Horizontal FDI: Part I

This version: September 6, 2016

[*An earlier version of this note was missing the "$\rho = 0.9$" in the last numerical example.*]

In these notes we will build a model to help us understand foreign direct investment that is undertaken to provide market access. Although we will introduce some of the multinational firm concepts in this note, we begin by studying a world without multinationals.

The world economy consists of two countries: $i = 1, 2$. Each country's total expenditure is E_i. We will consider two kinds of firms: 1) *domestic firms*, which only produce in their home country and serve foreign markets by exporting and 2) *multinational firms*, which produce in the home country and in the foreign country. There are many firms of each type in each country. Let m_i be the number of multinational firms producing in country i and let n_i be the number of domestic firms operating in i.

We begin by holding fixed the number of each type of firm in each country and study how prices and quantities are determined. Later, we will study how the number of firms of each type is determined.

Shares, profits, and prices

We assume that firms are identical: Each has a constant marginal cost of production (that may vary across countries) w_i.[1] The firm faces an inverse demand function, $p(x_i)$ which is downward sloping: the more the firm chooses to produce (x_i), the lower is the price it receives.

A firm operating in country i, with marginal cost w_i, selling x_i earns profit

$$\pi_i = (p_i(x_i) - w_i) \times x_i - w_i f_i, \tag{1}$$

where f_i are the fixed costs the firm must pay. The fixed cost may differ depending on the firm type (multinational or domestic) or it could depend on the country. A firm must pay the headquarters fixed cost f^h. For each country in which it produces, it must pay a production fixed cost, f^p. This means that a domestic firm would pay $f_i = f^h + f^p$ while a multinational operating in two countries would pay $f_i = f^h + f^p + f^p$. [What is the difference between a fixed cost and a marginal cost?]

How much should the firm produce? What price is associated with this amount? The standard approach would be to take the derivative of the profit function and set it to zero. If we did this, we would find that the marginal revenue of the firm should equal its marginal cost. [This should sound very familiar.] This condition implies that

$$p_i\left(1 - 1/\epsilon_i\right) = w_i, \tag{2}$$

where ϵ_i is the price elasticity of demand. For now, we will just assume a value for the elasticity ϵ. If we specified a demand curve, we could compute the elasticity.

[1] We will relax this assumption later. As we have seen in the data, multinational firms are often different from their domestic counterparts. [In what ways?]

Elasticity refresher. The price elasticity of demand is — holding everything else constant — the proportional change in quantity sold relative to the proportional change in price. Mathematically, this is

$$\epsilon = \frac{\Delta x/x}{\Delta p/p} \text{ or } \frac{\partial x}{\partial p} \times \frac{p}{x}. \tag{3}$$

This value is usually negative, since demand curves usually slope down. The larger is the elasticity, the more responsive are quantities to a change in price. The price elasticity of gasoline, for example, is small: When gas prices rise, people still have to drive places. The price elasticity of a Big Mac is large: When the price of a Big Mac rises, people can eat tacos, or Whoppers.

Substituting (2) into (1), we can write the firm's profit as

$$\begin{aligned}
\pi_i &= [p_i(x_i) - p_i(x_i)\,(1 - 1/\epsilon_i)] \times x_i - w_i f_i \\
\pi_i &= p_i(x_i)\,[1 - (1 - 1/\epsilon_i)] \times x_i - w_i f_i \\
\pi_i &= p_i(x_i)\,[1/\epsilon_i] \times x_i - w_i f_i. && (4)
\end{aligned}$$

We can simplify things further by defining $s_i = (p_i \times x_i)/E_i$ as the firm's share of country i's total expenditures. Substituting this share definition into (4) gives us

$$\pi_i = \frac{s_i E_i}{\epsilon_i} - w_i f_i. \tag{5}$$

We know ϵ_i, E_i, w_i, and f_i. If we knew what share of total spending the firm captured, s_i, we would know the firm's profits. [Are firm profits increasing or decreasing in ϵ_i? What is the intuition for this?]

A one-country closed economy

We begin by studying a country that is closed to trade and multinational production, $m_i = 0$ and $n_i > 0$. If there are n_i identical firms operating in the country, then each one must have the same share of the country's total expenditure,

$$s_i = \frac{1}{n_i}. \tag{6}$$

Using (5), we can express the firm's profits as

$$\pi_i = \frac{(1/n_i)E_i}{\epsilon_i} - w_i f_i. \tag{7}$$

How do a firm's profits change with n_i? As we increase the number of firms in the economy, each earns a smaller share of the country's total expenditure — we can see this from (6). Since the fixed costs stay the same, but the firm's revenues decrease, profits decrease.

Numerical example. Let $w_1 = 2$, $E_1 = 100$, $\epsilon_1 = 2$, $f^h = 0.5$, $f^p = 0.25$. What are the firm's profits when $n_1 = 10$?

Each firm captures 0.10 share of the economy's total spending. Using (5) we find that $\pi_1 = 3.50$.

What are profits when $n_1 = 20$ and all the other parameters are unchanged?

Each firm now captures 0.05 share of the economy's total spending and $\pi_1 = 1.0$.

We have doubled the number of firms competing in the country, but each firm's profits fell by more than one-half. Why?

A two-country economy with trade

The closed economy model helped us understand the relationship between the number of firms operating in a country and a firm's profits. What happens when we allow domestic firms to export?

In this model, there are two countries, $i = 1, 2$. There are only domestic firms in each country, $n_1 > 0$, $n_2 > 0$, $m_1 = m_2 = 0$, but the domestic firms can serve the other country by exporting. When a firm exports, we assume that it must pay some costs to do so. These costs could be tariffs or transportation costs — or any other costs that the firm faces when exporting that it does not face when selling to its domestic market.

Since exporters face higher costs, they will have lower market shares in the foreign country. If s_i is the market share of a domestic firm in country i, then $s_i \times \rho$ is the market share of firms exporting from the other country. We set $\rho \leq 1$, so that the exporters have smaller market shares than the domestic firms.

How do we determine the market shares in this model? We know that all the market shares in a country must sum to one. (That's the point of expressing them as shares.) In country 1 there are n_1 domestic firms who each have share s_1. There are also n_2 firms from country 2 that are exporting to country 1. Each of the exporting firms receives share $s_1\rho$. These must all add up to one,

$$1 = n_1 \times s_1 + n_2 \times s_1\rho. \tag{8}$$

Solving this expression for s_1 yields the market share for the domestic firms in country 1,

$$s_1 = \frac{1}{n_1 + n_2\rho}. \tag{9}$$

As in the closed economy model, we can compute the profits for a firm, but we now have to take into account the profits that a firm earns in the other country. For a country-1 firm, this is

$$\pi_1 = \frac{s_1 E_1}{\epsilon_1} + \frac{\rho s_2 E_2}{\epsilon_2} - w_1 f_1. \tag{10}$$

The first term on the right-hand side is the profit earned by selling in the home country. The second term is the profit earned by selling in the foreign country — exporting. The last term is the fixed costs. Since the firm is still only producing in its home country, its fixed costs are the same as they would be in the closed economy model, $f_1 = f^h + f^p$.

Numerical example, continued. Suppose we open our closed economy to trade with an identical foreign country. Let $w_1 = w_2 = 2$, $E_1 = E_2 = 100$, $\epsilon_1 = \epsilon_2 = 2$, $f^h = 0.5$, $f^p = 0.25$, and $\rho = 0.9$. What are the country 1 firm's profits when $n_1 = 10$ and $n_2 = 10$?

We find that the domestic market share is $s_1 = 0.0526$*, the firm's market share in the foreign country is* $\rho s_2 = 0.0474$*, and the firm's total profits is* $\pi_1 = 3.50$.

In this model with trade, 20 firms supply country 1 — half are domestic firms and half are exporting from country 2. In the closed economy model in the previous section, we also solved a model in which 20 domestic firms served country 1. Why is s_1 different in the two models?