The concept of ‘externality’ has been the focus of much of theoretical welfare economics since the late 1800’s. Yet, in their 1962 piece “Externality”, Buchanan and Stubblebine develop the first mathematically rigorous and operational definition of the term. Their conceptual development of the term highlighted several nuances to the standard definition and allowed them to distinguish between marginal and inframarginal externalities, potentially relevant and irrelevant externalities, and finally Pareto-relevant and Pareto-irrelevant externalities. The authors note that, as pecuniary externalities are considered welfare neutral, their analysis only applies to technological externalities, or those possessing welfare distorting effects. The work of Buchanan and Stubblebine helped to better operationalize the theoretical developments through the early 1900’s, paving the way for the development of environmental economics, among other economic disciplines.

To begin, they define an external effect, from here on externality, as represented by the utility function

\[ u^A = u^A(X_1, X_2, ..., X_m, Y_1). \] (1)

This function suggests that the utility of an individual, A, is affected not only by the activities they have direct control over \((X_1, X_2, ..., X_m)\), but also upon an external factor, \(Y_1\), which is under the authority of another individual, B. This simple formulation captures interdependence among utility functions when externalities are present, and begins to rigorously redefine the typical definition of their time, that being “external effects exist in consumption whenever the shape or position of a man’s indifference curve depends on the consumption of other men.” (p. 371) The authors note that, for simplicity, they restrict any external effects on individual A to one factor, \(Y_1\), though the introduction of further factors is possible.
Assuming standard utility maximizing behavior, the authors present their first definition of externality, that being a “marginal externality” as existing when

$$u_{Y_1}^A \neq 0. \quad (2)$$

Here, small u’s represent partial derivatives of the utility function with respect to the variable designated in the subscript. Thus, a marginal externality exists when small changes in $Y_1$, the external action taken by individual B, either positively or negatively effect individual A’s utility. For positive values (1), the authors define this as a marginal external economy, and for negative values, they suggest a marginal external diseconomy. These definitions are akin to the more contemporary positive and negative externalities, respectively.

Developing the definition further, the authors define an inframarginal external (dis)economy as

$$u_{Y_1}^A = 0, \text{ and } \int_0^{Y_1} u_{Y_1}^A dY_1 \neq 0, \quad (3)$$

where the marginal effects of individual B’s activity, $Y_1$, has no affect on A’s utility, yet the total effect of B’s decision has changed A’s utility. The integral of an inframarginal external economy would be positive, suggesting that the total effect of B’s action increased A’s utility, whereas the integral for an inframarginal external diseconomy would be negative. As the authors note about inframarginal externalities, “small changes in B’s activity do not change A’s level of satisfaction, but the total effect of B’s undertaking the activity in question is helpful (harmful) to A.” (p. 373)

Further distinguishing between types of externalities, the authors develop definitions for relevant and irrelevant externalities economies. As noted, the existence of relevant externalities depends upon whether $Y_1$ is actually carried out by individual B. The authors define a potentially relevant marginal externality as
where positive values represent potentially relevant marginal external economies (i.e. potentially relevant positive externalities) and negative values potentially relevant marginal external diseconomies. As the authors suggest, when the action is carried out, a potentially relevant externality exists when it “generates any desire on the part of the externally benefited (damaged) party (A) to modify the behavior of the party empowered to take action (B) through trade or some other collective action.” (p. 374) And conversely, any externality that influences no such desire on the part of A is then considered irrelevant.

Finally, the authors introduce the concept of Pareto relevance and irrelevance by expanding upon their definition of relevant and irrelevant externalities, as defined above. Incorporating Pareto criterion into their definition, they define a Pareto-relevant externality as when “the extent of the activity may be modified in such a way that the externality affected party, A, can be made better off without the acting party, B, being made worse off.” (p. 374) From this, the authors define a Pareto-relevant marginal externality as

\[ \left( - \frac{u_{Y_1}^A}{u_{X_j}^A} \right) > \left[ \frac{u_{Y_1}^B}{u_{Y_j}^B} - \frac{f_{Y_1}^B}{f_{Y_j}^B} \right]_{Y_1 = Y_j}. \]  

(5)

This rigorous definition suggests that, using the same variables to represent individuals (A and B) and actions (X_i and Y_i), “gains from trade would not exist between A and B” (p. 375), as the marginal rate of substitution for individual A between activities Y and X would be larger than the net marginal rate of substitution between the affecting activity, Y_1, and some numeraire activity, Y_j. The authors note that Pareto-relevant externalities vanish in Pareto equilibrium, though marginal externalities may continue to exist.

The authors then present a simple diagrammatic illustration using the example of two neighbors who each value privacy and the decision they face at what height to build a fence between their properties. Individual B’s desire for privacy is almost unlimited, so they are
willing to build a fence up to some reasonably high level. But for individual A, construction of a fence over a certain height will obstruct the view of a mountain beyond individual B’s house. Thus, B’s decision to build a fence, who which has the legal right to build a fence, thus has an external effect on A’s utility. The author’s depict this situation using the Edgeworth Box diagram in Figure 1 below.

The indifference curves for individual A are depicted as the lines aa and a’a’, whereas those for B are depicted as the lines bb and b’b’. As depicted in Figure 1, we see that A’s utility is increasing over the range of fence height $0H_1$, suggesting their desire for privacy. Individual A is then indifferent over the fence heights $H_1H_2$. But in the range $H_2H_4$, we see that A’s utility begins to decrease with fence height, suggesting this height is where their view of the mountain becomes obscured. And beyond $H_4$, A is again indifferent as their view of the mountain has been entirely blocked. Over the range from $0H_1$, B’s indifference curves take on the same decreasing marginal utility shape, suggesting that his desire for privacy is satiated and his view of the mountain remains unblocked.

Figure 2 below then depicts the marginal evaluation curves for A and B over the fence height range $0H_5$, simply representing slopes along the preference fields in Figure 1. For individual A, we can see their marginal evaluation is positive over some range ($0H_1$), zero over the second range, negative over the third, and returns to positive after the fence passes a height of $H_4$, directly following the utility preference from Figure 1. Similarly for individual B, we see that their marginal evaluation curves over the entire fence height range is positive, until it hits $H_5$, when it becomes zero.

This simple example can be directly related to the rigorous definitions of externality presented earlier. That is, over the range $0H_1$, B’s action exerts a marginal external economy (i.e. positive externality). Over the ranges $H_1H_2$, B’s action exerts infra-marginal external economies as incrementally changing the fence height over that range has no effect on A’s utility, but a discrete choice to build the fence or not would effect A’s utility. Third, over the range $H_2H_4$, B’s action exerts marginal external diseconomies (i.e. negative externalities),
as it begins to affect A’s view of the mountain. Finally, beyond $H_4$, B’s action exerts again infra-marginal external economies, depending on upon A’s preferences for privacy and views of the mountain.

Finally, the authors note that their work distinguishing between types of externalities has significant policy implications, as some marginal external (dis)economies may be Pareto-optimal. As the authors note, “the observation of external effects, taken alone, cannot provide a basis for judgment concerning the desirability of some modification in an existing state of affairs.” (p. 381) That is, based on first impression, there is not always a case for intervention when an externality exists. The authors also make the connection with traditional Pigovian understanding of externalities, in that there exists a divergence between private and social marginal costs, thus suggesting a marginal externality exists. Again, because this marginal externality exists does not necessarily mean action needs to be taken to close this gap, as that divergence may prove Pareto-optimal. And finally, the authors make an appeal to Coase’s “reciprocal nature” externality, by suggesting that “The Pigovian terminology tends to be misleading, however, in that it deals with the acting party to the exclusion of the externally affected party. It fails to take into account that there are always two parties involved in a single externality relationship.” (p. 381) Thus, instead of solely focusing on the behavior of the acting party alone, the affected party must in some way compensate the acting party for changing their behavior.
Figure 1: Utility functions for Individuals A and B from Buchanan and Stubblebine (1962)
Figure 2: Marginal Evaluation Curves for Individuals A and B from Buchanan and Stubblebine (1962)