

## A Simple Armington Model

[My notes are in beta. If you see something that doesn't look right, I would greatly appreciate a heads-up.]

Our first model of trade is a simple one that combines goods differentiated by their country of origin and constant elasticity of substitution preferences. This kind of model is still used often. We will also take this as an opportunity to reacquaint ourselves with constant elasticity of substitution preferences, which are everywhere in international economics. (Why? International models have lots of goods, and the CES functional form is a convenient way to aggregate them.)

The economy is made up of  $N$  countries (indexed  $i$  or  $j$ ), each endowed with a country-specific good,  $y_i$ . Each country is populated by a representative consumer. The consumer in country  $j$  has preferences

$$U_j = \left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right)^{1/\rho} \quad (1)$$

where  $\gamma_{ij}$  is a preference parameter that is related to the share of expenditure by  $j$  spent on the good from  $i$ , with  $\sum_{i=1}^N \gamma_{ij}^\rho = 1$ . Consumption of the good endowed to country  $i$  and consumed in country  $j$  is  $c_{ij}$ . Typically, we will assume  $0 < \rho < 1$ .

Feasibility in the world says that the total consumption of a good is equal to its endowment

$$\sum_{j=1}^N c_{ij} = y_i, \quad i = 1 \dots N \quad (2)$$

That's it. It's meant to be a simple model. Whenever you come across a new model, it's worth stopping to answer a basic question: Why is there trade? In this case, it's because each country is endowed with a unique kind of good, and preferences imply that consumers in each country want to consume that good. It's not a very deep theory of trade (international, or otherwise), but it's a place to start. This type of setup — national product differentiation — is known as the *Armington assumption* after Armington (1969).<sup>1</sup>

The household's problem is:

$$\max_{c_{ij}} U_j = \left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right)^{1/\rho} \quad (3)$$

$$\text{s.t.} \quad \sum_{i=1}^N p_{ij} c_{ij} = p_{jj} y_j. \quad (4)$$

<sup>1</sup>This setup is also the jumping off point for the international real business cycle literature: Just put time subscripts on everything, make the endowments stochastic, and modify preferences to include discounting and intertemporal substitution. This is Backus, Kehoe, and Kydland (1994).

### Properties of demand: aggregation

Let  $\lambda_j$  be the Lagrange multiplier on the country- $j$  budget constraint. The first order condition with respect to  $c_{kj}$  is

$$\left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right)^{1/\rho-1} c_{kj}^{\rho-1} \gamma_{kj}^\rho = \lambda_j p_{kj}, \quad k = 1 \dots N. \quad (5)$$

multiply (5) by  $c_{kj}$  and sum over all  $k$  to yield

$$\left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right)^{1/\rho-1} \sum_{k=1}^N c_{kj}^\rho \gamma_{kj}^\rho = \lambda_j \sum_{k=1}^N p_{kj} c_{kj}. \quad (6)$$

Notice that the term on the left side is  $U_j$  and (using the budget constraint) the summation term on the right side is total income. Rewriting this as

$$\lambda_j^{-1} U_j = p_{jj} y_j \quad (7)$$

gives the interpretation of  $\lambda_j^{-1}$  as being the price of one unit of country- $j$  utility, which we denote  $P_j$ .  $U_j$  is the total utility that the household can afford given its income.

Now let's solve for  $P_j$ . Arrange (5) so that

$$c_{kj}^{\rho-1} = \lambda_j p_{kj} \left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right)^{1-1/\rho} \gamma_{kj}^{-\rho}, \quad k = 1 \dots N, \quad (8)$$

then raise each side to the power  $\rho/(\rho-1)$  and multiply by  $\gamma_{kj}^\rho$

$$\gamma_{kj}^\rho c_{kj}^\rho = \lambda_j^{\frac{\rho}{\rho-1}} p_{kj}^{\frac{\rho}{\rho-1}} \left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right) \gamma_{kj}^{-\frac{\rho}{\rho-1}}, \quad k = 1 \dots N. \quad (9)$$

Now sum over  $k$ ,

$$\sum_{k=1}^N \gamma_{kj}^\rho c_{kj}^\rho = \lambda_j^{\frac{\rho}{\rho-1}} \left( \sum_{i=1}^N \gamma_{ij}^\rho c_{ij}^\rho \right) \sum_{k=1}^N \gamma_{kj}^{-\frac{\rho}{\rho-1}} p_{kj}^{\frac{\rho}{\rho-1}}, \quad (10)$$

and solve for  $\lambda_j$ ,

$$\lambda_j = \left( \sum_{k=1}^N \gamma_{kj}^{\frac{\rho}{1-\rho}} p_{kj}^{\frac{-\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}}. \quad (11)$$

The price of utility from (7) is

$$P_j = \lambda_j^{-1} = \left( \sum_{k=1}^N \gamma_{kj}^{\frac{\rho}{1-\rho}} p_{kj}^{\frac{-\rho}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}}. \quad (12)$$

### Properties of demand: love of variety

Consider a symmetric world, in which  $p_{kj} = p$  and  $\gamma_{kj} = 1$  for all  $k, j$ . (Earlier I specified the  $\gamma$  as weights that sum to one, but in this example, it's easier to do it this way. The allocations will be the same; we are just multiplying the utility function by a constant.) The price index is now

$$P_j = \left( \sum_{k=1}^N p^{\frac{-\rho}{1-\rho}} \right)^{-\frac{1-\rho}{\rho}} = N^{-\frac{1-\rho}{\rho}} p. \quad (13)$$

Notice that, if  $\rho < 1$ , the price of a unit of utility is decreasing in the number of goods available. Holding fixed the household's income, the household would like to have more varieties of goods available, since it increases the amount of utility it can buy. This property is often expressed as the *love of variety*. The love of variety generates trade: Rather than concentrate spending on one kind of good, the household would rather buy (i.e., import) goods from all the countries.

### Properties of demand: elasticities

Now that we have  $P_j$ , the demand for  $c_{kj}$  follows directly from (5),

$$c_{kj} = \gamma_{kj}^{\frac{\rho}{1-\rho}} \left( \frac{p_{kj}}{P_j} \right)^{-\frac{1}{1-\rho}} \left( \sum_{i=1}^N \gamma_{ij}^{\rho} c_{ij}^{\rho} \right)^{1/\rho}, \quad k = 1 \dots N. \quad (14)$$

The demand function works the way you would expect. 1) The higher is the price of the good ( $p_{kj}$ ) relative to the price of a unit of utility ( $P_j$ ), the less the good is demanded. 2) Goods with greater weight in utility (higher  $\gamma$ ) are demanded in higher quantities. 3) For a given  $P_j$ , increasing income leads to increased consumption of all goods.

The relative demand for  $c_{kj}$  and  $c_{mj}$  is

$$\frac{c_{kj}}{c_{mj}} = \left( \frac{\gamma_{kj}}{\gamma_{mj}} \right)^{\frac{\rho}{1-\rho}} \left( \frac{p_{kj}}{p_{mj}} \right)^{-\frac{1}{1-\rho}}. \quad (15)$$

To calculate the elasticity of substitution, take logs of (15) and differentiate it with respect to the relative price. The elasticity of substitution between two goods is  $\sigma = 1/(1 - \rho)$ . This elasticity is referred to as the *Armington Elasticity* in the context of a model like this. This elasticity does not vary with the quantities of the goods consumed — this is the constant elasticity being referenced when people refer to (1) as constant elasticity of substitution preferences.

We can see one more thing from (14). The own-price elasticity of demand — take logs and differentiate with respect to  $p_{kj}$  — depends on the impact of a change in  $p_{kj}$  on the aggregate price  $P_j$  and the induced change in consumption in the term on the right side. If there are lots of countries that are endowed with relatively small amounts of their good, so that these effects are small, then the own-price elasticity of demand will be close to  $1/(1 - \rho)$ . If there were a continuum of countries (replace the summations with integrals) the own-price elasticity is exactly  $1/(1 - \rho)$ .

### Calibration

It is sometimes easier to work with (14) in value terms rather than quantity terms. Multiply (14) by  $p_{kj}/P_j$  to yield

$$p_{kj}c_{kj} = \gamma_{kj}^{\frac{\rho}{1-\rho}} \left( \frac{p_{kj}}{P_j} \right)^{-\frac{\rho}{1-\rho}} \left( \sum_{i=1}^N \gamma_{ij}^{\rho} c_{ij}^{\rho} \right)^{1/\rho} P_j, \quad k = 1 \dots N, \quad (16)$$

and note that from (7), we have

$$\begin{aligned} p_{kj}c_{kj} &= \gamma_{kj}^{\frac{\rho}{1-\rho}} \left( \frac{p_{kj}}{P_j} \right)^{-\frac{\rho}{1-\rho}} p_{jj}y_j, \quad k = 1 \dots N \\ \frac{p_{kj}c_{kj}}{p_{jj}y_j} &= \gamma_{kj}^{\frac{\rho}{1-\rho}} \left( \frac{p_{kj}}{P_j} \right)^{-\frac{\rho}{1-\rho}}, \quad k = 1 \dots N \end{aligned} \quad (17)$$

which gives us the share of total spending (income) devoted to good  $k$ . This gives us an easy way to calibrate this model to generate trade shares that match those in the data. Suppose that  $\rho = 0.8$ ; an elasticity of 5 is plausible. Normalize units so that all prices are equal to 1. If the share of spending by U.S. household on goods from Canada is 0.05, then  $\gamma_{CAN,USA} = 0.473$ .

### Extensions

**Extension 1:** Suppose there are tariffs in the model. How does our exposition change? Let  $\tau_{ij}$  be the tariff rate, so  $p_{ij} = (1 + \tau_{ij})p_{ii}$ . The new budget constraint for the household in  $j$  is

$$\sum_{i=1}^N p_{ij}c_{ij} = p_{jj}y_j + T_j,$$

where  $T_j$  is the tariff revenue rebate (which the household takes as given) defined as

$$\sum_{i=1}^N p_{ii}c_{ij}\tau_{ij} = T_j.$$

**Extension 2:** Suppose there are iceberg transportation costs. How does the exposition change? Let  $\xi_{ij}$  be the transportation cost, so that if country  $j$  consumes  $c_{ij}$  units of good  $i$ ,  $c_{ij}(1 + \xi_{ij})$  units must be shipped from  $i$ . This means  $p_{ij} = p_{ii}(1 + \xi_{ij})$  and the feasibility constraint is

$$\sum_{i=1}^N c_{ij}(1 + \xi_{ij}) = y_i, \quad i = 1 \dots N.$$

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Armington, Paul S. (1969). "A Theory of Demand for Products Distinguished by Place of Production." *IMF Staff Papers* 16 (1), pp. 159–178.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1994). "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?" *American Economic Review* 84 (1), pp. 84–103.