Bidding Wars for Multi-Establishment Firms: An Application of Multi-Unit Auctions with Endogenous Entry

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Abstract: We propose a model of competition between local governments to attract new investment. The main contribution of this paper is to consider firms as multi-establishment firms, an oversight in the current literature. We make two arguments. First, by splitting its production, the firm can extract higher tax holidays from the governments in competition, by taking advantage of infra-marginal competition for the larger establishment. Second, we consider the possibility for regions to make prior investment in infrastructure, modelling this choice as an entry cost for the bidding war. The firm's investment allocation affects the magnitude of this cost, thus adding an additional trade-off for the firm: a larger establishment increases the subsidies, but may drive competition out of the bidding war.

Keywords: Fiscal competition, Auctions, Firm location.

JEL Classification Numbers: D44, D21, H71, H77.

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1 Introduction

Tax incentives given out to firms in exchange for new investments represent an appreciable amount of government spending each year. In the United States alone, state and local governments award approximately $80 billion in tax incentives each year to companies.[1] These subsidies are often the result of bidding wars between many governments. Owing to their importance, economists have investigated the behaviour of firms and governments participating in these bidding wars. However, they have generally considered a single firm opening a single establishment. In fact, the firms running these bidding wars are frequently multinationals, or at least multi-establishment companies. For example, between 2007 and 2012, Boeing received at least $327 million in incentives from 11 US states. In the same period, Procter & Gamble received at least $128 million from 10 states.[2] These examples illustrate how firms make multiple investments in potentially short periods of time. Consequently, these bidding wars are not necessarily independent.

In this paper, we propose a model in which regional governments are competing against each other to attract a firm's investment. Importantly, the firm in our model invests in more than one location, essentially making multiple plants available for bidding. We model this competition as an auction, but with multiple "units" for sale. We find that bidding wars affect the firm's structure. Indeed, by having a bidding war, the firm over-invests in one of the plants to create differentiation between the investments. In doing so, she creates incentives for the regional governments to offer larger tax breaks.

Another aspect of this competition is that regional governments often make investments in infrastructure even before these bidding wars take place. McAfee *et al.* (1993) consider this infrastructure in their model, arguing that regions can make these investments to become more attractive for firms. However, they consider a firm making an investment of a exogenous size. In our model, the firm can choose how to split the total investment in two establishments. Therefore, the size of the investments is endogenous, and may thus also affect the level of necessary infrastructure to host the firm. In other words, the entry cost to participate in the bidding war depends on the firm's choice of investment allocation.

Considering this feature, our model finds that when the firm chooses to put almost all

[1]The New York Times, "As Companies Seek Tax Deals, Governments Pay High Price," available at the following address: http://www.nytimes.com/2012/12/02/us/how-local-taxpayers-bankroll-corporations.html.

[2]Other examples are available from the New York Times, at the following URL: http://www.nytimes.com/interactive/2012/12/01/us/government-incentives.html

investment in one location, she may drive some regional governments out of the bidding war, thus reducing competition and the subsidies she receives. The firm can therefore face a trade-off between increasing subsidies by concentrating her investment, due to the infra-marginal competition induced, and the potential decrease in subsidies if the investment is too large for most regions to consider bidding for it.

Some may see these bidding wars as wasteful, but they can also play an important role in eliciting private information and improving allocation efficiency (Menezes, 2003). In fact, despite paying subsidies to the firm, the winning region may actually benefit from the presence of the new plant. In fact, Greenstone and Moretti (2003) compare the outcomes for winning and losing counties in contests for "million dollar plants", and find that winning counties experience greater increases in land value as well as in the total wage bill of other firms in the industry of the new plant.[3]

Many authors have previously highlighted the resemblance between this type of competition and auctions (e.g., King, McAfee and Welling, 1993; Menezes, 2003; Ferrett and Wooton, 2010a, 2010b). Indeed, auctions are often used by sellers who do not know the value potential buyers place on the product sold.[4] Black and Hoyt (1989) were, to our knowledge, the first to explicitly model the firm's location choice as an auction. They also highlight the fact that this competition need not be a zero-sum game; the bids offered by government can promote the efficient location of production.

Black and Hoyt (1989) had highlighted some caveats to their analysis. One caveat was the lack of dynamic considerations. King and Welling (1992) explore the consequences of allowing the firm to relocate in later periods. They consider a two-period model, in which the firm conducts an auction to decide on its location in each period. They find that when players cannot commit to second-period actions, the firm can re-locate to the region that lost in the first period.

King, McAfee and Welling (1993) generalise the model of King and Welling (1992), but with a continuum of local productivities. They also consider, as mentioned above, an extension in which regions can invest in infrastructure in a previous stage, thus increasing their productivity

[3]Economists are divided on whether these bidding wars are actually beneficial. A poll by the IGM 2015 found that about half of top economists surveyed were uncertain whether these bidding wars were beneficial for society as a whole.

[4]As suggested by Klemperer (2004, Chapter 2), auction theory can provide a rich set of tools to study a number of problems in economics and social sciences. Location contests are a good example of a context in which auctions are a useful theoritcal tool; many bidders (governments) place some private value on a good (investment), and a seller (the firm) does not know how to price it, thus choosing to accept bids (subsidies).

potential. They find that in equilibrium, regions tend to choose different levels of infrastructure, thus endogenously creating the productivity continuum described in their main model.

Later authors applied other models in auction theory to study specific aspects of that problem. One example is Taylor (1992), who investigated the role of infrastructure in the competition for investment. He built a model in which jurisdictions compete for new firms by investing in new infrastructure, in a way that resembles an all-pay auction. He finds that the investment race can potentially be wasteful, and that regions with a lower initial stock of infrastructure may be less willing to enter the competition. In this situation, infrastructure inequality between regions could rise over time. Another example is the paper by Ferrett and Wooton (2010a), who analyse the question of firm ownership. Intuition may have us believe that regional governments would offer higher subsidies to firms owned in part by shareholders residing locally. However, their model shows that the tax or subsidy offers are independent of the ownership structure of the firm.

The papers cited thus far all consider a firm making a single investment. However, as highlighted previously, many bidding wars take place for investment by multinationals. Haaparanta (1996), in a paper closely related to ours, applies a menu auction model to study a firm with divisible investment. However, the nature of a menu auction model limits his results to settings under complete information. Our model will take into account the fact that the firm does not know which regions put high value on her investment, one of the important aspects of these bidding wars. These different assumptions modify the conclusions, showing the importance of information asymmetry; in his model, the firm invests equal amounts in both regions.

Furthermore, Furusawa, Hori and Wooton (2010) show that the bidding mechanism can affect the results of the model. In their paper, they show that English auctions lead to more aggressive bidding, or to a “race beyond the bottom," compared to bidding in sealed bid auctions. Martin (2000) cites case studies that claim bidding wars resemble more closely open ascending auctions. Our results show that a model with an open ascending auction, in addition to accounting for information asymmetry, also changes the results.

The remainder of the paper is organised as follows. The next section presents the basic construction of the model, while Section 3 solves this simple model. In Section 4, we add endogenous entry to the model. Regional governments can decide, before the bidding war takes place, to invest in infrastructure (in other words, pay an entry cost for the auction). Section 5 discusses some implications of the model on social welfare. The last section concludes.

2 The Model

The players in this incomplete information game are the firm and the n regional governments indexed by $i \in 1, ..., n$. The firm wants to increase its production capacity, but is unsure of the optimal location for the new facilities. She may split her production in multiple locations, either in symmetric or asymmetric establishments. We will assume that the total investment is fixed at a certain exogenous amount, and that the firm chooses the proportion to allocate in each location. For simplicity and tractability, we limit the model to the case of two establishments, indexed by $j \in 1, 2$.

2.1 The Firm

The firm wants to invest a certain exogenous amount of capital K.[5] She can invest in two establishments, choosing a production split $\vec{\alpha} = (\alpha_1, \alpha_2)$, where α_k is the proportion of production in establishment k. Since $\alpha_1 + \alpha_2 = 1$, we will set $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$. Without loss of generality, we label the establishments such that $\frac{1}{2} \leqslant \alpha \leqslant 1$. In other words, the first establishment is the largest.

In each establishment, the firm produces and makes profits depending on the amount of capital invested $\pi(\alpha_k \cdot K)$. For simplicity, we choose to normalize the exogenous quantity K to 1, so that profits only depend on the proportion of production in each establishment: $\pi(\alpha_k)$. We assume that this function is positive ($\pi(\alpha_k) > 0$) on the interval $[0, 1]$. The functional form of $\pi(\cdot)$ describes what we call benefits from concentration. The sign of the second derivative will be determinant for the returns to concentration. If $\pi''(\cdot) > 0$, the firm has increasing marginal returns to concentration, while $\pi''(\cdot) < 0$ implies decreasing marginal returns to concentration. In the latter case, a larger α might bring more profits from the larger establishment, but, in return, less profits from the smaller establishment. To help the understanding of that profit function, the following remark describes the behaviour of the firm without a bidding war.

Remark 1. *Assume the firm chooses production sites without a bidding war. If $\pi''(\cdot) < 0$, the firm chooses $\alpha = \frac{1}{2}$. If $\pi''(\cdot) > 0$, the firm chooses $\alpha = 1$.*

Proof. The firm's maximisation problem in the absence of a bidding war is simply

$$\max_{\alpha} \pi(\alpha) + \pi(1 - \alpha)$$

[5] In another paper (Lapointe and Morand, forthcoming), we investigate the effect of the endogenous choice of total capital invested, by allowing the firm to choose two quantities of capital to invest.

The first-order condition is

$$\pi'(\alpha) - \pi'(1-\alpha) = 0$$

In words, the firm chooses α to equalize profits in both establishments: $\alpha = \frac{1}{2}$. First assume that $\pi''(\cdot) < 0$. Then, the second-order condition satisfies $\pi''(\alpha) + \pi''(1-\alpha) < 0$, giving us a maximum. However, if $\pi''(\cdot) > 0$, then the second-order condition is positive, thus indicating that $\alpha = \frac{1}{2}$ is a minimum. In this case, we have a corner solution at $\alpha = 1$. □

Without a bidding war, then, a firm with $\pi''(\cdot) < 0$ would choose to have two equally-sized establishments. In the opposite case ($\pi''(\cdot) > 0$), the firm would choose to produce in only one large establishment. The assumption that $\pi''(\cdot) < 0$ would correspond, for example, to a case where the production function of the firm is concave (i.e., $f''(k) < 0$). Note, however, that we do not make assumptions on the first derivative.

The firm's total *ex post* revenues are equal to

$$\Pi(\alpha) = s_1^* + s_2^* + \pi(\alpha) + \pi(1-\alpha) \tag{1}$$

In other words, the total profits of the firm are equal to the sum of the two winning bids, and the operating profits in each establishment $\pi(\alpha_k)$, which themselves depend on the production split chosen by the firm. The objective of the firm is to maximise $\Pi(\alpha)$.

2.2 The Regions

Each of the $n > 2$ (symmetric) regional governments has private benefits of hosting the firm, b_i. These private benefits capture, for example, an increase in labor taxation from workers who will be employed by the firm, as well as spillovers to domestic firms, but also the compatibility of the firm for the region. Indeed, if the industry of the firm has a bad reputation in one region, the regional government would receive only small benefits from the firm's investment (due to, for example, re-election concerns). The signals are identically and independently distributed according to a distribution $f(\cdot)$ on some interval $\left[\underline{b}, \overline{b}\right]$ (with $\underline{b} \geq 0$). The *ex post* valuation of the region for establishment k is then equal to

$$V_{ik} = \alpha_k b_i - s_{ik} \tag{2}$$

where s_{ik} is the subsidy paid by region i (or the tax holiday offered by region i) for establishment k.

2.3 The Auction Process

The firm then conducts an auction (the bidding war), with both establishments available simultaneously. It is an open ascending auction. In particular, the firm runs an ascending clock, representing the current price for the lowest-value establishment still available (the one with the lowest proportion of production). Regional governments still in the auction are ready to offer a bid equal to the price currently on the clock. The winning bid is determined from the price on the clock when the previous bidder dropped from the auction. In this kind of auction, it is optimal for regional governments to drop from the auction when the value on the clock reaches their own valuation, $\alpha_k b_i$. The bid can be interpreted as a total "fiscal package" offered to the firm.[6] At the end of the auction, the two winning regions offer bids s_1^* (for establishment α_1) and s_2^* (for the second establishment), receive their respective parts of the firm's investment, and the firm produces its goods, incurring profits.

We can already see that there will be a difference between the symmetric and asymmetric cases. In the symmetric case, both establishments have the same value, so the winning bid is determined for both at the same time. In the asymmetric case, the two remaining regions will continue to compete once the value of the lowest subsidy is determined.

We can summarize the timing of the game as follows:

Stage 0: Nature picks the set of $\{b_i\}_{i=1,\ldots,n}$. Regional governments learn their b_i.

Stage 1: The firm chooses and commits to an allocation of capital α, anticipating the subsidies offered by governments resulting from the auction in Stage 2.

Stage 2: The multi-unit auction takes place. Winning regions offer s_1^* and s_2^*.

Stage 3: The firm invests capital as determined in Stage 1 in the winning region(s), and production takes place.

[6] In effect, our model assumes that all regional governments have the same basic tax rate, but differentiate themselves with targeted tax holidays that may differ.

3 A Simple Model Without Entry Costs in Infrastructure

We start by calculating the equilibrium in a simple benchmark model without entry costs. In that case, all n regions enter the auction. We solve the model by backwards induction, to find the subgame-perfect Nash equilibrium. In the last stage, production takes place, and the firm's operating profits are simply determined by $\pi(\alpha) + \pi(1-\alpha)$.

3.1 Equilibrium subsidies

To calculate equilibrium bids, we first define the order statistics on the signals, $\{b_{(i)}\}_{i=1,\dots n}$, where $b_{(1)} > b_{(2)} > \dots > b_{(n)}$. Recall that every region has valuation $V_{ik} = \alpha_k b_i - s_i$. The following lemma describes the equilibrium subsidies:

Lemma 1. *The equilibrium subsidies for the first and second establishments are equal to*

$$s_1^* = (2\alpha - 1)b_{(2)} + (1-\alpha)b_{(3)} \tag{3}$$

$$s_2^* = (1-\alpha)b_{(3)} \tag{4}$$

Proof. Regions stay in the auction only as long as their valuation is positive at the current price. In other words, $V_{ik} \geqslant 0$, and thus $\alpha_k b_i \geqslant s_i$. The highest bid that will be offered by region i for establishment k will be equal to $\alpha_k b_i$. As the clock price increases, regions gradually drop from the auction. The price s_2^* for the second establishment is determined when there are only two regions left in the bidding. Since the region with the third-highest signal will be the third-to-last to stop bidding, s_2^* will be equal to the value of the smallest establishment to that region:

$$s_2^* = (1-\alpha)b_{(3)} \tag{5}$$

At this point, the remaining two regions will continue to bid until one of them exits to determine who among the two remaining bidders will receive the largest establishment. If the firm selected $\alpha = \frac{1}{2}$, the solution is easy: both remaining regions obtain one establishment for the same price (equal to s_2^*). However, by having asymmetric establishments, the firm can take advantage of the infra-marginal competition between the two remaining regions and increase the bid on the largest plant. The region with signal $b_{(2)}$ will leave first. To determine s_1^*, we calculate the bid at which that region is indifferent between the two establishments: $\alpha_1 b_{(2)} - s_1^* = \alpha_2 b_{(2)} - s_2^*$. From this point on, the region stops bidding because she receives a higher payoff by winning the small establishment. This leads to the following equilibrium bid:

$$s_1^* = (2\alpha - 1)b_{(2)} + (1-\alpha)b_{(3)} \tag{6}$$

□

Note that if $\alpha = \frac{1}{2}$, the previous equation implies that $s_1^* = s_2^*$.

3.2 The Firm's Revenue Maximization Problem

As mentioned earlier, the firm wants to choose a production split $(\alpha, 1-\alpha)$ to maximize the sum of the bids received for the establishments and her profits from production. She decides on that production split (α) before the auction takes place, committing to it.

Proposition 1. *With a bidding war, the firm always chooses $\alpha > \frac{1}{2}$. In other words, she chooses to invest in two asymmetric establishments, even choosing reduced operating profits in favour of larger subsidies.*

Proof. Recall that the firm's (ex-post) objective function is:

$$\Pi(\alpha) = s_1^* + s_2^* + \pi(\alpha_1) + \pi(\alpha_2)$$

The firm expects the equilibrium subsidies determined in Stage 2, and knows the distribution of the signals. Her expected revenues are thus equal to

$$E(\Pi) = \int_{\underline{b}}^{\bar{b}} \int_{\underline{b}}^{b_{(2)}} \Pi(\alpha) \cdot g_{2,3}(b_{(2)}, b_{(3)}, n) db_{(3)} db_{(2)} \tag{7}$$

where $g_{2,3}(b_{(2)}, b_{(3)}, n)$ is the joint distribution of the second and third-highest signals ($\underline{b} < b_{(3)} < b_{(2)} < \bar{b}$).[7] This distribution is equal to:

$$g_{2,3}(b_{(2)}, b_{(3)}, n) = n(n-1)(n-2) \cdot \left[1 - F(b_{(2)})\right] \left[F(b_{(3)})\right]^{n-3} f(b_{(2)}) f(b_{(3)})$$

[7] In general, the joint distribution of two order statistics $j < k$ is equal to:

$$g_{j,k}(b_{(j)}, b_{(k)}, n) = \binom{n}{k}\binom{k}{j-1} \left[1 - F(b_{(j)})\right]^{j-1} \left[F(b_{(j)}) - F(b_{(k)})\right]^{k-1-j} F(b_{(k)})^{n-k} f(b_{(j)}) f(b_{(k)})$$

where $F(\cdot)$ is the cumulative distribution of the signals.

We can use this expression in the equation for the expected revenues, obtaining:

$$E(\Pi) = \int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b_{(2)}} \Big[(2\alpha - 1)b_{(2)} + 2(1-\alpha)b_{(3)} + \pi(\alpha) + \pi(1-\alpha)\Big] \cdot$$
$$n(n-1)(n-2) \cdot \Big[1 - F(b_{(2)})\Big] \Big[F(b_{(3)})\Big]^{n-3} f(b_{(2)}) f(b_{(3)}) db_{(3)} db_{(2)} \quad (8)$$

The firm chooses $\alpha \in [\frac{1}{2}, 1]$ to maximize this function. The interior solutions[8] can be described by the following first-order condition with respect to α:

$$\frac{\partial E(\Pi)}{\partial \alpha} = 0 = \int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b_{(2)}} \Big[(2b_{(2)} - 2b_{(3)} + \pi'(\alpha) - \pi'(1-\alpha)\Big]$$
$$\cdot\, n(n-1)(n-2) \cdot \Big[1 - F(b_{(2)})\Big] \Big[F(b_{(3)})\Big]^{n-3} f(b_{(2)}) f(b_{(3)}) db_{(3)} db_{(2)} \quad (9)$$

The optimal value of α will therefore depend on the expected values of the signals, through the probability distribution function $f(\cdot)$, and on the functional form for the profits. We can simplify the first-order condition as such:

$$\begin{aligned} \pi'(1-\alpha) - \pi'(\alpha) &= \frac{2 \int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b_{(2)}} (b_{(2)} - b_{(3)}) \Big[1 - F(b_{(2)})\Big] \Big[F(b_{(3)})\Big]^{n-3} f(b_{(2)}) f(b_{(3)}) db_{(3)} db_{(2)}}{\int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b_{(2)}} \Big[1 - F(b_{(2)})\Big] \Big[F(b_{(3)})\Big]^{n-3} f(b_{(2)}) f(b_{(3)}) db_{(3)} db_{(2)}} \\ &= 2E(b_{(2)} - b_{(3)}) = B \end{aligned} \quad (10)$$

From the definition of order statistics, we know that B is always positive, so the FOC implies

$$\pi'(1-\alpha) > \pi'(\alpha)$$

This inequality has different implications depending on the sign of the second derivative of $\pi(\cdot)$. First, assume that $\pi''(\cdot) > 0$. In that case, the second-order condition

$$\begin{aligned} \frac{\partial E(\Pi)^2}{\partial^2 \alpha} &= \int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b2} (n-2)(n-1)n \left[1 - F(b2)\right] F(b3)^{n-3} f(b_{(2)}) f(b_{(3)}) \left(\pi''(1-\alpha) + \pi''(\alpha)\right) db_{(3)} db_{(2)} \\ &= \pi''(1-\alpha) + \pi''(\alpha) \end{aligned} \quad (11)$$

is always positive, implying that the first-order condition describes a minimum. Furthermore,

[8] On the $]0, \infty[$ interval, we assume $\pi(\cdot)$ and $F(\cdot)$ are continuous and twice differentiable. Therefore, $E(\Pi)$ is well-behaved. It is possible that we find a find extrema outside the $[\frac{1}{2}, 1]$ range. In that case, we'd have a corner solution at $\alpha = \frac{1}{2}$ or $\alpha = 1$. However, we can still describe the extremum, and subsequently find if it lies inside or outside the range.

the inequality $\pi'(1-\alpha) > \pi'(\alpha)$ along with the fact that $\pi''(\cdot) > 0$ implies that the minimum α_0 is:

$$1 - \alpha_0 > \alpha_0$$
$$\alpha_0 < \frac{1}{2}$$

Therefore, with economies of scale, $E(\Pi)$ has a minimum at α_0 smaller than $\frac{1}{2}$. In that case, we know that the optimal split of the firm is to concentrate all production in one location, or to choose $\alpha^* = 1$.

If the firm has decreasing marginal returns to concentration instead, ($\pi''(\cdot) < 0$), the second derivative of expected revenues is negative ($\frac{\partial E(\Pi)^2}{\partial^2 \alpha} < 0$). In that case, the first-order condition describes a maximum. As in the previous case, we can describe the optimal α_0 as such:

$$1 - \alpha_0 < \alpha_0$$
$$\alpha_0 > \frac{1}{2}$$

With decreasing marginal returns to concentration, $E(\Pi)$ has a maximum at α_0 strictly greater than $\frac{1}{2}$, so the firm will choose α^* in the interval $]\frac{1}{2}, 1]$. In other words, in these conditions, the firm always either differentiate her establishments or concentrates all production in one location. □

This result highlights the role of the auction in the firm's decision process. Without it, in the case that $\pi''(\cdot) < 0$,[9] she would always split in two symmetrical establishments. With the auction, she differentiates her production sites, or, in extreme cases, decides to have a single establishment ($\alpha^* = 1$).

This result captures the fact that by running an auction, the firm can take advantage of infra-marginal competition. She does so only when it is already optimal, technologically, to split in multiple locations (i.e., only when production exhibits decreasing marginal returns to concentration). In that case, the difference between $\pi'(1-\alpha)$ and $\pi'(\alpha)$ at the equilibrium, which is equal to the distortion from the auction, is increasing in the expected value of $b_{(2)} - b_{(3)}$. Put differently, as the firm expects a greater difference between the regions' signals, she increases the differentiation between her establishments. Intuitively, the firm expects that the bid for

[9] If $\pi''(\cdot) > 0$, the auction does not affect the firm's behaviour.

the large establishment will increase by an amount more than large enough to compensate for the reduction in the value of, and thus the bid for, the second establishment. In doing so, she expects an increase in her total profits (including subsidies).

Another interesting observation is that the first-order condition depends on n. One might wonder what happens when n increases. Intuitively, we anticipate that the expected value of the difference between the second- and third-highest signal will decrease with n. Indeed, as we take a greater number of draws from a bounded distribution, intuitively we'd expect the higher draws to be closer to the higher bound. Such a result would be difficult to prove in the general sense. We prove it here for the uniform distribution.

Remark 2. *Suppose that the regions' signals are distributed uniformly on $[\underline{b}, \overline{b}]$. The distortion in the firm's location decision arising from the auction decreases with n, such that the firm chooses a split closer to the symmetrical split.*

Proof. Assume the private benefits are distributed according to a uniform distribution on $[\underline{b}, \overline{b}]$. Then, we know that (by using $F(x) = \frac{x-\underline{b}}{\overline{b}-\underline{b}}$):

$$\begin{aligned} B = & \quad 2E(b_{(2)} - b_{(3)}) \\ = & \quad \frac{2(\overline{b} - \underline{b})}{n+1} \end{aligned}$$

Taking the limit of the last expression as n approaches infinity, we see that $2E(b_{(2)} - b_{(3)})$ goes to zero. In other words, the distortion disappears, and the firm would act the same with and without an auction. Perhaps more interesting than what happens at very large n, however, is the behavior of the distortion as n increases. The relation between B and n is not linear, although an increase in n always leads to a decrease in B. More formally, the first derivative of B with respect to n is equal to

$$\frac{\partial B}{\partial n} = \frac{2(\underline{b} - \overline{b})}{(n+1)^2}$$

which is always negative. In other words, as n increases, the distortion from the auction decreases, thus reducing the difference between $\pi'(1-\alpha)$ and $\pi'(\alpha)$. □

This result actually holds for distributions other than the uniform. One important condition is that as n increases, the order statistics $b_{(2)}$ and $b_{(3)}$ move closer together. Even very skewed distributions, such as a χ^2 $(k = 3)$, satisfy such a condition.

3.3 A Numerical Illustration

As a simple but practical form for $\pi(\alpha_k)$, assume that the profits are simply equal to an exogenous parameter π multiplied by a function $h(\alpha_k)$ of the share of production in that establishment:

$$\pi(\alpha_k) = \pi \cdot h(\alpha_k)$$

The parameter π captures the potential profits of the firm, which does not depend on how she allocates capital. and its functional form describes the marginal returns to concentration. In the simplest case, $h(\alpha) = \alpha$ and the firm is always indifferent between producing in one place or two. A flexible functional form for that function is $h(\alpha, \lambda) = \alpha^\lambda$, where λ determines if the firm experiences increasing ($\lambda > 1$) or decreasing ($\lambda < 1$) marginal returns to concentration. If $\lambda = 1$, we have the simple function $h(\alpha) = \alpha$.

First, assume $\lambda = 2$, so that the firm benefits from concentrating its production. With this value for λ, we obtain a single extremum at:

$$\alpha_0 = \frac{1}{2} - \frac{1}{2(1+n)\pi}$$

Since the second derivative is positive, we know that it is a minimum. We also see that it is always lower than $\frac{1}{2}$, so revenues are always increasing between $\alpha = \frac{1}{2}$ and $\alpha = 1$. With $\lambda = 2$, we thus find that the firm optimally concentrates its whole production in the largest establishment ($\alpha^* = 1$).

If instead we assume that $\lambda = \frac{1}{2}$, solving the first order condition gives us:

$$\alpha^* = \frac{1}{16}\left(8 + \sqrt{64 - 2(1+n)^2\pi^2\left(8 + (1+n)\pi\left(\pi + n\pi - \sqrt{16 + (1+n)^2\pi^2}\right)\right)}\right)$$

As shown above, the first-order condition describes a maximum. As a numerical example, taking $n = 3$ and $\pi = 1$, we obtain $\alpha^* \approx 0.78$.

To give a more general idea of the relation between the optimal values of α and different values of λ, we can calculate α^* for all λ, using some value of n and π. The following figure illustrates a stylized version of that relation for $n = 3$ and $\pi = 1$. Unsurprisingly, for $\lambda > 1$, the firm always concentrates in a single establishment. For values of λ below 1, however, the firm never splits in two equal establishments. The shape of the curve for $\lambda < 1$ obviously depends on the functional forms chosen in solving the problem. However, it will be interesting to compare

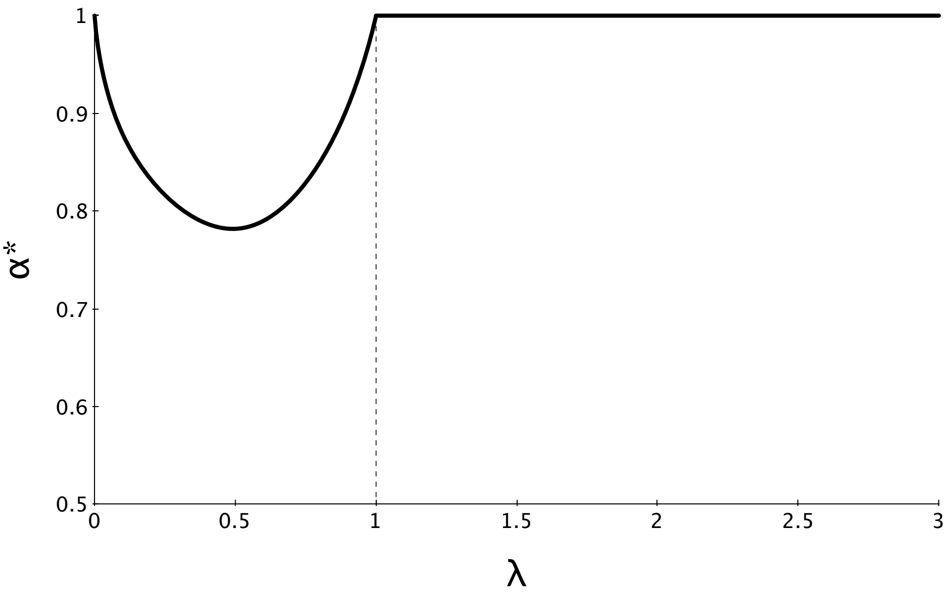

Figure 1: Optimal α given λ

the shape of that curve with the one obtained in the more complete model that includes entry costs (solved with the same functional form).

4 Endogenous Entry: Prior Investment in Infrastructure

We now turn to the more complete model, by introducing endogenous entry choice. We'll assume that regions must make some investment before they can be considered as a potential host region by the firm. In terms of auction theory, this investment corresponds to an entry cost. In our model, regions learn their signals b_i before making their entry decision. They also know the production split chosen by the firm. In this section, we will assume that $\pi(\alpha) = \pi \cdot h(\alpha) = \pi \cdot \alpha^\lambda$ to keep the model tractable. In the simple model, we saw that the value of λ determined whether the firm, without a bidding war, preferred to concentrate all production ($\lambda > 1$) or to split in multiple establishments ($\lambda < 1$). We also assume that the regions' signals are distributed uniformly on $[0, 1]$.

The regions all have the same initial level of infrastructure, which is sufficient to host the smaller establishment. In other words, it is sufficient for establishments of sizes up to $\frac{1}{2}$. However, if they also want to compete for the larger establishment, they need to make additional investment in infrastructure, which increases with α (in other words, larger establishments require more infrastructure).[10] To solve the model, we propose a flexible functional form for the

[10] Setting the entry costs for the small establishment to zero is basically equivalent to a standardization of the

entry costs:

$$c(\alpha) = d \cdot (\alpha - \frac{1}{2})^2 \tag{12}$$

This function captures the fact that entry costs should increase with α. In particular, this function assumes that they increase quadratically: slower for lower production, and increasingly faster as the establishment becomes larger. Entry costs are also null when the firm splits in two equal establishments ($\alpha = \frac{1}{2}$). The parameter d captures the magnitude of the costs.

This entry decision modifies the timing of the model, by adding another stage:

Stage 0: Nature picks the set of $\{b_i\}_{i=1,\ldots,n}$. Regional governments learn their b_i.

Stage 1: The firm chooses and commits to an allocation of capital α, anticipating entry decisions and the subsidies offered by governments resulting from the auction in Stage 3.

Stage 2: The regions decide whether they want to be considered for the large establishment. If so, they pay the entry cost $c(\alpha)$.

Stage 3: The multi-unit auction takes place. Winning regions offer s_1^* and s_2^*.

Stage 4: The firm invests capital, as determined in Stage 1, in the winning region(s).

As in the previous section, we solve the model by backwards induction.

By adding an endogenous entry decision, we also add the possibility that no region will wish to participate in the auction for the large establishment. In that case, we need to consider the possible courses of action for the firm. Figure 2 shows the different possibilities. After entry costs are made (or not), a certain number of regions m are in the competition for the largest establishment, while all $n > 3$ regions are competing for the smaller establishment.

If $m = 0$, and no regions pay the entry cost, then, if she decides to not change anything, the firm will only invest a total of $1 - \alpha$. She can obviously do better. In particular, she can renege on her previous commitment, and change her allocation to $\alpha = \frac{1}{2}$, ensuring every region participates in the bidding war, and allocating the entire amount of capital she wanted to. This modified auction using $\alpha = \frac{1}{2}$ is labeled as Stage 3' in Figure 2. Obviously, the regional governments expect this possibility, and will consider it when choosing to enter or not.[11]

costs. Indeed, if they were greater than zero but still equal for everyone, it could reduce the total number of regions competing, thus changing the exact solution of the model, but we suspect that it would not change our discussion, or the main findings of our paper. Another more interesting extension would consider regions that have different initial levels of infrastructure. In this case, some regions may already reach the level needed for the large establishment, thus removing entry costs for them. We leave this possibility for future research.

[11]We could also solve the model using the more realistic assumption that the firm makes the investment in

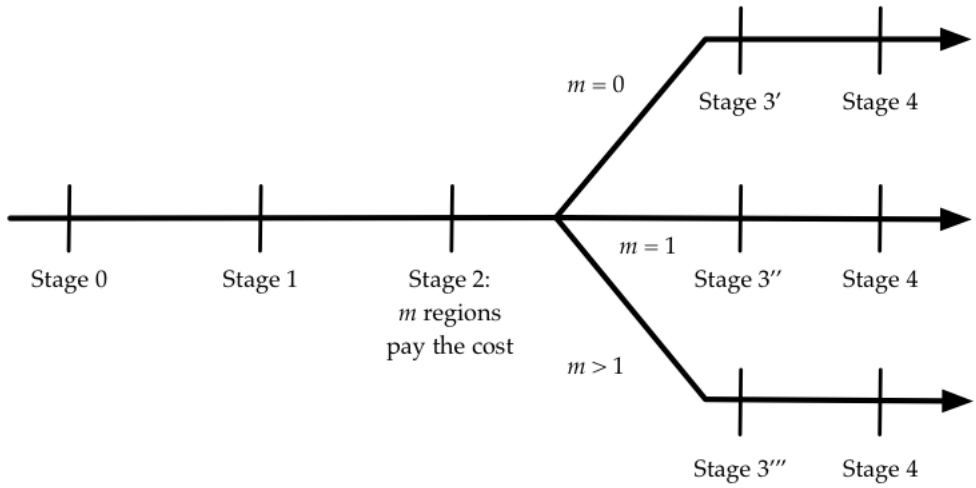

Figure 2: Timing with different number of entries

If $m = 1$, then only one region is suitable for the large investment of the firm, and that region can "win" that investment without further subsidies. In this case, the auction can carry on, but equilibrium subsidies will have to take into account the nature of the competition for the large establishment. This scenario is considered in Stage 3" of Figure 2.

If $m > 1$, then there are enough participants in the bidding war and the auction takes place as in the previous section, although only 2 regions will be able to bid for the large establishment. This scenario is considered in Stage 3"' of Figure 2.

4.1 Bidding Behavior

First assume that there are $n = 3$ regions.[12] We start by describing the winning bids, taking entry decisions as given. After entry costs are made (or not), a certain number of regions m are in the competition for the largest establishment, while all n regions are competing for the smaller establishment. The equilibrium bids will be similar to the ones in the simple model of the previous section, although some special cases exist. Table 1 summarises the possibilities for the bids.

infrastructure herself (i.e., installation costs). We could also imagine that the firm endogenously chooses a $\hat{\alpha}$ when faced with such a case. Alternatively, the firm could cancel the auction and restart the whole process, choosing a slightly lower α using the new information she gained from the non-entry decisions, repeating this process until at least one region participates. In all cases, the regions must anticipate that behavior by the firm. We leave a complete discussion of dynamic or multi-period models to a future extension to the paper.

[12] As will be clear in the following paragraphs, additional regions would act exactly like the third.

Table 1: Possible Cases for the Winning Bids

Number of regions incurring entry cost	s_1^*	s_2^*
0	$\frac{1}{2}b_{(3)}$	$\frac{1}{2}b_{(3)}$
1	0	$(1-\alpha)b_{(3)}$
2	$(2\alpha-1)b_{(2)}+(1-\alpha)b_{(3)}$	$(1-\alpha)b_{(3)}$
3	$(2\alpha-1)b_{(2)}+(1-\alpha)b_{(3)}$	$(1-\alpha)b_{(3)}$

First, if $m = 0$, then there is no region with the infrastructure required to host the large establishment. As explained above, we assume that, when faced with no demand for her large plant, the firm reneges on her commitment and instead announces a $(\frac{1}{2}, \frac{1}{2})$ split. In that case, all n regions are participants in the bidding war for both plants. Since the establishments are of equal sizes, the winning bid for both will thus simply be equal to the third-highest valuation: $s_2^* = (\frac{1}{2})b_{(3)}$. Indeed, these bids are simply a special case of the bids derived in the benchmark model, with $\alpha = \frac{1}{2}$. Since the establishments have the same value, there is no competition among the last two regions remaining in the bidding war, and they both pay the same subsidy. This case is summarized in the first row of Table 1.

If $m = 1$, there is the only region suitable for the large investment. In this case, the region can bid as low as zero, and still win the investment with certainty. The winning bid for the largest establishment will thus be equal to zero: $s_1^* = 0$. The winning bid for the smaller establishment is determined by the competition among the remaining regions, and will be equal to the second-highest valuation among those regions, which is the third-highest valuation among all regions: $s_2^* = (1-\alpha)b_{(3)}$.[13]

If $m > 1$, the regions' behaviour looks similar to the benchmark model in which there was no entry cost. In particular, the bid for the smaller establishment will be equal to the third-highest valuation: $s_2^* = (1-\alpha)b_{(3)}$. The bid for the larger establishment will be determined by the infra-marginal competition occurring between the two regions that have paid the entry cost and have the highest two signals. As shown in Section 3, the equilibrium bid is $s_1^* = (2\alpha-1)b_{(2)} + (1-\alpha)b_{(3)}$.

[13] As we will see in the next section, all regions with a signal higher than the threshold pay the entry cost, and no region with signals lower than the threshold do. Therefore, the only region incurring the entry cost necessarily has the highest signal.

4.2 The Regions' Entry Decision

After observing the investment split chosen by the firm in Stage 1 (α), regional governments decide whether they want to participate in the bidding war for the larger establishment. We define b_t as the minimum signal (threshold) for which a region decides to pay the entry cost and compete for both the large and small establishments. Our goal is then to characterize this threshold value.

Take an arbitrary region i. For this region, there are three interesting outcomes: either she has the highest signal, the second-highest signal, or her signal is lower than at least two other regions. The probability for the region i to be in k^{th} position is equal to

$$P_k(b_i) = \frac{(n-1)!}{(k-1)!\,(n-k)!} \cdot F(b_i)^{n-k}\left[1 - F(b_i)\right]^{k-1}$$

Now consider a region that is just at the threshold. In other words, this region is indifferent between incurring the entry cost or not. Her payoff varies according to her position in the list and whether she paid entry or not. The following table summarises the different cases, their associated probabilities, and associated payoffs.

Table 2: Decomposition of Possible Cases for the Threshold Region

Position	Pays entry cost	Payoff to region
First	Yes	$\alpha b_t - c(\alpha)$
	No	$(\frac{1}{2})b_t - (\frac{1}{2})b_{(3)}$
Second	Yes	$(1-\alpha)b_t - (1-\alpha)b_{(3)} - c(\alpha)$
	No	$(1-\alpha)b_t - (1-\alpha)b_{(3)}$
Third	Yes	$-c(\alpha)$
	No	0

If the threshold region has the highest signal, she will always win the large establishment if she pays the entry cost. The reason is that all other regions have a lower signal, and thus a signal lower than the threshold; they do not enter the auction for the large plant. Therefore, in this case, if she pays the entry cost, she will receive a payoff of $\alpha b_t - c(\alpha)$. As noted earlier, she does not have to pay a bid. Indeed, since there is no competition, she can submit a bid of zero and still win the auction. In effect, this corresponds to an absence of tax breaks. If she decides not to pay the entry cost, the firm splits in two equal establishments, thus the threshold region receives one establishment and pays a bid equal to the third-highest valuation, corresponding to a payoff of $(\frac{1}{2})b_t - (\frac{1}{2})b_{(3)}$.

If the threshold region has the second-highest signal, she will always win the smaller establishment. If she pays the entry costs, the region with a higher signal will win the auction for the large establishment, and the threshold region will win the small one, paying a bid equal to the third-highest valuation, corresponding to a payoff of $(1-\alpha)b_t - (1-\alpha)b_{(3)} - c(\alpha)$. If the threshold region does not pay the entry cost, she will still win the small plant, pay the same bid, but her payoff will be higher since she did not invest in infrastructure. Her payoff is thus equal to $(1-\alpha)b_t - (1-\alpha)b_{(3)}$. Note that in this case, the large plant is awarded without competition and without tax breaks.

Finally, if the signal of threshold region is the third-highest or lower, she will never win any of the establishments. Therefore, if she pays the entry cost, she receives a payoff of $-c(\alpha)$, while she receives a payoff of 0 if she does not incur the infrastructure cost.

Recall that we defined the threshold signal as the signal at which a region is indifferent between paying the entry cost, and not paying it. To determine the value for the threshold, we can therefore calculate the expected payoff from both decisions using the values described above. In particular, the expected payoff when paying the entry cost is determined by the probability of being first, second, or lower, and the payoff in each case. The expected payoff when not incurring entry costs is determined in a similar way. We start by calculating the expected payoff when investing in infrastructure, which is equal to:

$$W^c = \int_0^{b_t}\int_0^{y} (\alpha b_t - c(\alpha)) f_{x,y}(x,y)dxdy + \int_{b_t}^{1}\int_0^{b_t} ((1-\alpha)b_t - (1-\alpha)x - c(\alpha)) f_{x,y}(x,y)dxdy + \int_{b_t}^{1}\int_{b_t}^{1} (-c(\alpha)) f_{x,y}(x,y)dxdy \quad (13)$$

Similarly, we calculate the threshold region's expected payoff when not paying the entry cost:

$$W^0 = \int_0^{b_t}\int_0^{y} ((\frac{1}{2})b_t - (\frac{1}{2})x) f_{x,y}(x,y)dxdy + \int_{b_t}^{1}\int_0^{b_t} ((1-\alpha)b_t - (1-\alpha)x) f_{x,y}(x,y)dxdy + \int_{b_t}^{1}\int_{b_t}^{1} (0) f_{x,y}(x,y)dxdy \quad (14)$$

Setting $W^c = W^0$, we can find $b_t(d,\alpha)$: the signal at which a region is just indifferent between paying the entry cost and not paying it. Regions with a lower signal will not pay the cost, thus only competing on the small plant, while regions with a higher signal will pay the entry cost and compete on both establishments.

A complete analytical solution to $b_t(d, \alpha)$ is possible, although not informative. We instead describe the behaviour of that function with respect to both arguments. Figure 3 shows the value of the threshold signal for different values of d and α. The value of the threshold signal increases with α, and is unsurprisingly higher for higher d. The figure also clearly shows that for high values of d, the firm would actually drive all regions out of the competition if she decided to concentrate her production in a large establishment (remember that for simplicity, we assumed that signals are distributed in the $[0, 1]$ interval).

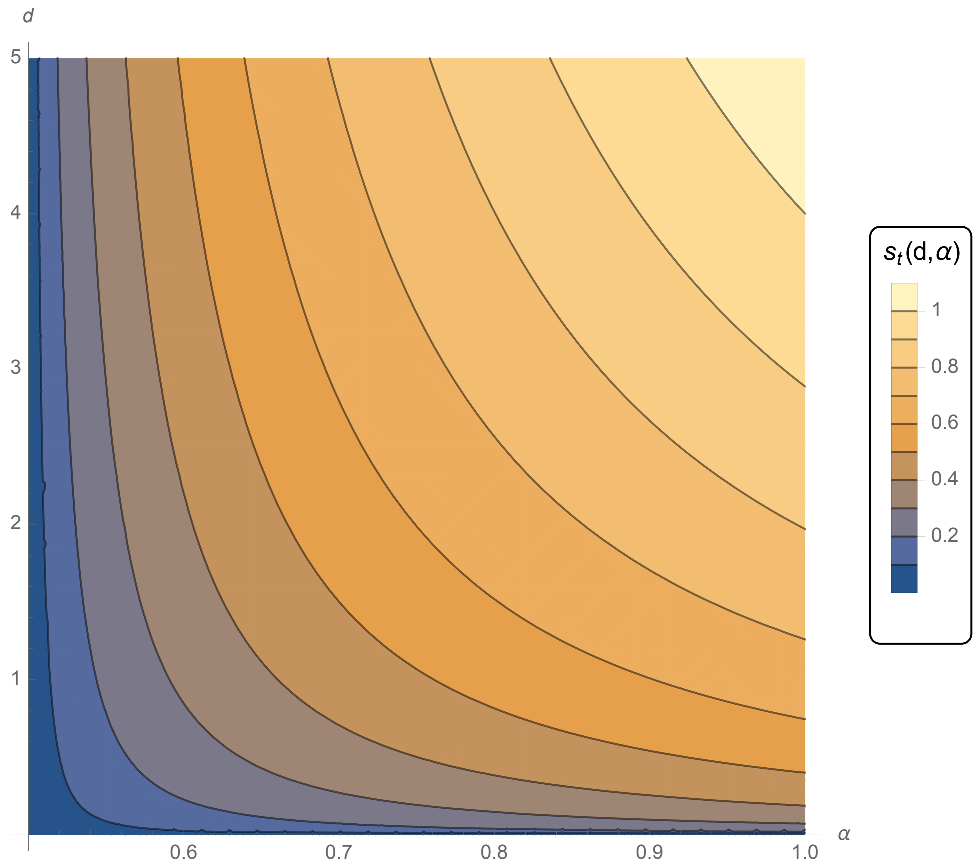

Figure 3: Threshold Signal Value for Combinations of d and α

Lemma 2. *There exists some values of d for which the firm can drive out all competition for her investment if she chooses a high α.*

Proof. Private benefits b_i are distributed on the interval $[0, 1]$. If $b_t(d, \alpha) > 1$, then the threshold for entry is too high for any region to pay the entry cost. Figure 3 illustrates such a case. For example, for $d = 5$, $\alpha > 0.93 \implies b_t(d, \alpha) > 1$. Therefore, for these values, the firm drives out all competition for her large establishment. $\square$

4.3 The Firm's Revenue Maximisation Problem

The firm wants to maximise the sum of the bids received for the establishments and her profits from production.[14] Before the auction takes places, the firm has to decide on a production split (α). The chosen split has consequences on the magnitude of the entry cost, the profits of the firm, as well as on the valuation of the regions. She chooses α by maximising the expected value of her revenues.

For the firm, there are a few cases possible, depending on the number of regions who enter the competition. Indeed, as seen above, the number of regions competing for the large establishment will determine the winning bids received by the firm. Moreover, if no region competes for that establishment, she will not receive profits from it. Expressed in terms of these four case, the expected revenues of the firm are equal to:

$$\begin{aligned} E(R) = & \int_0^{b_t} \int_0^x \int_0^y [z + (h(\frac{1}{2}) + h(\frac{1}{2}))\pi] g_{x,y,z}(x,y,z) \; dzdydx \\ & + \int_{b_t}^1 \int_0^{b_t} \int_0^y [(h(\alpha) + h(1-\alpha))\pi + (1-\alpha)z] g_{x,y,z}(x,y,z) \; dzdydx \\ & + \int_{b_t}^1 \int_{b_t}^x \int_0^{s_t} [(h(\alpha) + h(1-\alpha))\pi + 2(1-\alpha)z + (2\alpha-1)y] g_{x,y,z}(x,y,z) \; dzdydx \\ & + \int_{b_t}^1 \int_{b_t}^x \int_{b_t}^y [(h(\alpha) + h(1-\alpha))\pi + 2(1-\alpha)z + (2\alpha-1)y] g_{x,y,z}(x,y,z) \; dzdydx \end{aligned} \tag{15}$$

Revenues come from three sources: profits from the small establishment, profits from the large establishment, and winning bids. We can already see that the firm always receives the profits from the small establishment, and the profits from the larger one in three out of four cases. It may be informative to calculate the total revenues from each source separately. Let's consider the expected revenues accruing from the profits of the small establishment:

$$E(R_s) = \int_{b_t}^1 \int_0^x \int_0^y h(1-\alpha)\pi \cdot g_{x,y,z}(x,y,z) \; dzdydx \tag{16}$$

$$+ \int_0^{b_t} \int_0^x \int_0^y h(1/2)\pi \cdot g_{x,y,z}(x,y,z) \; dzdydx$$

$$= \pi \left((1 - b_t^3) \cdot h(1-\alpha) + b_t^3 \cdot h(1/2) \right) \tag{17}$$

[14]Note that in this section, the α that maximises the firm's revenues may not be the actual α realized. Indeed, we assumed that the firm may renege on her commitment. This behaviour, however, is anticipated by the regions in their entry choice. Moreover, their bids will be on the realized α.

First, we should note that the revenues from this plant can either increase or decrease with α, depending on the value of $\pi'(\alpha)$. If $\pi'(\cdot) = h'(\cdot) > 0$, the first derivative of $E(R_s)$ with respect to α is negative (or equal to zero if, for example, $\pi = 0$). Therefore, a lower production share in this plant translates in lower revenues. However, increasing α also increases the probability of no regions paying the entry cost, and the firm reverting to $\alpha = \frac{1}{2}$, reneging on her commitment. We can see from the equation above that the expected revenues from the small establishment can take two distinct values, depending on the value of α, and that the probability of realisation of each value depends on the threshold signal (b_t^3 and $1 - b_t^3$).

Next, let's consider the expected revenues accruing from the profits in the large establishment.

$$
\begin{aligned}
E(R_l) = & \int_{b_t}^{1} \int_0^x \int_0^y h(\alpha)\pi \cdot g_{x,y,z}(x,y,z)\ dzdydx \qquad (18) \\
& + \int_0^{b_t} \int_0^x \int_0^y h(1/2)\pi \cdot g_{x,y,z}(x,y,z)\ dzdydx \\
& = \pi \left((1 - b_t^3) \cdot h(\alpha) + b_t^3 \cdot h(1/2) \right) \qquad (19)
\end{aligned}
$$

If $\pi'(\cdot) = h'(\cdot) > 0$ ($\pi'(\cdot) = h'(\cdot) < 0$), increasing α translates in higher (lower) revenues from the large plant. However, a higher α also increases the threshold signal, and thus the probability that the firm will revert to a symmetric production split. Therefore, the choice of the firm has an ambiguous effect on her revenues accruing from the large establishment. As one would expect, high entry costs either through a large d or higher α decreases the expected revenues from this establishment, by increasing the threshold signal. However, a large α also has the opposite effect on expected revenues by increasing profits.

To illustrate, Figure 4 plots the previous equation for $\pi = 1$, and for $h(\alpha) = \alpha^\lambda$, with various values of λ and d. The figure clearly shows the trade-off facing the firm. When costs are high, she tends to receive higher revenues from splitting her production, but a higher λ has the opposing effect.

The last portion of expected revenues accrues from the bids made by the region to the firm.

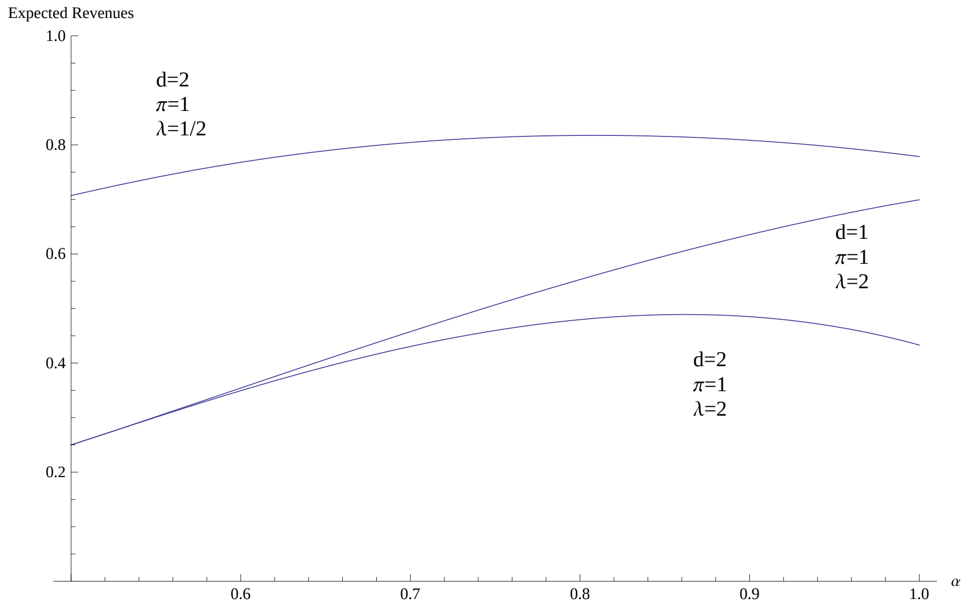

Figure 4: Expected Revenues from Large Establishment vs. α

It is separated in four distinct cases:

$$\begin{aligned} E(R_b) = & \int_0^{b_t}\int_0^{x}\int_0^{y}[z]\cdot g_{x,y,z}(x,y,z)\ dzdydx \\ & + \int_{b_t}^{1}\int_0^{b_t}\int_0^{y}[(1-\alpha)z]\cdot g_{x,y,z}(x,y,z)\ dzdydx \\ & + \int_{b_t}^{1}\int_{s_t}^{x}\int_0^{b_t}[2(1-\alpha)z+(2\alpha-1)y]\cdot g_{x,y,z}(x,y,z)\ dzdydx \\ & + \int_{b_t}^{1}\int_{b_t}^{x}\int_{b_t}^{y}[2(1-\alpha)z+(2\alpha-1)y]\cdot g_{x,y,z}(x,y,z)\ dzdydx \quad (20) \\ & = b_t^3 - \frac{3b_t^4}{4} + \frac{1}{2}\alpha(1+b_t^3(-6+5b_t)) \quad (21) \end{aligned}$$

This portion of expected revenues vary with α and the magnitude of the entry costs (d), since these parameters affect the threshold signal and thus the number of entrants. When the firm introduces high entry costs, the threshold increases and fewer regions pay the cost. In turn, the bids are lower, or even null for the large establishment. However, the firm can also increase the value of the bids by differentiating the two establishments. Indeed, this differentiation introduces infra-marginal competition for the largest establishment. Figure 5 illustrates the trade-off at play. We observe an inverse U-shaped relationship between α and the expected revenues from the bids. Moreover, as d increases, the magnitude of the costs increases, and the expected bids decrease as well. The firm's optimal split thus also decreases towards $\alpha^* = \frac{1}{2}$.

Combining the three portions we have discussed above, we can describe the total expected

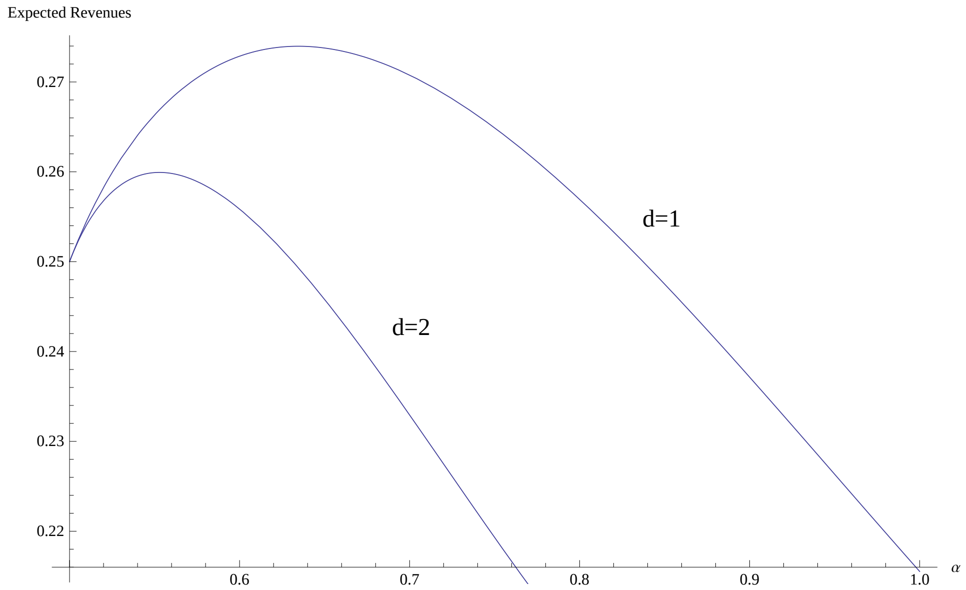

Figure 5: Expected Revenues from Bids, Depending on α

revenues of the firm.

Proposition 2. *There exist some parameters d (magnitude of infrastructure costs) and π (magnitude of potential profits) such that even if the firm has increasing marginal returns to concentration ($\lambda > 1$), she decides to locate her production in two asymmetrical establishments. This behaviour is non-existent when a bidding war does not take place.*

Proof. We do not provide the full analytical solution for α^*, since the solution is not informative. Figure 6 illustrates a case in which the firm, although benefiting from increasing marginal returns to concentration ($\lambda = 2$), is better off splitting her production in two establishments (with $d = 2$ and $\pi = 2$). The example also assumes a uniform distribution of private benefits on $[0, 1]$

□

In general, higher entry costs (higher d) tend to favour a production split closer to $\alpha^* = \frac{1}{2}$. Like in the simple example of the previous section, increasing marginal returns to concentration (larger λ) tend to favour concentration. In fact, in the simple example, we found that the firm always concentrated her production as $\lambda \geq 1$. In the presence of entry costs, however, it is possible that the firm chooses to split her production even if she would benefit from increasing marginal returns to concentration.

In the case of the simple model without entry costs, Figure 1 showed a stylised version of the optimal split for a range of values for λ. Figure 7 repeats the exercise for the full model we just described. We take a value of $d = 1$, and look at the optimal split for various values of

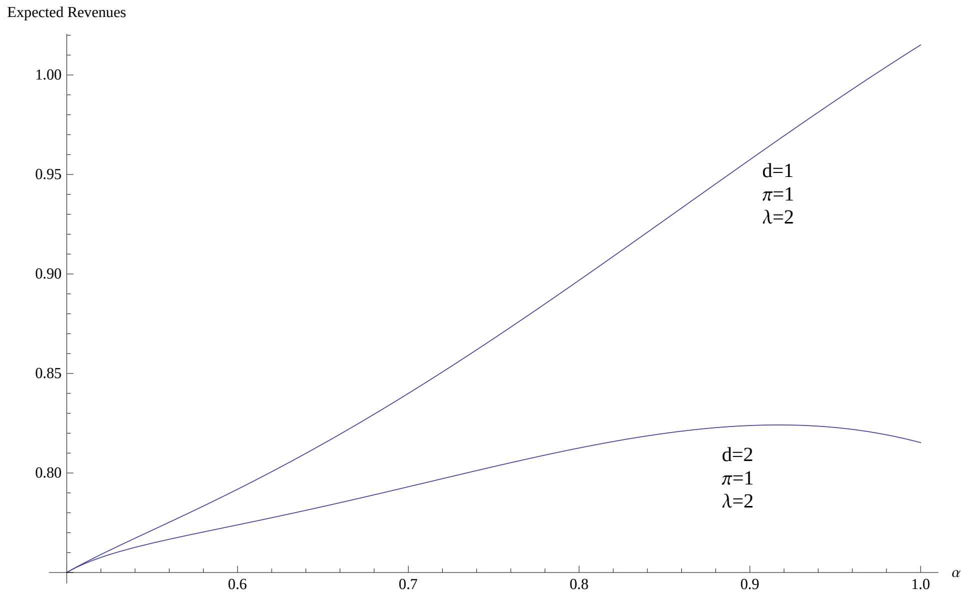

Figure 6: Total Expected Revenues, Depending on α

λ. Most strikingly, we find that with this magnitude of entry costs, the firm can benefit from increasing marginal returns to concentration but still decide to split in multiple establishments.

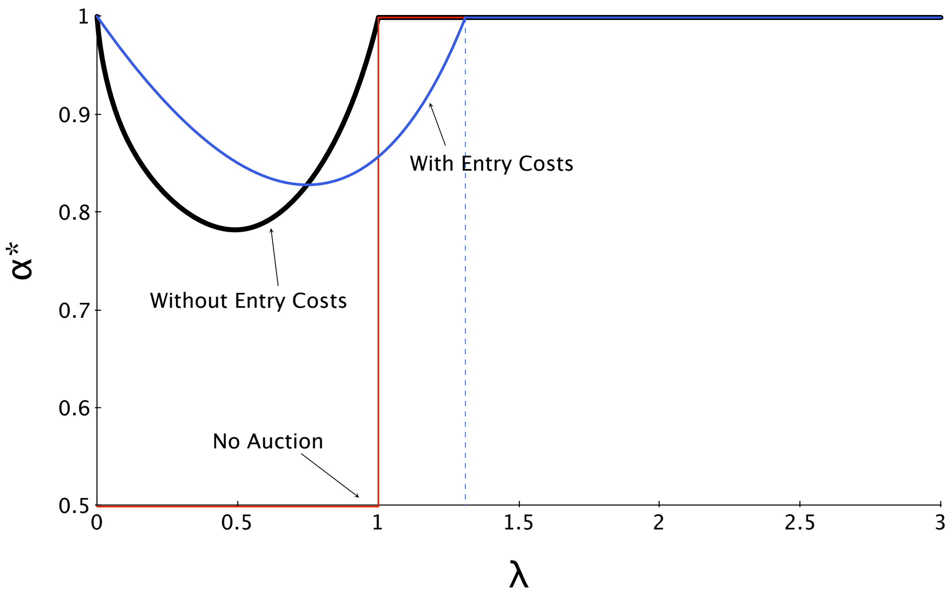

Figure 7: Optimal α for various values of λ, with entry costs

5 Welfare

We have showed that a bidding war may affect the firm's allocation of investment across production sites. One remaining question is how these bidding wars may affect social welfare. Let's first suppose that a social planner has complete information about the regions' signals, and maximises a social welfare function defined as

$$W^{inf} = \pi(\alpha_{inf}) + \pi(1-\alpha_{inf}) + \alpha_{inf} b_{(1)} + (1-\alpha_{inf}) b_{(2)} \tag{22}$$

We assume, first, that $c(\alpha) = 0$ (no necessary investment in infrastructure). The first-order condition is equal to

$$\begin{aligned} \pi'(\alpha_{inf}) - \pi'(1-\alpha_{inf}) + b_{(1)} - b_{(2)} &= 0 \\ \pi'(1-\alpha_{inf}) - \pi'(\alpha_{inf}) &= b_{(1)} - b_{(2)} \end{aligned} \tag{23}$$

Since $b_{(1)} - b_{(2)} > 0$, the FOC implies that $\pi'(\alpha_{inf}) < \pi'(1-\alpha_{inf})$. Therefore, with decreasing marginal returns to concentration ($\pi''(\cdot) < 0$), the optimal solution is always at $a_{inf} > \frac{1}{2}$. In other words, the social planner always chooses differentiated establishments, just like the firm would herself do by using a bidding war. This departure from symmetric establishments (when $\pi''(\cdot) < 0$), however, is different from the departure that existed in the auction.

Proposition 3. *Assume private benefits are distributed according to a uniform distribution. Then, a perfectly-informed social planner differentiates the firm's establishments, but less than the firm would using a bidding war.*

Proof. In a bidding war, the firm chooses an optimal split according to the inequality $\pi'(1-\alpha) - \pi'(\alpha) = 2E(b_{(2)} - b_{(3)})$. In general, $b_{(1)} - b_{(2)}$ is obviously not equal to $2E(b_{(2)} - b_{(3)})$. In fact, in expected values and with a uniform distribution (so $E(b_{(k)} - b_{(k+1)}) = E(b_{(n)})$), $2E(b_{(2)} - b_{(3)})$ is twice as large as $b_{(1)} - b_{(2)}$. In other words, the bidding war introduces a larger distortion than a perfectly-informed social planner would. □

If $\pi''(\cdot) > 0$, then the social planner's solution is to concentrate all production into one large establishment. The firm chooses the same α in a bidding war under this condition.

This result shows that a bidding war and an informed social planner leads to a similar allocation of production. In both cases, the establishments are differentiated. However, the firm and the social planner get to that allocation through very different mechanisms. In the bidding war, the firm chooses a level of differentiation to maximise the expected bids, through infra-marginal competition. Therefore, she takes into account the distribution of the second- and third-highest signals. The social planner, on the other hand, chooses a level of differentiation such that the increased benefits to the regions with largest b_i counterbalance the reduced operating profits.

To illustrate the difference, let's assume a uniform distribution on $[0,1]$. In this case, with 3 regions, $2E(b_{(2)} - b_{(3)}) = \frac{1}{2}$. For the social planner to choose the same level of differentiation as the firm, there would need to be a difference of $\frac{1}{2}$ between the realizations of the first two signals, or twice the expected difference of $\frac{1}{4}$. Obviously, though, any combination of realised signals is possible.

If the social planner did not know the exact signals, but only their distribution like the firm does, we would have the following first-order condition:

$$\pi'(\alpha_{un}) - \pi'(1-\alpha_{un}) + E(b_i) - E(b_j) = 0 \tag{24}$$

with i, j the two regions hosting the investment. The social planner, in this case, can do no better than to choose two regions randomly. Since $E(b_i) = E(b_j)$, the social planner chooses α such that the marginal profits in the two establishments are equal. In other words, he chooses the no-bidding war solution that we described earlier: $\alpha = \frac{1}{2}$ with decreasing marginal returns to concentration, and $\alpha = 1$ with increasing marginal returns to concentration.

To see how the bidding war may affect social welfare, let's define social welfare under a bidding war as such:

$$W^{bw} = \pi(\alpha_{bw}) + \pi(1-\alpha_{bw}) + \alpha_{bw} b_{(1)} + (1-\alpha_{bw}) b_{(2)} - m \cdot (s_1 + s_2) \tag{25}$$

where m is the marginal cost of raising public funds, the deadweight loss incurred by the regional governments in raising revenues to pay the subsidies. Note that the subsidies themselves are simply transfers. Here, α_{bw} is chosen by the firm, as in the previous sections.

Lemma 3. *For low enough marginal costs of public funds, expected social welfare is higher under a bidding war than under an uninformed social planner.*

Proof. If we can prove that $\forall \alpha$, $W^{bw} > W^{un}$, then we know that at the respective optimal values of α, $W^{bw}(\alpha_{bw}) > W^{un}(\alpha_{un})$.

Take an arbitrary α. Then, $E\left[W^{bw}(\alpha)\right] > E\left[W^{un}(\alpha)\right]$ if and only if:

$$\begin{aligned}
E\left[\pi(\alpha) + \pi(1-\alpha) + \alpha b_{(1)} + (1-\alpha)b_{(2)} - m \cdot (s_1(\alpha) + s_2(\alpha))\right] &> E\left[\pi(\alpha) + \pi(1-\alpha)\right] + E(b) \\
E\left[\alpha b_{(1)} + (1-\alpha)b_{(2)} - m \cdot ((2\alpha-1)b_{(2)} + 2(1-\alpha)b_{(3)})\right] &> E(b) \\
\alpha \cdot (\frac{1}{2} - m) + \frac{1}{2} &> \frac{1}{2} \\
m &< \frac{1}{2}
\end{aligned}$$

In the third step, we assumed, as before, that the private benefits are distributed according to a uniform distribution on $[0,1]$. Therefore, for $m < \frac{1}{2}$, social welfare is higher, in expected terms, under a bidding war than under an uninformed social planner. □

This result is intuitive. Suppose we must decide, as an uninformed social planner, whether to maximise social welfare using our limited information, or let the firm conduct a bidding war. If raising revenues is not too costly for the government (in terms of administrative costs or distortions), then it is worthwhile to do so and achieve a better allocation of establishments to regions. However, that result might not hold if the social planner observes the private benefits of every region.

Lemma 4. *Expected social welfare is always lower under a bidding war than under a perfectly-informed social planner.*

Proof. As previously, we only need to prove that $\forall\alpha$, $W^{bw} < W^{inf}$. Take an arbitrary α. Then, $E\left[W^{bw}(\alpha)\right] < E\left[W^{inf}(\alpha)\right]$ if and only if:

$$\begin{aligned}
E\left[\pi(\alpha) + \pi(1-\alpha) + \alpha b_{(1)} + (1-\alpha)b_{(2)} - m \cdot (s_1(\alpha) + s_2(\alpha))\right] &< E\left[\pi(\alpha) + \pi(1-\alpha) + \alpha b_{(1)} + (1-\alpha)b_{(2)}\right] \\
E\left[\alpha b_{(1)} + (1-\alpha)b_{(2)} - m \cdot ((2\alpha-1)b_{(2)} + 2(1-\alpha)b_{(3)})\right] &< E(\alpha b_{(1)} + (1-\alpha)b_{(2)}) \\
E[-m \cdot ((2\alpha-1)b_{(2)} + 2(1-\alpha)b_{(3)})] &< 0
\end{aligned}$$

Since $m > 0$ and $s_1(\alpha)+s_2(\alpha) \geq 0$, the previous is always true. Therefore, $\forall\alpha$, $W^{bw} < W^{inf}$. □

This result is not very surprising. A social planner with all the information can achieve the allocation of establishments to the regions that value it the most, without the added cost of raising public funds to pay subsidies to the firm. The previous two lemmas thus show that social welfare under a bidding war falls under the two extreme assumptions of information for the social planner.

Proposition 4. *Social welfare under a bidding war is lower than social welfare under a perfectly-informed social planner, but higher than under an uninformed social planner.*

Proof. See Lemmas 3 and 4. □

As an example, assume that $\pi(\alpha)$ takes the same functional form: $\pi(\alpha) = \alpha^{\frac{1}{2}}$. Also assume a uniform distribution on $[0, 1]$ Figure 8 shows social welfare under the three possibilities.

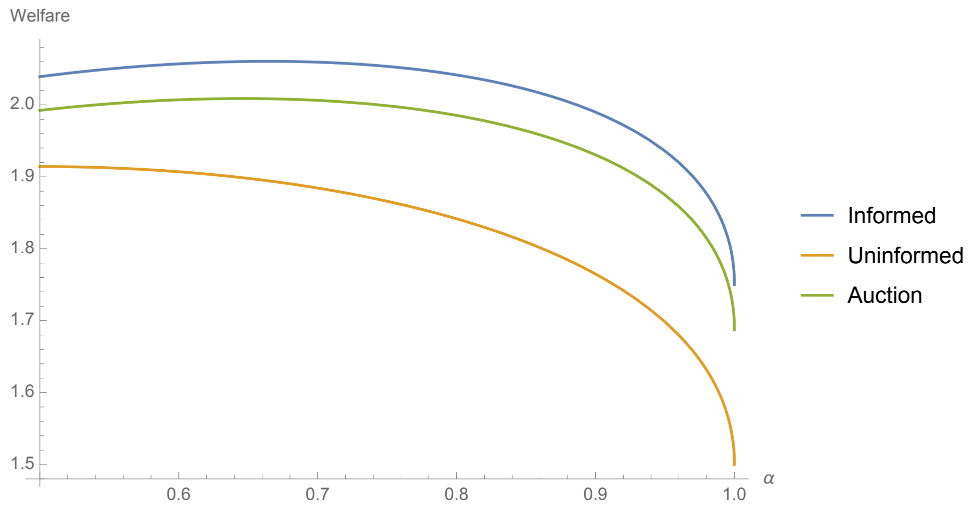

Figure 8: Social Welfare Under Three Possibilities

6 Conclusion

To be completed.

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