

MATHEMATICAL VECTOR ADDITION

Part One: The Basics

When combining two vectors that act at a right angle to each other, you are able to use some basic geometry to find the magnitude and direction of the resultant. What is a resultant? A resultant is the sum of the addition of two vectors. Since the vectors both have magnitude and direction, so does the resultant.

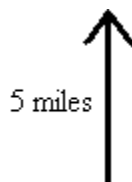
When adding vectors, it is important to make a sketch of the vectors involved so as to visualize the problem. There are some special rules for adding vectors that make it easier to work with them.

1. First of all you can only add vectors that measure the same type of quantity. For example you can add two velocity vectors together or two acceleration vectors together, but you cannot add a velocity vector with an acceleration vector. This is the old adding apples and oranges dilemma.
2. Always draw your vectors as arrows with the point in the direction that the vector is going. Also try to draw your vectors to relative scale. A 4-meter vector should not be longer than a 20-meter vector.
3. You can always move a vector around so long as you follow two important rules.
 - A. You cannot change the magnitude of the vector.
 - B. You cannot change the direction of the vector.
4. When drawing your diagram, always draw the vectors tail to head.
5. The resultant is drawn from the tail of then first vector to the head of the last vector.

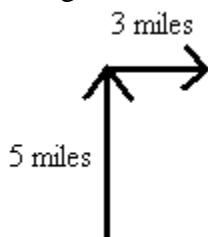
Let's do an example.

A man walks 5 miles north and 3 miles east.

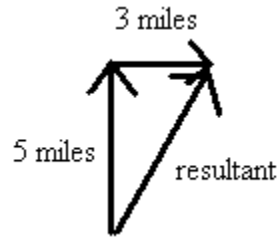
1. Draw the first vector.



2. Draw the second vector starting at the head of the first.



3. Draw the resultant, starting at the tail of the first and ending at the head of the last.



Now you try some and check your diagrams with the pictures at the bottom of the page.

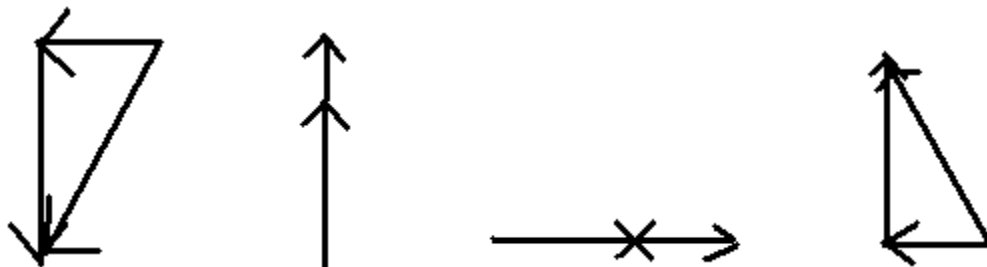
1. 15 m/s west and 28 m/s south

2. 2.3 m/s² west and 4.2 m/s² north

3. 3.5 m/s north and 8.5 m/s north

4. 35 miles east and 10 miles west

Answers

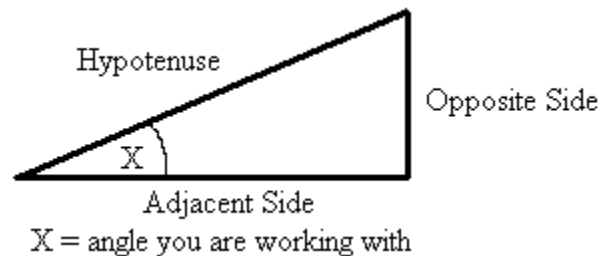


Part Two: Math and Vectors

If two vectors are in the same direction the resultant is simply the addition of the two magnitudes and the direction of the resultant is the direction they are both heading. This is like an airplane with a tail wind (the wind is blowing in the same direction the plane is traveling) or a boat heading downstream. If the vectors are in opposite directions, the magnitudes are subtracted and the direction of the resultant is the direction of the larger vector. This is like an airplane flying into a head wind (the wind is blowing into the front of the plane) or a boat heading upstream.

You may notice that the other case of vector combination is when the diagram forms a right triangle. Vectors don't always form right triangles, but let's leave that for later – much later. A right triangle is a triangle in which there is a right angle (90°). There are some neat properties of right triangles that make them easy to work with mathematically. So let's look at right triangles.

First, you need to know some definitions. The longest side of the triangle is called the hypotenuse. It is always across from the right angle. Of the two remaining angles, you will pick one to work with. The side that is not a part of that angle is called the opposite side and the side that is a part of the angle is called the adjacent side. Check out the diagram below.



Part Three: Pythagoras and Vectors

If your vectors form a right triangle in the diagram, you can find the magnitude of the resultant by using the Pythagorean Theorem. The Pythagorean Theorem is $a^2 + b^2 = c^2$, where c is the hypotenuse and a and b are the opposite and adjacent sides. Since you are looking for the magnitude of the hypotenuse, you need to rearrange the equation to $c = \sqrt{a^2 + b^2}$. Use the steps below to use this equation.

- Step 1. Square the length of the opposite side.
- Step 2. Square the length of the adjacent side.
- Step 3. Add the answers from Steps 1 and 2.
- Step 4. Take the square root of the sum from Step 3.

Now try some examples. Find the magnitudes or the resultants for the following pairs of vectors.

1. 12 miles east and 6 miles north
2. 8 miles south and 5 miles west
3. 11 m/s west and 5 m/s north
4. 3.5 m/s^2 west and 6.5 m/s^2 south

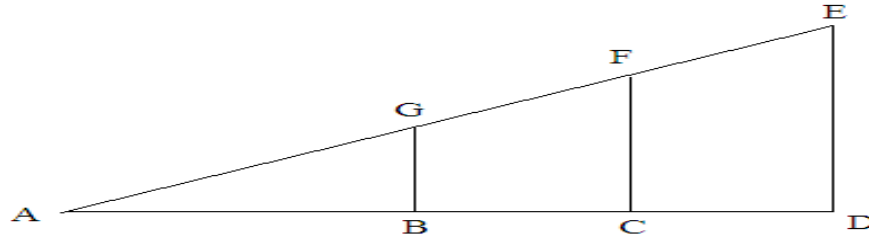
Answers:

12.1 17.6 9.5 7.4 13.4 10.5

Part Four: Finding your direction

Since the vectors you combine both have magnitude AND direction, you will need to know both the magnitude and the direction of the resultant. Part Three showed you how to find the magnitude and now you are going to see how to find the angle which describes the direction.

This section will involve a little Trigonometry (but not too much). It all has to do with another neat property of all right triangles. As it turns out, if you draw a right triangle with a particular angle, then divide the length of the opposite side by the length of the adjacent side, you will always get the same answer. It doesn't matter how long the opposite side and adjacent side are, so long as the angle is the same. Try it out. Fill in the charts below by measuring accurately between the two points indicated. Then perform the indicated division.



GB		AG		$GB \div AG$	
FC		AF		$FC \div AF$	
ED		AE		$ED \div AE$	
AB		AG		$AB \div AG$	
AC		AF		$AC \div AF$	
AD		AE		$AD \div AE$	
AB		GB		$GB \div AB$	
AC		FC		$FC \div AC$	
AD		ED		$ED \div AD$	

Pretty neat, huh? This relationship is called the TANGENT of the angle. We can use this number to find the degrees of the angle. The degrees will help us determine the exact direction of the resultant. To find the degrees you will use the TAN button on your calculator (you need a scientific calculator if you don't want to use the ugly table at the end of this packet). There are two steps.

Step One: Divide the opposite side by the adjacent side (be sure to press =)

Step Two: Take the inverse tangent of this quotient. This depends a lot on your specific calculator. It usually involves pressing the 2nd or INV button. If you don't have a scientific calculator, look up the answer to Step One in the Tangent column in the table at the end of this packet and estimate the degrees.

Now try some. Use the first vector listed as the adjacent side and the second vector as the opposite side. Find the angle of the resultant.

1. 12 miles east and 6 miles north
2. 8 miles south and 5 miles west
3. 11 m/s west and 5 m/s north
4. 3.5 m/s² west and 6.5 m/s² south

Answers: 19.5° 24.4° 26.6° 6.8° 32.0° 61.7°

NAME: _____

DATE: _____

Add or subtract the following pairs of vectors mathematically. Make a sketch for each problem. Write your answers on the blank lines on this page. Include magnitude and direction.

1.) 42.6 m north and 50.3 m west

2.) 8.33 miles east and 5.15 miles south

3.) 89.6 cm south and 113 cm east

4.) 0.532 ft west and 1.51 ft north

5.) 16.5 cm east and 13.4 cm north

Answers (not in order)

21.2 65.9 144 3.15 1.60 9.79 49.7 88.0 39.1 51.6 31.7 70.6

NAME: _____

DATE:

Add the following pairs of vectors. For each problem make a rough sketch and show all work. Write the magnitude and direction of the resultant on the line provided.

1.) 6 miles north and 13 miles west

2.) 83 m/s east and 51 m/s south

3.) 9.6 meters south and 8.4 meters east

4.) 2351 cm west and 962 cm north

5.) 0.06 km/hr east and 0.04 km/hr north

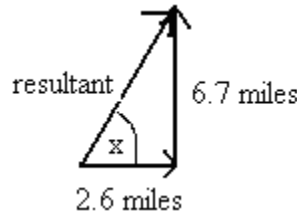
Part Five: Putting It All Together

Now this is where your diagram becomes REALLY important. Be sure to draw your vectors as arrows or the exact direction of the resultant becomes difficult to determine. Be sure to know which directional system you are using (90° , 180° or 360°). Use your diagram and the angle calculation to give the exact direction.

Here are two examples.

1. A man walks 2.6 miles east then turns and walks 6.7 miles north. Where is he in relation to his starting point?

Vector Diagram



Magnitude Calculation

$$c = \sqrt{6.7^2 + 2.6^2} = \sqrt{44.89 + 6.76} = \sqrt{51.65} = 7.19 \text{ miles}$$

Angle Calculation

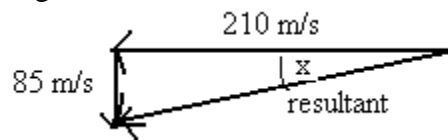
$$\tan^{-1} = \frac{6.7}{2.6} = 2.58 = 68.8^\circ$$

Final Answer

7.19 miles @ 68.8° north of east

2. A plane is flying west at 210 m/s with a crosswind blowing south at 85 m/s. What is the resultant velocity of the plane?

Vector Diagram



Magnitude Calculation

$$c = \sqrt{85^2 + 210^2} = \sqrt{7225 + 44100} = \sqrt{51325} = 227 \text{ m/s}$$

Angle Calculation

$$\tan^{-1} = \frac{85}{210} = 0.405 = 22.0^\circ$$

Final Answer

227 m/s @ 22° south of west

Now you try some.

- 1.) A family drives 63.6 miles west and 11.2 miles south. What is their resultant displacement?

Vector Diagram

Magnitude Calculation

Angle Calculation

Final Answer

- 2.) A boat crosses a river at 6.65 mph north through a current going 9.87 mph east. What is the resultant velocity of the boat?

Vector Diagram

Magnitude Calculation

Angle Calculation

Final Answer

3.) A plane flies 226 m/s west with a crosswind blowing 142 m/s north. What is the resultant velocity of the plane?

Vector Diagram

Magnitude Calculation

Angle Calculation

Final Answer

4.) A woman runs 4.5 miles south and 3.7 miles east. What is her resultant displacement?

Vector Diagram

Magnitude Calculation

Angle Calculation

Final Answer

Answers:

266	187	11.9	64.6	5.8	93.7
12°	32°	56°	10°	34°	40°