Multinationals and the Globalization of Production

Optimization

Penn State // Fall 2017

Roadmap

- Past: Where do firms locate?
  - FDI for market access (Horizontal/Export platform)
  - FDI for factor cost savings (Vertical)
  - FDI for tax motives
- Future: **Why** do firms own affiliates?
  - Why not purchase from another firm?
  - Hold-up problems and contracts
  - Why not license to another firm?
  - Technology diffusion
- These are models are models of firm optimization
  - Today: optimization refresher

Optimization

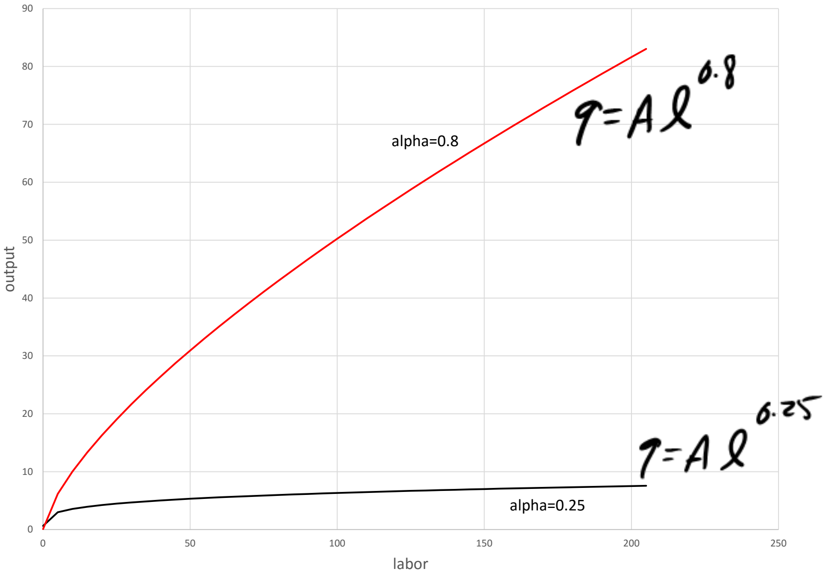
- Intermediate micro recap
- Given a production function, how should the firm choose inputs?
  1. Set up the profit function
  2. Construct the first-order condition
  3. Solve first-order condition to find input value
- This is just a step in solving more interesting problems…

Example

- Production function uses labor ℓ, paid wage w

$$q = A\ell^{\alpha}$$

- α controls the returns to labor


$q = A\ell^{0.8}$
$q = A\ell^{0.25}$
alpha=0.8
alpha=0.25
output
labor
0
10
20
30
40
50
60
70
80
90
0
50
100
150
200
250

Profit function

6.

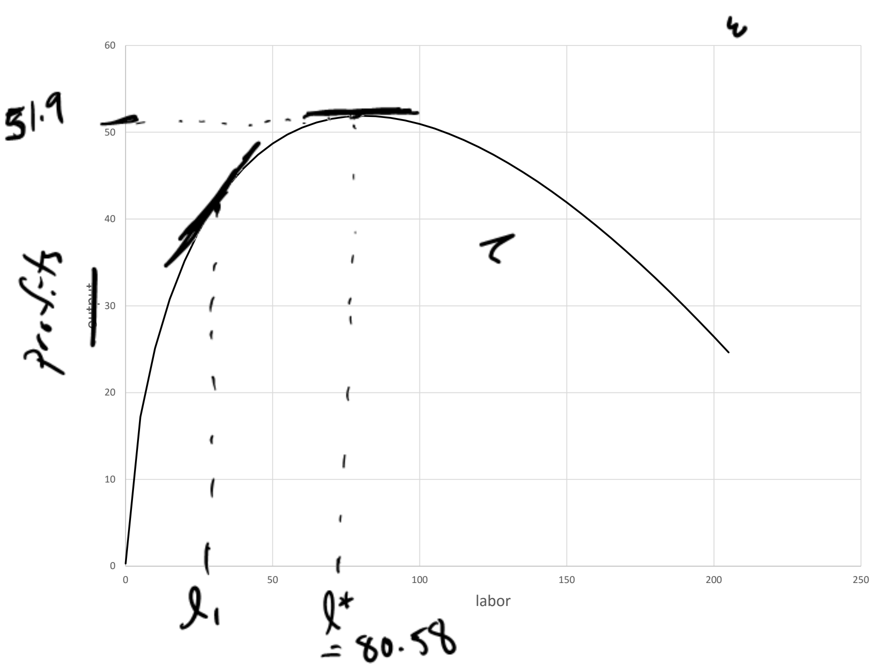
- Revenue is price of good times quantity of good
- Cost is wage times labor hired
- Profit is revenue minus cost

$$\pi = p \times q - w \times \ell$$

$$\pi = p \times A\ell^{\alpha} - w \times \ell$$

- Choose ℓ to maximize profits   Know A, α, p, w

$$\max_{\ell} \; p \times A\ell^{\alpha} - w \times \ell$$


51.9
profits
output
labor
l_1
$l^* = 80.58$
60
50
40
30
20
10
0
0
50
100
150
200
250

Finding the maximum

- First-order condition: derivative of profit function = 0

$$= w\ell'$$

$$\max_{\ell} \; p \times A\ell^{\alpha} - w \times \ell$$

$$pA\alpha\ell^{\alpha-1} - w\,1\,\cancel{\ell^{1-1}} \quad 1$$

- First-order condition is

$$\underset{\text{MR}}{\alpha p A\ell^{\alpha-1}} - \underset{\text{MC}}{w} = 0$$

- Two derivative rules
  1. Derivative of x^{α} is $\alpha x^{\alpha-1}$
  2. Compute derivative of each piece separately

First-order condition

- At the maximum: marginal revenue = marginal cost

$$\alpha p A \ell^{\alpha-1} - w = 0$$

- Find the ℓ that makes this true

$$\alpha p A \ell^{\alpha-1} - w = 0$$
$$\alpha p A \ell^{\alpha-1} = w$$
$$\ell^{\alpha-1} = \frac{w}{\alpha p A}$$
$$\ell = \left(\frac{w}{\alpha p A}\right)^{\frac{1}{\alpha-1}}$$

\# workers firm hires to max π $\longrightarrow$

$$\ell = \left(\frac{\alpha p A}{w}\right)^{\frac{1}{1-\alpha}}$$

r

Example

- $\alpha = 0.7, w = 1.5, A = 2, p = 4$
- Profit function

$$\max_{\ell}\ 4 \times 2\ell^{0.7} - 1.5 \times \ell$$

- First-order condition

$$4 \times 2 \times 0.7\, \ell^{-0.3} - 1.5 = 0$$

$$\ell^* = \left(\frac{pA\alpha}{w}\right)^{\frac{1}{1-\alpha}} = \left(\frac{4 \times 2 \times 0.7}{1.5}\right)^{\frac{1}{0.3}}$$

- $\ell^* =$ $\quad \ell^* = 86.58$
- Profit is $\pi^* =$ $\quad pA\ell^\alpha - w\ell$

$$= 4 \times 2 \times (80.58)^{0.7} - 1.5\,(80.58) = 51.9$$