforms.

Figure 6: Mass of exporters

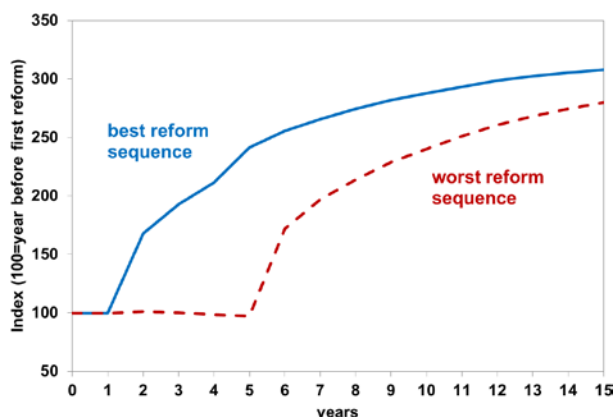
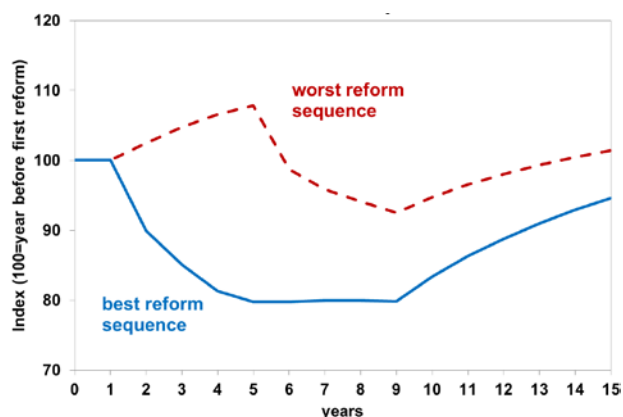


Figure 7: Mass of non-exporters



5. Conclusion

In this paper, we construct a two-country dynamic general equilibrium model with three potential policy distortions: entry costs, trade costs, and poor enforcement of contracts. We calibrate the model to the United States and subsequently create a benchmark distorted economy in which all three distortions are present. We use the model to quantitatively study the optimal sequencing of reforms. Our findings indicate that the order in which reforms are conducted has an impact on the gains from these reforms. In particular, we find that if a country undertakes three reforms, then it should first reduce trade costs. Furthermore, we find that the sequencing of reforms has an impact on the distribution of firms for a significant number of years. If a country is going to eventually reform foreign trade and investment, it is better to do so early in its industrialization

process so that the economy has a composition of firms that are more suited to competing in international markets.

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Appendix A: Additional tables

Table 9: Change in varieties from removing a second distortion

reform #1	reform #2	percent change			
		total varieties	domestic non-exporters	domestic exporters	foreign exporters
entry costs	enforcement	22.5	23.2	-0.4	-0.4
trade costs	enforcement	18.8	20.9	-1.1	-1.1
enforcement	entry costs	18.0	19.3	-0.6	-0.6
trade costs	entry costs	14.5	17.3	-1.4	-1.4
entry costs	trade costs	-2.4	-34.3	15.9	15.9
enforcement	trade costs	-2.5	-33.2	15.3	15.3

Notes: Columns report the percent change, from the initial balanced growth path with one reform to the new balanced growth path, when a second reform is implemented.

Table 10: Change in varieties from removing a third distortion

reform #1	reform #2	percent change			
		total varieties	domestic non-exporters	domestic exporters	foreign exporters
trade costs / entry costs	enforcement	19.8	23.1	-1.7	-1.7
trade costs / enforcement	entry costs	15.5	19.2	-1.9	-1.9
entry costs / enforcement	trade costs	-4.6	-28.6	12.0	12.0

Notes: Columns report the percent change, from the initial balanced growth path with two reforms to the new balanced growth path, when a third reform is implemented.

Appendix B: Proofs

Proof of lemma 1:

The enforcement constraint does not bind for a firm with no debt. Thus, we will derive an expression for $V_{i,t+k}^n(b_{i,k-1,t+k}(x), x)$ of an age k firm that holds debt. The value of a firm that never exports (12) can be written as

$$V_{i,t+k}^n(b_{i,k-1,t+k}(x), x) = -b_{i,k-1,t+k}(x) + \sum_{m=k}^{\infty} \left(\prod_{s=k+1}^m (1-\delta) q_{it+s} \right) \pi_{it+m}^d(x). \quad (50)$$

The optimal bond policy of a domestic firm that never exports is given by

$$b_{i,k-1,t+k}(x) = \max \left\{ \frac{w_{it} \kappa_i^d - \sum_{m=1}^{k-1} \left(\prod_{s=1}^m (1-\delta) q_{it+s} \right) \pi_{it+m}^d(x)}{\prod_{s=1}^k (1-\delta) q_{i,t+s}}, 0 \right\}. \quad (51)$$

This means that the firm will use all profits to pay down debt, and once all debts are paid off, the firm will pay all profits in dividends. Because firms discount future dividends by exactly the price at which firms issue debt, they are indifferent about the sequence of paying down debt and issuing dividends. However, this bond policy relaxes the enforcement constraints the most, since it involves the earliest repayment of debt. Substituting (51) into (50), we obtain

$$V_{i,t+k}^n(b_{i,k-1,t+k}(x), x) = \frac{1}{\prod_{s=1}^k (1-\delta) q_{i,t+s}} \left(\sum_{m=1}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{it+s} \right) \pi_{it+m}^d(x) - w_{it} \kappa_i^d \right) \quad (52)$$

Hence the enforcement constraint in (30) can be written as

$$\sum_{m=1}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{i,t+s} \right) \pi_{i,t+m}^d(x) - w_{it} \kappa_i^d \geq (1-\theta_i) \sum_{m=k}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{i,t+s} \right) \pi_{i,t+m}^d(x). \quad (53)$$

The left side of (53) does not depend on age k , while the right side is decreasing in age k . Thus, the constraint can only hold with equality at $k=1$. \square

Proof of lemma 2:

The enforcement constraint does not bind for a firm with no debt. Thus, we will derive an expression for $V_{i,h,t-k+h}^e(b_{i,h-1,t-k+h}(x), x)$ of an age h exporting firm that holds debt. The problem in (11) can be written as

$$V_{i,h,t-k+h}^e(b_{i,h-1,t-k+h}(x), x) = -b_{i,h-1,t-k+h}(x) + \sum_{m=h}^{\infty} \left(\prod_{s=h+1}^m (1-\delta) q_{i,t-k+s} \right) (\pi_{i,t-k+m}^d(x) + \pi_{i,t-k+m}^e(x)). \quad (54)$$

The optimal bond policy of an age h firm that pays the trade cost at age $\ell < h$ is given by

$$b_{i,h-1,t-k+h}(x) = \max \left\{ 0, \frac{1}{\prod_{s=1}^h (1-\delta) q_{i,t-k+s}} \left[\begin{aligned} & w_{i,t-k} \kappa_i^d + w_{j,t-k+\ell} \kappa_i^e \prod_{s=1}^{\ell} (1-\delta) q_{i,t-k+s} \\ & - \sum_{m=1}^{\ell} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) \pi_{i,t-k+m}^d(x) \\ & - \sum_{m=\ell+1}^{h-1} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) (\pi_{i,t-k+m}^d(x) + \pi_{i,t-k+m}^e(x)) \end{aligned} \right] \right\}. \quad (55)$$

This means that the firm will use all profits to pay down debt, and once all debts are paid off, the firm will pay all profits in dividends. Because firms discount future dividends by exactly the price at which firms issue debt, they are again indifferent about the sequence of paying down debt and issuing dividends. However, this bond policy relaxes the enforcement constraints the most, since it involves the earliest repayment of debt. Substituting (55) into (54), we obtain

$$V_{i,h,t-k+h}^e(b_{i,h-1,t-k+h}(x), x) = \frac{1}{\prod_{s=1}^h (1-\delta) q_{i,t-k+s}} \left(\begin{aligned} & \sum_{m=1}^{\ell} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) \pi_{i,t-k+m}^d(x) \\ & + \sum_{m=\ell+1}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) (\pi_{i,t-k+m}^d(x) + \pi_{i,t-k+m}^e(x)) \\ & - w_{i,t-k} \kappa_i^d - w_{j,t-k+\ell} \kappa_i^e \prod_{s=1}^{\ell} (1-\delta) q_{i,t-k+s} \end{aligned} \right) \quad (56)$$

Hence, the enforcement constraint for age $h \geq \ell + 1$ in (34) can be written as

$$\begin{aligned}
& \sum_{m=1}^{\ell} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) \pi_{i,t-k+m}^d(x) + \sum_{m=\ell+1}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) \left(\pi_{i,t-k+m}^d(x) + \pi_{i,t-k+m}^e(x) \right) \\
& - w_{i,t-k} \kappa_i^d - w_{j,t-k+\ell} \kappa_i^e \prod_{s=1}^{\ell} (1-\delta) q_{i,t-k+s} \geq (1-\theta) \sum_{m=h}^{\infty} \left(\prod_{s=1}^m (1-\delta) q_{i,t-k+s} \right) \left(\pi_{i,t-k+m}^d(x) + \pi_{i,t-k+m}^e(x) \right).
\end{aligned} \tag{57}$$

The left side of (57) does not depend on age h , while the right side is decreasing in age h . Thus, the constraint can only hold with equality at $h = \ell + 1$. \square

Proof of proposition 1:

The proof of proposition 1 involves guessing and verifying the existence of an equilibrium with a balanced growth path.

From the first order condition of the household and applying the balanced growth path conditions, we obtain $q_{t+1} = \beta/g$.

Next, using (5) and (8), we can derive

$$\left(\frac{w_{it}}{P_{it}} \right)^{\frac{\rho}{1-\rho}} = \frac{\rho^{\frac{\rho}{1-\rho}} \gamma(1-\rho)}{\gamma(1-\rho) - \rho} \sum_{k=1}^{\infty} (1-\delta)^k g^{\gamma(t-k)} \left(\mu_i \underline{x}_i^{\gamma} \hat{x}_{ikt}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + \mu_j \underline{x}_j^{\gamma} \left(\frac{w_{jt}}{w_{it}} \right)^{\frac{-\rho}{1-\rho}} \hat{x}_{jkt}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right). \tag{58}$$

Using (43), we find that

$$\begin{aligned}
& \frac{A_{it}}{w_{it}} + \lambda_{it}^d \kappa_i^d + \frac{w_{jt}}{w_{it}} \lambda_{it}^e \kappa_i^e = \\
& (1-\rho) Y_{it} \left(\frac{w_{it}}{P_{it}} \right)^{\frac{-1}{1-\rho}} \frac{\rho^{\frac{\rho}{1-\rho}} \gamma(1-\rho)}{\gamma(1-\rho) - \rho} \mu_i \underline{x}_i^{\gamma} \sum_{k=1}^{\infty} (1-\delta)^k g^{\gamma(t-k)} \left\{ \hat{x}_{ikt}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} + \left(\frac{P_{jt}}{P_{it}} \right)^{\frac{1}{1-\rho}} \frac{Y_{jt}}{Y_{it}} \hat{x}_{jkt}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} \right\}.
\end{aligned} \tag{59}$$

Using the expression for cutoffs from (31), (36), (38), and (40), and the balanced growth path conditions, $w_{it} = gw_{it-1}$, $P_{it} = P_i$, and $A_{it} = gA_{it-1}$, we obtain

$$\left(\frac{w_{it}}{P_{it}} \right)^{\gamma} = \rho^{\gamma} \frac{\gamma(1-\rho)}{\gamma(1-\rho) - \rho} g^{\gamma t} \left[(1-\rho) \left(L_i + \frac{A_{it}}{w_{it}} \right) \right]^{\frac{\gamma(1-\rho) - \rho}{\rho}} v_i, \tag{60}$$

and

$$\frac{A_{it}(\kappa, \theta)}{w_{it}(\kappa, \theta)} = \frac{L_i \xi_i}{1 - \xi_i}, \tag{61}$$

where

$$\xi_i = (1-\rho) \frac{v_i^d + (\Delta_i^P)^{\frac{1}{1-\rho}} \Delta_i^Y v_i^e - \frac{\gamma(1-\rho) - \rho}{\gamma(1-\rho)} \tilde{\xi}_i}{v_i}, \quad (62)$$

$$v_i = v_i^d + (\Delta_i^w)^{\frac{-\rho}{1-\rho}} \left(\frac{\Delta_i^P \Delta_i^Y}{\Delta_i^w} \right)^{\frac{\gamma(1-\rho) - \rho}{\rho}} \left(\frac{\Delta_i^w}{\Delta_i^P} \right)^{\frac{\rho - \gamma(1-\rho)}{1-\rho}} v_j^e, \quad (63)$$

$$v_i^d = \sum_{k=1}^{\infty} (1-\delta)^k g^{-k \frac{\rho}{1-\rho}} \mu_i \underline{x}_i^\gamma \tilde{\kappa}_i^d \frac{\rho - \gamma(1-\rho)}{\rho}, \quad (64)$$

$$v_j^e = \sum_{k=1}^{\infty} (1-\delta)^k g^{-k \frac{\rho}{1-\rho}} \mu_j \underline{x}_j^\gamma \tilde{\kappa}_j^e \frac{\rho - \gamma(1-\rho)}{\rho}, \quad (65)$$

$$\tilde{\xi}_i = \mu_i \underline{x}_i^\gamma \left\{ \kappa_i^d \tilde{\kappa}_i^d \frac{-\gamma(1-\rho)}{\rho} + \Delta_i^w \kappa_i^e \left[\tilde{\kappa}_{i,0}^e \frac{-\gamma(1-\rho)}{\rho} + \sum_{k=1}^{\hat{n}_i} (1-\delta)^k \left(\tilde{\kappa}_{i,k}^e \frac{-\gamma(1-\rho)}{\rho} - \tilde{\kappa}_{i,k-1}^e \frac{-\gamma(1-\rho)}{\rho} \right) \right] \right\}. \quad (66)$$

By substituting (61) into (60) we obtain

$$\frac{w_{it}}{P_{it}} = g^t \rho \left[\frac{\gamma(1-\rho)}{\gamma(1-\rho) - \rho} v_i \right]^{\frac{1}{\gamma}} \left[\frac{(1-\rho)L_i}{1-\xi_i} \right]^{\frac{\gamma(1-\rho) - \rho}{\gamma\rho}}. \quad (67)$$

Finally, using the balanced trade condition in (25), we obtain the relative prices:

$$\left(\Delta_i^P \right)^{\frac{-\rho}{1-\rho}} = \left(\Delta_i^P \right)^{\frac{1}{1-\rho}} \Delta_i^Y \frac{v_j v_i^e}{v_i v_j^e}. \quad (68)$$

Thus, our guess has been verified and all optimality conditions are satisfied. \square