Lecture notes on the Kehoe-Ruhl (2009) model of sudden stops. A MatLab/Octave library.

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Contents

1 Introduction

These lecture notes provide the basic programs to replicate the numerical experiments carried out by Kim Ruhl and Timothy Kehoe on sudden stops titled, "Sudden Stops, Sectoral Reallocations, and the Real Exchange Rate," published in the Journal of Development Economics, 89(2), 235-249. This paper can be freely downloaded from the NBER site as a working paper or directly from the web page of their authors. The second link provided below, also contains all the data required for the calibration.

- http://www.nber.org/papers/w14395
- http://www.econ.umn.edu/~tkehoe/publications.html
- http://www.kimjruhl.com/research/

The notation used in these notes is the same as the notation used in the original paper and in the MatLab programs to increase their readability. Program names will be referred to in bold face like in **program.m**, and the program bodies will be written in

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verbatim
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font.

The MatLab library provided with these notes is divided in as many folders as sections are in the notes. The name of the sections coincide with the name of the folder containing the programs. Notice that programs with the same name in different folders can be different.

The folders with the MatLab library are the following:

- Steady_State0: A group of MatLab programs that solve any steady state depending of the parameter values given at the beginning of the main scrip file called **Steady_State.m**. In this folder a program called **secant.m** written by Carlos Urrutia performs Newton's iterations over a function file called **SScpo.m** where the steady state equations are written. Exercise 1, should be solved using this set of programs.
- Calibration: are essentially the same programs that run in the folder Steady_State0. The particularity of this group of programs is that they read from an Excel file, present in that folder, where the input-output matrix is written. That Excel file is called **HojaCalibracion.xlsx.** In that input-output matrix you can write the economy you like, provided that the numbers are consistent (in a National Accounts sense) and that the fields are the same. Notice that the main script (**Sudden_Stops.m** again) invokes the following MatLab command: **xlsread('HojaCalibracion.xlsx', 'A1:I10');** The text you introduce will not be used in the program, and therefore you are free to write the labels of rows and columns with the text you like. With that set of programs you can see how different input-output values are transmitted into the parameter values of your calibrated

economy. By default, the values in the calibration coincide with the values in the paper. Exercise 2, shall be solved with this set of programs, were you are asked to anticipate the effects on each parameter value, after a change in the input-output table.

- ComparativeStatics: This set of programs provides a tool to think about the effects on the steady state values of the variables after a change in the parameter values. The paper has the challenging task of explaining the evolution of sectoral reallocation of productive resources, such as sectoral employment and capital utilization in a context of a sudden stop. A fall in TFP and output is not an easy task because some natural equilibrating mechanisms have to be inhibited. The set of programs in this folder provide a tool to visualize the expected effects on the steady state of the model economy when a parameter value changes. Exercises 3 and 7 should make use of this library.

- Dynamics3Periods: This set of programs is quite auxiliary. Its aim is to show that the most appropriate way of constructing a model is by parts. Once you have the set of equations characterizing a steady state of a model economy, the most secure way of building the dynamics is trough a simple 3 periods dynamic model. It is easier to generalize to a n periods model once you have a clear picture of how to solve it with the 3 periods in your computer.

- DynamicsNperiods: Is the straightforward generalization of the set of equations written in the Dynamics3Periods model. This program is the base for exercises 4, 5 and 6.

The pages that follow try to be self explanatory and exhaustive in the mechanics of the paper. Read carefully the introduction of the paper to understand the goals pursued therein and complete the exercises to come to a complete understanding of the challenge that debt crises pose to modern economic theory.

2 The model

2.1 Households

Households maximize their lifetime utility given by:

$$\sum_{t=1}^{\infty} \beta^t \left(\left(\varepsilon \left(\frac{c_{Tt}}{n_t} \right)^{\rho} + (1-\varepsilon) \left(\frac{c_{Nt}}{n_t} \right)^{\rho} \right)^{\frac{\eta \Psi}{\rho}} \left(\frac{\bar{l}_t - l_t}{\bar{l}_t} \right)^{(1-\eta)\Psi} - 1 \right) / \Psi$$

Subject to a budget constraint [Eq 13 of the paper]:

$$p_{Tt} c_{Tt} + p_{Nt} c_{Nt} + q_t i_t + b_{t+1} = w_t l_t + (1+r_t) b_t + r_{kt} k_t + T_t \tag{1}$$

and a law of motion for capital [Eq 14 of the paper]:

$$k_{t+1} = k_t(1-\delta) + i_t \tag{2}$$

together with given data for k_1, b_1 and a constraint [Eq 15 of the paper] that rules out Ponzi schemes and guarantees the fulfillment of the appropriate transversality condition for b_t.

$$b_t \geq -n_t g^t B \tag{3}$$

Consumers choose their labor supply l_t, consumption of traded and non traded goods, c_{Tt} and c_{Nt}, investment on capital i_t, and bond holdings, b_t, to maximize utility. The adult equivalent population is [Eq 11 of the paper]

$$n_t = \bar{l}_t + \frac{1}{2}\left(\widetilde{n}_t - \bar{l}_t\right) \tag{4}$$

Where $\bar{l}_t$ is working age population and $\widetilde{n}_t$ denotes total population.

The Lagrange function associated to the utility maximization program is:

$$\begin{aligned} L_H \quad = \quad & \sum_{t=1}^{\infty} \beta^t \left[\left(\left(\varepsilon \left(\frac{c_{Tt}}{n_t} \right)^{\rho} + (1-\varepsilon) \left(\frac{c_{Nt}}{n_t} \right)^{\rho} \right)^{\frac{\eta\Psi}{\rho}} \left(\frac{\bar{l}_t - l_t}{\bar{l}_t} \right)^{(1-\eta)\Psi} - 1 \right) / \Psi + \right. \\ & \left. \lambda_{t+1} \left(w_t l_t + (1+r_t) b_t + r_{kt} k_t + T_t - p_{Tt} c_{Tt} - p_{Nt} c_{Nt} - q_t \left(k_{t+1} - k_t(1-\delta) \right) - b_{t+1} \right) \right] \end{aligned}$$

And the set of first order conditions is:

$$\begin{aligned} \frac{\partial L_H}{\partial c_{Tt}} \quad &= \quad \frac{\beta^t}{\Psi} \left(\frac{\bar{l}_t - l_t}{\bar{l}_t} \right)^{(1-\eta)\Psi} \left(\varepsilon \left(\frac{c_{Tt}}{n_t} \right)^{\rho} + (1-\varepsilon) \left(\frac{c_{Nt}}{n_t} \right)^{\rho} \right)^{\frac{\eta\Psi}{\rho}-1} \frac{\eta\Psi}{\rho} \frac{\varepsilon\rho}{n_t^{\rho}} c_{Tt}^{\rho-1} - \beta^t \lambda_{t+1} p_{Tt} = 0 \\ \frac{\partial L_H}{\partial c_{Nt}} \quad &= \quad \frac{\beta^t}{\Psi} \left(\frac{\bar{l}_t - l_t}{\bar{l}_t} \right)^{(1-\eta)\Psi} \left(\varepsilon \left(\frac{c_{Tt}}{n_t} \right)^{\rho} + (1-\varepsilon) \left(\frac{c_{Nt}}{n_t} \right)^{\rho} \right)^{\frac{\eta\Psi}{\rho}-1} \frac{\eta\Psi}{\rho} \frac{(1-\varepsilon)\rho}{n_t^{\rho}} c_{Nt}^{\rho-1} - \beta^t \lambda_{t+1} p_{Nt} = 0 \\ \frac{\partial L_H}{\partial l_t} \quad &= \quad -\frac{\beta^t}{\Psi} \left(\varepsilon \left(\frac{c_{Tt}}{n_t} \right)^{\rho} + (1-\varepsilon) \left(\frac{c_{Nt}}{n_t} \right)^{\rho} \right)^{\frac{\eta\Psi}{\rho}} (1-\eta)\Psi \left(\frac{\bar{l}_t - l_t}{\bar{l}_t} \right)^{(1-\eta)\Psi-1} \frac{1}{\bar{l}_t} + \beta^t \lambda_{t+1} w_t = 0 \\ \frac{\partial L_H}{\partial k_t} \quad &= \quad \beta^t \lambda_{t+1} \left(r_{kt} + q_t(1-\delta) \right) - \beta^{t-1} \lambda_t q_{t-1} = 0 \\ \frac{\partial L_H}{\partial b_t} \quad &= \quad \beta^t \lambda_{t+1} (1+r_t) - \beta^{t-1} \lambda_t = 0 \end{aligned}$$

Simplify to:

$$\begin{aligned}
\frac{\partial L_H}{\partial c_{Tt}} &= \left(\frac{\bar{l}_t - l_t}{\bar{l}_t}\right)^{(1-\eta)\Psi} \left(\varepsilon\left(\frac{c_{Tt}}{n_t}\right)^{\rho} + (1-\varepsilon)\left(\frac{c_{Nt}}{n_t}\right)^{\rho}\right)^{\frac{\eta\Psi}{\rho}-1} \frac{\eta\varepsilon}{n_t^{\rho}} c_{Tt}^{\rho-1} - \lambda_{t+1} p_{Tt} = 0 \\
\frac{\partial L_H}{\partial c_{Nt}} &= \left(\frac{\bar{l}_t - l_t}{\bar{l}_t}\right)^{(1-\eta)\Psi} \left(\varepsilon\left(\frac{c_{Tt}}{n_t}\right)^{\rho} + (1-\varepsilon)\left(\frac{c_{Nt}}{n_t}\right)^{\rho}\right)^{\frac{\eta\Psi}{\rho}-1} \frac{\eta(1-\varepsilon)}{n_t^{\rho}} c_{Nt}^{\rho-1} - \lambda_{t+1} p_{Nt} = 0 \\
\frac{\partial L_H}{\partial l_t} &= -\left(\varepsilon\left(\frac{c_{Tt}}{n_t}\right)^{\rho} + (1-\varepsilon)\left(\frac{c_{Nt}}{n_t}\right)^{\rho}\right)^{\frac{\eta\Psi}{\rho}} (1-\eta)\left(\frac{\bar{l}_t - l_t}{\bar{l}_t}\right)^{(1-\eta)\Psi-1} \frac{1}{\bar{l}_t} + \lambda_{t+1} w_t = 0 \\
\frac{\partial L_H}{\partial k_t} &= \beta\lambda_{t+1}\left(r_{kt} + q_t(1-\delta)\right) - \lambda_t q_{t-1} = 0 \\
\frac{\partial L_H}{\partial b_t} &= \beta\lambda_{t+1}(1+r_t) - \lambda_t = 0
\end{aligned}$$

Define:

$$\begin{aligned}
U_{mt} &= \varepsilon\left(\frac{c_{Tt}}{n_t}\right)^{\rho} + (1-\varepsilon)\left(\frac{c_{Nt}}{n_t}\right)^{\rho} \\
\pounds_t &= \frac{\bar{l}_t - l_t}{\bar{l}_t} = 1 - \frac{l_t}{\bar{l}_t}
\end{aligned}$$

The system of equations for the household can be written as:

$$\frac{\partial L_H}{\partial c_{Tt}} = \left(\pounds_t^{(1-\eta)\Psi}\right)\left(U_{mt}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta\varepsilon}{n_t^{\rho}} c_{Tt}^{\rho-1} - \lambda_{t+1} p_{Tt} = 0 \tag{5}$$

$$\frac{\partial L_H}{\partial c_{Nt}} = \left(\pounds_t^{(1-\eta)\Psi}\right)\left(U_{mt}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta(1-\varepsilon)}{n_t^{\rho}} c_{Nt}^{\rho-1} - \lambda_{t+1} p_{Nt} = 0 \tag{6}$$

$$\frac{\partial L_H}{\partial l_t} = -\left(U_{mt}^{\frac{\eta\Psi}{\rho}}\right)(1-\eta)\,\pounds_t^{(1-\eta)\Psi-1}\frac{1}{\bar{l}_t} + \lambda_{t+1} w_t = 0 \tag{7}$$

$$\frac{\partial L_H}{\partial k_t} = \beta\lambda_{t+1}\left(r_{kt} + q_t(1-\delta)\right) - \lambda_t q_{t-1} = 0 \tag{8}$$

$$\frac{\partial L_H}{\partial b_t} = \beta\lambda_{t+1}(1+r_t) - \lambda_t = 0 \tag{9}$$

Then, dividing (6) by (7), we get,

$$\frac{\partial L_H/\partial c_{Nt}}{\partial L_H/\partial l_t} = \frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_t}{n_t^{\rho}}\frac{\pounds_t}{U_{mt}} c_{Nt}^{\rho-1} = \frac{p_{Nt}}{w_t}.$$

Combining equations (8) and (9) we obtain a new relation [Eq 16 of the paper]:

$$r_{kt} + q_t(1-\delta) = q_{t-1}(1+r_t) \tag{10}$$

Interpreted as a non arbitrage condition. It says that the return to one unit of capital r_{kt} plus the market value of that unit after production $q_t(1-\delta)$, has to be equal to the purchase cost accrued at the international lending rate.

Combining equations (5) and (6) we obtain:

$$\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{Tt}}{c_{Nt}}\right)^{\rho-1} = \frac{p_{Tt}}{p_{Nt}} \tag{11}$$

This equation says that the optimal choice of consumption is such that the ratio of marginal utilities has to equate the ratio of market prices. Equivalently, it says that:

$$\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{Tt}}{c_{Nt}}\right)^{\rho} = \frac{p_{Tt}c_{Tt}}{p_{Nt}c_{Nt}}$$

The ratio of expenditures in tradables to non tradables is a function of the ratio of consumption on each good, with the exception where $\rho = 0$ (Cobb-Douglas utility), in which case the ratio of expenditures is constant.

2.2 Firms

There are five types of goods in this economy: a domestically produced traded good, y_{Dt}, an imported good m_t, a composite traded good y_{Tt}, made up of domestic goods and imports, a non traded good y_{Nt}, and an investment good y_{It}.

2.2.1 The domestic traded good

The domestic traded good is produced using capital k_{Dt}, labor, l_{Dt}, a composite traded good z_{TDt} and the non traded good z_{NDt} according to [Eq 5 of the paper]

$$y_{Dt} = \min\left[z_{TDt}/a_{TD}, z_{NDt}/a_{ND}, A_D k_{Dt}^{\alpha_D}(g^t l_{Dt})^{1-\alpha_D}\right] \tag{12}$$

Profits are zero due to perfect competition and constant returns to scale, therefore: [Eq 7 of the paper]

$$p_{Dt}y_{Dt} - r_{kt}k_{Dt} - w_t l_{Dt} - p_{Tt}z_{TDt} - p_{Nt}z_{NDt} = 0 \tag{13}$$

The cost minimization problem is

$$\begin{aligned}
& \min_{\{z_{TD}, z_{ND}, k_D, l_D\}} r_{kt}k_{Dt} + w_t l_{Dt} + p_{Tt}z_{TDt} + p_{Nt}z_{NDt} \\
& s.t. \\
\overline{y}_{Dt} \quad \geq \quad & \min\left[z_{TDt}/a_{TD}, z_{NDt}/a_{ND}, A_D k_{Dt}^{\alpha_D}(g^t l_{Dt})^{1-\alpha_D}\right]
\end{aligned}$$

The Lagrange function is:

$$\begin{aligned}
L_D \quad = \quad & r_{kt}k_{Dt} + w_t l_{Dt} + p_{Tt}z_{TDt} + p_{Nt}z_{NDt} + \\
& \lambda_D\left[\overline{y}_{Dt} - A_D k_{Dt}^{\alpha_D}(g^t l_{Dt})^{1-\alpha_D}\right] + \\
& \mu_{D1}\left[\overline{y}_{Dt} - z_{TDt}/a_{TD}\right] + \\
& \mu_{D2}\left[\overline{y}_{Dt} - z_{NDt}/a_{ND}\right]
\end{aligned}$$

First order conditions are:

$$\frac{\partial L_D}{\partial k_{Dt}} = r_{kt} - \lambda_D \alpha_D A_D k_{Dt}^{\alpha_D - 1}(g^t l_{Dt})^{1-\alpha_D} = 0 \tag{14}$$

$$\frac{\partial L_D}{\partial l_{Dt}} = w_t - \lambda_D (1-\alpha_D) A_D k_{Dt}^{\alpha_D}(g^t)^{1-\alpha_D} l_{Dt}^{-\alpha_D} = 0 \tag{15}$$

$$\frac{\partial L_D}{\partial z_{TDt}} = p_{Tt} - \mu_{D1}\frac{1}{a_{TD}} = 0 \tag{16}$$

$$\frac{\partial L_D}{\partial z_{NDt}} = p_{Nt} - \mu_{D2}\frac{1}{a_{ND}} = 0 \tag{17}$$

To obtain the value of the Lagrange multiplier, we use the zero profit condition after performing the following operations

Multiply $\frac{\partial L_D}{\partial k_{Dt}}$	by k_{Dt}	to obtain	$r_{kt} k_{Dt} = \lambda_D \alpha_D y_{Dt}$
Multiply $\frac{\partial L_D}{\partial l_{Dt}}$	by l_{Dt}	to obtain	$w_t l_{Dt} = \lambda_D (1-\alpha_D) y_{Dt}$
Multiply $\frac{\partial L_D}{\partial z_{TDt}}$	by z_{TDt}	to obtain	$p_{Tt} z_{TDt} = \mu_{D1} y_{Dt}$
Multiply $\frac{\partial L_D}{\partial z_{NDt}}$	by z_{NDt}	to obtain	$p_{Nt} z_{NDt} = \mu_{D2} y_{Dt}$

Summing up we have

$$r_{kt} k_{Dt} + w_t l_{Dt} + p_{Tt} z_{TDt} + p_{Nt} z_{NDt} = (\lambda_D + \mu_{D1} + \mu_{D2}) y_{Dt}$$

But zero profits implies:

$$r_{kt} k_{Dt} + w_t l_{Dt} + p_{Tt} z_{TDt} + p_{Nt} z_{NDt} = p_{Dt} y_{Dt}$$

We conclude that

$$p_{Dt} = \lambda_D + \mu_{D1} + \mu_{D2}$$

Equations (16) and (17) imply that $\mu_{D1} = p_{Tt} a_{TD}$ and $\mu_{D2} = p_{Nt} a_{ND}$. Finally, we obtain the following pricing equations for capital and labor [Equation 6 of the paper]:

$$\begin{aligned} r_{kt} &= (p_{Dt} - p_{Tt} a_{TD} - p_{Nt} a_{ND})\, \alpha_D A_D k_{Dt}^{\alpha_D - 1}(g^t l_{Dt})^{1-\alpha_D} \\ w_t &= (p_{Dt} - p_{Tt} a_{TD} - p_{Nt} a_{ND})\, (1-\alpha_D) A_D k_{Dt}^{\alpha_D}(g^t)^{1-\alpha_D} l_{Dt}^{-\alpha_D} \end{aligned}$$

2.2.2 The nontraded good

The nontraded good is obviously domestic and is produced using capital k_{Nt}, labor, l_{Nt}, a composite traded good z_{TNt} and the non traded good z_{NNt} according to

$$y_{Nt} = \min\left[z_{TNt}/a_{TN}, z_{NNt}/a_{NN}, A_N k_{Nt}^{\alpha_N}(g^t l_{Nt})^{1-\alpha_N}\right] \tag{18}$$

Profits are, therefore, (we will check that this is true substituting the non-arbitrage condition in the profits function)

$$p_{Nt} y_{Nt} - r_{kt} k_{Nt} - w_t l_{Nt} - p_{Tt} z_{TNt} - p_{Nt} z_{NNt} = 0 \tag{19}$$

The cost minimization problem is

$$
\begin{aligned}
& \min_{\{z_{TN}, z_{NN}, k_N, l_N\}} r_{kt}k_{Nt} + w_t l_{Nt} + p_{Tt}z_{TNt} + p_{Nt}z_{NNt} \\
& s.t. \\
\overline{y}_{Nt} \quad \geq \quad & \min\left[z_{TNt}/a_{TN}, z_{NNt}/a_{NN}, A_N k_{Nt}^{\alpha_N}(g^t l_{Nt})^{1-\alpha_N}\right]
\end{aligned}
$$

The Lagrange function is:

$$
\begin{aligned}
L_N \quad = \quad & r_{kt}k_{Nt} + w_t l_{Nt} + p_{Tt}z_{TNt} + p_{Nt}z_{NNt} + \\
& \lambda_N \left[\overline{y}_{Nt} - A_N k_{Nt}^{\alpha N}(g^t l_{Nt})^{1-\alpha N}\right] + \\
& \mu_{N1}\left[\overline{y}_{Nt} - z_{TNt}/a_{TN}\right] + \\
& \mu_{N2}\left[\overline{y}_{Nt} - z_{NNt}/a_{NN}\right]
\end{aligned}
$$

First order conditions are:

$$
\frac{\partial L_N}{\partial k_{Nt}} = r_{kt} - \lambda_N \alpha_N A_N k_{Nt}^{\alpha_N - 1}(g^t l_{Nt})^{1-\alpha_N} = 0 \tag{20}
$$

$$
\frac{\partial L_N}{\partial l_{Nt}} = w_t - \lambda_N (1-\alpha_N) A_N k_{Nt}^{\alpha_N}(g^t)^{1-\alpha_N} l_{Nt}^{-\alpha_N} = 0 \tag{21}
$$

$$
\frac{\partial L_N}{\partial z_{TNt}} = p_{Tt} - \mu_{N1}\frac{1}{a_{TN}} = 0 \tag{22}
$$

$$
\frac{\partial L_N}{\partial z_{NNt}} = p_{Nt} - \mu_{N2}\frac{1}{a_{NN}} = 0 \tag{23}
$$

To obtain the value of the Lagrange multiplier, we use the zero profit condition after performing the following operations

Multiply $\frac{\partial L_N}{\partial k_{Nt}}$	by k_{Nt}	to obtain	$r_{kt}k_{Nt} = \lambda_N \alpha_N y_{Nt}$
Multiply $\frac{\partial L_N}{\partial l_{Nt}}$	by l_{Nt}	to obtain	$w_t l_{Nt} = \lambda_N (1-\alpha_N) y_{Nt}$
Multiply $\frac{\partial L_N}{\partial z_{TNt}}$	by z_{TNt}	to obtain	$p_{Tt}z_{TNt} = \mu_{N1} y_{Nt}$
Multiply $\frac{\partial L_N}{\partial z_{NNt}}$	by z_{NNt}	to obtain	$p_{Nt}z_{NNt} = \mu_{N2} y_{Nt}$

Summing up we have

$$
r_{kt}k_{Nt} + w_t l_{Nt} + p_{Tt}z_{TNt} + p_{Nt}z_{NNt} = (\lambda_N + \mu_{N1} + \mu_{N2}) y_{Nt}
$$

But zero profits imply:

$$
r_{kt}k_{Nt} + w_t l_{Nt} + p_{Tt}z_{TNt} + p_{Nt}z_{NNt} = p_{Nt} y_{Nt}
$$

We conclude that

$$
p_{Nt} = \lambda_N + \mu_{N1} + \mu_{N2}
$$

Equations (22) and (23) imply that $\mu_{N1} = p_{Tt}a_{TN}$ and $\mu_{N2} = p_{Nt}a_{NN}$. Finally, we obtain the following pricing equations for capital and labor:

$$
\begin{aligned}
r_{kt} &= (p_{Nt} - p_{Tt}a_{TN} - p_{Nt}a_{NN})\, \alpha_N A_N k_{Nt}^{\alpha_N - 1}(g^t l_{Nt})^{1-\alpha_N} \\
w_t &= (p_{Nt} - p_{Tt}a_{TN} - p_{Nt}a_{NN})\, (1-\alpha_N) A_N k_{Nt}^{\alpha_N}(g^t)^{1-\alpha_N} l_{Nt}^{-\alpha_N}
\end{aligned}
$$

2.2.3 The composite traded good

The traded good is a composite of two goods: the domestic traded good and the imported good. It is produced using an Armington aggregator [Equation 8 of the paper]:

$$y_{Tt} = M\left(\mu x_{Dt}^{\zeta} + (1-\mu)m_t^{\zeta}\right)^{\frac{1}{\zeta}}$$

The imported good is the numeraire in the model and its f.o.b. price is 1. Imports are subject to a domestically levied tariff, τ_t. The Armington aggregator combines imperfect substitutes to produce a new good with the minimum expenditure. Profit maximization of

$$\max_{\{x_{Dt},m_t\}} p_{Tt}M\left(\mu x_{Dt}^{\zeta} + (1-\mu)m_t^{\zeta}\right)^{\frac{1}{\zeta}} - p_{Dt}x_{Dt} - (1+\tau_t)m_t \tag{24}$$

Yields:

$$\begin{aligned} p_{Tt}M\left(\mu x_{Dt}^{\zeta} + (1-\mu)m_t^{\zeta}\right)^{\frac{1}{\zeta}-1} \mu x_{Dt}^{\zeta-1} &= p_{Dt} && (25)\\ p_{Tt}M\left(\mu x_{Dt}^{\zeta} + (1-\mu)m_t^{\zeta}\right)^{\frac{1}{\zeta}-1} (1-\mu)m_t^{\zeta-1} &= (1+\tau_t) && (26) \end{aligned}$$

Dividing the first two f.o.c. we obtain [Eq 26 of the paper]:

$$\frac{p_{Dt}}{1+\tau_t}\left(\frac{m_t}{x_{Dt}}\right)^{\zeta-1} = \frac{\mu}{1-\mu} \tag{27}$$

The domestic traded good satisfies foreign demand with a downwards sloping curve, since the economy is semi-small [Equation 10 of the paper].

$$x_{Ft} = D_t\left((1+\tau_{Ft})\,p_{Tt}\right)^{\frac{-1}{1-\zeta}}$$

Notice that the demand of exports is similar to the demand of imports from the rest of the world, therefore the rest of the world use a similar Armington aggregator.

2.2.4 The investment good

The investment good is made out of the transformation of the traded and the nontraded good. The technology is a Cobb-Douglas technology according to empirical studies referred to in the paper.[Equation 9 of the paper].

$$y_{It} = Gz_{TIt}^{\gamma}z_{NIt}^{1-\gamma}$$

The cost minimization problem is

$$\begin{aligned} &\min_{\{z_{TIt},z_{NIt}\}} p_{Tt}z_{TIt} + p_{Nt}z_{NIt}\\ &s.t.\\ y_{It} \quad \leq \quad &\overline{y}_{It} \end{aligned}$$

The Lagrange function for this problem is:

$$L_I = p_{Tt} z_{TIt} + p_{Nt} z_{NIt} + \lambda_I \left[\overline{y}_{It} - G z_{TIt}^{\gamma} z_{NIt}^{1-\gamma} \right]$$

The set of F.O.C. is

$$\begin{aligned} \frac{\partial L_I}{\partial z_{Tt}} &= p_{Tt} - \lambda_I G \gamma z_{TIt}^{\gamma-1} z_{NIt}^{1-\gamma} = 0 && (28) \\ \frac{\partial L_I}{\partial z_{Nt}} &= p_{Nt} - \lambda_I G (1-\gamma) z_{TIt}^{\gamma} z_{NIt}^{-\gamma} = 0 && (29) \end{aligned}$$

To determine the value of λ_I, we perform the following operations

Multiply $\frac{\partial L_I}{\partial z_{TIt}}$	by z_{TIt}	to obtain	$p_{Tt} z_{TIt} = \lambda_I \gamma y_{It}$
Multiply $\frac{\partial L_I}{\partial z_{NIt}}$	by z_{NIt}	to obtain	$p_{Nt} z_{NIt} = \lambda_I (1-\gamma) y_{It}$

Summing up, we finally obtain:

$$p_{Tt} z_{TIt} + p_{Nt} z_{NIt} = \lambda_I y_{It}$$

Imposing perfect competition and consequently, zero profits:

$$q_t y_{It} - p_{Tt} z_{TIt} - p_{Nt} z_{NIt} = 0$$

We conclude that $\lambda_I = q_t$, and we can write equations. (28) and (29) as:

$$\begin{aligned} q_t G \gamma z_{TIt}^{\gamma-1} z_{NIt}^{1-\gamma} &= p_{Tt} \\ q_t G (1-\gamma) z_{TIt}^{\gamma} z_{NIt}^{-\gamma} &= p_{Nt} \\ \frac{\gamma}{1-\gamma} \frac{z_{NIt}}{z_{TIt}} &= \frac{p_{Tt}}{p_{Nt}} \end{aligned}$$

The investment good accrues into new capital.

$$k_{t+1} = (1-\delta) k_t + y_{It}$$

therefore we state the following identity $y_{It} = i_t$.

2.3 Market clearing conditions

Some of the market clearing conditions where already stated or are implicit in the notation. The full list of market clearing conditions is: [Equations 17 to 23 of the paper]:

$$\begin{aligned} x_{Dt} &= y_{Dt} && (30) \\ c_{Nt} + z_{NIt} + z_{NDt} + z_{NNt} &= y_{Nt} && (31) \\ c_{Tt} + z_{TIt} + z_{TDt} + z_{TNt} + x_{Ft} &= y_{Tt} && (32) \\ i_t &= y_{It} && (33) \\ k_{Dt} + k_{Nt} &= k_t && (34) \\ l_{Dt} + l_{Nt} &= l_t && (35) \\ m_t + b_{t+1} &= p_{Tt} x_{Ft} + (1+r_t) b_t && (36) \end{aligned}$$

Equation (36) can be stated in different forms. Written as $\mathbf{b}_{t+1} - (\mathbf{1} + \mathbf{r}_t)\mathbf{b}_t = p_{Tt}x_{Ft} - m_t$ is the balance of trade, whereas $\mathbf{b}_{t+1} - \mathbf{b}_t = p_{Tt}x_{Ft} - m_t + r_t b_t$ is the current account.

To provide a picture of the market clearing conditions we can look at the following input-output type of representation.

	I. demands		Final dem + exports			Total demand
Intermediate Inputs	z_{TD}	z_{TN}	c_{Tt}	z_{TI}	x_{Ft}	y_{Tt}
	z_{ND}	z_{NN}	c_{Nt}	z_{NI}	–	y_{Nt}
	k_D	k_N	–	y_{It}	–	–
	l_D	l_N	–	–	–	–
Gross O..	y_{Dt}	y_{Nt}				
imports	m_t	–				
Final output	y_{Tt}	y_{Nt}				

2.4 Walras Law

We start with the budget constraint. We will assume all taxes are zero, therefore there are no lump sum transfers and we set $T_t = 0$. The budget constraint is therefore

$$p_{Tt}c_{Tt} + p_{Nt}c_{Nt} + q_t i_t + b_{t+1} = w_t l_t + (1 + r_t)b_t + r_{kt}k_t$$

1. Consider equation (10)

$$r_{kt} + q_t(1 - \delta) = q_{t-1}(1 + r_t) \tag{37}$$

   And multiply by k_{Dt}, to get

$$r_{kt}k_{Dt} + q_t(1 - \delta)k_{Dt} = q_{t-1}(1 + r_t)k_{Dt} \tag{38}$$

   Substitution in the zero profit condition of the domestic traded good:

$$p_{Dt}y_{Dt} - r_{kt}k_{Dt} - w_t l_{Dt} - p_{Tt}z_{TDt} - p_{Nt}z_{NDt} = 0$$

   Yields

$$\underbrace{p_{Dt}y_{Dt} + q_t(1 - \delta)k_{Dt}}_{\substack{\text{Revenues from production} \\ \text{and capital sales at t}}} \quad \underbrace{-w_t l_{Dt} - p_{Tt}z_{TDt} - p_{Nt}z_{NDt} - q_{t-1}(1 + r_t)k_{Dt}}_{\substack{\text{Labor costs, interm. costs and cost of} \\ \text{purchasing capital at t-1}}} = 0.$$

   Which makes sense as a profit function of a firm that purchases and sells capital in every period.

2. Let us construct the proof of Walras Law with the set of equations given by the first order conditions, market clearing, and zero profit equations. The budget constraint is therefore

$$p_{Tt}c_{Tt} + p_{Nt}c_{Nt} + q_t i_t + b_{t+1} = w_t l_t + (1 + r_t)b_t + r_{kt}k_t$$

And

$$p_{Tt}c_{Tt}+p_{Nt}c_{Nt}+q_t i_t+b_{t+1}=w_t\left(l_{Dt}+l_{Nt}\right)+(1+r_t)b_t+r_{kt}\left(k_{Dt}+k_{Nt}\right)$$

Arranging terms

$$p_{Tt}c_{Tt}+p_{Nt}c_{Nt}+q_t i_t+b_{t+1}=(w_t l_{Dt}+r_t k_{Dt})+(w_t l_{Nt}+r_{kt}k_{Nt})+(1+r_t)b_t$$

Zero profits state that:

$$p_{Dt}y_{Dt}-r_{kt}k_{Dt}-w_t l_{Dt}-p_{Tt}z_{TDt}-p_{Nt}z_{NDt}=0 \tag{39}$$

and

$$p_{Nt}y_{Nt}-p_{Tt}z_{TNt}-p_{Nt}z_{NNt}=r_{kt}k_{Nt}+w_t l_{Nt}$$

Substituting:

$$\begin{aligned} p_{Tt}c_{Tt}+p_{Nt}c_{Nt}+q_t i_t+b_{t+1} &= (p_{Dt}y_{Dt}-p_{Tt}z_{TDt}-p_{Nt}z_{NDt})+ \\ &\quad (p_{Nt}y_{Nt}-p_{Tt}z_{TNt}-p_{Nt}z_{NNt})+(1+r_t)b_t \end{aligned} \tag{40}$$

Market clearing says:

$$c_{Tt}+z_{TIt}+z_{TDt}+z_{TNt}+x_{Ft}=y_{Tt}$$

And

$$c_{Nt}+z_{NIt}+z_{NDt}+z_{NNt}=y_{Nt}$$

In particular

$$\begin{aligned} p_{Tt}c_{Tt}+p_{Tt}z_{TIt}+p_{Tt}z_{TDt}+p_{Tt}z_{TNt}+p_{Tt}x_{Ft} &= p_{Tt}y_{Tt} \\ p_{Nt}c_{Nt}+p_{Nt}z_{NIt}+p_{Nt}z_{NDt}+p_{Nt}z_{NNt} &= p_{Nt}y_{Nt} \end{aligned}$$

Then

$$\begin{aligned} p_{Tt}c_{Tt}+p_{Tt}z_{TDt}+p_{Tt}z_{TNt} &= p_{Tt}y_{Tt}-p_{Tt}z_{TIt}-p_{Tt}x_{Ft} \\ p_{Nt}c_{Nt}+p_{Nt}z_{NDt}+p_{Nt}z_{NNt} &= p_{Nt}y_{Nt}-p_{Nt}z_{NIt} \end{aligned}$$

Substitute in (40) to obtain:

$$\begin{aligned} p_{Tt}y_{Tt}-p_{Tt}z_{TIt}-p_{Nt}z_{NIt}-p_{Tt}x_{Ft}+q_t i_t+b_{t+1} &= p_{Dt}y_{Dt}+ \\ &\quad +(1+r_t)b_t \end{aligned} \tag{41}$$

Zero profits in the investment sector yields a new relation:

$$p_{Tt}y_{Tt}-p_{Tt}x_{Ft}+b_{t+1}=p_{Dt}y_{Dt}+(1+r_t)b_t$$

Since $x_{Dt}=y_{Dt}$ by market clearing condition given by eq. (30), combined with eq. (24) we have:

$$p_{Tt}y_{Tt}-p_{Tt}x_{Ft}+b_{t+1}=p_{Tt}y_{Tt}-m_t+(1+r_t)b_t$$

Finally:

$$m_t+b_{t+1}=p_{Tt}x_{Ft}+(1+r_t)b_t$$

But this equation is the balance of payments equation (36). We conclude that Walras Law is satisfied.

2.5 Calibration

From the data appendix we obtain the required data for the calibration. Here we have an integer approximation to the reduced input-output matrix for the Mexican economy in 1988. We will assume (not like in the paper) that this economy is in a steady state. That implies that population growth is zero and that investment replaces capital consumption. We denote the steady state value of variable v as v_{ss}.

Input-Output matrix

	traded	n. traded	tot. inter	C+G	I	X	$\sum$	total
traded	33 z_{TDt}	11 z_{TNt}	44	27 c_{Tt}	10 z_{TIt}	19 x_{Ft}	56	100 y_{Tt}
nontraded	14 z_{NDt}	22 z_{NNt}	36	51 c_{Nt}	13 z_{NIt}	0.00	64	100 y_{Nt}
tot. inter	47	33	80	78	23	19	120	200
wages	18 $w_t l_{Dt}$	45 $w_t l_{Nt}$	63					63
capital	15 $r_{kt} k_{Dt}$	22 $r_{kt} k_{Nt}$	37					37
val. added	33	67	100					100
imports	18 m_t	0.00	18					18
tariffs	2	0.00	2					2
total	100	100	200	78	23	19	120	320

The calibration assumes that all goods prices are equal to 1. We can use the set of first order conditions of the investment sector and divide eq. (28) by (29) to obtain:

$$\frac{p_{Tss}}{p_{Nss}} = \frac{\gamma}{1-\gamma}\frac{z_{Nss}}{z_{Tss}}$$

If we assume $p_{Tss} = p_{Nss} = 1$, we can obtain the value of γ as

$$\gamma = \frac{z_{TIss}}{z_{NIss} + z_{TIss}} = \frac{10}{13+10} = 0.43478260869565$$

Once we have the value of γ, we can obtain, from the production function of the investment sector the scale parameter G, combining the production function of new capital with the aggregated input-output matrix, as:

$$G = \frac{y_{Iss}}{z_{TIss}^{\gamma} z_{NIss}^{1-\gamma}} = \frac{23}{10^{0.434} 13^{(1-0.434)}} = 1.98301076427603$$

To calibrate consumption parameters we use

$$\begin{aligned}
\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{Tss}}{c_{Nss}}\right)^{\rho-1} &= \frac{p_{Tss}}{p_{Nss}} \\
\varepsilon &= \frac{1}{\left(\frac{c_{Tss}}{c_{Nss}}\right)^{\rho-1}+1} \\
\varepsilon &= \frac{1}{\left(\frac{27}{51}\right)^{-2}+1} = 0.21891891891892
\end{aligned}$$

To calibrate the stock of capital we can use the no arbitrage condition and multiply by k_{ss} to get:

$$r_{kss}k_{ss} + q_{ss}(1-\delta)k_{ss} = q_{ss}(1+r_{ss})k_{ss} \tag{42}$$

Assume that $q_{ss} = 1$, then

$$r_{kss}k_{ss} - \delta k_{ss} = r_{ss}k_{ss} \tag{43}$$

so that

$$k_{ss} = \frac{\text{Gross capital earnings - depreciation}}{r_{ss}} = \frac{37-23}{0.1574} = 88.94536213468868$$

Where $\delta k_{ss} = 23$ is taken from the input output table. Then

$$\delta = \frac{23}{88.94536213468868} = 0.25858571428571$$

The value for $r_{ss} = 0.1574$ is the sum of Mexican interest rate premium $\sigma = 0.1174$ in the base year over a value of $r^* = 0.04$, that corresponds to a yearly accrued international riskless bond.

Nontraded good production parameters are obtained in a similar way. From equations (20) and (21) we get the following ratio:

$$\frac{r_{kss}}{w_{ss}} = \frac{\alpha_N}{1-\alpha_N}\frac{l_{Nss}}{k_{Nss}}$$

Therefore:

$$\alpha_N = \frac{r_{kss}k_{Nss}}{r_{kss}k_{Nss} + w_{ss}l_{Nss}} = \frac{\text{nontraded gross capital earnings}}{\text{nontraded total income}} = \frac{22}{67} = 0.32835820895522$$

Similarly:

$$\alpha_D = \frac{r_{kss}k_{Dss}}{r_{kss}k_{Dss} + w_{ss}l_{Dss}} = \frac{\text{traded gross capital earnings}}{\text{traded total income}} = \frac{15}{33} = 0.45454545454545$$

From the same set of conditions we have

$$\begin{aligned}
p_{Tss}z_{TNss} &= a_{TN}y_{Nss} \Rightarrow a_{TN} = \frac{z_{TNss}}{y_{Nss}} = \frac{11}{100} = 0.1100 \\
p_{Nss}z_{NNss} &= a_{NN}y_{Nss} \Rightarrow a_{NN} = \frac{z_{NNss}}{y_{Nss}} = \frac{22}{100} = 0.2200 \\
y_{Dss} &= z_{TDss}/a_{TD} \Rightarrow a_{TD} = \frac{z_{TDss}}{y_{Dss}} = \frac{33}{80} = 0.4125 \\
y_{Dss} &= z_{NDss}/a_{ND} \Rightarrow a_{ND} = \frac{z_{NDss}}{y_{Dss}} = \frac{14}{80} = 0.1750
\end{aligned}$$

Total factor productivity is computed to match total output. Since $r_{kss} = r+\delta = 0.1574+0.25858571428571 = 0.41598571428571$. Therefore if $r_{kss}k_{Nss} = 22$, then $k_{Nss} = 52.88643153954461$ and $k_{Dss} = 36.05893059514406$. Since $w_{ss}l_{Nss} = 45$, and wages are 1, then $l_{Nss} = 45$, and $l_{Dss} = 18$. Combining production functions with these results we get:

$$\begin{aligned}
AN &= \frac{y_{Nss}}{k_{Nss}^{\alpha_N}l_{Nss}^{1-\alpha_N}} = \frac{100}{52.886^{0.328}45^{1-0.328}} = 2.10745907710161 \\
AD &= \frac{y_{Dss}}{k_{Dss}^{\alpha_D}l_{Dss}^{1-\alpha_D}} = \frac{80}{36.058^{0.454}18^{1-0.454}} = 3.24087874847995
\end{aligned}$$

The elasticity of substitution between imports and exports is chosen to be $1/(1-\zeta) = 2$. From the equilibrium conditions of the composite traded good, considering that $x_{Dss} = y_{Dss}$, and that $\tau = 2/18$ (see the implicit tax in the input-output table).

$$\begin{aligned}
\frac{p_{Dss}}{1+\tau_{ss}}\left(\frac{m_{ss}}{x_{Dss}}\right)^{\zeta-1} &= \frac{\mu}{1-\mu} \\
\frac{1}{1+0.111}\left(\frac{18}{80}\right)^{-0.5} &= 1.89736659610103 \\
\mu &= 0.65485900149961 \\
M &= \frac{y_{Tss}}{\left(\mu x_{Dss}^{\zeta} + (1-\mu)m_{ss}^{\zeta}\right)^{\frac{1}{\zeta}}} \\
M &= \frac{100}{(0.654 * 80^{0.5} + (1-0.654) * 18^{0.5})^2} = 1.86549626493379
\end{aligned}$$

The population parameter is obtained as:

$$D_{ss} = \frac{x_{Fss}}{(1+\tau_F)^{\frac{-1}{1-\zeta}}} = \frac{19}{1.01^{-2}} = 19.3819$$

Provided that $\tau_F = 0.01$. And

$$b_{ss} = \frac{m_{ss} - x_{Fss}}{r_{ss}} = \frac{18-19}{0.1574} = -6.35324015247776$$

Finally,consider a steady state value for the ratio $l_{ss}/\bar{l}$ as the one used in the paper, where $l_{ss}/\bar{l} = 0.267$. From the input-output matrix we have that when wages are $w_{ss} = 1$, $l_{ss} = 63$, therefore $\bar{l} = 63/0.267 = 235.9550561797753$. Using this information and the equilibrium conditions of the household:

$$
\begin{aligned}
\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_{ss}}{n_{ss}^{\rho}}\frac{\pounds}{U_m}c_{Nss}^{\rho-1} &= 1 \\
\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_{ss}}{n_{ss}^{\rho}}\pounds c_{Nss}^{\rho-1} &= U_m \\
\frac{\eta}{1-\eta}(1-\varepsilon)\bar{l}_{ss}\pounds c_{Nss}^{\rho-1} &= \varepsilon c_{Tss}^{\rho} + (1-\varepsilon)\, c_{Nss}^{\rho} \\
\frac{\eta}{1-\eta}\bar{l}_{ss}\pounds c_{Nss}^{-1} &= \frac{\varepsilon}{(1-\varepsilon)}\frac{c_{Tss}^{\rho}}{c_{Nss}^{\rho}} + 1 \\
\frac{\eta}{1-\eta}\bar{l}_{ss}\pounds c_{Nss}^{-1} &= \frac{c_{Tss}}{c_{Nss}} + 1 \\
\frac{\eta}{1-\eta}\bar{l}_{ss}\pounds &= c_{Tss} + c_{Nss} \\
\frac{\eta}{1-\eta} &= \frac{c_{Tss}+c_{Nss}}{\bar{l}_{ss}\pounds} = \frac{c_{Tss}+c_{Nss}}{\bar{l}_{ss}-l_{ss}} \\
\eta &= \frac{1}{1+\frac{\bar{l}_{ss}-l_{ss}}{c_{Tss}+c_{Nss}}} = 0.31081262592344
\end{aligned}
$$

Collecting all parameters we have a table similar to table 2 of the paper.

Consumer parameters

Parameter	**Value**	**Statistic**	**Target**
b_{1988}	-6.353	Trade balance to GDP in 1988, in percent	1.390
k_{1988}	88.945	Real interest rate in 1988, in percent	15.740
β	0.864	U.S. real interest rate, in percent	4.000
ε	0.218	Traded good share in consumption in 1988	0.356
ρ	-1.000	Elasticity of substitution: traded nontraded	0.500
η	0.311	Ratio of hours worked to available hours in 1988	0.267
Ψ	-1.000	Intertemporal elasticity of substitution	0.500
δ	0.258	Depreciation to GDP in 1988, in percent	23
τ_{k1988}	0.201	Investment	23

Producer parameters

Parameter	Value	Statistic	Target
a_{TD}	0.412	Share of traded inputs in domestic trade in 1988	0.412
a_{ND}	0.175	Share of nontraded inputs in domestic trade in 1988	0.175
a_{TN}	0.110	Share of traded inputs in domestic nontraded in 1988	0.110
a_{NN}	0.220	Share of nontraded inputs in domestic nontraded in 1988	0.220
A_D	3.241	Traded gross output in 1988	80
A_N	2.107	Nontraded gross output in 1988	100
α_D	0.454	Capital's share of domestic traded value added in 1988	0.454
α_N	0.328	Capital's share of nontraded value added in 1988	0.328
γ	0.435	Share of traded inputs in investment good production in 1988	0.435
G	1.983	Investment in 1988	22
g	1.000	Growth rate of U.S. GDP per working age person, in percent	1.000

Trade parameters

Parameter	Value	Statistic	Target
M	1.866	Total traded goods in 1988	99.955
μ	0.653	Ratio of imports to domestic traded good in 1988	0.233
ζ	0.500	Elasticity of substitution: domestic traded to imports	2.000
D_{1988}	19.382	Exports in 1988	19.928

Exogenous processes

$\bar{l}_t$	Mexican working age population data and projections
n_t	Mexican adult equivalent population data and projections
σ_t	Mexican interest premia
D_t	U.S. working age population data and projections
τ_t	Mexican tariffs on U.S. imports
τ_{Ft}	U.S. tariffs on Mexican imports

3 Steady state

This economy has an ever growing population. To find a steady state, we write the set of conditions with $g = 1$. The set of equations characterizing a steady state are:

$$\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{Tss}}{c_{Nss}}\right)^{\rho-1} = \frac{p_{Tss}}{p_{Nss}} \tag{44}$$

$$r_{kss}+q_{ss}(1-\delta) = q_{ss}(1+r_{ss}) \tag{45}$$

$$k_{ss} = \delta i_{ss} \tag{46}$$

$$\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_{ss}}{n_{ss}^{\rho}}\frac{£}{U_m}c_{Nss}^{\rho-1} = \frac{p_{Nss}}{w_{ss}} \tag{47}$$

$$\left(p_{Dss}-p_{Tss}a_{TD}-p_{Nss}a_{ND}\right)\alpha_D A_D k_{Dss}^{\alpha_D-1}l_{Dss}^{1-\alpha_D} = r_{kss} \tag{48}$$

$$\left(p_{Dss}-p_{Tss}a_{TD}-p_{Nss}a_{ND}\right)(1-\alpha_D)A_D k_{Dss}^{\alpha_D}l_{Dss}^{-\alpha_D} = w_{ss} \tag{49}$$

$$A_D k_{Dss}^{\alpha_D}l_{Dss}^{1-\alpha_D} = y_{Dss} \tag{50}$$

$$z_{TDss}/a_{TD} = y_{Dss} \tag{51}$$

$$z_{NDss}/a_{ND} = y_{Dss} \tag{52}$$

$$\left(p_{Nss}-p_{Tss}a_{TN}-p_{Nss}a_{NN}\right)\alpha_N A_N k_{Nss}^{\alpha_N-1}l_{Nss}^{1-\alpha_N} = r_{kss} \tag{53}$$

$$\left(p_{Nss}-p_{Tss}a_{TN}-p_{Nss}a_{NN}\right)(1-\alpha_N)A_N k_{Nss}^{\alpha_N}l_{Nss}^{-\alpha_N} = w_{ss} \tag{54}$$

$$A_N k_{Nss}^{\alpha_N}l_{Nss}^{1-\alpha_N} = y_{Nss} \tag{55}$$

$$z_{TNss}/a_{TN} = y_{Nss} \tag{56}$$

$$z_{NNss}/a_{NN} = y_{Nss} \tag{57}$$

$$\frac{\mu}{1-\mu}\left(\frac{x_{Dss}}{m_{ss}}\right)^{\zeta-1} = \frac{p_{Dss}}{1+\tau_{ss}} \tag{58}$$

$$M\left(\mu x_{Dss}^{\zeta}+(1-\mu)m_{ss}^{\zeta}\right)^{\frac{1}{\zeta}} = y_{Tss} \tag{59}$$

$$\frac{\gamma}{1-\gamma}\frac{z_{NIss}}{z_{TIss}} = \frac{p_{Tss}}{p_{Nss}} \tag{60}$$

$$q_{ss}G\gamma z_{TIss}^{\gamma-1}z_{NIss}^{1-\gamma} = p_{Tss} \tag{61}$$

$$G z_{TIss}^{\gamma}z_{NIss}^{1-\gamma} = y_{Iss} \tag{62}$$

$$D_{ss}\left((1+\tau_{Fss})\,p_{Tss}\right)^{\frac{-1}{1-\zeta}} = x_{Fss} \tag{63}$$

Plus the set of market clearing conditions

$$x_{Dss} = y_{Dss} \tag{64}$$

$$c_{Nss}+z_{NIss}+z_{NDss}+z_{NNss} = y_{Nss} \tag{65}$$

$$c_{Tss}+z_{TIss}+z_{TDss}+z_{TNss}+x_{Fss} = y_{Tss} \tag{66}$$

$$i_{ss} = y_{Iss} \tag{67}$$

$$k_{Dss}+k_{Nss} = k_{ss} \tag{68}$$

$$l_{Dss}+l_{Nss} = l_{ss} \tag{69}$$

$$m_{ss} = p_{Tss}x_{Fss}+r_{ss}b_{ss} \tag{70}$$

3.1 Solving the steady state

The above system of equations is a Cramer's system of 27 unknowns in 27 linearly independent equations. In principle, we could use Newton's algorithm directly providing a seed of dimension 27, writing down the whole system in a MatLab function file and letting the algorithm to solve it. However, that would not be a wise procedure, since much of the success of Newton's algorithm is based on the dimension of the system and the quality of the seed. The latter is not a problem in this particular case, since we know in advance the value of all quantities and prices, but it will be when we are dealing with the dynamics of the system. As a general rule, we should always try to find the minimal system of equations where Newton's algorithm has to iterate. In the next lines we show that the 27 equations system can be reduced to an equivalent one of four equations in four unknowns.

For it, assume that we have knowledge of k_{Dss}, k_{Nss}, m_{ss} and l_{ss}. By eq. (68) we get k_{ss}. Then by eq. (46), we know i_{ss}, which in turn, by eq. (67) yields a value for y_{Iss}.
Dividing (48) by (49) we get:

$$\frac{\alpha_D}{(1-\alpha_D)}\frac{l_{Dss}}{k_{Dss}} = \frac{r_{kss}}{w_{ss}}$$

Similarly, dividing (53) by (54) we get:

$$\frac{\alpha_N}{(1-\alpha_N)}\frac{l_{Nss}}{k_{Nss}} = \frac{r_{kss}}{w_{ss}}$$

Equating the last two expressions:

$$\begin{aligned}\frac{\alpha_D}{(1-\alpha_D)}\frac{k_{Nss}}{k_{Dss}} &= \frac{\alpha_N}{(1-\alpha_N)}\frac{l_{Nss}}{l_{Dss}} \\ \frac{l_{Nss}}{l_{Dss}} &= \frac{(1-\alpha_N)}{\alpha_N}\frac{\alpha_D}{(1-\alpha_D)}\frac{k_{Nss}}{k_{Dss}}\end{aligned}$$

So the ratio l_{Nss}/l_{Dss} is known. Then, by eq. (69)

$$l_{Dss} = \frac{l_{ss}}{1+\frac{l_{Nss}}{l_{Dss}}}$$

and from here, we have that $l_{Nss} = (l_{Nss}/l_{Dss})\, l_{Dss}$. And from the above equations we also know the ratio r_{kss}/w_{ss}.
Once we have all labor and capital utilizations we can get output. From eq. (50) we have

$$A_D k_{Dss}^{\alpha_D} l_{Dss}^{1-\alpha_D} = y_{Dss}$$

And from here we have by eqs. (51) and (52):

$$z_{TDss} = a_{TD} y_{Dss} \text{ and } z_{NDss} = a_{ND} y_{Dss}$$

Similarly, using eq. (55)

$$A_N k_{Nss}^{\alpha_N} l_{Nss}^{1-\alpha_N} = y_{Nss}$$

And from here, by eqs. (56) and (57):

$$z_{TNss} = a_{TN} y_{Nss} \text{ and } z_{NNss} = a_{NN} y_{Nss}$$

From equation (59) we get y_{Tss}.

$$M\left(\mu x_{Dss}^{\zeta} + (1-\mu) m_{ss}^{\zeta}\right)^{\frac{1}{\zeta}} = y_{Tss}$$

and from eq. (58) we have

$$(1+\tau_{ss}) \frac{\mu}{1-\mu} \left(\frac{x_{Dss}}{m_{ss}}\right)^{\zeta-1} = p_{Dss}$$

From eq. (70) we get

$$p_{Tss} x_{Tss} = m_{ss} - r_{ss} b_{ss}$$

From eq. (63)

$$D_{ss}\left((1+\tau_F)\, p_{Tss}\right)^{\frac{-1}{1-\zeta}} = x_{Fss}$$

multiplying by p_{Tss} we get:

$$D_{ss}\left(1+\tau_F\right)^{\frac{-1}{1-\zeta}} p_{Tss}^{\frac{\zeta}{\zeta-1}} = p_{Tss} x_{Fss}$$

And from here we obtain the value of p_{Tss} as

$$p_{Tss} = \left(\frac{p_{Tss} x_{Fss}}{D_{ss}\left(1+\tau_F\right)^{\frac{-1}{1-\zeta}}}\right)^{\frac{\zeta-1}{\zeta}}$$

It is immediate to get

$$x_{Fss} = \left(p_{Tss} x_{Fss}\right) \frac{1}{p_{Tss}}$$

Once we know p_{Dss}, and p_{Tss} from eqs. (48) and (53)

$$\begin{aligned}
\left(p_{Dss} - p_{Tss} a_{TD} - p_{Nss} a_{ND}\right) \alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D} &= r_{kss} \\
\left(p_{Nss} - p_{Tss} a_{TN} - p_{Nss} a_{NN}\right) \alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N} &= r_{kss}
\end{aligned}$$

Division by p_{Tss} and equalization

$$
\begin{aligned}
\left(\frac{p_{Dss}}{p_{Tss}} - a_{TD} - \frac{p_{Nss}}{p_{Tss}} a_{ND}\right) \alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D} &= \frac{r_{kss}}{p_{Tss}} \\
\left(\frac{p_{Nss}}{p_{Tss}} - a_{TN} - \frac{p_{Nss}}{p_{Tss}} a_{NN}\right) \alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N} &= \frac{r_{kss}}{p_{Tss}} \\
\left(\frac{p_{Dss}}{p_{Tss}} - a_{TD} - \frac{p_{Nss}}{p_{Tss}} a_{ND}\right) \alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D} &= \left(\frac{p_{Nss}}{p_{Tss}} - a_{TN} - \frac{p_{Nss}}{p_{Tss}} a_{NN}\right) \alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N} \\
\left(\frac{p_{Dss}}{p_{Tss}} - a_{TD} - \frac{p_{Nss}}{p_{Tss}} a_{ND}\right) &= \frac{\alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N}}{\alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}} \left(\frac{p_{Nss}}{p_{Tss}} - a_{TN} - \frac{p_{Nss}}{p_{Tss}} a_{NN}\right) \\
\frac{p_{Dss}}{p_{Tss}} - a_{TD} + a_{TN} \frac{\alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N}}{\alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}} &= \left(\frac{\alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N}}{\alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}} (1 - a_{NN}) + a_{ND}\right) \frac{p_{Nss}}{p_{Tss}} \\
\frac{p_{Nss}}{p_{Tss}} &= \frac{\frac{p_{Dss}}{p_{Tss}} - a_{TD} + a_{TN} \frac{\alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N}}{\alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}}}{\frac{\alpha_N A_N k_{Nss}^{\alpha_N - 1} l_{Nss}^{1-\alpha_N}}{\alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}} (1 - a_{NN}) + a_{ND}}
\end{aligned}
$$

Once we have the ratio p_{Nss}/p_{Tss} we get p_{Nss} as

$$
p_{Nss} = \left(\frac{p_{Nss}}{p_{Tss}}\right) p_{Tss}
$$

And from eq. (60)

$$
\begin{aligned}
\frac{\gamma}{1-\gamma} \frac{z_{NIss}}{z_{TIss}} &= \frac{p_{Tss}}{p_{Nss}} \\
\frac{z_{NIss}}{z_{TIss}} &= \frac{p_{Tss}}{p_{Nss}} \frac{1-\gamma}{\gamma}
\end{aligned}
$$

From eq. (62)

$$
\begin{aligned}
G z_{TIss}^{\gamma} z_{NIss}^{1-\gamma} &= y_{Iss} \\
G \left(\frac{z_{NIss}}{z_{TIss}}\right)^{1-\gamma} &= \frac{y_{Iss}}{z_{TIss}} \\
z_{TIss} &= \frac{y_{Iss}}{G \left(\frac{z_{NIss}}{z_{TIss}}\right)^{1-\gamma}}
\end{aligned}
$$

Therefore:

$$
z_{NIss} = \left(\frac{z_{NIss}}{z_{TIss}}\right) z_{TIss}
$$

From eq. (65)

$$
\begin{aligned}
c_{Nss} + z_{NIss} + z_{NDss} + z_{NNss} &= y_{Nss} \\
y_{Nss} - z_{NIss} - z_{NDss} - z_{NNss} &= c_{Nss}
\end{aligned}
$$

Therefore, from eq. (44)

$$\begin{aligned}
\frac{\varepsilon}{1-\varepsilon} c_{Tss}^{\rho-1} &= \frac{p_{Tss}}{p_{Nss}} c_{Nss}^{\rho-1} \\
c_{Tss} &= \left(\frac{1-\varepsilon}{\varepsilon} \frac{p_{Tss}}{p_{Nss}} \right)^{\frac{1}{\rho-1}} c_{Nss}
\end{aligned}$$

From eq. (48)

$$r_{kss} = (p_{Dss} - p_{Tss} a_{TD} - p_{Nss} a_{ND})\, \alpha_D A_D k_{Dss}^{\alpha_D - 1} l_{Dss}^{1-\alpha_D}$$

And from eq. (49)

$$w_{ss} = (p_{Dss} - p_{Tss} a_{TD} - p_{Nss} a_{ND})\, (1-\alpha_D) A_D k_{Dss}^{\alpha_D} l_{Dss}^{-\alpha_D}$$

Finally, from eq. (45)

$$\begin{aligned}
r_{kss} + q_{ss}(1-\delta) &= q_{ss}(1 + r_{ss}) \\
r_{kss} - \delta q_{ss} &= r_{ss} q_{ss} \\
q_{ss} &= \frac{r_{kss}}{r_{ss} + \delta}
\end{aligned}$$

We are left with four unused equations to iterate Newton's algorithm.

$$\begin{aligned}
f(1) &= \frac{\eta}{1-\eta} \frac{(1-\varepsilon)\bar{l}_{ss}}{n_{ss}^{\rho}} \frac{\pounds}{U_m} c_{Nss}^{\rho-1} - \frac{p_{Nss}}{w_{ss}} \\
f(2) &= y_{Tss} - (c_{Tss} + z_{TIss} + z_{TDss} + z_{TNss} + x_{Fss}) \\
f(3) &= q_{ss} G \gamma z_{TIss}^{\gamma-1} z_{NIss}^{1-\gamma} - p_{Tss} \\
f(4) &= p_{Tss} M \left(\mu x_{Dss}^{\zeta} + (1-\mu) m_{ss}^{\zeta} \right)^{\frac{1}{\zeta}-1} \mu x_{Dss}^{\zeta-1} - p_{Dss}
\end{aligned}$$

The whole system is non-singular and converges nicely to the unique steady state from the seed provided in the calibration.

3.2 A MatLab program

The following code reads from an Excel spread sheet where a given economy is numerically represented by an aggregated input-output table. The program **Sudden_Stops.m** provides a calibration and solves the steady state. To see the effects of a different input-output table on the parameters, just change the values of the table in a consistent way. If prices are different from 1, something went wrong. There are two function files, one with the Newton algorithm called **secant.m** that was written by Carlos Urrutia, the other function file is **SS-cpo.m** and contains the system of equations that characterize the steady state of the economy.

```
%Sudden_Stop.m solves the baseline Kehoe-Ruhl model of sudden stops.
%               Tr     Ntr    Int   C+G     I      X     C+I+X  Tot
%Traded         33     11     44     27     10     19     56    100
%Nontraded      14     22     36     51     13      0     64    100
%Intermediate   47     33     80     78     23     19    120    200
%Wages          18     45     63      0      0      0      0     63
%Capital        15     22     37      0      0      0      0     37
%Value Added    33     67    100      0      0      0      0    100
%Imports        18      0     18      0      0      0      0     18
%Tariffs        2       0      2      0      0      0      0      2
%Total          100    100   200     78     23     19    120    320

clear all
[data, texto]=xlsread('HojaCalibracion.xlsx', 'A1:I10');

%Trade parameters
zeta    =   0.5;
tau     =   data(8,1)/data(7,1);
mu      =   1/(1 + (1+tau) * (data(7,1)/(data(3,1)+data(6,1)))^(1-zeta));
M       =   data(9,1)/(mu*(data(3,1)+data(6,1))^zeta + ...
            (1-mu)*data(7,1)^zeta)^(1/zeta);
tauF    =   0.01;
D1988   =   data(1,6)/(1+tauF)^(-1/(1-zeta));
r_star  =   0.04;
sigma   =   0.1174;
rss     =   r_star+sigma;
bss     =   (data(7,1)-data(1,6))/rss;
Tparam  =   [M mu zeta D1988 r_star bss];

%Consumer parameters
beta    =   1/(1+rss);
rho     =   -1;
epsilon =   1/((data(1,4)/data(2,4))^(rho-1)+1);
K       =   (data(5,3)-data(3,5))/rss;
delta   =   data(3,5)/K;
l_bar   =   data(4,3)/0.267;
eta     =   1/(1+(l_bar-data(4,3))/(data(3,4)));
Psi     =   -1;
tauk    =   0; %0.201;
Cparam  =   [beta epsilon rho eta Psi delta tauk];

%Producer parameters
aTD     =   data(1,1)/(data(3,1)+data(6,1));
aND     =   data(2,1)/(data(3,1)+data(6,1));
aTN     =   data(1,2)/data(9,2);
aNN     =   data(2,2)/data(9,2);
```

```
alphaD  =   data(5,1)/data(6,1);
alphaN  =   data(5,2)/data(6,2);
rkss    =   rss+delta;
kNss    =   data(5,2)/rkss;
kDss    =   data(5,1)/rkss;
AD      =   (data(3,1)+data(6,1))/(kDss^alphaD*data(4,1)^(1-alphaD));
AN      =   data(9,2)/(kNss^alphaN*data(4,2)^(1-alphaN));
gamma   =   data(1,5)/data(3,5);
G       =   data(3,5)/(data(1,5)^gamma*data(2,5)^(1-gamma));
g       =   1;
Pparam  =   [aTD aND aTN aNN AD AN alphaD alphaN gamma G g];

%Time series parameters
n       =   1; %(1/2)*l_bar;
Dusa    =   1;
TSparam =   [l_bar n sigma Dusa tau tauF];

%Newton's algorithm parameters
crit    =   1e-10;
maxit   =   1000;
param   =   [Cparam Pparam Tparam TSparam];
x0      =   [36.0589 52.8864 63 18]';
sol     =   secant('SScpo', x0, param, crit, maxit);

%Variables allocation
kDss        =   sol(1);
kNss        =   sol(2);
lss         =   sol(3);
mss         =   sol(4);

%System
rss         =   r_star+sigma;
lnOverlD    =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kNss/kDss;
lDss        =   lss/(1+lnOverlD);
lNss        =   lnOverlD*lDss;
yDss        =   AD*kDss^alphaD*lDss^(1-alphaD);
zTDss       =   aTD*yDss;
zNDss       =   aND*yDss;
yNss        =   AN*kNss^alphaN*lNss^(1-alphaN);
zTNss       =   aTN*yNss;
zNNss       =   aNN*yNss;
yTss        =   M*(mu*yDss^zeta+(1-mu)*mss^zeta)^(1/zeta);
yIss        =   delta*(kDss+kNss);
pDss        =   (1+tau)*(mu/(1-mu))*(yDss/mss)^(zeta-1);
pTxF        =   mss-rss*bss;
pTss        =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
```

```
xFss        =   pTxF/pTss;
pmgkN       =   alphaN*AN*(kNss/lNss)^(alphaN-1);
pmgkD       =   alphaD*AD*(kDss/lDss)^(alphaD-1);
pmglD       =   (1-alphaD)*AD*(kDss/lDss)^alphaD;
pmglN       =   (1-alphaN)*AN*(kNss/lNss)^alphaN;
ratPmgs     =   pmgkN/pmgkD;
pNoverpT    =   (pDss/pTss-aTD+aTN*ratPmgs)/(ratPmgs*(1-aNN)+aND);
pNss        =   pNoverpT*pTss;
zNIoverzTI  =   (1-gamma)/(gamma*pNoverpT);
zTIss       =   yIss/(G*zNIoverzTI^(1-gamma));
zNIss       =   zNIoverzTI*zTIss;
rkss        =   (pDss-pTss*aTD-pNss*aND)*pmgkD;
wss         =   (pDss-pTss*aTD-pNss*aND)*pmglD;
qss         =   rkss/(rss+delta);
cNss        =   yNss-(zNIss+zNDss+zNNss);
cTss        =   cNss*((1-epsilon)*pTss/(epsilon*pNss))^(1/(rho-1));

U           =   epsilon*(cTss/n)^rho+(1-epsilon)*(cNss/n)^rho;
pound       =   (l_bar-lss)/l_bar;

%Check
C1  =   G*zTIss^gamma*zNIss^(1-gamma)-yIss;
C2  =   pTss*cTss+pNss*cNss+qss*yIss-wss*lss-rkss*(kDss+kNss)-...
        rss*bss-tau*mss;
C3  =   pDss*yDss-rkss*kDss-wss*lDss-pTss*zTDss-pNss*zNDss;
C4  =   pNss*yNss-rkss*kNss-wss*lNss-pTss*zTNss-pNss*zNNss;
C5  =   pTss*M*(mu*yDss^zeta+(1-mu)*mss^zeta)^(1/zeta)-pDss*yDss-...
        (1+tau)*mss;
C6  =   xFss-D1988*((1+tauF)*pTss)^(-1/(1-zeta));
C7  =   qss*yIss-pTss*zTIss-pNss*zNIss;

[C1 C2 C3 C4 C5 C6 C7]
```

The function file that contains the system of equations is **SScpo.m**

```
function f = SScpo(x0, param)
%Parameter allocation
beta        =   param(1);
epsilon     =   param(2);
rho         =   param(3);
eta         =   param(4);
Psi         =   param(5);
delta       =   param(6);
tauk        =   param(7);
```

```
aTD         =   param(8);
aND         =   param(9);
aTN         =   param(10);
aNN         =   param(11);
AD          =   param(12);
AN          =   param(13);
alphaD      =   param(14);
alphaN      =   param(15);
gamma       =   param(16);
G           =   param(17);
g           =   param(18);

M           =   param(19);
mu          =   param(20);
zeta        =   param(21);
D1988       =   param(22);
r_star      =   param(23);
bss         =   param(24);

l_bar       =   param(25);
n           =   param(26);
sigma       =   param(27);
Dusa        =   param(28);
tau         =   param(29);
tauF        =   param(30);

%Variables allocation
kDss        =   x0(1);
kNss        =   x0(2);
lss         =   x0(3);
mss         =   x0(4);

%System
rss         =   r_star+sigma;
lnOverlD    =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kNss/kDss;
lDss        =   lss/(1+lnOverlD);
lNss        =   lnOverlD*lDss;
yDss        =   AD*kDss^alphaD*lDss^(1-alphaD);
zTDss       =   aTD*yDss;
zNDss       =   aND*yDss;
yNss        =   AN*kNss^alphaN*lNss^(1-alphaN);
zTNss       =   aTN*yNss;
zNNss       =   aNN*yNss;
yTss        =   M*(mu*yDss^zeta+(1-mu)*mss^zeta)^(1/zeta);
yIss        =   delta*(kDss+kNss);
```

```
pDss        =   (1+tau)*(mu/(1-mu))*(yDss/mss)^(zeta-1);
pTxF        =   mss-rss*bss;
pTss        =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
xFss        =   pTxF/pTss;
pmgkN       =   alphaN*AN*(kNss/lNss)^(alphaN-1);
pmgkD       =   alphaD*AD*(kDss/lDss)^(alphaD-1);
pmglD       =   (1-alphaD)*AD*(kDss/lDss)^alphaD;
pmglN       =   (1-alphaN)*AN*(kNss/lNss)^alphaN;
ratPmgs     =   pmgkN/pmgkD;
pNoverpT    =   (pDss/pTss-aTD+aTN*ratPmgs)/(ratPmgs*(1-aNN)+aND);
pNss        =   pNoverpT*pTss;
zNIoverzTI  =   (1-gamma)/(gamma*pNoverpT);
zTIss       =   yIss/(G*zNIoverzTI^(1-gamma));
zNIss       =   zNIoverzTI*zTIss;
rkss        =   (pDss-pTss*aTD-pNss*aND)*pmgkD;
wss         =   (pDss-pTss*aTD-pNss*aND)*pmglD;
qss         =   rkss/(rss+delta);
cNss        =   yNss-(zNIss+zNDss+zNNss);
cTss        =   cNss*((1-epsilon)*pTss/(epsilon*pNss))^(1/(rho-1));

U           =   epsilon*(cTss/n)^rho+(1-epsilon)*(cNss/n)^rho;
pound       =   (l_bar-lss)/l_bar;

f(1)        =   ((eta*(1-epsilon)*l_bar)/((1-eta)*n^rho))*...
                (pound/U)*cNss^(rho-1) / (pNss/wss)-1;
f(2)        =   (cTss+zTIss+zTDss+zTNss+xFss)/yTss-1;
f(3)        =   (qss*G*gamma*(zNIss/zTIss)^(1-gamma))/pTss-1;
f(4)        =   (pTss*M*(mu*yDss^zeta+(1-mu)*mss^zeta)^(1/zeta-1)*...
                mu*yDss^(zeta-1))/pDss-1;

f=f';
```

4 Comparative statics and robustness

The program **Comp_Statics.m** displays the effect of a variation in steady state values of prices and quantities as a consequence of a variation in a given parameter. For example, imagine that we are interested in the effect of an increase in 90% in the value of M (TFP in the composite traded good). Then we would invoke at the MatLab prompt the name of the program to get:

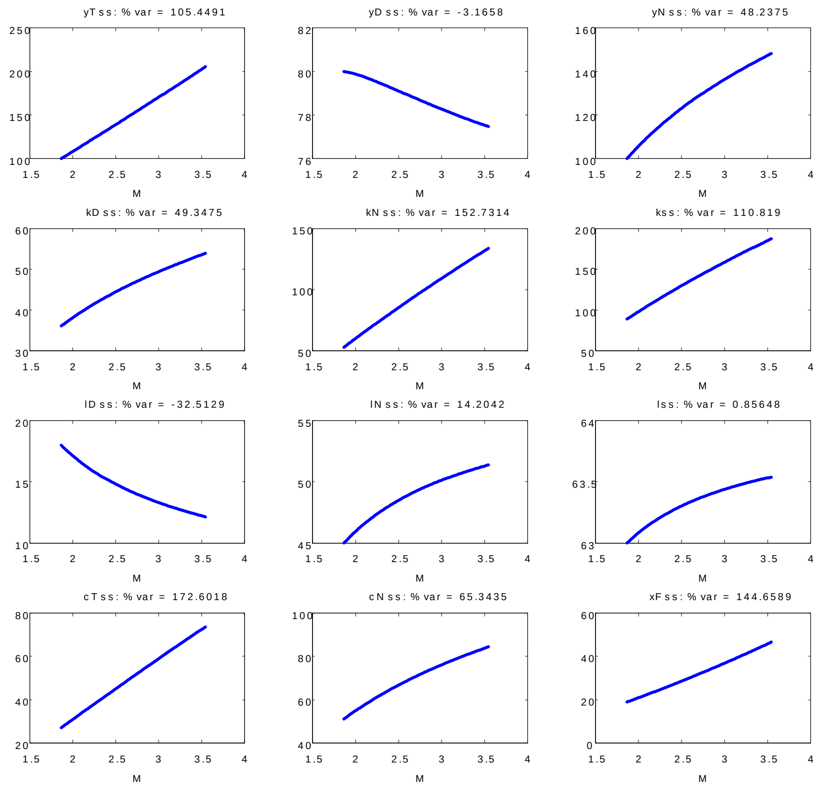
```
>> Comp_Statics
Enter a parameter name:
```

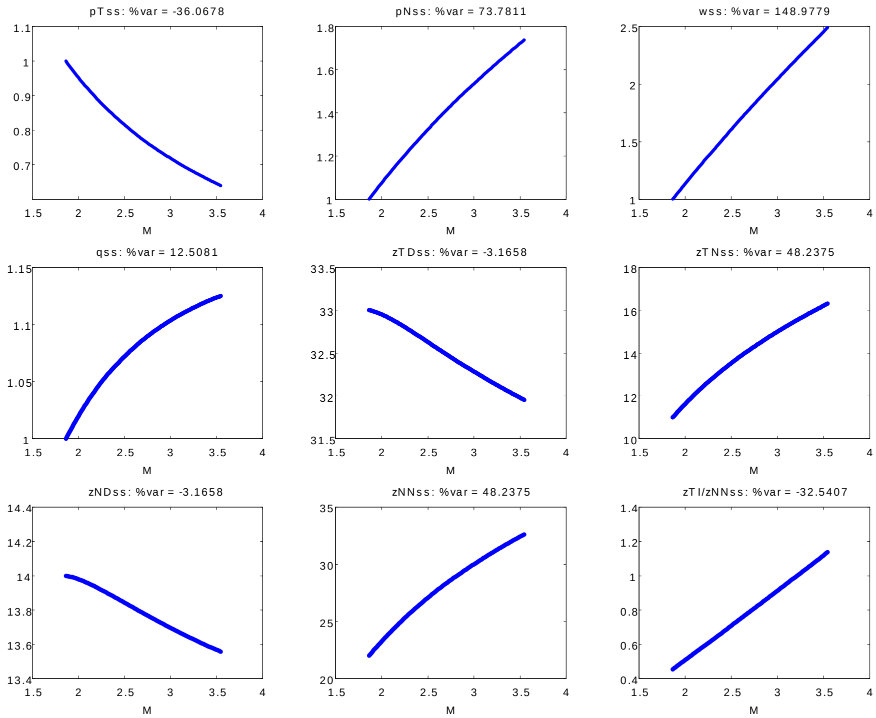
Then we would enter M without quotes to get

```
>> Comp_Statics
```

```
Enter a parameter name: M
% variation:
```

Then, enter the percentage, in percentage units. In our case, we want a 90% increase in M, then we enter 90. The program will compute, and display in a graph the steady state values of the variables for each value of M, starting at the calibration value. The title of each panel contains the variable name and the total percent variation between the steady state value and the final value, which corresponds to the steady state value with an increase in 90% in M.


yT ss: % var = 105.4491
yD ss: % var = -3.1658
yN ss: % var = 48.2375
kD ss: % var = 49.3475
kN ss: % var = 152.7314
kss: % var = 110.819
lD ss: % var = -32.5129
lN ss: % var = 14.2042
lss: % var = 0.85648
cT ss: % var = 172.6018
cN ss: % var = 65.3435
xF ss: % var = 144.6589
M



5 Dynamics

In this section we will show how to proceed to compute the dynamics of the model given a set of initial conditions for the state variables. In computing the steady state we assumed no population growth. The purpose of this simplification of the model is to convince us that we have the correct set of equations and the correct numerical implementation in our programs.

Here we will assume that the economy is not too far away from the steady state, so that we can spell the whole system of equations to be solved by an economy that reaches the steady state in just one period. That is, we departure from a set of initial conditions, the dynamics are completed in one period, and in the next period our model economy reaches a steady state. Once we have this system of equations solved, we will increase the time span where the dynamics occur.

We will write down all the equations for $t = 1, t = 2$ and $t = 3$ with their appropriate indexing. The identities $y_{It} = i_t$ and $x_{Dt} = y_{Dt}$ are incorporated in the notation.

5.1 The three periods equations

$t = 1$

Households

$$\begin{aligned}
\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{T1}}{c_{N1}}\right)^{\rho-1} &= \frac{p_{T1}}{p_{N1}} \\
\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_1}{n_1^{\rho}}\frac{\pounds_1}{U_1}c_{N1}^{\rho-1} &= \frac{p_{N1}}{w_1} \\
r_{k1} + q_1(1-\delta) &= q_0(1+r_1) \\
k_1(1-\delta) + y_{I1} &= k_2
\end{aligned}$$

Firms

$$\left(p_{D1} - p_{T1}a_{TD} - p_{N1}a_{ND}\right)\alpha_D A_D k_{D1}^{\alpha_D - 1} l_{D1}^{1-\alpha_D} = r_{k1} \tag{71}$$
$$\left(p_{D1} - p_{T1}a_{TD} - p_{N1}a_{ND}\right)(1-\alpha_D) A_D k_{D1}^{\alpha_D} l_{D1}^{-\alpha_D} = w_1 \tag{72}$$
$$A_D k_{D1}^{\alpha_D} l_{D1}^{1-\alpha_D} = y_{D1} \tag{73}$$
$$z_{TD1}/a_{TD} = y_{D1} \tag{74}$$
$$z_{ND1}/a_{ND} = y_{D1} \tag{75}$$
$$\left(p_{N1} - p_{T1}a_{TN} - p_{N1}a_{NN}\right)\alpha_N A_N k_{N1}^{\alpha_N - 1} l_{N1}^{1-\alpha_N} = r_{k1} \tag{76}$$
$$\left(p_{N1} - p_{T1}a_{TN} - p_{N1}a_{NN}\right)(1-\alpha_N) A_N k_{N1}^{\alpha_N} l_{N1}^{-\alpha_N} = w_1 \tag{77}$$
$$A_N k_{N1}^{\alpha_N} l_{N1}^{1-\alpha_N} = y_{N1} \tag{78}$$
$$z_{TN1}/a_{TN} = y_{N1} \tag{79}$$
$$z_{NN1}/a_{NN} = y_{N1} \tag{80}$$
$$\frac{\mu}{1-\mu}\left(\frac{x_{D1}}{m_1}\right)^{\zeta-1} = \frac{p_{D1}}{1+\tau_1} \tag{81}$$
$$M\left(\mu x_{D1}^{\zeta} + (1-\mu)m_1^{\zeta}\right)^{\frac{1}{\zeta}} = y_{T1} \tag{82}$$
$$\frac{\gamma}{1-\gamma}\frac{z_{NI1}}{z_{TI1}} = \frac{p_{T1}}{p_{N1}} \tag{83}$$
$$q_1 G\gamma z_{TI1}^{\gamma-1} z_{NI1}^{1-\gamma} = p_{T1} \tag{84}$$
$$G z_{TI1}^{\gamma} z_{NI1}^{1-\gamma} = y_{I1} \tag{85}$$
$$D\left((1+\tau_{F1})\, p_{T1}\right)^{\frac{-1}{1-\zeta}} = x_{F1} \tag{86}$$

Markets

$$
\begin{aligned}
c_{N1}+z_{NI1}+z_{ND1}+z_{NN1} &= y_{N1} \quad (87)\\
c_{T1}+z_{TI1}+z_{TD1}+z_{TN1}+x_{F1} &= y_{T1} \quad (88)\\
k_{D1}+k_{N1} &= k_1 \quad (89)\\
l_{D1}+l_{N1} &= l_1 \quad (90)\\
p_{T1}x_{F1}+r_1b_1 &= m_1 \quad (91)
\end{aligned}
$$

$t=2$
Households

$$
\begin{aligned}
\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{T2}}{c_{N2}}\right)^{\rho-1} &= \frac{p_{T2}}{p_{N2}}\\
\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_2}{n_2^{\rho}}\frac{\pounds_2}{U_2}c_{N2}^{\rho-1} &= \frac{p_{N2}}{w_2}\\
k_2(1-\delta)+y_{I2} &= k_{ss}
\end{aligned}
$$

Firms

$$
\begin{aligned}
(p_{D2}-p_{T2}a_{TD}-p_{N2}a_{ND})\,\alpha_D A_D k_{D2}^{\alpha_D-1}l_{D2}^{1-\alpha_D} &= r_{k2} \quad (92)\\
(p_{D2}-p_{T2}a_{TD}-p_{N2}a_{ND})\,(1-\alpha_D)A_D k_{D2}^{\alpha_D}l_{D2}^{-\alpha_D} &= w_2 \quad (93)\\
A_D k_{D2}^{\alpha_D}l_{D2}^{1-\alpha_D} &= y_{D2} \quad (94)\\
z_{TD2}/a_{TD} &= y_{D2} \quad (95)\\
z_{ND2}/a_{ND} &= y_{D2} \quad (96)\\
(p_{N2}-p_{T2}a_{TN}-p_{N2}a_{NN})\,\alpha_N A_N k_{N2}^{\alpha_N-1}l_{N2}^{1-\alpha_N} &= r_{k2} \quad (97)\\
(p_{N2}-p_{T2}a_{TN}-p_{N2}a_{NN})\,(1-\alpha_N)A_N k_{N2}^{\alpha_N}l_{N2}^{-\alpha_N} &= w_2 \quad (98)\\
A_N k_{N2}^{\alpha_N}l_{N2}^{1-\alpha_N} &= y_{N2} \quad (99)\\
z_{TN2}/a_{TN} &= y_{N2} \quad (100)\\
z_{NN2}/a_{NN} &= y_{N2} \quad (101)\\
\frac{\mu}{1-\mu}\left(\frac{x_{D2}}{m_2}\right)^{\zeta-1} &= \frac{p_{D2}}{1+\tau_2} \quad (102)\\
M\left(\mu x_{D2}^{\zeta}+(1-\mu)m_2^{\zeta}\right)^{\frac{1}{\zeta}} &= y_{T2} \quad (103)\\
\frac{\gamma}{1-\gamma}\frac{z_{NI2}}{z_{TI2}} &= \frac{p_{T2}}{p_{N2}} \quad (104)\\
q_2 G\gamma z_{TI2}^{\gamma-1}z_{NI2}^{1-\gamma} &= p_{T2} \quad (105)\\
G z_{TI2}^{\gamma}z_{NI2}^{1-\gamma} &= y_{I2} \quad (106)\\
D\left((1+\tau_{F2})\,p_{T2}\right)^{\frac{-1}{1-\zeta}} &= x_{F2} \quad (107)
\end{aligned}
$$

Markets

$$
\begin{aligned}
c_{N2} + z_{NI2} + z_{ND2} + z_{NN2} &= y_{N2} && (108)\\
c_{T2} + z_{TI2} + z_{TD2} + z_{TN2} + x_{F2} &= y_{T2} && (109)\\
k_{D2} + k_{N2} &= k_2 && (110)\\
l_{D2} + l_{N2} &= l_2 && (111)\\
p_{T2} x_{F2} + r_2 b_2 &= m_2 && (112)
\end{aligned}
$$

Euler

$$
\beta(1+r_2)\underbrace{\left(\pounds_2^{(1-\eta)\Psi}\right)\left(U_{m2}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta\varepsilon}{n_2^{\rho}}\frac{c_{T2}^{\rho-1}}{p_{T2}}}_{\lambda_3} = \underbrace{\left(\pounds_1^{(1-\eta)\Psi}\right)\left(U_{m1}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta\varepsilon}{n_1^{\rho}}\frac{c_{T1}^{\rho-1}}{p_{T1}}}_{\lambda_2}
$$

$t = 3$
Households

$$
\begin{aligned}
\frac{\varepsilon}{1-\varepsilon}\left(\frac{c_{T3}}{c_{N3}}\right)^{\rho-1} &= \frac{p_{T3}}{p_{N3}}\\
\frac{\eta}{1-\eta}\frac{(1-\varepsilon)\bar{l}_3}{n_3^{\rho}}\frac{\pounds_3}{U_3}c_{N3}^{\rho-1} &= \frac{p_{N3}}{w_3}\\
k_2(1-\delta) + y_{I3} &= k_3
\end{aligned}
$$

Firms

$$
\begin{aligned}
(p_{D3}-p_{T3}a_{TD}-p_{N3}a_{ND})\,\alpha_D A_D k_{D3}^{\alpha_D-1} l_{D3}^{1-\alpha_D} &= r_{k3} && (113)\\
(p_{D3}-p_{T3}a_{TD}-p_{N3}a_{ND})\,(1-\alpha_D) A_D k_{D3}^{\alpha_D} l_{D3}^{-\alpha_D} &= w_3 && (114)\\
A_D k_{D3}^{\alpha_D} l_{D3}^{1-\alpha_D} &= y_{D3} && (115)\\
z_{TD3}/a_{TD} &= y_{D3} && (116)\\
z_{ND3}/a_{ND} &= y_{D3} && (117)\\
(p_{N3}-p_{T3}a_{TN}-p_{N3}a_{NN})\,\alpha_N A_N k_{N3}^{\alpha_N-1} l_{N3}^{1-\alpha_N} &= r_{k3} && (118)\\
(p_{N3}-p_{T3}a_{TN}-p_{N3}a_{NN})\,(1-\alpha_N) A_N k_{N3}^{\alpha_N} l_{N3}^{-\alpha_N} &= w_3 && (119)\\
A_N k_{N3}^{\alpha_N} l_{N3}^{1-\alpha_N} &= y_{N3} && (120)\\
z_{TN3}/a_{TN} &= y_{N3} && (121)\\
z_{NN3}/a_{NN} &= y_{N3} && (122)\\
\frac{\mu}{1-\mu}\left(\frac{x_{D3}}{m_3}\right)^{\zeta-1} &= \frac{p_{D3}}{1+\tau_3} && (123)\\
M\left(\mu x_{D3}^{\zeta}+(1-\mu)m_3^{\zeta}\right)^{\frac{1}{\zeta}} &= y_{T3} && (124)\\
\frac{\gamma}{1-\gamma}\frac{z_{NI3}}{z_{TI3}} &= \frac{p_{T3}}{p_{N3}} && (125)\\
q_3 G\gamma z_{TI3}^{\gamma-1} z_{NI3}^{1-\gamma} &= p_{T3} && (126)\\
G z_{TI3}^{\gamma} z_{NI3}^{1-\gamma} &= y_{Iss} && (127)\\
D_3\left((1+\tau_{F3})\,p_{T3}\right)^{\frac{-1}{1-\zeta}} &= x_{F3} && (128)
\end{aligned}
$$

Markets

$$
\begin{aligned}
c_{N3}+z_{NI3}+z_{ND3}+z_{NN3} &= y_{N3} && (129)\\
c_{T3}+z_{TI3}+z_{TD3}+z_{TN3}+x_{F3} &= y_{T3} && (130)\\
k_{D3}+k_{N3} &= k_3 && (131)\\
l_{D3}+l_{N3} &= l_3 && (132)\\
p_{T3}x_{F3}+r_3 b_3 &= m_3 && (133)
\end{aligned}
$$

Euler

$$
\beta(1+r_3)\underbrace{\left(\pounds_3^{(1-\eta)\Psi}\right)\left(U_{m3}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta\varepsilon}{n_3^{\rho}}\frac{c_{T3}^{\rho-1}}{p_{T3}}}_{\lambda_{ss}} = \underbrace{\left(\pounds_2^{(1-\eta)\Psi}\right)\left(U_{m2}^{\frac{\eta\Psi}{\rho}-1}\right)\frac{\eta\varepsilon}{n_2^{\rho}}\frac{c_{T2}^{\rho-1}}{p_{T2}}}_{\lambda_3}
$$

We have $4+3*2=10$ equations from the household, $16*3=48$ equations from the firms and $5*3=15$ market clearing equations, summing up to 73 equations plus 2 Euler equations makes 75. The list of variables is:

- Output: $\{y_{T1}, y_{D1}, y_{N1}, y_{I1}, y_{T2}, y_{D2}, y_{N2}, y_{I2}, y_{T3}, y_{D3}, y_{N3}, y_{I3}\}$ #12

- Consumption: $\{c_{T1}, c_{N1}, c_{T2}, c_{N2}, c_{T3}, c_{N3}, x_{F1}, x_{F2}, x_{F3}\}$ #9
- Intermediate inputs t=1: $\{z_{TD1}, z_{TN1}, z_{ND1}, z_{NN1}, z_{TI1}, z_{NI1}\}$ #6
- Intermediate inputs t=2: $\{z_{TD2}, z_{TN2}, z_{ND2}, z_{NN2}, z_{TI2}, z_{NI2}\}$ #6
- Intermediate inputs t=3: $\{z_{TD3}, z_{TN3}, z_{ND3}, z_{NN3}, z_{TI3}, z_{NI3}\}$ #6
- Trade: $\{m_1, m_2, m_3, b_2, b_3\}$ #5 since b_1 is given
- Labor: $\{l_{D1}, l_{N1}, l_1, l_{D2}, l_{N2}, l_2, l_{D3}, l_{N3}, l_3\}$ #9
- Capital: $\{k_{D2}, k_{N2}, k_2, k_{D3}, k_{N3}, k_3\}$ #6 since k_{D1} and k_{N1} are given
- Prices$\{p_{T1}, p_{N1}, w_1, r_{k1}, p_{T2}, p_{N2}, w_2, r_{k2}, p_{T3}, p_{N3}, w_3, r_{k3}\}$ #12
- Price of capital: $\{q_0, q_1, q_2, q_3\}$ #4
- Sum of number of variables: $12 + 9 + 6 * 3 + 5 + 9 + 6 + 12 + 4 = 75$.

5.2 A n periods program

Next, we produce a guess for $\{k_{D1}, k_{N1}, k_{D2}, k_{N2}, k_{D3}, k_{N3}, l_1, l_2, l_3, m_1, m_2, m_3, b_1, b_2, b_3\}$. It is easy to check, that you can obtain any other variable value as a residual of this guess using the set of equations above. But 15 equations would not be used. This set of unused equations are used to iterate Newton's algorithm. The system is:

$$\begin{aligned}
f(1) &= kD(1) - kD0; \\
f(2) &= kN(1) - kN0; \\
f(3) &= b(1) - b0; \\
f(4) &= cT(1) + zTI(1) + zTD(1) + zTN(1) + xF(1) - yT(1); \\
f(5) &= beta * (1 + r(2)) * lambda(3) - lambda(2); \\
f(6) &= (pT(1) * M * (mu * yD(1)\hat{}zeta + (1 - mu) * m(1)\hat{}zeta)\hat{}(1/zeta - 1) * ... \\
&\quad mu * yD(1)\hat{}(zeta - 1)) - pD(1);
\end{aligned}$$

Notice that the first three equations come from the household problem, were k_{D0}, k_{N0} and b_0 were given. Equations $f(1), f(2)$ and $f(3)$ are saying the Newton's algorithm that the economy starts at that point.

$$
\begin{aligned}
f(7) &= cT(2)+zTI(2)+zTD(2)+zTN(2)+xF(2)-yT(2);\\
f(8) &= beta*(1+r(3))*lambdass-lambda(3);\\
f(9) &= (pT(2)*M*(mu*yD(2)\hat{\ }zeta+(1-mu)*m(2)\hat{\ }zeta)\hat{\ }(1/zeta-1)*...\\
&\quad mu*yD(2)\hat{\ }(zeta-1))-pD(2);\\
f(10) &= (q(2)*G*gamma*(zNI(2)/zTI(2))\hat{\ }(1-gamma))/pT(2)-1;\\
f(11) &= ((eta*(1-epsilon)*l\_bar)/((1-eta)*n\hat{\ }rho))*...\\
&\quad (pound(2)/U(2))*cN(2)\hat{\ }(rho-1)/(pN(2)/w(2))-1;
\end{aligned}
$$

This part going from $f(7)$ to $f(11)$ is the system that will be indexed when we extend the program to an n periods program.

$$
\begin{aligned}
f(12) &= ((eta*(1-epsilon)*l\_bar)/((1-eta)*n\hat{\ }rho))*...\\
&\quad (pound(3)/U(3))*cN(3)\hat{\ }(rho-1)/(pN(3)/w(3))-1;\\
f(13) &= (cT(3)+zTI(3)+zTD(3)+zTN(3)+xF(3))/yT(3)-1;\\
f(14) &= (q(3)*G*gamma*(zNI(3)/zTI(3))\hat{\ }(1-gamma))/pT(3)-1;\\
f(15) &= (pT(3)*M*(mu*yD(3)\hat{\ }zeta+(1-mu)*m(3)\hat{\ }zeta)\hat{\ }(1/zeta-1)*...\\
&\quad mu*yD(3)\hat{\ }(zeta-1))/pD(3)-1;
\end{aligned}
$$

This part last part, from $f(12)$ to $f(15)$ is the same system that we solved for the steady state and calibration program.

This way to proceed is the most secure method, since the system is still under control and it is very easy to generalize to a n periods program once the three periods program is written.

Next we provide the n periods program. It uses the already known **SScpo.m** and **secant.m**. Here we provide the script for the n periods program and the function file called SScpodin.m, where the dynamic system is written. File **SScpo.m** is the same that we used for the steady state program.

The following programs can be found in the folder called DynamicNperiods.

```
%Sudden_Stop.m solves the baseline Kehoe-Ruhl model of sudden stops.
%               Tr     Ntr    Int  C+G     I      X     C+I+X  Tot
%Traded         33     11     44    27     10     19     56    100
%Nontraded      14     22     36    51     13      0     64    100
%Intermediate   47     33     80    78     23     19    120    200
%Wages          18     45     63     0      0      0      0     63
%Capital        15     22     37     0      0      0      0     37
%Value Added    33     67    100     0      0      0      0    100
%Imports        18      0     18     0      0      0      0     18
%Tariffs        2       0      2     0      0      0      0      2
```

```
%Total          100   100   200    78    23    19   120   320
clear all           %Input-output table
data    =   [33    11    44    27    10    19    56   100;
             14    22    36    51    13     0    64   100;
             47    33    80    78    23    19   120   200;
             18    45    63     0     0     0     0    63;
             15    22    37     0     0     0     0    37;
             33    67   100     0     0     0     0   100;
             18     0    18     0     0     0     0    18;
             2      0     2     0     0     0     0     2;
             100   100   200    78    23    19   120   320];

%Trade parameters
zeta    =   0.5;
tau     =   data(8,1)/data(7,1);
mu      =   1/(1 + (1+tau) * (data(7,1)/(data(3,1)+data(6,1)))^(1-zeta));
M       =   data(9,1)/(mu*(data(3,1)+data(6,1))^zeta + ...
            (1-mu)*data(7,1)^zeta)^(1/zeta);
tauF    =   0.01;
D1988   =   data(1,6)/(1+tauF)^(-1/(1-zeta));
r_star  =   0.04;
sigma   =   0.1174;
rss     =   r_star+sigma;
bss     =   (data(7,1)-data(1,6))/rss;
Tparam  =   [M mu zeta D1988 r_star bss];

%Consumer parameters
beta    =   1/(1+rss);
rho     =   -1;
epsilon =   1/((data(1,4)/data(2,4))^(rho-1)+1);
K       =   (data(5,3)-data(3,5))/rss;
delta   =   data(3,5)/K;
l_bar   =   data(4,3)/0.267;
eta     =   1/(1+(l_bar-data(4,3))/(data(3,4)));
Psi     =   -1;
tauk    =   0; %0.201;
Cparam  =   [beta epsilon rho eta Psi delta tauk];

%Producer parameters
aTD     =   data(1,1)/(data(3,1)+data(6,1));
aND     =   data(2,1)/(data(3,1)+data(6,1));
aTN     =   data(1,2)/data(9,2);
aNN     =   data(2,2)/data(9,2);
alphaD  =   data(5,1)/data(6,1);
alphaN  =   data(5,2)/data(6,2);
rkss    =   rss+delta;
```

```
kNss    =   data(5,2)/rkss;
kDss    =   data(5,1)/rkss;
AD      =   (data(3,1)+data(6,1))/(kDss^alphaD*data(4,1)^(1-alphaD));
AN      =   data(9,2)/(kNss^alphaN*data(4,2)^(1-alphaN));
gamma   =   data(1,5)/data(3,5);
G       =   data(3,5)/(data(1,5)^gamma*data(2,5)^(1-gamma));
g       =   1;
Pparam  =   [aTD aND aTN aNN AD AN alphaD alphaN gamma G g];

%Time series parameters
n       =   1; %(1/2)*l_bar;
Dusa    =   1;
TSparam =   [l_bar n sigma Dusa tau tauF];

%Newton's algorithm parameters
crit    =   1e-10;
maxit   =   1000;
param   =   [Cparam Pparam Tparam TSparam];
x0      =   [36.0589 52.8864 63 18]';
sol     =   secant('SScpo', x0, param, crit, maxit);

%Variables allocation
kDss        =   sol(1);
kNss        =   sol(2);
lss         =   sol(3);
mss         =   sol(4);

%System
rss         =   r_star+sigma;
lnOverlD    =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kNss/kDss;
lDss        =   lss/(1+lnOverlD);
lNss        =   lnOverlD*lDss;
yDss        =   AD*kDss^alphaD*lDss^(1-alphaD);
zTDss       =   aTD*yDss;
zNDss       =   aND*yDss;
yNss        =   AN*kNss^alphaN*lNss^(1-alphaN);
zTNss       =   aTN*yNss;
zNNss       =   aNN*yNss;
yTss        =   M*(mu*yDss^zeta+(1-mu)*mss^zeta)^(1/zeta);
yIss        =   delta*(kDss+kNss);
pDss        =   (1+tau)*(mu/(1-mu))*(yDss/mss)^(zeta-1);
pTxF        =   mss-rss*bss;
pTss        =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
xFss        =   pTxF/pTss;
pmgkN       =   alphaN*AN*(kNss/lNss)^(alphaN-1);
pmgkD       =   alphaD*AD*(kDss/lDss)^(alphaD-1);
```

```
pmglD       =   (1-alphaD)*AD*(kDss/lDss)^alphaD;
pmglN       =   (1-alphaN)*AN*(kNss/lNss)^alphaN;
ratPmgs     =   pmgkN/pmgkD;
pNoverpT    =   (pDss/pTss-aTD+aTN*ratPmgs)/(ratPmgs*(1-aNN)+aND);
pNss        =   pNoverpT*pTss;
zNIoverzTI  =   (1-gamma)/(gamma*pNoverpT);
zTIss       =   yIss/(G*zNIoverzTI^(1-gamma));
zNIss       =   zNIoverzTI*zTIss;
rkss        =   (pDss-pTss*aTD-pNss*aND)*pmgkD;
wss         =   (pDss-pTss*aTD-pNss*aND)*pmglD;
qss         =   rkss/(rss+delta);
cNss        =   yNss-(zNIss+zNDss+zNNss);
cTss        =   cNss*((1-epsilon)*pTss/(epsilon*pNss))^(1/(rho-1));
U           =   epsilon*(cTss/n)^rho+(1-epsilon)*(cNss/n)^rho;
pound       =   (l_bar-lss)/l_bar;

%Dyamics
%Newton's algorithm parameters
crit    =   1e-10;
maxit   =   1000;
kD0     =   kDss*0.9;
kN0     =   kNss*0.8;
b0      =   bss;
T       =   30;
param   =   [Cparam Pparam Tparam TSparam kD0 kN0 b0 T];
kDseed  =   kDss*ones(size(1:T));
kNseed  =   kNss*ones(size(1:T));
lseed   =   lss*ones(size(1:T));
mseed   =   mss*ones(size(1:T));
bseed   =   bss*ones(size(1:T));
x0      =   [kDseed kNseed lseed mseed bseed]';
sol     =   secant('SScpodin', x0, param, crit, maxit);

for t=1:T
    kD(t)   =   sol(0*T+t);
    kN(t)   =   sol(1*T+t);
    l(t)    =   sol(2*T+t);
    m(t)    =   sol(3*T+t);
    b(t)    =   sol(4*T+t);
end


for t=1:T-1
    k(t)        =   kD(t)+kN(t);
    r(t)        =   r_star+sigma;
    lnOverlD(t) =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kN(t)/kD(t);
```

```
    lD(t)       =   l(t)/(1+lnOverlD(t));
    lN(t)       =   lnOverlD(t)*lD(t);
    yD(t)       =   AD*kD(t)^alphaD*lD(t)^(1-alphaD);
    zTD(t)      =   aTD*yD(t);
    zND(t)      =   aND*yD(t);
    yN(t)       =   AN*kN(t)^alphaN*lN(t)^(1-alphaN);
    zTN(t)      =   aTN*yN(t);
    zNN(t)      =   aNN*yN(t);
    yT(t)       =   M*(mu*yD(t)^zeta+(1-mu)*m(t)^zeta)^(1/zeta);
    k(t+1)      =   kD(t+1)+kN(t+1);
    yI(t)       =   k(t+1)-(1-delta)*k(t);
    pD(t)       =   (1+tau)*(mu/(1-mu))*(yD(t)/m(t))^(zeta-1);
    pTxF        =   m(t)-r(t)*b(t);
    pT(t)       =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
    xF(t)       =   pTxF/pT(t);
    pmgkN(t)    =   alphaN*AN*(kN(t)/lN(t))^(alphaN-1);
    pmgkD(t)    =   alphaD*AD*(kD(t)/lD(t))^(alphaD-1);
    pmglD(t)    =   (1-alphaD)*AD*(kD(t)/lD(t))^alphaD;
    pmglN(t)    =   (1-alphaN)*AN*(kN(t)/lN(t))^alphaN;
    ratPmgs(t)  =   pmgkN(t)/pmgkD(t);
    pNoverpT(t) =   (pD(t)/pT(t)-aTD+aTN*ratPmgs(t))/(ratPmgs(t)*(1-aNN)+aND);
    pN(t)       =   pNoverpT(t)*pT(t);
    zNIoverzTI(t)=   (1-gamma)/(gamma*pNoverpT(t));
    zTI(t)      =   yI(t)/(G*zNIoverzTI(t)^(1-gamma));
    zNI(t)      =   zNIoverzTI(t)*zTI(t);
    rk(t)       =   (pD(t)-pT(t)*aTD-pN(t)*aND)*pmgkD(t);
    w(t)        =   (pD(t)-pT(t)*aTD-pN(t)*aND)*pmglD(t);
    if t==1
        q(t)        =   pT(t)/(G*gamma*(zNI(t)/zTI(t))^(1-gamma));
    else
        q(t)        =   (q(t-1)*(1+r(t))-rk(t))/(1-delta);
    end
    q0          =   (rk(t)+q(t)*(1-delta))/(1+r(t));
    cN(t)       =   yN(t)-(zNI(t)+zND(t)+zNN(t));
    cT(t)       =   cN(t)*((1-epsilon)*pT(t)/(epsilon*pN(t)))^(1/(rho-1));
    U(t)        =   epsilon*(cT(t)/n)^rho+(1-epsilon)*(cN(t)/n)^rho;
    pound(t)    =   (l_bar-l(t))/l_bar;
    lambda(t+1) =   (pound(t)^(1-eta)*Psi)*U(t)^(eta*Psi/rho-1)*...
                    (eta*epsilon/n^rho)*(cT(t)^(rho-1)/pT(t));
end
k(T)        =   kD(T)+kN(T);
r(T)        =   r_star+sigma;
lnOverlD(T) =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kN(T)/kD(T);
lD(T)       =   l(T)/(1+lnOverlD(T));
lN(T)       =   lnOverlD(T)*lD(T);
yD(T)       =   AD*kD(T)^alphaD*lD(T)^(1-alphaD);
```

```
zTD(T)       =   aTD*yD(T);
zND(T)       =   aND*yD(T);
yN(T)        =   AN*kN(T)^alphaN*lN(T)^(1-alphaN);
zTN(T)       =   aTN*yN(T);
zNN(T)       =   aNN*yN(T);
yT(T)        =   M*(mu*yD(T)^zeta+(1-mu)*m(T)^zeta)^(1/zeta);
yI(T)        =   k(T)-(1-delta)*k(T);
pD(T)        =   (1+tau)*(mu/(1-mu))*(yD(T)/m(T))^(zeta-1);
pTxF         =   m(T)-r(T)*b(T);
pT(T)        =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
xF(T)        =   pTxF/pT(T);
pmgkN(T)     =   alphaN*AN*(kN(T)/lN(T))^(alphaN-1);
pmgkD(T)     =   alphaD*AD*(kD(T)/lD(T))^(alphaD-1);
pmglD(T)     =   (1-alphaD)*AD*(kD(T)/lD(T))^alphaD;
pmglN(T)     =   (1-alphaN)*AN*(kN(T)/lN(T))^alphaN;
ratPmgs(T)   =   pmgkN(T)/pmgkD(T);
pNoverpT(T) =    (pD(T)/pT(T)-aTD+aTN*ratPmgs(T))/(ratPmgs(T)*(1-aNN)+aND);
pN(T)        =   pNoverpT(T)*pT(T);
zNIoverzTI(T)=    (1-gamma)/(gamma*pNoverpT(T));
zTI(T)       =   yI(T)/(G*zNIoverzTI(T)^(1-gamma));
zNI(T)       =   zNIoverzTI(T)*zTI(T);
rk(T)        =   (pD(T)-pT(T)*aTD-pN(T)*aND)*pmgkD(T);
w(T)         =   (pD(T)-pT(T)*aTD-pN(T)*aND)*pmglD(T);
q(T)         =   (q(T-1)*(1+r(T))-rk(T))/(1-delta);
q0           =   (rk(T)+q(T)*(1-delta))/(1+r(T));
cN(T)        =   yN(T)-(zNI(T)+zND(T)+zNN(T));
cT(T)        =   cN(T)*((1-epsilon)*pT(T)/(epsilon*pN(T)))^(1/(rho-1));
U(T)         =   epsilon*(cT(T)/n)^rho+(1-epsilon)*(cN(T)/n)^rho;
pound(T)     =   (l_bar-l(T))/l_bar;
lambda(T+1) =    (pound(T)^(1-eta)*Psi)*U(T)^(eta*Psi/rho-1)*...
                 (eta*epsilon/n^rho)*(cT(T)^(rho-1)/pT(T));
```

The dynamic system of equations is found in **SScpodin.m**

```
function f = SScpodin(x0, param)
%Parameter allocation
beta    =   param(1);
epsilon =   param(2);
rho     =   param(3);
eta     =   param(4);
Psi     =   param(5);
delta   =   param(6);
tauk    =   param(7);
```

```
aTD     =   param(8);
aND     =   param(9);
aTN     =   param(10);
aNN     =   param(11);
AD      =   param(12);
AN      =   param(13);
alphaD  =   param(14);
alphaN  =   param(15);
gamma   =   param(16);
G       =   param(17);
g       =   param(18);

M       =   param(19);
mu      =   param(20);
zeta    =   param(21);
D1988   =   param(22);
r_star  =   param(23);
bss     =   param(24);

l_bar   =   param(25);
n       =   param(26);
sigma   =   param(27);
Dusa    =   param(28);
tau     =   param(29);
tauF    =   param(30);

kD0     =   param(31);
kN0     =   param(32);
b0      =   param(33);
T       =   param(34);

%Variables allocation
for t=1:T
    kD(t)   =   x0(0*T+t);
    kN(t)   =   x0(1*T+t);
    l(t)    =   x0(2*T+t);
    m(t)    =   x0(3*T+t);
    b(t)    =   x0(4*T+t);
end

for t=1:T-1
    k(t)        =   kD(t)+kN(t);
    r(t)        =   r_star+sigma;
    lnOverlD(t) =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kN(t)/kD(t);
    lD(t)       =   l(t)/(1+lnOverlD(t));
    lN(t)       =   lnOverlD(t)*lD(t);
```

```
    yD(t)       =   AD*kD(t)^alphaD*lD(t)^(1-alphaD);
    zTD(t)      =   aTD*yD(t);
    zND(t)      =   aND*yD(t);
    yN(t)       =   AN*kN(t)^alphaN*lN(t)^(1-alphaN);
    zTN(t)      =   aTN*yN(t);
    zNN(t)      =   aNN*yN(t);
    yT(t)       =   M*(mu*yD(t)^zeta+(1-mu)*m(t)^zeta)^(1/zeta);
    k(t+1)      =   kD(t+1)+kN(t+1);
    yI(t)       =   k(t+1)-(1-delta)*k(t);
    pD(t)       =   (1+tau)*(mu/(1-mu))*(yD(t)/m(t))^(zeta-1);
    pTxF        =   m(t)-r(t)*b(t);
    pT(t)       =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
    xF(t)       =   pTxF/pT(t);
    pmgkN(t)    =   alphaN*AN*(kN(t)/lN(t))^(alphaN-1);
    pmgkD(t)    =   alphaD*AD*(kD(t)/lD(t))^(alphaD-1);
    pmglD(t)    =   (1-alphaD)*AD*(kD(t)/lD(t))^alphaD;
    pmglN(t)    =   (1-alphaN)*AN*(kN(t)/lN(t))^alphaN;
    ratPmgs(t)  =   pmgkN(t)/pmgkD(t);
    pNoverpT(t) =   (pD(t)/pT(t)-aTD+aTN*ratPmgs(t))/(ratPmgs(t)*(1-aNN)+aND);
    pN(t)       =   pNoverpT(t)*pT(t);
    zNIoverzTI(t)=   (1-gamma)/(gamma*pNoverpT(t));
    zTI(t)      =   yI(t)/(G*zNIoverzTI(t)^(1-gamma));
    zNI(t)      =   zNIoverzTI(t)*zTI(t);
    rk(t)       =   (pD(t)-pT(t)*aTD-pN(t)*aND)*pmgkD(t);
    w(t)        =   (pD(t)-pT(t)*aTD-pN(t)*aND)*pmglD(t);
    if t==1
        q(t)        =   pT(t)/(G*gamma*(zNI(t)/zTI(t))^(1-gamma));
    else
        q(t)        =   (q(t-1)*(1+r(t))-rk(t))/(1-delta);
    end
    q0          =   (rk(t)+q(t)*(1-delta))/(1+r(t));
    cN(t)       =   yN(t)-(zNI(t)+zND(t)+zNN(t));
    cT(t)       =   cN(t)*((1-epsilon)*pT(t)/(epsilon*pN(t)))^(1/(rho-1));
    U(t)        =   epsilon*(cT(t)/n)^rho+(1-epsilon)*(cN(t)/n)^rho;
    pound(t)    =   (l_bar-l(t))/l_bar;
    lambda(t+1) =   (pound(t)^(1-eta)*Psi)*U(t)^(eta*Psi/rho-1)*...
                    (eta*epsilon/n^rho)*(cT(t)^(rho-1)/pT(t));
end
k(T)        =   kD(T)+kN(T);
r(T)        =   r_star+sigma;
lnOverlD(T) =   ((1-alphaN)/alphaN)*(alphaD/(1-alphaD))*kN(T)/kD(T);
lD(T)       =   l(T)/(1+lnOverlD(T));
lN(T)       =   lnOverlD(T)*lD(T);
yD(T)       =   AD*kD(T)^alphaD*lD(T)^(1-alphaD);
zTD(T)      =   aTD*yD(T);
zND(T)      =   aND*yD(T);
```

```
yN(T)        =   AN*kN(T)^alphaN*lN(T)^(1-alphaN);
zTN(T)       =   aTN*yN(T);
zNN(T)       =   aNN*yN(T);
yT(T)        =   M*(mu*yD(T)^zeta+(1-mu)*m(T)^zeta)^(1/zeta);
yI(T)        =   k(T)-(1-delta)*k(T);
pD(T)        =   (1+tau)*(mu/(1-mu))*(yD(T)/m(T))^(zeta-1);
pTxF         =   m(T)-r(T)*b(T);
pT(T)        =   ((pTxF/D1988)^((zeta-1)/zeta))*(1+tauF)^(-1/zeta);
xF(T)        =   pTxF/pT(T);
pmgkN(T)     =   alphaN*AN*(kN(T)/lN(T))^(alphaN-1);
pmgkD(T)     =   alphaD*AD*(kD(T)/lD(T))^(alphaD-1);
pmglD(T)     =   (1-alphaD)*AD*(kD(T)/lD(T))^alphaD;
pmglN(T)     =   (1-alphaN)*AN*(kN(T)/lN(T))^alphaN;
ratPmgs(T)  =    pmgkN(T)/pmgkD(T);
pNoverpT(T) =    (pD(T)/pT(T)-aTD+aTN*ratPmgs(T))/(ratPmgs(T)*(1-aNN)+aND);
pN(T)        =   pNoverpT(T)*pT(T);
zNIoverzTI(T)=    (1-gamma)/(gamma*pNoverpT(T));
zTI(T)       =   yI(T)/(G*zNIoverzTI(T)^(1-gamma));
zNI(T)       =   zNIoverzTI(T)*zTI(T);
rk(T)        =   (pD(T)-pT(T)*aTD-pN(T)*aND)*pmgkD(T);
w(T)         =   (pD(T)-pT(T)*aTD-pN(T)*aND)*pmglD(T);
q(T)         =   (q(T-1)*(1+r(T))-rk(T))/(1-delta);
q0           =   (rk(T)+q(T)*(1-delta))/(1+r(T));
cN(T)        =   yN(T)-(zNI(T)+zND(T)+zNN(T));
cT(T)        =   cN(T)*((1-epsilon)*pT(T)/(epsilon*pN(T)))^(1/(rho-1));
U(T)         =   epsilon*(cT(T)/n)^rho+(1-epsilon)*(cN(T)/n)^rho;
pound(T)     =   (l_bar-l(T))/l_bar;
lambda(T+1) =    (pound(T)^(1-eta)*Psi)*U(T)^(eta*Psi/rho-1)*...
                 (eta*epsilon/n^rho)*(cT(T)^(rho-1)/pT(T));

f(1)    =   kD(1)-kD0;
f(2)    =   kN(1)-kN0;
f(3)    =   b(1)-b0;
f(4)    =   cT(1)+zTI(1)+zTD(1)+zTN(1)+xF(1)-yT(1);
f(5)    =   beta*(1+r(2))*lambda(3)-lambda(2);
f(6)    =   (pT(1)*M*(mu*yD(1)^zeta+(1-mu)*m(1)^zeta)^(1/zeta-1)*...
            mu*yD(1)^(zeta-1))-pD(1);

for t=2:T-1
f(5*t-3)=   cT(t)+zTI(t)+zTD(t)+zTN(t)+xF(t)-yT(t);
f(5*t-2)=   beta*(1+r(t+1))*lambda(t+2)-lambda(t+1);
f(5*t-1)=   (pT(t)*M*(mu*yD(t)^zeta+(1-mu)*m(t)^zeta)^(1/zeta-1)*...
            mu*yD(t)^(zeta-1))-pD(t);
f(5*t)  =   (q(t)*G*gamma*(zNI(t)/zTI(t))^(1-gamma))/pT(t)-1;
f(5*t+1)=   ((eta*(1-epsilon)*l_bar)/((1-eta)*n^rho))*...
                (pound(t)/U(t))*cN(t)^(rho-1) / (pN(t)/w(t))-1;
```

```
end

f(5*T-3)=   ((eta*(1-epsilon)*l_bar)/((1-eta)*n^rho))*...
                (pound(T)/U(T))*cN(T)^(rho-1) / (pN(T)/w(T))-1;
f(5*T-2)=   (cT(T)+zTI(T)+zTD(T)+zTN(T)+xF(T))/yT(T)-1;
f(5*T-1)=   (q(T)*G*gamma*(zNI(T)/zTI(T))^(1-gamma))/pT(T)-1;
f(5*T)  =   (pT(T)*M*(mu*yD(T)^zeta+(1-mu)*m(T)^zeta)^(1/zeta-1)*...
             mu*yD(T)^(zeta-1))/pD(T)-1;
f=f';
```

Notice that the last four equations of the Newton's algorithm coincide with those that conform the system of equations when we compute the steady state. Any sequence of prices and quantities that satisfy the above system of equations has to end up in a state that is a steady state of the economy.

6 Exercises

1) The value of b_{ss} used to compute the steady state can be changed freely. Use the set of programs provided in folder SteadyState0 and increase or decrease the value of b_{ss} by a factor of 2 in line 34 of the main script **Sudden_Stops.m**.

- What happens with steady state quantities and prices?
- What is the interpretation of equation $\frac{\partial L_H}{\partial b_t} = 0$ in the household's set of first order conditions?
- The steady state level of b_{ss} seems to be somehow arbitrary. Can you set up a simple one sector model of a small open economy to determine the optimal value of b_{ss} by hand?

2) Open **HojaCalibracion.xlsx** file in folder Calibration and change the value of intermediate inputs in a way that the input output table is consistent (in a National Accounts sense) and run the program **Sudden_Stops.m**.

- What happens with steady state prices and quantities?
- What parameter values are changed?

3) Open the folder ComparativeStatics and perform an experiment on δ. Run the file **Comp_Statics.m,** type delta (without quotes) and then type 50.

- What has happened with prices and output?
- Is it a reasonable theory for the drop in output after a sudden stop?
- Read carefully section **6.3 Variable capital utilization** of the Kehoe-Ruhl paper on sudden stops.

4) Labor adjustment costs. Consider the following production function

$$y_{Dt} = \min\left[z_{TDt}/a_{TDt}, z_{NDt}/a_{NDt}, A_D k_{Dt}^{\alpha_D} (g^t l_{Dt})^{1-\alpha_D}\right] - g^t \theta \left(\frac{l_{Dt}}{l_{Dt-1}} - 1\right)^2 l_{Dt-1}$$

- What equations have to be modified in the steady state computations?
- What equations have to be modified in the dynamics?
- Solve the problem. Hint: Notice that you have a lagged value for l_{Dt} in the new specification. Solve the original problem as written in these notes and use the dynamics as a seed for the appended set equations with the modification.

5) Variable capital utilization. Consider the following specification.

$$\begin{aligned} k_{Dt+1} &= (1 - \delta(u_{Dt}))k_{Dt} + i_{Dt} \\ \delta(u_{Dt}) &= \overline{\delta} + \frac{\chi}{\omega}(u_{Dt}^{\omega} - 1) \\ y_{Dt} &= A_D (u_{Dt} k_{Dt})^{\alpha_D} (g^t l_{Dt})^{1-\alpha_D} \end{aligned}$$

- What values of u_{Dt}^{ω} would not have any effect on the steady state?
- If you were free to choose a sequence for $\{u_{Dt}^{\omega}\}_{t=1}^{T}$ what values would you use before, during and after a sudden stop, based on the results of exercise 3?
- Can you incorporate those modifications in the program?

6) Quasilinear utility. In some small open economies models quasilinear utility is specified to prevent income effects to take place.

$$u(c_{Tt}, c_{Nt}, l_t) = \frac{1}{\Psi}\left(\left(\left(\varepsilon\left(\frac{c_{Tt}}{n_t}\right)^{\rho} + (1-\varepsilon)\left(\frac{c_{Nt}}{n_t}\right)^{\rho}\right)^{\frac{1}{\rho}} - \lambda g^t \left(\frac{l_t}{\overline{l}_t}\right)^{\eta}\right)^{\Psi} - 1\right)$$

- Is this a major change in our model? What do you expect to happen?
- Modify the programs to incorporate this utility specification and compare the results.

7) Exogenous TFP. Use again the library contained in folder ComparativeStatics to produce a change in TFP for the traded, the non-traded and the composite good.

- Change the values of these parameters to obtain a target output change in both sectors.
- Introduce A_D and A_N as exogenous processes in the program **Sudden_Stops.m** in folder DynamicsNperiods to get the target changes in output.