The Impact of Bidding Wars on the Optimal Investment Decisions of Multi-Establishment Firms

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Abstract: This paper studies the competition between regional governments to attract one of a firm's new plants. The goal of this analysis is to study the strategic behaviour of the firm in such competitions or location contests. Indeed, in contrast to the existing literature on the subject that considers only firms producing in a single location, the paper shows that the firm can modify its allocation of production across sites by differentiating the plants, thus attracting larger subsidies.

Keywords: Fiscal competition, Auctions, Firm's location choice.

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1 Introduction

Tax incentives offered to firms in exchange for new investments represent an appreciable amount of government spending each year. In the United States alone, state and local governments award approximately $80 billion in tax incentives each year to companies.[1] These subsidies are often the result of bidding wars between many local or regional governments. Owing to their prominence, economists investigated the behaviour of firms and governments participating in these location contests. However, they have generally considered a single firm opening a single establishment. In fact, the firms running these bidding wars are frequently multinationals, or at least multi-establishment companies. For example, between 2007 and 2012, Boeing received at least $327 million in incentives from 11 US states. In the same period, Procter & Gamble received at least $128 million from 10 states.[2] These examples illustrate how firms make multiple investments in short periods of time. Consequently, these bidding wars are not necessarily independent.

In this paper, the objective is to investigate the strategic behaviour of a firm that is conducting a bidding war for multiple establishments. The main question is whether the firm can allocate investment across its production sites strategically, in order to increase the subsidies she receives from regional governments. The focus is thus mostly on the firm's strategic behaviour, instead of the governments'. To do so, we propose a model in which regional governments are competing against each other to attract one of a firm's investments. The main originality is twofold. First, the firm endogenously decides how much to invest, and her decision can affect the bidding behaviour of the regions. Second, we allow the firm to invest in more than one location, essentially making multiple plants available for bidding. Formally, we model this competition as a multi-unit auction. We find that such a bidding war affects the firm's structure. Indeed, the firm invests more in one of the plants, creating differentiation between them. In doing so, she creates incentives for the regional governments to offer larger tax breaks, through infra-marginal competition between the last two remaining bidding regions for the largest plant in the auction. Cowie *et al.* (2007) previously considered infra-marginal competition in the context of an auction. They analyse how a seller can divide the units for sale in multiple lots in order to receive higher offers from the bidders. They find that differentiating the lots can

[1] The New York Times, "As Companies Seek Tax Deals, Governments Pay High Price," available at the following address: http://www.nytimes.com/2012/12/02/us/how-local-taxpayers-bankroll-corporations.html.

[2] Other examples are available from the New York Times, at the following URL: http://www.nytimes.com/interactive/2012/12/01/us/government-incentives.html

lead to higher bids due to the infra-marginal competition for the largest lot. We have a similar reasoning in the auction stage of our model.

This paper contributes mainly to the literature on fiscal competition, but also to the analysis of the location decision of multinationals. To the literature on fiscal competition, this paper is particularly related to the subset of papers that consider competition for a single large firm. Keen and Konrad (2014) offer a short overview of this literature, which includes early contributions by, e.g., Black and Hoyt, (1989), Doyle and van Wijnbergen, (1994), and King *et al.*, (1993). This is in contrast to the larger stream of that literature that considers the competition between regions or countries to attract units of homogeneous and perfectly divisible capital. Wilson (1999) and Keen and Konrad (2014) offer extensive surveys of these models. Moreover, in contrast to many of these papers, we are primarily interested in how bidding wars affect the strategy and the behaviour of the firm, instead of governments.

As in this paper, many papers that investigate these bidding wars for a single large firm use models from, or similar to, auctions. Indeed, auctions are a useful tool for sellers who do not know the value potential buyers place on the product sold. Moreover, as suggested by Klemperer (2004, Chapter 2), auction theory can also provide a rich set of tools to study a number of problems in economics and social sciences. Location contests are a good example of a context in which auctions are a useful theoretical tool; many bidders (governments) place some private value on a good (investment), and a seller (the firm) does not know how to price it, thus choosing to accept bids (subsidies).

Our model is particularly related to the analysis of Haaparanta (1996), who uses a menu auction model. This author considers two regions competing for investment from a firm, under the assumption that this investment is divisible. However, while Haaparanta (1996) considers a model under perfect information, we assume that the regions' private benefits from hosting the firm are private knowledge. In fact, such information asymmetry is a justification to use a mechanism similar to an auction in the first place.

As the model will show formally, analysing the question under an open ascending auction instead of a menu auction (as in Haaparanta's paper) will reveal new insights about the bidding war and the allocation of investment. First, when establishments are asymmetric, infra-marginal competition takes place between the last two remaining bidders, increasing the subsidy on the large plant, and allowing the firm to benefit from higher total subsidies. Consequently, at the equilibrium, the firm modifies her allocation of production to take advantage of this phenomenon. Cowie et al. (2007) previously considered infra-marginal competition in the context of an auction. They analyse how a seller can divide the units for sale in multiple lots in order to receive higher offers from the bidders. They find that differentiating the lots can lead to higher bids due to the infra-marginal competition for the largest lot. I have a similar reasoning in the auction stage of the model. The second new insight results from the presence of information asymmetry. Under a menu auction, Haaparanta (1996) finds that the firm captures the whole rent from the regions. In this paper, the information asymmetry curbs the firm's ability to extract rents from the regions.

Another closely related paper is that of Martin (1999). This author studies two firms in the same industry who use bidding wars sequentially to decide where to locate. Martin (1999) shows that agglomeration effects incite regions to overbid in the first auction, expecting it will increase their probability of winning in the second period. Indeed, winning the investment in the first period from the first firm increases the attractiveness of the region to other firms in the same industry. In this paper, we also find that regions offer greater subsidies for one plant. However, we consider how a single firm can entice greater subsidies by modifying her allocation of production between two plants. In addition, we do so without considering agglomeration economies.

Other related papers include Black and Hoyt (1989) who were, to our knowledge, the first to explicitly model the firm's location choice as an auction. They highlight the fact that this competition need not be a zero-sum game; the bids offered by government can promote the efficient location of production. In their model, they also consider that smaller, already established firms may move once a new firm is opened in one of the regions. Indeed, small firms will relocate to the winning region, thus increasing its potential gains. This multiplier effect can explain why regions may seem to "overbid" for the large firm.

Before Black and Hoyt (1989), Doyle and van Wijnbergen (1994), in a paper first published in 1984, considered a bargaining game between one firm and a government over taxation. Doyle and van Wijnbergen (1994) assumed that firms negotiate with a single government at a time. In their solution, the host government initially sets a low tax rate, but gradually increases it until it reaches a limit. The government has some bargaining power due to the fact that the multinational must incur a positive cost if it relocates to a new location. However, firms have no reason not to negotiate simultaneously with multiple governments. Recognising this fact, Bond and Samuelson (1986) investigate a situation in which a firm has to decide between two locations. In their model, tax holidays are used as a signal of productivity by the governments. An important feature of their model is information asymmetry. It allows for the presence of tax holidays even if there are no fixed costs, in contrast to Doyle and Van Wijnbergen (1984). Similarly to Bond and Samuelson (1986), information asymmetry is an important of our model, although our results are derived without productivity differences between the regions.

Black and Hoyt (1989) had highlighted some caveats to their analysis. One caveat was the lack of dynamic considerations. King and Welling (1992) explore the consequences of allowing the firm to relocate in later periods. They consider a two-period model, in which the firm conducts an auction to decide on its location in each period. They find that when players cannot commit to second-period actions, the firm can re-locate to the region that lost in the first period. This possibility modifies the first-period bids, thus changing the outcome even if the relocation threat is not materialised. The authors also show that the firm would prefer a world with commitment, but that without commitment, total social welfare is higher.

King, McAfee and Welling (1993) generalise the model of King and Welling (1992), but with a continuum of local productivities. They also consider an extension in which regions can invest in infrastructure in a previous stage, thus increasing their productivity potential. They find that in equilibrium, regions tend to choose different levels of infrastructure, thus endogenously creating the productivity continuum described in their main model. King et al. (1993) assume some information asymmetry, but it is the firm who does not know its productivity in each region. In this paper, we instead assume (like Martin, 1999) that the regions hold some private information, while productivity is the same everywhere. This modelling choice reflect the fact that not all regions value the firm's presence identically.

In this paper, we also do not consider a two-period model. The firm installs new production facilities in one period, but we do not model the interactions in the following periods. We do so deliberately, to focus instead on how the firm decides to allocate across regions in multiple establishments in a single period. If we did consider many periods, our results could be related to those of Janeba (2000), for example, who considers a firm that installs excess production capacity in multiple regions in order to avoid the problem of hold-up by the regions. Indeed, in subsequent periods, regions could increase taxes or renege on their commitment to tax breaks (i.e., subsidies). By having excess capacity, the firm could credibly threaten to decrease production, and thus employment in the region that increased taxes, to increase it in the other.

Furusawa, Hori and Wooton (2010) show that the bidding mechanism can affect the results of the model. In their paper, they show that English auctions lead to more aggressive bidding, or to a “race beyond the bottom,” compared to bidding in sealed bid auctions. Martin (2000), however, cites case studies that claim bidding wars resemble more closely open ascending auctions, as in our model. Menezes (2003) describes the basic competition for investment under several auction mechanisms, and shows that the expected amount paid to the firm is the same, which is not surprising given the Revenue Equivalence theorem. In this paper, we show that the open ascending auction we use implements the optimal mechanism (under some conditions).

Other examples of papers on bidding wars for a specific firm include Ferrett and Wooton (2010), who analyse the question of firm ownership, showing that tax or subsidy offers are independent of the ownership country of the firm. In another paper, Martin (2000) applies auctions with favouritism to study these contests when firms have explicit preferences for a region. Finally, Scoones (2001) studies bidding wars for firms when the value of the investment has two components: common and private. In other words, part of the investment’s value is the same in every region, while another is specific to producing in a given region. He shows that if the common share increases, then the subsidies increase as well, eventually transferring all value to the firm.

Some may see these bidding wars as wasteful, but they can also play an important role in eliciting private information and improving allocation efficiency (Menezes, 2003). In fact, despite paying subsidies to the firm, the winning region may actually benefit from the presence of the new plant. Greenstone and Moretti (2003) compare the outcomes for winning and losing counties in contests for “million dollar plants”, and find that winning counties experience greater increases in land value as well as in the total wage bill of other firms in the industry of the new plant. In our model, we show that this bidding war ensures that regions that value the investment the most are those receiving it, a favourable outcome in terms of social welfare. We also show that the differentiation between the plants is optimal from the point of view of a social planner.

In addition to the literature on fiscal competition, this paper is related to the analysis of the multinational. Indeed, one of our contribution over most papers cited above is to allow the firm to have multiple establishments, and investigate how a bidding war affects the firm’s choice of production locations. Ekholm and Forslid (2001) explain how Core-Periphery models argued that firms prefer to concentrate in a single location, as long as trade costs are low enough. These two authors then depart from the usual Core-Periphery models and investigate firms that have multi-plant economies of scale, and thus produce in many regions. They cite the example of soft drink and beer manufacturers, who usually operate in numerous locations. Their model points to less agglomeration than previous models did. Hanink (1984) offers another potential justification for multi-plant firms: risk diversification. He does so by comparing firms to investors. In the same way that investors prefer holding a diverse portfolio, firms can hold a diverse "geographical" portfolio to increase their overall profits. These papers show that firms can have incentives to operate in many plants. In this paper, we take as given the existence of multi-plant firms, focusing instead of the location decision of these firms, and how it is affected by a bidding war.

More closely related to this paper, Behrens and Picard (2008) study the choice of firms to become multinationals, in a model that includes subsidies to location. They find that bidding wars and subsidies affect the choice of firms to become multinationals, in effect increasing the number of multinationals. In their model, they consider a continuum of firms deciding to locate in one or two countries. In this paper, we instead study the choice of a single firm, taking its decision to be a multinational as a given. However, their result underlines the importance to study bidding wars in the context of multinationals.

Our main results are as follows. First, we show that when a firm conducts a bidding war for multiple establishments, she can increase her total profits (operating profits plus subsidies) by differentiating her establishments. Second, we show that under certain assumptions on the production function, the firm invests more in total and receives larger subsidies under such a bidding war than she would if she allocated production without relying on a bidding war. Third, we show that total investment and subsidies would be over-estimated if we did not consider the linkages between the bidding wars between the multiple establishments of a single firm. Fourth, we show that the multi-unit auction under which we derive our main results is equivalent to the optimal mechanism from the firm's viewpoint. Similarly, we show that a social planner would also choose the same allocation and payment rules, although the conditions under which the social planner chooses not to impose reserve prices are looser than the conditions from the firm's viewpoint. Finally, although our model is derived with a commitment to investment amounts by the firm in the first period, we show that in expected value, our model is equivalent *ex ante* to a more general model without commitment.

The next section presents the framework of the model, including the timing of the game. Section 3 solves the three stages of the game, while Section 4 compares the results with those of an alternative model with only one plant, but with an endogenous amount of investment. This model shows how both endogenous investment and multiple investments affect the firm's behaviour. Section 5 derives the optimal mechanism, first from the viewpoint of the firm, and then from that of a social planner. Then, it derives the optimal mechanism without prior commitment to investment amounts, showing that it is *ex ante* equivalent in expected value to the constrained optimal mechanism. The last section concludes.

2 The Model

Consider a firm that plans to build new production facilities in two of n regions, indexed by $i \in 1, ..., n$. To decide the location of these plants, the firm puts the n regional governments in competition against each other. The governments submit offers of subsidies to attract the firm to their territory. In contrast to most of the previous literature, however, the firm can divide her production in multiple locations, either in symmetric or asymmetric establishments. For simplicity and tractability, we limit the model to the case of two establishments, indexed by $j \in 1, 2$. Without loss of generality, we label the largest plant by $j = 1$, so that $K_1 \geq K_2$.

2.1 The Firm

We consider a multinational firm that already produces elsewhere, and wants to increase production by installing new establishments among the n regions. Once she decided where to install the new plants, the firm produces, in each establishment, according to the production function $f(K_j, L_j)$, with K_j the capital invested in location j, and L_j the labour employed in that establishment. We make the usual assumptions that the production function exhibits decreasing returns to scale in both inputs ($\frac{\partial f(K_j,L_j)}{\partial K_i} > 0$, $\frac{\partial f(K_j,L_j)}{\partial L_i} > 0$ and $\frac{\partial f^2(K_j,L_j)}{\partial K_i^2} < 0$, $\frac{\partial f^2(K_j,L_j)}{\partial L_i^2} < 0$).[3] The firm sells the product on a global market for a price p, acting as a price-taker. We deliberately do not model the goods market explicitly, to instead focus on the firm's location decision and the bidding war between regions. The production costs are identical in every region (w, r). Therefore, the firm's operating profits in each establishment $j = 1, 2$ are equal to

$$\pi_j = pf(K_j, L_j) - wL_j - rK_j \tag{2.1}$$

[3] This assumption implies, in the model, that the firm has incentives to produce in more than one establishment.

In addition to the profits from production, the firm also receives subsidies from the regions (resulting from the bidding war), so that her total *ex post* profits are equal to:

$$\Pi = s_1^* + s_2^* + \pi_1 + \pi_2 \tag{2.2}$$

where s_j^* is the equilibrium subsidy for establishment j.

2.2 The Regions

These subsidies depend on the regions' valuation of the firm's investments. In particular, if regional government i wins establishment j, it receives a payoff equal to

$$V_{ij} = L_j \cdot b_i - s_{ij} \tag{2.3}$$

where L_j is the number of persons employed by the firm in establishment j, b_i is the level of private benefits from hosting the firm for region i's government, and s_{ij} is the subsidy (bid) offered to the firm by region i when winning establishment j. The subsidy can be interpreted as a total "fiscal package" offered to the firm.[4]

A region's private benefits b_i are private knowledge, and they capture, for example,[5] an increase in labour taxation from workers who will be employed by the firm, as well as spillovers to domestic firms, but also the compatibility of the firm for the region. Indeed, if the industry of the firm has a bad reputation in one region, the regional government would put only a small value on the firm's investment (due to, for example, re-election concerns).[6] The private benefits are identically and independently distributed according to a distribution $g(\cdot)$ on some interval $\left[\underline{b}, \bar{b}\right]$ (with $\underline{b} \geq 0$).

2.3 The Auction Process

The equilibrium subsidies are then determined by an auction in which the firm takes the role of the auctioneer, and the regional governments submit their bids to host the firm's plants.

[4]In effect, our model assumes that all regional governments have the same basic tax rate, but differentiate themselves with targeted tax holidays that may differ. This assumption may not be unreasonable in the case of sub-national jurisdictions. Even when considering countries, we are mostly interested in the competition taking place in subsidies, and abstracting from tax competition allows us to focus on our variables of interest.

[5]Ferrett and Wooton (2013) use a similar justification for private benefits, while Martin (2000:6) provides a more thorough list of potential explanation for these benefits.

[6]For example, Buts, Jegers, and Jottier (2012) find that subsidies to firms increase support for incumbent politicians.

Since there are two establishments available, the firm conducts a multi-unit auction, with both establishments available simultaneously.

The formal mechanism is an open ascending auction. More specifically, the firm runs an ascending clock, representing the current price for the lowest-value establishment still available (the one with the lowest investment). Regional governments still in the running are ready to offer a bid equal to the current price. The winning bid is determined from the price on the clock when the previous bidder withdrew from the auction. In particular, if the two establishments are still available, then when there are only two regions left bidding, the price for the lowest-valued establishment will be determined from the clock price at which the third-to-last region withdrew from the auction.[7] These two remaining regions will then continue bidding until one of them exits. The clock price at which the second-to-last region withdrew will be the price for the highest-valued establishment. Formally, this mechanism is a type of second-price auction.

2.4 Timing

We can summarize the timing of the whole game as follows.

Stage 0: Nature picks the set of $\{b_i\}_{i=1,\dots,n}$. Regional governments learn their b_i.

Stage 1: The firm chooses and commits to an allocation of capital (K_1, K_2), anticipating the subsidies offered by governments resulting from the auction in Stage 2, and the firm's own profit maximization in the last stage.

Stage 2: The multi-unit auction takes place. Winning regions offer s_1^* and s_2^*, based on their expectation of the labor that will be employed by the firm (from profit maximzation in the last stage).

Stage 3: The firm invests capital K_1 and K_2, as determined in Stage 1, in the winning regions. She then maximizes her profits, taking capital fixed, choosing L_1 and L_2.

In the first stage, the firm commits to a certain allocation of capital. One could reasonably argue that the firm has incentives to deviate from that allocation once she receives the subsidies from the region. However, in that case, regions would anticipate these deviations and bid accordingly. To facilitate the analysis, we make the assumption that the firm can credibly commit to her allocation.

[7] Note that regions who withdraw from the auction without winning one establishment do not pay anything.

3 Equilibrium Subsidies and Firm Location Choice

We will solve the game described in the previous section by backwards induction.

3.1 Stage 3: Production

We first solve the last stage of the game, to find the firm's optimal labour input demand in each firm for each level of capital invested. At this stage of the game, the firm already knows the identity of the winning regions, and invests the capital in these two regions as determined in the first stage. She also knows how the amount of the subsidies conditional on the amount of labour she will employ.

The firm thus maximizes her profits in each plant, choosing L. At this last stage of the game, the firm already decided on (K_1, K_2), so it is fixed. Her maximization problem in each plant is thus as follows.

$$\max_{L_j} \quad pf(K_j^*, L_j) - wL_j - rK_j^* \tag{3.1}$$

The first-order condition is

$$pf'(K_j^*, L_j) - w = 0$$

implying that the firm chooses L_j to equalize the marginal product that input, $f'(K_j^*, L_j)$, with the ratio of w and p. Therefore, the optimal L_j will depend on the amount of capital invested, K_j^*. We define the function $L(K_j)$, determining the amount of labour employed for each possible equilibrium level of capital invested in the first stage.

Since the regions' valuation depends on the amount of labour employed, we want to know how L varies with K. By totally differentiating the first-order condition, we can obtain the sign of $\frac{dL}{dK}$:

$$\frac{dL}{dK} = -\frac{\frac{\partial^2 f(K^*, L^*)}{\partial K^2}}{\frac{\partial^2 f(K^*, L^*)}{\partial K \partial L}} > 0$$

This derivative is greater than zero as long as the cross partial derivatives in K and L are positive (e.g., increasing capital increases the marginal product of labour). Therefore, a greater investment by the firm in an establishment translates into a greater valuation of that establishment by the regions.

As an example, take a simple Cobb-Douglas production function $f(K, L) = K^\alpha L^\beta$ with $\alpha + \beta < 1$. At that stage, K is fixed in each establishment and the firm already received the subsidies. Therefore, the firm chooses L in each plant to maximise her operating profits. In that case, for each level of K, she chooses an optimal amount of labour L equal to

$$L(K) = \left(\frac{p\beta}{w}\right)^{\frac{1}{1-\beta}} K^{\alpha/(1-\beta)} \tag{3.2}$$

In this example, larger investments by the firm translate in more labour employed ($L'(K) > 0$), but at a decreasing rate ($L''(K) < 0$).

3.2 Stage 2: Auction and Equilibrium Subsidies

In the auction stage, the firm puts up two plants for sale of sizes K_1 and K_2. The regional governments expect the firm to employ $L(K_1)$ and $L(K_2)$, respectively, and bid according to their valuation functions V_{ij}. The following lemma describes the equilibrium subsidies resulting from the auction.

Lemma 1. *The equilibrium bids for the two establishments will be equal to*

$$s_2^*(K_1, K_2) = L^*(K_2) \cdot b_{(3)} \tag{3.3}$$

$$s_1^*(K_1, K_2) = (L^*(K_1) - L^*(K_2))b_{(2)} + L^*(K_2)b_{(3)} \tag{3.4}$$

where $b_{(z)}$ is the z^{th}-highest signal among the n regions.

Proof. To see why these two bids are optimal, take a region i with private benefits b_i and assume that everyone else bids according to the following strategy: continue bidding until the clock reaches my private valuation. In that case, if the clock reaches L_2b_i and there are still 3 or more regions in the auction, then region i has no incentive to continue bidding. Indeed, if she does, whatever the stop price, she will need to pay more than her valuation if she wins. Therefore, at price L_2b_i, she prefers to leave the auction. Now consider prices lower than L_2b_i, for example L_2b_l. At that clock price, region i has a positive valuation and would like to win. Therefore, she has no incentive to leave the auction. Therefore, the equilibrium bid for the small establishment will be equal to

$$s_2^*(K_1, K_2) = L(K_2) \cdot b_{(3)}$$

where $b_{(3)}$ is the third-highest signal among the n regions.

If the two plants are of symmetric sizes (i.e., $K_1 = K_2$), then the two remaining regions each pay $s_2^*(K_1, K_2)$ and each receive the same investment.

However, if the two plants are asymmetric (i.e., $K_1 \neq K_2$), we still have to determine which region receives the largest investment. Both regions know that their possibilities are now to pay $s_2^*(K_1, K_2)$ and receive the small establishment, or to pay more and receive the large establishment. The bid for the largest establishment will thus be determined by the infra-marginal competition between the two remaining bidders. Since at that point, the auction becomes a simple second-price auction between two bidders, it is optimal for both regions to simply withdraw once the clock price reaches their valuation of the large plant. If they continue past that price, they either win and pay a price higher than their valuation, or they lose and pay the price for the second establishment, which was already determined.

Take the decision problem of the region with the second-highest private benefits.[8] It will be indifferent between the two establishments when

$$L(K_1)b_{(2)} - s_1^*(K_1, K_2) = L(K_2)b_{(2)} - s_2^*(K_1, K_2)$$

By rearranging this equation and substituting the value of $s_2^*(K_2)$ found earlier, we obtain the value of the highest bid

$$s_1^*(K_1, K_2) = (L(K_1) - L(K_2))b_{(2)} + L(K_2)b_{(3)}$$

□

Note that, as expected, if $K_1 = K_2$, this equation is equal to $s_2^*(K_1, K_2)$. In the more interesting case of asymmetric investments, however, the two last remaining regions continue to compete for the large establishment. We see, from equation 2.6, that an increase in K_1 for a given value of K_2 raises, through the infra-marginal competition, the subsidy offered for the most valuable establishment. A reduction in K_2 has a similar effect, while also reducing the bid received for the small investment.

[8]Given the monotonicity of the valuation function of the regions, for any level of private benefits, regions prefer the largest establishment to the small one.

3.3 Stage 1: The Firm's Optimal Location Choice

In the first stage, the firm's optimisation problem is the following:

$$\max_{K_1,K_2} E(s_1^* + s_2^* + \pi_1 + \pi_2) \tag{3.5}$$

where $\pi_j = pf(K_j, L_j) - wL_j - rK_j$ and s_j^* are, respectively, the operating profits in each establishment and the equilibrium subsidies as determined in Lemma 2.1. The firm thus chooses K_1 and K_2 to maximise her total expected profits, anticipating the bids of the regions, as well as her profit maximisation in the last stage. The solution to this optimisation problem leads to the following proposition.

Proposition 1. *When the firm allocates her production units through a multi-unit auction, she always chooses to differentiate the two establishments ($K_1 \neq K_2$).*

Proof. The firm does not know the private benefits of the regions in the competition, but knows that they are distributed according to $g(\cdot)$ on the interval $[\underline{b}, \overline{b}]$. Her objective function can thus be expressed as

$$\begin{aligned} E(\Pi) = \int_{\underline{b}}^{\overline{b}} \int_{\underline{b}}^{b_{(2)}} \Big[&(L(K_1) - L(K_2))b_{(2)} + 2L(K_2)b_{(3)} \\ &+ pf(K_1, L(K_1)) - wL(K_1) - rK_1 + pf(K_2, L(K_2)) - wL(K_2) - rK_2 \Big] \cdot \\ &h(b_{(2)}, b_{(3)}, n) db_{(3)} db_{(2)} \end{aligned} \tag{3.6}$$

where the last part $h(b_{(2)}, b_{(3)}, n) = n(n-1)(n-2) \cdot \Big[1 - G(b_{(2)})\Big] \Big[G(b_{(3)})\Big]^{n-3} g(b_{(2)})g(b_{(3)})$ is the joint distribution of $b_{(2)}$ and $b_{(3)}$, and $L(K_j)$ is the equilibrium amount of labour for a level of capital K_j. We obtain the following first-order conditions:

$$\begin{aligned} \frac{\partial E(\Pi)}{\partial K_1} = &L'(K_1)E(b_{(2)}) \\ &+ p(\frac{\partial f(K_1, L(K_1))}{\partial K_1} + \frac{\partial f(K_1, L(K_1))}{\partial L(K_1)} \cdot L'(K_1)) - wL'(K_1) - r = 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial E(\Pi)}{\partial K_2} = &-L'(K_2)E(b_{(2)}) + 2L'(K_2)E(b_{(3)}) \\ &+ p(\frac{\partial f(K_2, L(K_2))}{\partial K_2} + \frac{\partial f(K_2, L(K_2))}{\partial L(K_2)} \cdot L'(K_2)) - wL'(K_2) - r = 0 \end{aligned}$$

Since $L(K)$ represents equilibrium values, the FOCs can be simplified using the Envelope Theorem.We then obtain:

$$\frac{\partial E(\Pi)}{\partial K_1} = L'(K_1)E(b_{(2)}) + p\frac{\partial f(K_1, L(K_1))}{\partial K_1} - wL'(K_1) - r = 0 \tag{3.7}$$

$$\frac{\partial E(\Pi)}{\partial K_2} = -L'(K_2)E(b_{(2)}) + 2L'(K_2)E(b_{(3)}) + p\frac{\partial f(K_2, L(K_2))}{\partial K_2} - wL'(K_2) - r = 0 \tag{3.8}$$

Combining the two FOCs, we see that

$$p\left(\frac{\partial f(K_2, L(K_2))}{\partial K_2} - \frac{\partial f(K_1, L(K_1))}{\partial K_1}\right) = L'(K_2)\left(w + E(b_{(2)}) - 2E(b_{(3)})\right) - L'(K_1)\left(w + E(b_{(2)})\right)$$

We want to show that $K_1 \neq K_2$. Let's first assume that $E(b_{(2)}) \neq E(b_{(3)})$ (i.e., we focus on the interesting cases where the firm expects regions to have different valuations). To prove that the firm has to optimally split in asymmetric establishments, we first assume that she does not, and show that it leads to an inconsistency. Indeed, if $K_1 = K_2 = K$, the previous equation reduces to

$$0 = 2L'(K)\left(E(b_{(2)}) - E(b_{(3)})\right)$$

Since the regions have different expected private benefits, this equation is true only if $L'(K) = 0$. However, that derivative is always positive. Therefore, we conclude that $K_1 \neq K_2$.

□

Note that we can rearrange the first-order conditions as such:

$$p\frac{\partial f(K_1, L(K_1))}{\partial K_1} = L'(K_1)(w - E(b_{(2)})) + r \tag{3.9}$$

$$p\frac{\partial f(K_2, L(K_2))}{\partial K_2} = L'(K_2)(w + E(b_{(2)}) - 2E(b_{(3)})) + r \tag{3.10}$$

This formulation is informative of the trade-offs at play. In each establishment, the firm's choice of K_j reflects the usual trade-off of marginal revenues and marginal costs. However, the marginal cost of labour is not simply equal to the wages paid. In fact, the firm receives subsidies that depend on the level of employment, effectively lowering the firm's marginal labour costs. Denoting total equilibrium subsidies by s_t^*, we find that $\frac{\partial s_t^*}{\partial K_1} = E(b_{(2)})$ and $\frac{\partial s_t^*}{\partial K_2} = -E(b_{(2)}) + 2E(b_{(3)})$. Therefore, when the firm increases K_j, her labour costs increase not simply by $L'(K_j) \cdot w$, but by an amount with wages "adjusted" by the marginal subsidies.

Having solved all the stages of the game, we can describe the sub-game perfect Nash equilibrium. In it, the firm commits in Stage 1 to (K_1^*, K_2^*), defined by the first-order conditions (3.9) and (3.10). In Stage 2, the regions bid until the price on the clock passes their valuation. The region with the highest private benefits wins the largest establishment and offers subsidies of $s_1^*(K_1, K_2) = (L^*(K_1) - L^*(K_2))b_{(2)} + L^*(K_2)b_{(3)}$. The region with the second-highest private benefits wins the smaller establishment, paying subsidies equal to $s_2^*(K_1, K_2) = L^*(K_2) \cdot b_{(3)}$. In Stage 3, the firm invests the amounts (K_1^*, K_2^*), employs labour $L(K_j)$ in each establishment j, and produces according to $f(\cdot)$.

3.4 Equilibrium Amount of Investment: Bidding War vs. No Bidding War

For comparison purposes, without a bidding war, the firm chooses to invest an equal amount of capital in two random regions. Indeed, the firm's revenues are then simply equal to $\pi_1(K_1) + \pi_2(K_2)$. The first-order conditions are

$$p \cdot \frac{\partial f(K_1, L(K_1))}{\partial K_1} = wL'(K_1) + r$$
$$p \cdot \frac{\partial f(K_2, L(K_2))}{\partial K_2} = wL'(K_2) + r$$

Put differently, the firm's optimal allocation in this case simply results from equating marginal revenues and marginal costs in each establishment. The assumptions on the production function imply that the firm chooses an identical investment in both plants: K_{nbw}.

Since the firm has no information about the private benefits of the regions, and since regions are identical in terms of productive capacity, the firm chooses to invest an equal amount K_{nbw} in two regions. She can just choose two regions at random, since her production costs and profits will be identical with any set of two regions.

This comparison begs the question whether the firm invests more in total when allocating through a bidding war than when she randomly chooses two regions to invest in. Intuitively, one might suspect that the firm always chooses a larger K_1 when using a bidding war, since "adjusted wages" are lower than w. We prove this intermediary result in the following lemma.

Lemma 2. *The capital investment in the first establishment (K_1) is always greater under a bidding war than without a bidding war.*

Proof. As long as $E(b_{(2)}) > 0$, adjusted wages (the firm pays wages w, but the subsidy effectively lowers them) are lower than w. Indeed, $w > w - E(b_{(2)})$. Therefore, $K_1^* > K_{nbw}$. □

The intuition is less clear in the case of the second establishment. Indeed, $E(b_{(2)}) - 2E(b_{(3)})$ could be greater or smaller than zero, depending on the distribution of the private benefits. In turn, investment could be lower or higher than without a bidding war. With a uniform distribution, it is easy to see that K_2^* will be greater than (with $n > 3$) or equal to (with $n = 3$) K_{nbw}. In the following lemma, we prove that the opposite is possible.

Lemma 3. *There exists some distribution of private benefits for which the firm invests less in the second establishment under a bidding war than under a situation without a bidding war.*

Proof. We prove this lemma by constructing an example. Take the following cumulative distribution function: $G(b) = b^{1/3}$ on the interval $[0, 1]$. With such a function, $E(b_{(2)}) = \frac{n(n-1)}{(n+2)(n+3)}$ and $E(b_{(3)}) = \frac{n(n-1)(n-2)n!}{(n+3)!}$. Consequently, $E(b_{(2)}) - 2E(b_{(3)}) > 0$ if and only if:

$$\frac{n+1}{n-2} > 2$$

$$n < 5$$

For this distribution function, if $n < 5$, we have $w + E(b_{(2)}) - 2E(b_{(3)}) > w$, and the firm has larger effective marginal labour costs in the second establishment than she would under a situation with no bidding war. Consequently, she chooses a level of K_2^* lower than the no-bidding-war amount ($K_2^* < K_{nbw}$). □

This distribution function is strongly skewed to the right, giving more weight to values closer to zero. Therefore, for low values of n, $b_{(3)}$ is sufficiently close to the lower bound, and thus smaller than $b_{(2)}$, for the wage adjustment to be positive. In economic terms, such a distribution would translate in a situation where one or few regions put a great value on the firm's presence, while the great majority of regions put little to no value. In such a case, the firm might be able to extract a large subsidy from one government, but the differentiation comes at the cost of lower production in the second plant.

We are ultimately interested in the comparison of $K_1^* + K_2^*$ and $2 \cdot K_{nbw}$. From Lemma 2, we know that $K_1^* > K_{nbw}$, so for total investment to be lower under a bidding war, it is necessary that $K_2^* < K_{nbw}$ by an amount large enough to counter-balance the increase in the first establishment. Lemma 3 shows that it is possible that $K_2^* < K_{nbw}$.

We are unable to provide a general proof for a comparison of $K_1^* + K_2^*$ and $2 \cdot K_{nbw}$. However, we follow Haaparanta (1996) and prove it here for a specific functional form, namely a Cobbs-

Douglas production function. We show that in this case, the decrease in K_2 is never large enough to counter-balance the increase in K_1. In other words, total investment is always larger when using a bidding war than under the benchmark without a bidding war.

Proposition 2. *Assuming a Cobbs-Douglas production function with decreasing returns to scale ($\alpha + \beta < 1$), the total amount invested by the firm under a bidding war is always larger than the amount she would invest without a bidding war.*

Proof. The first-order condition for profit maximisation in one arbitrary establishment is:

$$p\frac{\partial f(K, L(K))}{\partial K} = L'(K)(w - x) + r \tag{3.11}$$

where x can be zero or the adjustment on marginal labour costs arising from subsidies. If $f(K, L) = K^{\alpha} L^{\beta}$, and using the function $L(K)$ as in equation (3.2), we find that:

$$\frac{p(\frac{p\beta}{w})^{\frac{\beta}{1-\beta}}(\frac{\alpha}{1-\beta}) \cdot K^{\frac{\alpha}{1-\beta}-1} - r}{(\frac{p\beta}{w})^{\frac{1}{1-\beta}}(\frac{\alpha}{1-\beta})K^{\frac{\alpha}{1-\beta}-1}} = w - x$$

This equation can be expressed as (with $A > 0$ and $B > 0$):

$$A - B \cdot K^{\frac{1-\alpha-\beta}{1-\beta}} = w - x \tag{3.12}$$

Since $0 < \frac{1-\alpha-\beta}{1-\beta} < 1$, Figure 1 illustrates a stylised version of the left-hand side of Equation (3.12).

In particular, since the second derivative is positive, a decrease in the right-hand side of a given amount (e) increases K by more than an identical increase in the right-hand side would decrease K. Since we know that $\left|E(b_{(2)}) - 2E(b_{(3)})\right| < \left|-E(b_{(2)})\right|$, the possible increase in the left-hand side (in the case of K_2) is always lower than the decrease in the left-hand side (in the case of K_1),[9] Therefore, we can conclude that the increase in K_1 due to a bidding war is always larger than the decrease in K_2. Consequently, the total amount invested is always larger in a bidding war, under specific assumptions on the production function. □

This proposition implies that bidding wars actually increase the firm's total investment. Note that in the proof above, we made no assumption on the distribution of the regions' private benefits, other than they are always non-negative. While the proof was for a Cobbs-Douglas

[9] Or they are both decreases, in which case total investment is certainly increased

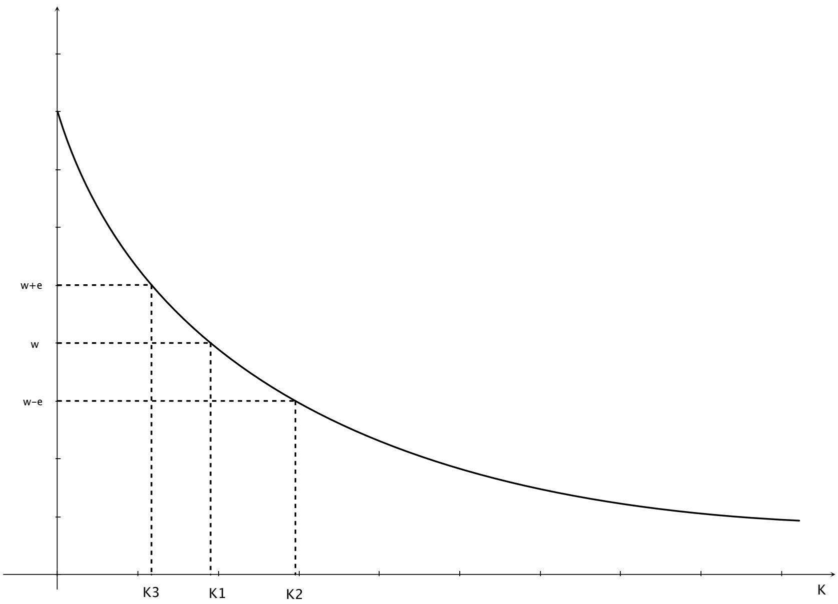

Figure 1: A stylised illustration comparing an upwards adjustment of wages to a downwards adjustment and their effects on the amount of capital invested.

function, the result should hold in many situations. In fact, Proposition 2 would be reversed only when two conditions are met: the distribution of private benefits respects Lemma 3 (so that $K_2^* < K_{nbw}$), and the production function has to be of a different shape than described in Proposition 2. Moreover, Proposition 2 does provide a general condition on the production function for which total investment increases. In particular, any function with a similar shape (first derivative negative, second derivative positive) for the left-hand side should provide the same result:

$$\frac{\partial}{\partial K}\left(\frac{p\frac{\partial f(K,L(K))}{\partial K} - r}{L'(K)}\right) < 0$$
$$\frac{\partial^2}{\partial K^2}\left(\frac{p\frac{\partial f(K,L(K))}{\partial K} - r}{L'(K)}\right) > 0$$

3.4.1 Are Regional Governments Better Off with a Bidding War?

Given the results above, one may wonder if it's in the regions' best interests that such a bidding war takes place. Without a bidding war, region i has the following expected utility:

$$E(W_{nbw,i}) = \frac{2}{n}L(K_{nbw}^*) \cdot b_i \tag{3.13}$$

where K_{nbw}^* is the investment from the firm in one establishment, without a bidding war.

With a bidding war, the same region has the following expected utility

$$E(W_{bw,i}) = \int_{\underline{b}}^{b_i} \int_{\underline{b}}^{b_{(1),-i}} (L(K_1^*)b_i - s_1^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1)} + \int_{b_i}^{\bar{b}} \int_{\underline{b}}^{b_i} (L(K_2^*)b_i - s_2^*)h(b_{(1),-i}, b_{(2),-i}, n-1)db_{(2),-i}db_{(1),-i} \quad (3.14)$$

where for region i, $db_{(k),-i}$ is the $k-th$ highest benefit among the $n-1$ other regions. The expression $E(W_{nbw,i}) = E(W_{bw,i})$ defines a level of b_i over which a region prefers a bidding war. Conversely, it also defines a level of b_i under which regional governments are made worse off by a bidding war.

This preference results from 2 factors. First, with a bidding war, regions with large private benefits expect to win more often. Second, under a bidding war, the regions expect the firm to choose a higher level of capital K_1, and, as seen in the discussion on Proposition 2, a higher level of capital K_2 as well, at least in some cases.

To illustrate, with a Cobb-Douglas production function of parameters $\alpha = \beta = 1/3$, a uniform distribution of benefits on $[0, 1]$, and $p = w = r = 1$, to illustrate, we find that a given region i prefers that the firm uses a bidding war as long as

$$b_i > 0.227$$

3.5 A Numerical Example

To illustrate the results of the model, let's continue with the simple Cobbs-Douglas production function introduced previously: $f(K, L) = K^\alpha L^\beta$, with $\alpha + \beta < 1$. With specific functional forms, we can find the optimal investment allocation given a set of parameters $\{\alpha, \beta, p, w, r, n\}$ The analytical solutions are omitted here, as they are not informative. Instead, we describe graphically how the firm behaves facing different conditions.

One interesting question is whether the number of regions in the bidding war affects the firm's investment choices. In the more general model, note that when $E(b_{(2)})$ and $E(b_{(3)})$ are closer together, the differentiation between K_1 and K_2 diminishes. In the extreme case of $E(b_{(2)}) = E(b_{(3)})$, we have that $K_1^* = K_2^*$. In turn, the number of regions n participating in the bidding war affects the difference between $E(b_{(2)})$ and $E(b_{(3)})$. For example, if the distribution of private benefits is uniform on $[0, 1]$, then a low number of regions (e.g., 3 regions) will translate in a large difference between the expected private benefits of the regions, while a larger number of regions will translate in lower differences. For that reason, we should see decreasing differentiation with an increasing number of competitors. Figure 2 illustrates this relationship for specific values of the parameters and a uniform distribution. It also shows how both K_1^* and K_2^* are larger than K_{nbw}^*.

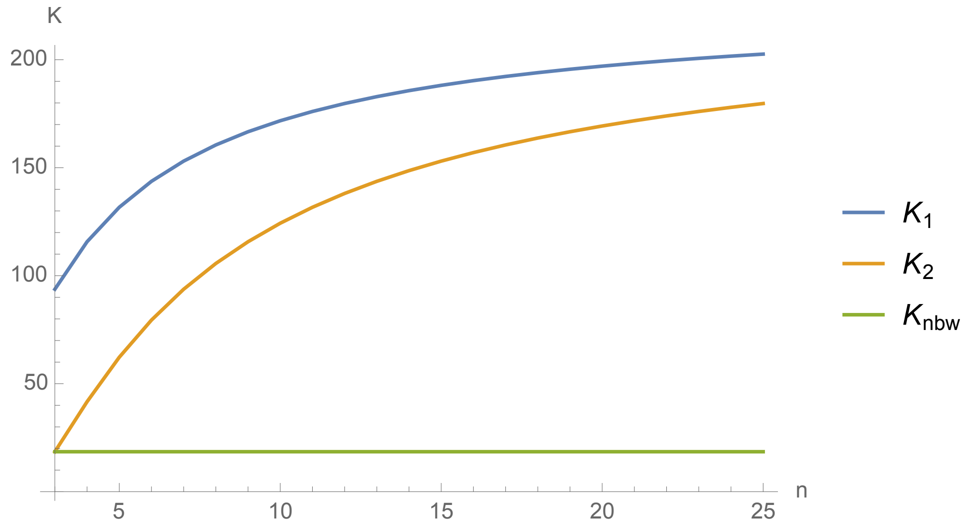

Figure 2: K_1 vs K_2, with a uniform distribution

Lemma 3 showed that for some distributions of the private benefits, the value of K_2 may behave differently. Figure 3 shows how K_1 and K_2 vary with n for the distribution $b^{\frac{1}{3}}$ on the interval $[0, 1]$, along with the value of K_{nbw} as reference. It shows how investment in the second establishment may actually be lower than without a bidding war, but that even in this example, *total* investment is higher.

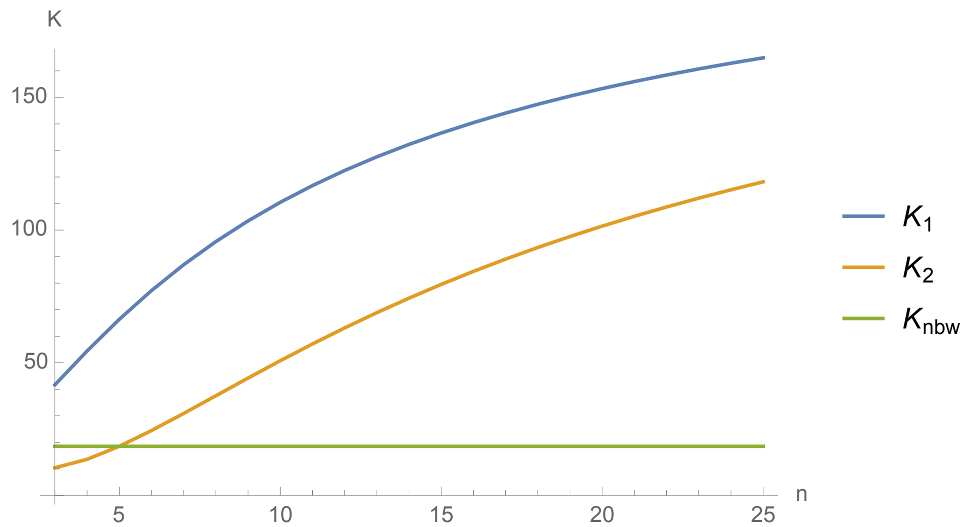

Figure 3: K_1 vs K_2, with distribution of private benefits respecting Lemma 3

4 Endogenous Plant Size: Comparison to a Single Plant Bidding War

The previous sections makes two additions to the usual discussion on bidding wars. First, the firm can choose the amount of capital to invest endogenously. Second, we consider the possibility for the firm to make multiple new investments. In this section, we aim to disentangle these two effects, investigating the endogenous investment decision of the firm when she is restricted to one plant. To that end, we consider the model, but restricted to only one new establishment. The set-up of the model is identical, except that the firm only decides on $K_1 = K_s$.

In this restricted model, the solution in Stage 3 is simple. The firm maximizes profits in her plant by choosing L, with a fixed K since it is chosen in Stage 1. Her maximization problem in the plant is as follows.

$$\max_{L} \quad pf(K^*, L) - wL - rK^* \tag{4.1}$$

The first-order condition is

$$pf'(K^*, L) - w = 0$$

This condition is standard, and defines the optimal choice of L given the amount of capital invested in the earlier stages. We define the function $L(K)$, determining the amount of labour employed for each possible equilibrium level of capital invested in the first stage.

At the auction stage, the equilibrium winning bid will be

$$s^*(K) = L(K_s) \cdot b_{(2)} \tag{4.2}$$

where $b_{(2)}$ is the second-highest private benefits among the n competing regions.

In the first stage, then, the firm's optimisation problem is the following:

$$\max_{K} \Pi = E(\pi(K) + s^*) = E\left[pf(K, L(K)) - wL(K) - rK + L(K) \cdot b_{(2)}\right] \tag{4.3}$$

where π and s^* are, respectively, the operating profits of the firm's plant and the equilibrium subsidy. The result of that problem leads to the following lemma.

Lemma 4. *A single bidding war for a new plant increases the firm's investment compared to a situation without a bidding war.*

Proof. The first-order condition is:

$$\frac{\partial E(\Pi)}{\partial K} = L'(K)E(b_{(2)}) + p(\frac{\partial f(K,L(K))}{\partial K} + \frac{\partial f(K,L(K))}{\partial L(K)} \cdot L'(K)) - wL'(K) - r = 0$$
$$\frac{\partial E(\Pi)}{\partial K} = L'(K)E(b_{(2)}) + p\frac{\partial f(K,L(K))}{\partial K} - wL'(K) - r = 0$$

It simplifies to

$$p\frac{\partial f(K,L(K))}{\partial K} = r + L'(K)(w - E(b_{(2)})) \tag{4.4}$$

The first-order condition is similar to the one for the largest establishment in the two-plant model. In particular, it implies that when using a bidding war, the firm chooses to invest an amount of capital K_s greater than would be invested without a bidding war. Indeed, the subsidies received effectively reduce the cost for the firm's labour ($w - E(b_{(2)}) < w$). □

Therefore, even simply allowing the firm to choose the amount to invest already affects her investment decision. In fact, the first-order condition (equation 4.4) is exactly the same as that for the largest establishment in the two-plant model. In turn, the amount invested in the single plant is the same as that invested in that largest establishment. Note that in the two-plant problem, we had $s_1^*(K_1, K_2) = (L(K_1) - L(K_2))b_{(2)} + L(K_2)b_{(3)}$. Assuming $K_2 = 0$, the equilibrium subsidy for the first establishment reduces to $s_1^*(K_1, 0) = L(K_1)b_{(2)}$, which is exactly the value of subsidies found in the one-plant problem.

We thus find that a single bidding war already increases the firm's investment (when it is endogenous to the model) compared to a situation without a bidding war. Another question is whether the assumption that the firm conducts two *simultaneous* bidding wars further modifies the allocation of investment.

To see this, we compare our results of the multi-establishment bidding war to a firm investing in multiple plants, but with unrelated bidding wars. Instead of a multi-unit auction as in the previous model, the firm would essentially conduct one auction, then a second one, with regions acting as though the contests are independent of each other. Comparing the investment and subsidies in this set-up with the findings of the multi-establishment bidding war, we find that:

Proposition 3. *Total investment and subsidies are larger when a single-establishment bidding war is repeated than in a two-plant bidding war.*

Proof. Define $\check{K}_1$ and $\check{K}_2$ as the amounts of investment chosen by the firm in these two bidding wars. From Lemma 4, we already know the investment in the first establishment, $\check{K}_1 = K_s$.

The related bid is thus $s^*(\check{K}_1) = L(\check{K}_1) \cdot b_{(2)}$. To keep a similar environment in both models, we assume that the second bidding war will take place among the $n-1$ remaining regions. The region with the largest private benefits among the $n-1$ remaining regions is $b_{(2)}$, so $s^*(\check{K}_2) = L(\check{K}_2) \cdot b_{(3)}$. The firm will decide to invest $\check{K}_2$ according to the following condition:

$$p\frac{\partial f(K, L(K))}{\partial K} = r + L'(K)(w - E(b_{(3)})) \tag{4.5}$$

Since $b_{(3)} < b_{(2)}$, $\check{K}_2 < \check{K}_1$. As in the multi-unit auction model, the plants are differentiated.

We start by comparing $\check{K}_1 + \check{K}_2$ and $K_1^* + K_2^*$. First, we know that $K_1^* = \check{K}_1$. For the second establishments, we can compare the respective first-order conditions:

$$\begin{aligned} p\frac{\partial f(K_2^*, L(K_2^*))}{\partial K_2^*} &= r + L'(K_2^*)(w + E(b_{(2)}) - 2E(b_{(3)})) \\ p\frac{\partial f(\check{K}_2, L(\check{K}_2))}{\partial \check{K}_2} &= r + L'(\check{K}_2)(w - E(b_{(3)})) \end{aligned}$$

Since $E(b_{(2)}) - 2E(b_{(3)} > -E(b_{(3)}$, the second establishment is larger with unrelated bidding wars ($\check{K}_2 > K_2^*$), so total capital invested is larger ($\check{K}_1 + \check{K}_2 > K_1^* + K_2^*$).

Subsidies are also larger. From equations 3.3, 3.4, and 4.2:

$$\begin{aligned} L(\check{K}_1) \cdot E(b_{(2)}) + L(\check{K}_2) \cdot E(b_{(3)}) &> 2L(K_2^*) \cdot E(b_{(3)}) + E(b_{(2)})(L(K_1^*) - L(K_2^*)) \\ E(b_{(3)}) \left[L(\check{K}_2) - 2L(K_2^*)\right] &> E(b_{(2)}) \left[(L(K_1^*) - L(K_2^*)) - L(\check{K}_1)\right] \\ \frac{2L(K_2^*) - L(\check{K}_2)}{L(K_2^*)} &< \frac{E(b_{(2)})}{E(b_{(3)})} \\ \frac{L(\check{K}_2)}{L(K_2^*)} &> 2 - \frac{E(b_{(2)})}{E(b_{(3)})} \end{aligned}$$

Since $L(\check{K}_2) > L(K_2^*)$, the left-hand side is greater than 1, and since $E(b_{(2)}) > E(b_{(3)})$, the right-hand side is smaller than 1. The last line in the previous calculation is thus always true. Therefore, subsidies are larger when the firm conducts two unrelated bidding wars. □

The assumptions in Proposition 3 may be unrealistic. When participating in the repeated bidding war of Proposition 3, regional governments may in fact expect that the firm will have multiple plants available, as we argued earlier. For that reason, a repeated bidding war is unreasonable in practice. However, it offers a good benchmark to compare the main model of this paper, namely the multi-establishment bidding war, to previous contributions in the literature. Indeed, previous papers implicitly assumed that bidding wars are unrelated. What Proposition 3 suggests is that single-plant models may overestimate the size of subsidies over many bidding wars.

This setup is essentially equivalent to two firms running separate bidding wars, under the assumption of single-unit demand from the regions.[10] In previous papers in this literature, authors consider a firm auctioning a single plant. In effect, in these models, if a second firm ran a bidding war, that second bidding war would be unrelated to the first. A notable exception is Martin (1999). He assumes that firms in the same industry benefit from agglomeration economies. In a model of sequential auctions for establishments from 2 different firms, he finds that regions overbid in the first auction, expecting to have an advantage in the second period auction.

In our multi-establishment bidding war, we consider a different case, where there are no agglomeration economies, but where the bidding wars are related since they are conducted by the same firm. Like Martin (1999), we find that bids are higher for one of the establishments, but the reason for this phenomenon in our model is different. Instead of being due to agglomeration economies, it results from the differentiation by the firm of the two establishment available who expects it will increase subsidies.

Finally, note that the simultaneous vs. sequential nature of our multi-establishment bidding war is not important for our results. Indeed, in a sequential auction for two establishments, but where the regions know that both auctions are conducted by the same firm and are thus related, the regional governments would bid differently in the first auction. Indeed, they would take into account that they can also participate in the second auction.

To see how the bids are equivalent, note that the optimal bid in the first auction is derived from the indifference between winning K_1, and losing K_1 but winning K_2.

$$L(K_1)b_i - \beta_1 = L(K_2)b_i - s_2^*(K_1, K_2)$$

with β_1 the bid from region for K_1 that makes it indifferent between the 2. We find $\beta_1 = (L_1 - L_2)b_i + E(s_2^*(K_1, K_2))$, with $E(s_2^*) = L(K_2)E(b_{(3)})$. The equilibrium subsidy is thus $s_1^*(K_1, K_2) = (L_1 - L_2)b_{(2)} + E(s_2^*(K_1, K_2))$ since the bids are monotonically increasing in b_i. The expected value of total subsidies for the firm are therefore equal to $(L(K_1) - L(K_2))E(b_{(2)}) +$

[10] This assumption ensures some continuity with the previous sections.

$2L(K_2)E(b_{(3)})$, which is exactly the same as in the open ascending auction.

5 Optimal Mechanism

So far, this paper considered that the firm allocated the plants using an open ascending auction. However, there could be other options available to the firm. Is the one in the model optimal? This section determines the optimal mechanism, comparing it to the open ascending auction.

5.1 Constrained Problem: Pre-Determined Investment Choices

First, we find the optimal mechanism when making the assumption that the firm previously chooses the values of K_1 and K_2, committing to them as in the model of previous sections. We have n regional governments, each willing to buy up to 1 unit of production from a firm. Each regional government i has a private valuation for each job created by the firm of b_i. The b_i are identically and independently distributed according to $g(\cdot)$ on the interval $[\underline{b}, \overline{b}]$. Define $b = (b_1, ..., b_n)$, $B_i = [\underline{b}, \overline{b}]$, and $B = \Pi_i B_i = [\underline{b}, \overline{b}]^n$. The firm has two units of production available, $j = 1, 2$. We define $x_i(b) = (x_{i,1}(b), x_{i,2}(b))$ as the allocation function vector, with $x_{i,j}(b) \in [0, 1]$. Then, the expected payoff to a regional government is equal to $V_i = x_{i,1}(b)L(K_1)b_i + x_{i,2}(b)L(K_2)b_i$, and the expected utility is:

$$EU_i(x_i, b, s_i) = \int_{B_{-i}} (x_{i,1}(b_i, b_{-i})L(K_1)b_i + x_{i,2}(b_i, b_{-i})L(K_2)b_i - s_i(b_i, b_{-i}))g_{-i}(b_{-i})db_{-i} \quad (5.1)$$

The firm wants to maximise

$$E(\Pi) = \pi_1(K_1) + \pi_2(K_2) + \sum_{i=1}^{n} \int_B s_i(b)g(b)db \quad (5.2)$$

She chooses (K_1, K_2) in a previous step, and then implements a mechanism to allocate these two plants while receiving subsidies $(s_i(b))$ from regional governments.

The firm's objective is to choose a mechanism to maximise her profits. By the revelation principle, we can restrict our attention to direct mechanism characterised by a set of functions $\{x_i(b), s_i(b)\}_{i=1,...n}$ where the x_i's reflect the allocation rule, the s_i's reflect the payment rule when b is the vector of types reported by the regions. Formally, the firm then solves the following problem:

$$
\begin{aligned}
\max_{x(b),s(b)} \quad & \int_B \left(\sum_{i=1}^{n} s_i(b) + \sum_{j=1}^{2} \left(\pi(K_j) \sum_{i=1}^{n} x_{ij} \right) \right) g(b) db \\
s.t. \quad & EU_i(x_i, b_i, s_i) \geq EU_i(x_i, \tilde{b_i}, b_{-i}, s_i) \quad \forall i & ICC \\
& EU_i(x_i, b_i, s_i) \geq 0 \quad \forall i \forall j & IRC \\
& \sum_{i=1}^{n} x_{ij} \leq 1 \quad \forall j = 1, 2 & FC1 \\
& x_{ij}(b) \geq 0 & FC2 \\
& x_{i1}(b) + x_{i2}(b) \leq 1 & FC3
\end{aligned}
$$

The Incentive Compatibility Constraint (ICC) states that it must be optimal for each region to report its true private benefits (b_i). The Individual Rationality Constraint (IRC) states that it must be optimal for each region to participate in the mechanism. The other three constraints are feasibility constraints. FC1 states that for each plant, the allocation probabilities for all regions must sum to one or less. FC2 states that these probabilities must be non-negative. FC3 states that regions can, at the equilibrium, receive only one plant. The solution to this optimisation problem leads to the following proposition.

Proposition 4. *Assume the firm first commits to the sale of (K_1, K_2), and subsequently chooses a mechanism to allocate these two plants. The optimal mechanism results in the same allocation $(x^*(b))$ and subsidies $(s^*(b))$ as the multi-unit open ascending auction.*

Proof. The solution to this problem in general is due to Myerson (1981). The solution here will follow Morand (2000), constrained to unit demand.

The incentive compatibility constraint (ICC) states that regional governments must have incentives to state their true private benefits. It has to be satisfied locally. Using the envelope theorem, it must be that

$$
\begin{aligned}
\frac{dEU_i(x_i, b_i, s_i)}{db_i} &= \frac{\partial EU_i(x_i, \tilde{b_i}, s_i, b_i)}{\partial b_i}\Big|_{\tilde{b_i}=b_i} \\
&= \int_{B_{-i}} (x_{i1}(b)L(K_1) + x_{i2}(b)L(K_2)) g_{-i}(b_{-i}) db_{-i} \qquad (5.3)
\end{aligned}
$$

Define the marginal probabilities as:

$$
\begin{aligned}
p_{i1}(x_i, b_i) &= \int_{B_{-i}} x_{i1}(b) g_{-i}(b_{-i}) db_{-i} \\
p_{i2}(x_i, b_i) &= \int_{B_{-i}} x_{i2}(b) g_{-i}(b_{-i}) db_{-i} \\
p_i &= (p_{i1}, p_{i2})
\end{aligned}
$$

With these, we can rewrite Equation 5.3 as

$$
\frac{dEU_i(x_i, b_i, s_i)}{db_i} = p_{i1}(x_i, b_i) L(K_1) b_i + p_{i2}(x_i, b_i) L(K_2) b_i \quad \forall i \tag{5.4}
$$

From Equation 5.4, we can find the expected utility of a regional government such that the incentive compatibility constraint is respected:

$$
\begin{aligned}
\int_{\underline{b}}^{b_i} \frac{dEU_i(x_i, t, s_i)}{dt} dt &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t) L(K_1) t + p_{i2}(x_i, t) L(K_2) t) dt \\
EU_i(x_i, b_i, s_i) - EU_i(x_i, \underline{b}, s_i) &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t) L(K_1) t + p_{i2}(x_i, t) L(K_2) t) dt \\
EU_i(x_i, b_i, s_i) &= \int_{\underline{b}}^{b_i} (p_{i1}(x_i, t) L(K_1) t + p_{i2}(x_i, t) L(K_2) t) dt + EU_i(x_i, \underline{b}, s_i)
\end{aligned} \tag{5.5}
$$

This expected utility is thus expressed in two terms. The first term depends on the marginal probabilities to win one of the production sites, while the second one is the expected utility of a regional government with the lowest private benefits ($\underline{b}$).

With the incentive compatibility constraint, we can also show that $p_{ij}(x_i, b_i)$ is non-decreasing $\forall i, j$. First, we can rewrite the expected utility of a region that announces private benefits $\tilde{b_i}$ when he actually has private benefits b_i, and conversely, as

$$
\begin{aligned}
EU_i(x_i, b_i, \tilde{b_i}, s_i) &= EU_i(x_i, \tilde{b_i}, s_i) - (b_i - \tilde{b_i}) \left[L(K_1) p_{i1}(x_i, \tilde{b_i}) + L(K_2) p_{i2}(x_i, \tilde{b_i}) \right] \\
EU_i(x_i, \tilde{b_i}, b_i, s_i) &= EU_i(x_i, b_i, s_i) - (\tilde{b_i} - b_i) \left[L(K_1) p_{i1}(x_i, b_i) + L(K_2) p_{i2}(x_i, b_i) \right]
\end{aligned}
$$

From the incentive compatibility constraint, we thus have that

$$EU_i(x_i, b_i, s_i) \geq EU_i(x_i, \tilde{b_i}, s_i) - (b_i - \tilde{b_i}) \left[L(K_1)p_{i1}(x_i, \tilde{b_i}) + L(K_2)p_{i2}(x_i, \tilde{b_i})\right]$$
$$EU_i(x_i, \tilde{b_i}, s_i) \geq EU_i(x_i, b_i, s_i) - (\tilde{b_i} - b_i) \left[L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)\right]$$

A few manipulations show that

$$EU_i(x_i, b_i, s_i) - EU_i(x_i, \tilde{b_i}, s_i) \geq (\tilde{b_i} - b_i) \left[L(K_1)p_{i1}(x_i, \tilde{b_i}) + L(K_2)p_{i2}(x_i, \tilde{b_i})\right]$$
$$EU_i(x_i, b_i, s_i) - EU_i(x_i, \tilde{b_i}, s_i) \leq (\tilde{b_i} - b_i) \left[L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)\right]$$
$$(\tilde{b_i} - b_i) \left[L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)\right] \geq (\tilde{b_i} - b_i) \left[L(K_1)p_{i1}(x_i, \tilde{b_i}) + L(K_2)p_{i2}(x_i, \tilde{b_i})\right]$$

Therefore, if $\tilde{b_i} > b_i$, $L(K_1)p_{i1}(x_i, b_i) + L(K_2)p_{i2}(x_i, b_i)$ is non-decreasing in b_i. Defining $L = (L(K_1), L(K_2))$, we can express this equation as $L \cdot p_i(x_i, b_i)$. With this property, we can also simplify the individual rationality constraint to a single one:

$$EU(x, \underline{b}, s) \geq 0 \tag{5.6}$$

The problem of the firm can now be simplified. From Equations (5.1) and (5.5), we know that

$$\int_{\underline{b}}^{b_i} (p_i(x_i, t)L)dt + EU_i(x_i, \underline{b}, s_i) = \int_{B_{-i}} (x_i(b_i, b_{-i})Lb_i - s_i(b_i, b_{-i}))g_{-i}(b_{-i})db_{-i}$$

Therefore,

$$
\begin{aligned}
E\Pi &= \int_B \sum_{i=1}^{n} s_i(b)g(b)db \\
&= \sum_{i=1}^{n} \left[\int_B b_i x_i(b)Lg(b)db - \int_{B_i}\int_{\underline{b}}^{b_i}\int_{B_{-i}} (x_i(t,b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot dt \cdot g(b_i)db_i - EU_i(x_i,\underline{b},s_i) \right] \\
&= \sum_{i=1}^{n} \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i,\underline{b},s_i) \right] - \sum_{i=1}^{n} \left[\int_{\underline{b}}^{\bar{b}}\int_{\underline{t}}^{\bar{b}}\int_{B_{-i}} (x_i(t,b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot g(b_i)db_i \cdot dt \right] \\
&= \sum_{i=1}^{n} \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i,\underline{b},s_i) \right] - \sum_{i=1}^{n} \left[\int_{\underline{b}}^{\bar{b}}\int_{B_{-i}} (x_i(t,b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot (1-G(t)) \cdot dt \right] \\
&= \sum_{i=1}^{n} \left[\int_B b_i x_i(b)Lg(b)db - EU_i(x_i,\underline{b},s_i) \right] - \sum_{i=1}^{n} \left[\int_{\underline{b}}^{\bar{b}}\int_{B_{-i}} (x_i(b_i,b_{-i})Lg_{-i}(b_{-i})db_{-i} \cdot (1-G(b_i)) \cdot db_i \right] \\
&= \sum_{i=1}^{n} \left[\int_B b_i x_i(b)Lg(b)db - \int_B (x_i(b)Lg(b)db \frac{(1-G(b_i))}{g_i(b_i)} \right] - \sum_{i=1}^{n} \left[EU_i(x_i,\underline{b},s_i) \right] \\
&= \sum_{i=1}^{n} \left[\int_B (b_i - \frac{(1-G(b_i))}{g_i(b_i)}) \cdot x_i(b)Lg(b)db - EU_i(x_i,\underline{b},s_i) \right]
\end{aligned}
$$

Define the virtual benefits of region i as $\beta_i(b_i) = b_i - \frac{1-G(b_i)}{g(b_i)}$. We make the usual assumption that the distribution function is regular: $\beta_i(b_i)$ is increasing in b_i. We can write the firm's expected revenues as

$$
\sum_i \int_B \beta_i(b_i) x_i(b) Lg(b)db \tag{5.7}
$$

In doing so, we assume that at the optimum, $EU_i(x_i,\underline{b},s_i) = 0$. From this assumption, and Equations (5.1) and (5.5), we then find:

$$
\begin{aligned}
EU_i(x_i,b_i,s_i) &= \int_{\underline{b}}^{b_i} p_i(x_i,t)Ldt \\
&= \int_{B_{-i}}\int_{\underline{b}}^{b_i} x_i(t,b_{-i}dtg_{-i}(b_{-i})db_{-i} \\
&= \int_{B_{-i}} \left[b_i(x_i(b_i,b_{-i}))L - s_i(b_i,b_{-i}) \right] g_{-i}(b_{-i})db_{-i}
\end{aligned}
$$

With this equation, we can express the equilibrium payments $s_i^*(b)$:

$$
\begin{aligned}
\int_B \left[b_i(x_i(b))L - s_i(b) \right] g(b)db &= \int_B \int_{\underline{b}}^{b_i} x_i(t,b_{-i})dtg(b)db \\
\int_B s_i(b)g(b)db &= \int_B b_i(x_i(b))Lg(b)db - \int_B \int_{\underline{b}}^{b_i} x_i(t,b_{-i})dtg(b)db \\
s_i^*(b) &= b_i(x_i^*(b))L - \int_{\underline{b}}^{b_i} x_i^*(t,b_{-i})dt
\end{aligned}
$$

Assuming $x^*(b)$ is the allocation function that solves the firm's problem, we can then find the optimal payment function $s_i^*(b)$:

$$s_i^*(b) = b_i x_i^*(b) \cdot L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt \tag{5.8}$$

The optimisation problem can therefore be expressed as follows. Let $x^*(b)$ be the solution to the following problem:

$$\begin{aligned}
\max_{x(b)} \quad & \sum_i \int_B \left[\beta_i(b_i) x_i(b) L + \sum_{j=1}^{2} \pi(K_j) x_{ij}(b) \right] g(b) db \\
s.t. \quad & EU_i(x_i, \underline{b}, s_i) = 0 \quad \forall i \\
& (\tilde{b_i} - b_i) \left[p_i(x_i, b_i) \cdot L \right] \geq (\tilde{b_i} - b_i) \left[p_i(x_i, \tilde{b_i}) \cdot L \right] \quad \forall b_i < \tilde{b_i} \\
& \sum_{i=1}^{n} x_{ij} \leq 1 \quad \forall j = 1, 2 \\
& x_{ij}(b) \geq 0 \quad \forall i, j \\
& x_{i1}(b) + x_{i2}(b) \leq 1 \quad \forall i
\end{aligned}$$

Let $s_i^*(b)$ be given by:

$$s_i^*(b) = b_i(x_i^*(b)) L - \int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) dt$$

Then, (x^*, t^*) is the optimal mechanism.

Similarly to standard problems in mechanism design, the optimal allocation function is deterministic: $x^*(b)$ takes value of 0 or 1. In particular, the firm will allocate the first production unit $(L(K_1))$ to the region with the highest virtual valuation (equivalently, to the one with highest private benefits), and the second one $(L(K_2))$ to the region with second-highest virtual valuation. Defining $b_{(k)}$ as the k-th highest private benefits, we thus have

$$x^*(b) = (x_1^*(b), x_2^*(b)) = \begin{cases} (1,0) & if \quad b = b_{(1)} \\ (0,1) & if \quad b = b_{(2)} \\ (0,0) & otherwise \end{cases} \tag{5.9}$$

The optimal payment rule depends on this allocation function. First note that the first term in $s_i^*(b)$ is simply the value to the region of hosting the firm. For the region hosting K_1, it is equal to $b_{(1)} L(K_1)$, while for the region hosting K_2 it is equal to $b_{(2)} L(K_2)$. The second term can be interpreted as the informational rent going to the regions. Define $z_{ij}(b_{-i})$ as the lowest value of private benefits that a region i can announce and still win establishment j. The integral in the second term then takes the following values:

$$\int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt = \begin{cases} L(K_1)b_i - L(K_1)b_{(2)} + L(K_2)b_{(2)} - L(K_2)b_{(3)} & if \quad b_i > z_{i1}(b_{-i}) \\ L(K_2)b_i - L(K_2)b_{(3)} & if \quad z_{i1}(b_{-i}) > b_i > z_{i2}(b_{-i}) \\ 0 & otherwise \end{cases}$$

The first case warrants some discussion. If a region's private benefits are greater than $z_{i1}(b_{-i})$, such that they win the first establishment, we also need to take into account the fact that by winning the first establishment, that region also renounces to the smaller establishment. We can see this more clearly when developing the expression to integrate: $\int_{\underline{b}}^{b_i} x_i^*(t, b_{-i}) L dt = \int_{\underline{b}}^{b_i} (x_{i1}^*(t, b_{-i}) L(K_1) + x_{i2}^*(t, b_{-i}) L(K_2)) dt$. When calculating the integral over the interval $[\underline{b}, b_i]$, we have to take into account that $x(b)$ takes non-null values not only over the interval in which the region wins the first establishment, but also over the interval over which the regions wins the second establishment. These results lead to the following payments

$$s_i^*(b) = \begin{cases} (L(K_1) - L(K_2))b_{(2)} + L(K_2)b_{(3)} & if \quad b_i > z_{i1}(b_{-i}) \\ L(K_2)b_{(3)} & if \quad z_{i1}(b_{-i}) > b_i > z_{i2}(b_{-i}) \\ 0 & otherwise \end{cases} \tag{5.10}$$

These are exactly the same payments found in the auction in the previous sections. Since that auction led to the same allocation and the same payments, we can conclude that the auction implemented the optimal mechanism (albeit without reserve prices), from the point of view of the firm. Moreover, we can see that the *ex ante* choice of K_1 and K_2 will be identical.

□

This proposition indicates that the open ascending auction chosen in the first part of the paper is actually optimal from the firm's point of view. In other words, she can do no better, when committing to K_1 and K_2 beforehand, than the open ascending auction.

In the solution to the problem, we saw that the firm allocated the plants to the regions with the highest *virtual valuations*. In our model, the regions have information on their own benefits while the firm does not. In turn, they receive some informational rents (as seen by the payments). In setting the optimal mechanism, the firm tries to extract some of that rent. In fact, the firm could decide not to allocate the plant at all even if it is efficient to do so. Indeed, at some positive level of b_i, $\beta_i(b_i)$ can be negative. If all signals are such that $\beta_i(b_i)$ is negative, the firm maximises her objective function by not allocating the plants.

In fact, the optimal mechanism should also define reserve prices: threshold values of the regions' private benefits under which the firm would not allocate her plants. In a simpler model, the reserve prices would simply be defined by the level of private benefits under which the virtual valuation is negative, $b_r = \beta^{-1}(0)$. Indeed, if the revealed b_i's are all lower, then the objective function would also be negative, thus choosing not to allocate the units at all.

In our model, however, the reserve prices must take the technological profits into account. Indeed, by not allocating the plants, the firm actually reduces her own profits. The intuition is similar. The firm selects the regions to allocate the plant by choosing $x(b)$, and her payoff must be positive: $\pi(K_1) + \beta(b_{r1})L(K_1) > 0$ and $\pi(K_2) + \beta(b_{r2})L(K_2) > 0$.

With reserve prices, the optimal mechanism would differ from the open ascending auction of the previous sections. Therefore, we find the conditions under which the reserve prices are non-binding.

Lemma 5. *For $K_j \in [0, \overline{K}]$ $\forall j$ and $p > \underline{p}$, the reserve prices in the optimal mechanism are non-binding.*

Proof. If the private benefits revealed through the mechanism are equal to $\underline{b}$, the lowest possible level, the payoff to the firm must respect the following condition:

$$\pi(K_j) + \beta(\underline{b})L(K_j) = pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j \geq 0$$

Notably, $K_j = 0$ respects this condition. Moreover, given the assumptions on the production function, we know that the slopes of $pf(K_j, L(K_j))$ and $L(K_j)(w - \beta(\underline{b})) + rK_j$ are positive. Therefore, two cases are possible at $K_j = \epsilon$ (i.e., an arbitrary small level of investment):

- The firm makes positive profits: $pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j > 0$
- The firm does not make profit: $pf(K_j, L(K_j)) - L(K_j)(w - \beta(\underline{b})) - rK_j < 0$

In the second case, we can conclude that for any $K_j > \epsilon$, she never makes profits if the winning region has private benefits $\underline{b}$. In the first case, we can conclude that the firm will make positive profits up to a certain point $\overline{K}$ (where $pf(\overline{K}, L(\overline{K})) = L(\overline{K})(w - \beta(\underline{b})) + r\overline{K})$. Assuming that p is large enough, we are always in the first situation.

Under these assumptions, the firm always makes profits at $\underline{b}$. If the private benefits revealed in the mechanism are higher, she also necessarily makes profits. Indeed, a larger b implies a lower $w - \beta(\underline{b})$, and thus higher profits at the same level of capital invested and prices.

Therefore, even if the firm sets reserve prices, they would never affect the decision if the pre-determined K_j are always in the interval $[0, \overline{K}]$. □

5.2 The Social Planner Problem

The previous discussion shows that the optimal mechanism (at least under some conditions on the price p and with the pre-determined $K_j \in [0, \overline{K}]\ \forall j$) is equivalent to the open ascending auction of the previous sections. An interesting question, then, is whether this mechanism is optimal in terms of social welfare. To investigate the social welfare question, we can replace the firm as the decision-maker by a social planner trying to find a mechanism to allocate K_1 and K_2 (decided ex ante) to maximise a social welfare function. Would the allocations and payments be the same? The set-up is similar to the one of Proposition 4. The differences is that the objective function considers not only the firm's welfare, but also that of the regions. It also considers the marginal cost or public funds (λ). We also assume that the social planner is uninformed about the regions' signals.[11]

$$
\begin{aligned}
\max_{x(b),s(b)} \quad & E(W) = \int_B \left[\alpha E(\Pi) + \gamma \sum_{i=1}^{n} EU_i - \lambda \sum_{i=1}^{n} s_i(b)\right] g(b)db \\
s.t. \quad & EU_i(x_i, b_i, s_i) \geq EU_i(x_i, \tilde{b_i}, b_{-i}, s_i) \quad \forall i && ICC \\
& EU_i(x_i, b_i, s_i) \geq 0 \quad \forall i \forall j && IRC \\
& \sum_{i=1}^{n} x_{ij} \leq 1 \quad \forall j = 1,2 && FC1 \\
& x_{ij}(b) \geq 0 && FC2 \\
& x_{i1}(b) + x_{i2}(b) \leq 1 && FC3
\end{aligned}
$$

The values for α and γ are the social weights placed on the welfare of the firm and the regions, respectively, and they sum to one ($\alpha + \gamma = 1$). In this section, we assume that the firm

[11] If the social planner has perfect information, the problem is trivial. It allocates the plants to the regions that value them the most, with no payment.

chooses the amounts of capital to invest in a previous step, and that the mechanism is used to allocate these amounts. Therefore, $E(\Pi) = \pi(K_1)\sum_{i=1}^n x_{i1}(b) + \pi(K_2)\sum_{i=1}^n x_{i2}(b) + \sum_{i=1}^n s_i(b)$. Since the firm has no information to reveal, she does not appear in the incentive compatibility constraints.

Lemma 6. *For a given K_1 and K_2, an uninformed social planner would locate the plants in the same regions as the firm.*

Proof. We can simplify the objective function as such:

$$E(W) = \int_B \left[\gamma \sum_{i=1}^n (x_i(b) b_i L) + (\alpha - \gamma - \lambda) \sum_{i=1}^n s_i(b) + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db \tag{5.11}$$

By using the same manipulations on the constraints as in the previous problem, we find that

$$E(W) = \sum_{i=1}^n \int_B \left[(\alpha - \lambda)(x_i(b) b_i L) - (\alpha - \gamma - \lambda)(x_i(b) L \frac{1 - G_i(b_i)}{g_i(b_i)}) + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db$$
$$E(W) = \sum_{i=1}^n \int_B \left[\left(b_i - \frac{\alpha - \gamma - \lambda}{\alpha - \lambda} \cdot \frac{1 - G_i(b_i)}{g_i(b_i)} \right) (\alpha - \lambda) x_i(b) L + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db$$
$$E(W) = \sum_{i=1}^n \int_B \left[\beta_i'(b_i)(\alpha - \lambda) x_i(b) L + \alpha \sum_{j=1}^2 x_{ij}(b) \cdot \pi(K_j) \right] g(b) db$$

The function $\beta_i'(b_i)$ differs from $\beta_i(b_i)$ by the multiplication of the inverse of the hazard function by a combination of the model parameters. We make the assumption that $\alpha - \gamma \geq \lambda$, implying also that $\alpha \geq \lambda$, so that welfare is non-negative. Intuitively, from Equation 5.11, this assumption implies that for the transfers from the regions to the firm, the additional social weight placed on the firm versus the regions (i.e., $\alpha - \gamma$) is large enough to cover the marginal cost of public funds (λ).[12]

Notably, with these assumptions, the social planner chooses the same deterministic $x^*(b)$:

$$x^*(b) = (x_1^*(b), x_2^*(b)) = \begin{cases} (1,0) & if \quad b = b_{(1)} \\ (0,1) & if \quad b = b_{(2)} \\ (0,0) & otherwise \end{cases} \tag{5.12}$$

[12]The assumption $\alpha - \gamma \geq \lambda$ has additional implications for the marginal cost of public funds. Indeed, if the weight placed on the firm and regions is the same ($\alpha = \gamma$), then if $\lambda > 0$, social welfare is negative. Conversely, if we allow $\alpha - \gamma < \lambda$, then the shape of $\beta_i'(b_i)$ is uncertain. For a uniform distribution, it is merely necessary that $\left| \frac{\alpha-\gamma-\lambda}{\alpha-\lambda} \right| \leq 1$, but this is not true in the general case.

□

While the allocation function may be the same as the firm, the social planner's optimal mechanism is not entirely identical to the firm's. Since $\frac{\alpha-\gamma-\lambda}{\alpha-\lambda} \leq 1$, then $\beta_i'(b_i) \geq \beta_i(b_i)\ \forall i$. We can see that this has implications for the reserve prices. In particular, the social planner allocates the plants more often.

As in the discussion on the optimal mechanism from the firm's viewpoint, we do not discuss reserve prices but the conditions under which the reserve prices are not binding. However, to illustrate how the reserve prices would change, we first look at an example. Abstracting from the technological profits, we would find the following b_r:

$$0 = \beta_i'(b_r'^*) \qquad\qquad 0 = \beta_i(b_r^*)$$

$$0 = \frac{\alpha-\gamma-\lambda}{\alpha-\lambda} \cdot \frac{1-G_i(b_r'^*)}{g_i(b_r'^*)} \qquad\qquad 0 = \frac{1-G_i(b_r^*)}{g_i(b_r^*)}$$

With a uniform distribution on $[0,1]$, for example,

$$b_r'^* = \frac{\alpha-\gamma-\lambda}{\alpha-\lambda} \cdot (1-b_r'^*) \qquad\qquad b_r^* = 1-b_r^*$$

$$b_r'^* = \frac{\frac{\alpha-\gamma-\lambda}{\alpha-\lambda}}{1+\frac{\alpha-\gamma-\lambda}{\alpha-\lambda}} < \frac{1}{2} \qquad\qquad b_r^* = \frac{1}{2}$$

Therefore, although the allocations under a social planner and the firm are the same, the social planner would allocate more often. We show now that this result translates in a looser condition on the possible investment interval ($[0, \overline{K}']$).

Proposition 5. *For any distribution such that $\beta(\underline{b}) < 0$, $\overline{K}' > \overline{K}$. In other words, the social planner allocates the plants more often.*

Proof. Similar to the firm's problem, we have that

$$\alpha\pi(K_j) + \beta'(\underline{b})(\alpha-\lambda)L(K_j) = \alpha p f(K_j, L(K_j)) - L(K_j)(\alpha w - (\alpha-\lambda)\beta'(\underline{b})) - \alpha r K_j \geq 0$$

Like in the firm's problem, if $K = 0$, then $\alpha\pi(K_j) + \beta'(\underline{b})(\alpha-\lambda)L(K_j) = 0$. Assuming that p is high enough so that with an arbitrary small amount of capital ϵ, $\alpha\pi(K_j)+\beta'(\underline{b})(\alpha-\lambda)L(K_j) > 0$, we want to find the maximum amount of investment such that profits are positive at the lowest level of private benefits $\underline{b}$.

$$\alpha p f(\overline{K}', L(\overline{K}')) - L(\overline{K}')(\alpha w - (\alpha - \lambda)\beta'(\underline{b})) - \alpha r\overline{K}' = 0$$
$$\alpha \left[p f(\overline{K}', L(\overline{K}')) - L(\overline{K}')(w - \frac{\alpha - \lambda}{\alpha}\beta'(\underline{b})) - r\overline{K}' \right] = 0$$
$$\alpha \left[p f(\overline{K}', L(\overline{K}')) - L(\overline{K}')(w - (1 - \frac{\gamma + \lambda}{\alpha}\beta(\underline{b})) - r\overline{K}' \right] = 0$$

In the third line, we used the definitions of $\beta'(\cdot)$ and $\beta(\cdot)$. This equation defines a level $\overline{K}'$. How does that amount differ from $\overline{K}$? The difference between the two conditions is in the multiplier in front of $L'(\cdot)$ (essentially the effective marginal cost of labour). We therefore compare these costs. $\overline{K}' > \overline{K}$ if and only if:

$$w - \beta(\underline{b}) > w - (1 - \frac{\gamma + \lambda}{\alpha}\beta(\underline{b}))$$
$$\beta(\underline{b}) < 1 - \frac{\gamma + \lambda}{\alpha}\beta(\underline{b})$$
$$\underline{b}\left(\frac{\lambda}{\alpha}\right) - \frac{1 - G(\underline{b})}{g(\underline{b})}\left(\frac{\gamma + \lambda}{\alpha}\right) < 0$$
$$\underline{b} - \frac{\gamma + \lambda}{\lambda}\frac{1 - G(\underline{b})}{g(\underline{b})} < 0$$

This last expression will be true for low levels of $\underline{b}$. Notably, it is always true for $\underline{b} = 0$. However, more generally, we can prove that for any distribution $f(\cdot)$,

$$\underline{b} - \frac{1 - G(\underline{b})}{g(\underline{b})} < 0 \implies \underline{b} - \frac{\gamma + \lambda}{\lambda}\frac{1 - G(\underline{b})}{g(\underline{b})} < 0$$

Therefore, if the virtual valuation, from the firm's point of view, of a region with private benefits $\underline{b}$ is negative, then the social planner's condition is looser: $\overline{K}' > \overline{K}$. □

The condition ($\beta(\underline{b}) < 0$) for this result is not very restrictive. Indeed, if from the firm's point of view, virtual valuations are all positive, the firm allocates all the time, so a discussion on welfare is not as interesting. Since it is inefficient socially to not allocate the plants, the looser conditions set by the social planner actually increases welfare. The intuition for this result is that the social planner puts less importance on the firm capturing the regions' informational rent.

5.3 Unconstrained Problem: Investment Choices Endogenous to the Mechanism

In the auction model and in the derivation of the optimal mechanism thus far, we have assumed that the firm commits to levels of capital investment (K_1, K_2). After the reveal of the private benefits, however, it is possible that the firm would like to modify her allocation of capital.

In this section, we investigate how the allocation and payments would differ under an unconstrained optimal mechanism, where the firm chooses the amounts to invest simultaneously with the allocation and payments. Also, we show that at least in expected values, the constrained optimal mechanism derived above leads to the same allocations as a more general, unconstrained mechanism.

To do so, we will modify the problem above slightly. Instead of only choosing a vector of probabilities $x_i(b)$, the firm chooses, in addition, a vector of investments $k_i(b) = (k_{i1}(b), k_{i2}(b))$. The firm's problem becomes:

$$
\begin{aligned}
\max_{x(b),k(b),s(b)} \quad & E(\Pi) = \sum_{i=1}^{n} \int_B \left[x_{i1}(b)\pi(k_{i1}(b)) + x_{i2}(b)\pi(k_{i2}(b)) + s_i(b)\right] g(b)db & \\
s.t. \quad & EU_i(k_i, b_i, s_i) \geq EU_i(k_i, \tilde{b_i}, b_{-i}, s_i) \quad \forall i & ICC \\
& EU_i(k_i, b_i, s_i) \geq 0 \quad \forall i \forall j & IRC \\
& \sum_{i=1}^{n} x_{ij} \leq 1 \quad \forall j = 1, 2 & FC1 \\
& x_{ij}(b) \geq 0 & FC2 \\
& x_{i1}(b) + x_{i2}(b) \leq 1 & FC3 \\
& k_{ij}(b) \geq 0 & FC4
\end{aligned}
$$

By using similar manipulations on the constraints as in the constrained mechanism problem, we can transform the firm's objective function as such

$$
E(\Pi) = \sum_{i=1}^{n} \int_B \left[x_{i1}(b)\pi(k_{i1}(b)) + x_{i2}(b)\pi(k_{i2}(b)) + \beta_i(b_i)\left(x_{i1}(b)L(k_{i1}(b)) + x_{i2}(b)L(k_{i2}(b))\right)\right] g(b)db \tag{5.13}
$$

As in the previous sections, the solution for $x(b)$ is deterministic:

$$x^*(b) = (x^*_{i1}(b), x^*_{i2}(b)) = \begin{cases} (1,0) & if \quad b = b_{(1)} \\ (0,1) & if \quad b = b_{(2)} \\ (0,0) & otherwise \end{cases} \tag{5.14}$$

What are the values of $k_1^*(b)$ and $k_2^*(b)$? The firm will choose these investment amounts after observing the signals. She only commits to a function $k(b)$. From the objective function, we can find the first-order condition, assuming the b_i's are observed, and the plants are assigned to the respective winners.

$$p\frac{\partial f(k_1^*, L(k_1^*))}{\partial k_1^*} = L'(k_1^*)(w - \beta(b_{(1)})) + r$$
$$p\frac{\partial f(k_2^*, L(k_2^*))}{\partial k_2^*} = L'(k_2^*)(w - \beta(b_{(2)})) + r$$

These conditions define functions $k * (b)$.[13] The actual values of capital investment are not decided until the end of the mechanism. However, as long as $b_{(1)} \neq b_{(2)}$, $\beta(b_{(1)}) \neq \beta(b_{(2)})$. In turn, we can conclude that $k_1^* \neq k_2^*$, just as in the constrained problem and in the auction model of the previous sections.

Obviously, since the firm does not commit to ex ante optimal values of investment, but chooses the amount only when observing the private benefits of the regions, the firm does better *ex post* in this unconstrained problem. However, on average, the constrained problem may lead, *ex ante* to the same solution.

Proposition 6. *If the regions' private benefits are uniformly distributed, then the constrained problem, on average, leads to the same amounts of investment from the firm.*

Proof. Assume that the b_i's are distributed uniformly on the interval $[0, \bar{b}]$. Then,

$$E(\beta(b_{(1)}) = E\left[b_{(1)} - (\bar{b} - (b_{(1)}))\right] = E[2b_{(1)} - \bar{b}] = \frac{n-1}{n+1}\bar{b} = E(b_{(2)})$$

[13]For this optimal mechanism, a discussion on reserve prices is unnecessary. Indeed, reserve prices will be endogenously determined in the $k^*(b)$ functions. The firm can simply set $k^*(b) = 0$ for some values of b, which is equivalent to a reserve price.

Similarly,

$$E(\beta(b_{(2)}) = E\left[b_{(2)} - (\bar{b} - (b_{(2)})\right] = E[2b_{(2)} - \bar{b}] = \frac{n-3}{n+1}\underline{b}$$

We also know that $E(b_{(3)}) = \frac{n-2}{n+1}\bar{b}$. Therefore,

$$-E(b_{(2)}) + 2E(b_{(3)}) = \frac{n-3}{n+1}\bar{b}$$

Consequently,

$$E(\beta(b_{(2)}) = -E(b_{(2)}) + 2E(b_{(3)})$$

Recall the first-order conditions from the auction model:

$$p\frac{\partial f(K_1, L(K_1))}{\partial K_1} = L'(K_1)(w - E(b_{(2)})) + r$$
$$p\frac{\partial f(K_2, L(K_2))}{\partial K_2} = L'(K_2)(w + E(b_{(2)}) - 2E(b_{(3)})) + r$$

On average, we thus have the exact same first-order conditions, leading to the same investment decisions from the firm. □

The previous result holds for other distributions as well. This finding is not surprising. Indeed, virtual valuations can be interpreted as the marginal revenues on the sale for the seller. In the auction model, the "adjustments" on the marginal cost of labour, $E(b_{(2)})$ and $-E(b_{(2)}) + 2E(b_{(3)})$, are simply the marginal revenues from the subsidies. Similarly, $\beta(b_{(1)})$ and $\beta(b_{(2)})$ are the marginal revenues from the sale of the 2 investments to the seller in the unconstrained optimal mechanism.

6 Conclusion

This paper investigates how a firm can allocate investment across multiple sites strategically to attract larger subsidies from regions who participate in a bidding war for these investments. It proposes a model in which a firm wishes to install new production facilities and puts regional governments in competition against each other to decide the location of those facilities. Regional governments submit bids, in the form of tax holidays or other financial packages, and the firm invests in the winning region(s). In contrast to previous models, the firm can split her production in two establishments. This split introduces new strategic choices for the firm, and modifies the bidding behaviour of the regional governments.

First, I find that equilibrium subsidies will depend on the firm's choice of capital amounts to invest. In particular, when she chooses asymmetric plants, total subsidies are larger. Second, I show that this bidding behaviour affects the optimal amounts of investment of the firm. More specifically, I find that she always chooses to differentiate her establishments. Therefore, the firm manipulates the bidding war, in order to attract larger subsidies. I then compare this result to a situation without a bidding war. I find that the effect of the bidding war is ambiguous in general, but when using a Cobbs-Douglas form for the production function, I find that total investment increases. Notably, this result is true for any distribution function. Moreover, total subsidies also increase.

I also discuss the optimal mechanism to allocate the establishments under three different sets of assumptions: first from the point of view of the firm, second from the point of view of a social planner, and finally under more relaxed assumptions on the firm's commitment. I find that the open ascending auction used in the model implements the optimal auction, under certain conditions. Moreover, I find that a social planner would optimally choose the same allocation and payment rules as the firm. Finally, I describe the optimal mechanism under the more general assumption that the firm chooses amounts to invest endogenously through the mechanism. I show that in expected value, this optimal mechanism without prior commitment from the firm results in the same *ex ante* allocation and payments.

To summarise, this paper can be interpreted as two successive additions to the usual literature on bidding wars for firms. First, instead of considering a fixed investment amount, this paper allows the firm to choose the amount of capital to invest and make available in a bidding war. This addition changes the strategy of the firm, inciting her to over-invest in comparison to a situation without a bidding war. Second, we add a multi-location component: the firm can allocate the total investment across two sites. This addition modifies the firm's behaviour further, by inciting her to differentiate the amounts of investment between the production sites. In doing so, she continues to over-invest in total.

In terms of social welfare, the paper shows that while the allocation of investment is distorted versus a situation without a bidding war, the positive effect on allocative efficiency resulting from bidding wars is preserved in a multi-plant bidding war. More specifically, the regions that value the investment the most win it. However, while total investment is increased, which may have positive or negative implications, the increase is (under some conditions) for both hosts of the plants. Therefore, the increase in investment is not skewed only towards one of the winners at the expense of the other.

To conclude, the paper shows that the strategic behaviour of firms has important implications on the bidding wars for plants between regions. This distinction is important, since many bidding wars involve multi-nationals, and that such firms receive many subsidies in short periods by many local governments.

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