

Multinationals and the Globalization of Production

Optimization

Penn State // Fall 2016

Optimization

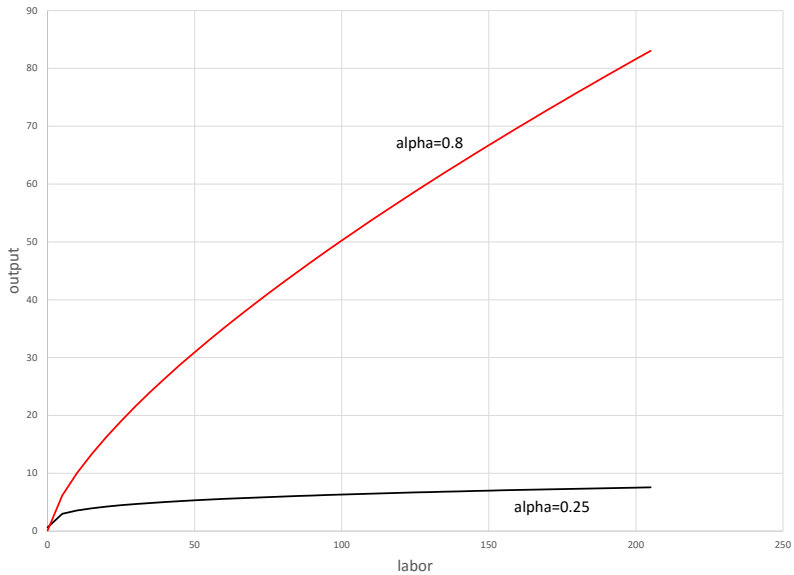
- ▶ Intermediate micro recap
- ▶ Given a production function, how should the firm choose inputs?
 1. Set up the profit function
 2. Construct the first-order condition
 3. Solve first-order condition to find input value

Example

- ▶ Production function uses labor ℓ , paid wage w

$$q = A\ell^\alpha$$

- ▶ α controls the returns to labor



Profit function

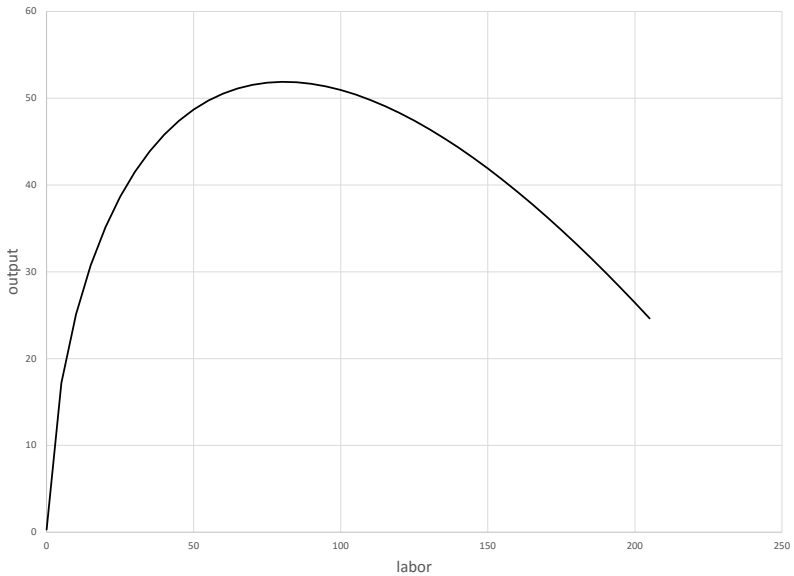
- ▶ Revenue is price of good times quantity of good
- ▶ Cost is wage times labor hired
- ▶ Profit is revenue minus cost

$$\pi = p \times q - w \times \ell$$

$$\pi = p \times A\ell^\alpha - w \times \ell$$

- ▶ Choose ℓ to maximize profits

$$\max_{\ell} p \times A\ell^\alpha - w \times \ell$$



Finding the maximum

- ▶ First-order condition: derivative of profit function = 0

$$\max_{\ell} p \times A\ell^{\alpha} - w \times \ell$$

- ▶ First-order condition is

$$\alpha p A \ell^{\alpha-1} - w = 0$$

- ▶ Two derivative rules

1. Derivative of x^{α} is $\alpha x^{\alpha-1}$
2. Compute derivative of each piece separately

First-order condition

- ▶ At the maximum: marginal revenue = marginal cost

$$\alpha p A \ell^{\alpha-1} - w = 0$$

- ▶ Find the ℓ that makes this true

$$\alpha p A \ell^{\alpha-1} - w = 0$$

$$\alpha p A \ell^{\alpha-1} = w$$

$$\ell^{\alpha-1} = \frac{w}{\alpha p A}$$

$$\ell = \left(\frac{w}{\alpha p A} \right)^{\frac{1}{\alpha-1}}$$

$$\ell = \left(\frac{\alpha p A}{w} \right)^{\frac{1}{1-\alpha}}$$

Example

▶ $\alpha = 0.7, w = 1.5, A = 2, p = 4$

▶ Profit function

$$\max_{\ell} 4 \times 2\ell^{0.7} - 1.5 \times \ell$$

▶ First-order condition

$$0.7 \times 4 \times 2\ell^{0.7-1} - 1.5 = 0$$

▶ $\ell^* = 80.72$

▶ Profit is $\pi^* = 172.9 - 121.08 = 51.89$