The Cost of Bad Parents: Evidence from the Effects of Parental Incarceration on Children’s Education

Carolina Arteaga*

January 7, 2019

Abstract

This paper provides evidence that parental incarceration increases children’s educational attainment. I collect criminal records for 90,000 low-income parents who have been convicted of a crime in Colombia, and combine it with administrative data on the educational attainment of their children. I exploit exogenous variation in parental incarceration resulting from the random assignment of defendants to judges with different propensities to convict and incarcerate. My identification strategy differs from the usual judge IV application because I model incarceration as a two-stage decision problem: First conviction, and then incarceration. I exploit judge leniency along these two different margins. Intuitively, I take advantage of the fact that I can compare children of parents who faced similar judge conviction leniency, but had different incarceration leniency. I derive a new expression that extends the Local Average Treatment Effect concept, to a setting with two sources of unobserved treatment heterogeneity. I find that conditional on conviction, parental incarceration increases education by 0.8 years for children whose parents are on the margin of incarceration. This positive effect is larger for boys, violent crimes, and cases in which the incarcerated parent is the mother.

JEL No. I24, J24, K42

---

*I am very grateful for the support, guidance, and encouragement of Adriana Lleras-Muney, Rodrigo Pinto, Sarah Reber, and Till von Wachter. I thank Stephen Billings, Leah Boustan, Moshe Buchinsky, Denis Chetverikov, Christian Dippel, David Eil, Paola Giuliano, Paul Goldsmith-Pinkham, Martin Hackman, Maurizio Mazzocco, Sam Norris, Rosa Matzkin, Matt Pecenco, Ricardo Perez-Truglia, Manisha Shah, and Jeff Weaver for their feedback. I also thank my colleagues Richard Domurat, Sepehr Ekbatani, Stefano Fiorin, Alex Fon, Keyoung Lee, Rustin Partow, Vitaly Titov, Lucia Yanguas and seminar participants at ALCAPONE, UCLA, CCPR, LACAE, Universidad de los Andes and the Central Bank of Colombia for insightful discussions. Natalia Cardenas, Mauricio Duran, and Juan D. Restrepo provided invaluable help in answering my questions about the institutional context. I gratefully acknowledge support from the Treiman Fellowship, CCPR, Colciencias, and the Central Bank of Colombia. Comments are greatly appreciated. Department of Economics, UCLA (caroartc@ucla.edu).
1 Introduction

Over one million children in EU countries, and 2.7 million children in the U.S., have a parent in prison (Sykes and Pettit, 2014). As a result, a very large number of children are growing up without a parent. Family environments during the early years, and especially parenting, are major determinants of human development (Heckman, 2013 and Almond et al., 2019), yet there is only a small literature investigating the effects of parental incarceration on children’s outcomes. A large body of correlation-based evidence finds negative associations between parental incarceration and a host of important variables such as mental health, education, and crime (Wakefield, 2015). However, households with incarcerated parents are disadvantaged along many dimensions. Therefore, simple comparisons of outcomes would lead to negatively biased estimates.

In this paper, I estimate the causal effects of parental incarceration on children’s educational attainment in Colombia. I exploit exogenous variation in parental incarceration resulting from the random assignment of defendants to judges with different propensities to convict and incarcerate defendants. I construct a new dataset that links several data sources: I link sociodemographic data on households with children from SISBEN, Colombia’s census of the low-income population, to publicly available criminal records for parents scraped from the internet. I find criminal records for approximately 90,000 parents for the years 2005 to 2016. Then, I link the educational outcomes of criminals’ children using administrative data on public school enrollment, and also web-scrape the children’s criminal records.

I estimate that on average, conditional on conviction, parental incarceration increases education by 0.8 years for children whose parents were on the margin of going to prison. With an average schooling of 6.8 years, this corresponds to an increase of 11.8%. The benefit of parental incarceration is larger for children of parents who were incarcerated by more lenient judges. Intuitively, those who are incarcerated even by lenient judges likely have worse unobserved characteristics, on average, than those incarcerated by the most strict judges. In terms of observed heterogeneity, point estimates suggest that the benefit of parental incarceration is larger when the child is a boy, incarceration was for a violent crime, or the incarcerated parent is

---

1Sykes and Pettit (2014) also estimate that for the U.S., 62% of black children born to high school dropouts will experience the imprisonment of a parent by age 17.

2Even prior to the incarceration event, these households are more likely to be poor and to experience domestic violence (Arditti, 2005; Arditti et al., 2012). In the US, Mumola (2000) finds that 60% of parents in prison reported that they used drugs in the month before their offense, 25% reported a history of alcohol dependence, and about 14% reported a mental illness. Western (2018) also documents that around 60% of parents in prison had experienced childhood trauma, such as domestic violence and sexual abuse.
the mother (as opposed to the father), though only the difference in the treatment effects by gender of the child is statistically different. I also find a U-shaped pattern in the age of the child at the time of the parent’s incarceration. Larger positive effects are estimated between ages 0 to 5 and 10 to 15, relative to 5 to 10.

My findings suggest that on average, parents who are on the margin of incarceration in Colombia are likely to reduce their child’s educational attainment if they instead remain in the household. Research shows that removing a violent parent or negative role model from the household can create a safer environment for a child (Jaffee et al., 2003; Johnson, 2009). Criminal parents may also deplete economic resources, and the economic contribution of defendants is likely to be small; Mueller-Smith (2015) finds that in the US, only one-third to two-fifths of incarcerated parents were employed before being charged. Parental incarceration may also reduce the intergenerational transmission of violence, substance abuse, and crime.\footnote{For example, using data from Sweden, Hjalmarsson and Lindquist (2007) report significant father-son correlations in criminal activity that begin to appear between the ages 7 and 12, and are fully established by the son’s teen years. This result also relates to findings in other fields that conclude that the positive effects of being raised by one’s parents depend on the quality of care that the parents can provide (Jaffee et al., 2003).}

Lastly, parental incarceration may result in the child being placed with an alternative caregiver who has better resources to care for the child. Indeed, I find that after an episode of parental incarceration, children often move in with their grandparents. They are also more likely to move to a household not in SISBEN, which suggests an improvement in economic conditions.

Previous papers in this literature use the random assignment of defendants to judges and their systematic differences in leniency to estimate the causal effects of incarceration on various outcomes.\footnote{For previous papers in the incarceration literature see Kling (2006); Aizer and Doyle (2013); Di Tella and Schargrodsky (2013); Mueller-Smith (2015); Bhuller et al. (2016); and Dobbie et al. (2018a), among others.} These papers omit the fact that there are two distinct margins on which judges are marking decisions. Specifically, judges usually first decide on conviction, and then, for those convicted, they decide on incarceration. In my setting this distinction is particularly important because I only observe defendants who are convicted, however, this is also relevant if those not convicted are actually observed in the data. Conviction is determined after random assignment of cases to judges, so the observed sample of convicted defendants is not necessarily balanced across judges. My paper is the first i) to show that under treatment effect heterogeneity, the resulting sample selection in the incarceration stage implies that the estimated treatment effect of incarceration does not have the standard local average treatment effect (LATE) interpretation, and ii) to derive an
easily interpretable expression of the resulting experimental estimates that extends the LATE concept to a setting with two sources of treatment heterogeneity.

I model this situation in a general framework in which treatment can take three values, and is decided upon crossing two thresholds along distinct margins of selection. The first determines sample selection, and the second defines the treatment of interest. In my case, crossing the first threshold decides conviction, and for those convicted, crossing the second threshold determines incarceration. The three treatment outcomes are i) not convicted, ii) convicted and not incarcerated, and iii) convicted and incarcerated. In this setting, given an instrument for each of the two decision margins, treatment effects related to the second margin (incarceration) can be identified by conditioning on crossing the first threshold given a fixed level of the first threshold’s instrument, and then exploiting further instrumental variation on the second margin. Unconditional treatment effects cannot be identified without further assumptions. This weaker result is, however, economically relevant: It allows me to estimate the causal effect of incarceration conditional on conviction under a specific conviction leniency.

Classical approaches to address sample selection omit discussion of treatment effect heterogeneity (Heckman (1978); Ahn and Powell (1993); Garren (1984)). In the presence of such heterogeneity, the estimated treatment effect is a function of the margin at which individuals were selected into the sample. In this paper, I combine the insights of Ahn and Powell (1993) and Angrist (1995), who use an instrument to account for the probability of selection into the sample, with the technology to identify treatment effects in a setting with multiple threshold-crossing rules of Lee and Salanie (2018). This allows me to identify a new counterfactual object that corresponds to a LATE for a given threshold of selection into the sample.

In my context, the estimated treatment effect corresponds to the effect of parental incarceration on children whose parents are on the margin of incarceration (the standard LATE), and who were found guilty under a specific level of evidence requirement. To understand the importance of this object, consider a situation in which DNA evidence or phone location records become available in court to decide on conviction. This may change the size and pool of individuals who are found guilty, and as a result treatment effects may also change. This result is also relevant outside the incarceration context. For example, when estimating the returns to STEM versus non-STEM majors, it is important to compare students who had the same underlying probability of attending college. In the framework of my model, we can think of this situation as first deciding on attending college, and then, conditional on college attendance, choosing STEM or non-STEM majors. The returns to STEM
may be a function of the margin at which students decide to attend college.

Contemporaneous to the writing of this article, three papers exploiting judge leniency as an instrument have provided different results using data from Norway, Sweden and Ohio in the US. Dobbie et al. (2018) and Bhuller et al. (2018) find imprecise null effects of parental incarceration on academic achievement for Sweden and Norway, respectively. For Cuyahoga County in Ohio, Norris et al. (2018) find large decreases in the probability of graduating from high school as a result of parental incarceration. These results are in contrast to the large positive effects I find for Colombia. Such heterogeneity points to the importance of understanding the settings and identifying the population at the margin. In my analysis, I find that the magnitude and sign of the effects is a function of the type of parent being removed from the home. Given the higher incarceration rate in the US, combined with the lower crime rates in both the Scandinavian countries and the US compared to Colombia, it is plausible that the marginal incarcerated parent in Colombia is more negatively selected than in the US, Norway, or Sweden.

An additional important difference is that unlike the other papers, my sample consists only of children who lived with their parent prior to the incarceration episode. In the US, half of the parents were not living with their children at the time of incarceration (Parke and Clarke-Stewart, 2002), and as a result the scope for positive effects from removing a parent is very limited. Consistent with this view, other papers that focus on parents living with their children in the US find similar results to mine. Cho (2009) finds that children in Chicago’s public schools whose mothers went to prison instead of jail for less than one week are less likely to experience grade retention. Using an event study design, Billings (2018) finds that incarceration improves end-of-grade exams and behavioral outcomes. He also finds, as I do, larger benefits when the mother is the incarcerated parent.

My paper also contributes to the literature on how parents affect their children’s outcomes. This includes a large body of papers on the intergenerational effects of human capital (Black et al., 2005; Oreopoulos et al., 2006), wealth (Black et al., 2015), and welfare receipt (Dahl et al., 2014), among other variables. Specifically, my paper contributes to the literature on household structure and children’s out-

5There are many differences between Colombia and Scandinavian countries, some of which may drive these different results. First, the size of the treatment is larger in Colombia, where on average prison sentences are 4.4 years, compared with three and eight months in Sweden and Norway, respectively. A second key difference is the potential size of the effects on schooling before college: In Colombia, 31% of the population between 25 and 34 years old has less than a high school degree, whereas this number is 17% for both Norway and Sweden (OECD, 2016). Finally, Norway and Sweden have very generous welfare programs and better education systems compared to those available in Colombia; these programs help insure disadvantaged children and would also point toward smaller treatment effects in the Scandinavian countries.
comes, and shows that living with a parent is not always better for children. Finlay and Neumark (2010) study whether marriage is good for children, and find that unobserved factors drive the negative relationship between never-married motherhood and child education. On the other hand, Doyle (2007, 2008) finds negative effects of removing children from their parents and placing them in foster care. My paper contributes to this body of literature with evidence that suggests that children may benefit from the absence of a convicted parent who is at the margin of incarceration.

Finally, my results highlight the importance of parenting, and specifically the costs of bad parents. This calls for a greater governmental role in assisting children from fragile households. Interventions that offer after-school activities or teach parenting guidelines can mitigate these costs. Early childhood interventions have been remarkably successful in complementing parental care in very disadvantaged populations (Heckman et al, 2010). These programs can be a starting point to both complement and improve the parenting skills of this population.

The rest of the paper is structured as follows. Section 2 provides background on the judicial system in Colombia, and Section 3 describes the data sources and provides summary statistics. Section 4 describes a model to identify causal effects in my setup, Section 5 presents my estimation and results, and Section 6 discusses the results, the mechanism and external validity. Section 7 concludes.

2 Background: The Colombian Court System

In this section, I describe the criminal justice system in Colombia: how defendants are processed, how cases are assigned to judges, the types of crimes involved, and the stages of a standard trial.

Figure 1 illustrates how defendants are processed in Colombia’s criminal justice system. A criminal record is created when an arrest is made. Once this happens, the police and a randomly assigned prosecutor must present the evidence that motivated the arrest in front of a judge within 36 hours. This judge, who is randomly assigned from the lowest tier of the judicial hierarchy, determines whether the arrest was legal and whether the defendant should await trial in prison. Next, the case is

---

6 There is also a literature in sociology on the effects of marital conflict and divorce on children’s well-being. Using longitudinal data, Amato et al. (1995) find that in high-conflict families, children have higher levels of well-being as young adults if their parents divorce rather than stay together.

7 Acuerdo CSJ, 3329.

8 A defendant will go to prison before trial when at least one of the following conditions holds: i) the defendant is a danger to society, ii) the defendant can interfere with the judicial investigation, or iii) there is reason to believe that the defendant will not appear in court for trial. Art 308. Criminal Proceedings Code.
randomly assigned to another judge who will preside over the trial—this is the judge
who provides the exogenous variation in conviction and incarceration I use in this
paper. In practice, once the first judge decides to continue with the prosecution of
a defendant, the case is entered immediately into a software program that assigns a
judge at random among the judges in the judicial district and at the court level that
the case is designated to; I refer to the district/court level as the “randomization
unit.”

Colombia is divided into 33 judicial districts. In the largest cities, a district
usually encompasses the city’s metropolitan area, and for the rest of the country, it
usually corresponds to a state. Depending on the severity of the charge(s), a case will
be randomized within one out of three possible court levels within the judicial district
in which the crime was committed. The first level, municipal courts, receive simple
cases, such as misdemeanors, property crimes involving small amounts, and simple
assault cases. These cases account for 38% of the data. More severe crimes, such
as violent crimes, drug- or gun-related crimes, and large property crimes are sent
to circuit courts (56%). Lastly, the most severe types of crime, such as aggravated
homicide or terrorism, are assigned to a specialized judge (6%). On average, there
are 20 judges per randomization unit, and the largest district—Bogota—has 55
judges.

Once the judge is assigned, the prosecutor and defense present their arguments
to the judge over the course of multiple hearings. The purpose of the first hearing is
to formally press charges. In the second hearing, the prosecutor and defense present
all relevant evidence. In the third hearing, the judge decides whether to convict; if
the defendant is found guilty, the judge holds a final hearing to determine sentence
length and incarceration. The Colombian Penal Code establishes minimum and
maximum sentences for each crime, but there is significant discretion on the part
of the judge. The general sentencing guidelines range is often quite broad. For
example, prison time for possession of 100 grams of cocaine is between five and nine
years (Penal Code, Art 376). The judge also determines the crime and severity of
the charge the defendant will ultimately be sentenced for—for example, murder
versus involuntary manslaughter.

The decision to send a defendant to prison is determined by the length of the
sentence. To deal with prison overcrowding, those convicted only serve time in
prison when the sentence is longer than a certain threshold. Currently, a sentence
equal to four years or less is not served in prison. As a result, the population

9Art 35-37, Criminal Proceedings Code.
10In these cases, the only consequence of being convicted is that for the duration of the sentence,
the judge must be notified of any change of address or if the convict plans to travel outside the
that faces a trial is divided into three groups: i) not convicted; ii) convicted and not incarcerated; and iii) convicted and incarcerated. The fact that a portion of the convicted population does not serve time in prison is not a special feature of the Colombian penal system; for example, it is comparable to a sentence of probation in the US.

In Colombia, judges are selected based on their performance on an exam from an open call of attorneys, with specific legal experience requirements for each category of judge. Appointments do not have term limits, and it is common that, over time, judges rise within the judicial hierarchy. The average tenure of a judge is six years, and on average, a judge presides over 344 cases.

While in prison, inmates can receive visits from adults once a week and from their children once a month. The government does not provide special welfare assistance to inmates’ families. Unlike in the US, being convicted of a crime does not change one’s eligibility for welfare benefits, and in the labor market, it is not common practice to ask about previous convictions, although this information is available online.

3 Data and Summary Statistics

3.1 Data sources

I collect data from several sources. First, I use two waves of Colombia’s census of potential beneficiaries of welfare (SISBEN). These data are collected by the government to characterize the country’s poor population and to target social programs to them. SISBEN has information on national identification numbers (NINs), household structure, age, gender, education, labor force participation of each household member, and a large set of variables on characteristics and assets of each house (e.g., refrigerator, stove, and floor material, among others). With this information, the government creates a score for each household that summarizes its level of wealth. The score is used to determine eligibility for most public programs—for example, free health insurance, conditional cash transfers, nutrition programs, subsidized housing, and college loans, among many others (Bottia et al., 2012). The first wave, conducted from 2003 to 2005, has data on 31.9 million citizens; the second wave, conducted from 2008 to 2010, has data on 25.6 million citizens.

From this database, I obtain two key elements for my analysis. First, I observe parent and child links when they live in the same household. Second, I use parents’
NINs to scrape criminal records that are public and available online. Anecdotal evidence for Colombia suggests that a large share of children with an incarcerated parent were not living with the parent at the time of the parent’s incarceration. My target population is, however, likely to be the most affected by parental incarceration.\[11\]

In Colombia, criminal records from defendants who are convicted are public and available online for 17 out of 33 judicial districts. These 17 districts represent 67% of the population, 69% of homicides, and 83% of property crimes; they include the largest cities in the country; and they are richer and more urban than the 16 districts without data online.\[12\] Each criminal record includes the name and NIN of the defendant, crime, date of crime, sentence information, and the court type and number that handled the case.\[13\]

I complement these data with individual-level, anonymized records from the Attorney General’s Office. This database has information on the universe of criminal cases (including cases that did not result in a conviction), along with courtroom identifiers, date of trial, final verdict, and gender and age of the defendant. I use this information to construct a measure of conviction stringency at the judge level. Finally, I use administrative records of public school enrollment for 2005-2016 with names and NINs to construct a measure of educational attainment. Children’s years of school are capped at 11, which is the last year of high school in Colombia.

### 3.2 Sample selection

To construct my sample, I proceed as follows: From SISBEN, I take the NINs of all parents living with their children in the 17 districts who have information online and web-scrape their criminal records. This adds up to 17 million adults. For computational reasons, I only search for records in the district where the person was living at the time of the SISBEN survey. To assess the number of records I miss due to this restriction, I take a 5% random sample and look for their criminal records in all 17 districts. From this, I estimate that I miss 8.6% of the sample due to crimes committed in districts different from the one found in SISBEN. My

---

\[11\] Given how my parent-to-child links are constructed, I focus on parents who are living with the children rather than the biological parents. This definition includes stepchildren when the parent identifies the child as his or her child instead of describing themselves as not being related to the child.

\[12\] The universe of judicial sentences is public; however, they are only available in the nation’s National Archives. Criminal records for Bogotá can be found at the following link: http://procesos.ramajudicial.gov.co/jepms/bogotajepms/conectar.asp

\[13\] I use information from court directories and court identifiers to link each record to a specific judge.
sample, therefore, includes only poor parents who, at the time of the SISBEN survey, lived with their children, lived in the largest districts of the country, and committed crimes in the district in which they were living.

I find 328,579 criminal records for 256,108 individuals, of which 63,654 have missing fields in at least one of the key variables, such as court identifier, crime, year, or sentence. Half of these records with missing data correspond to Medellin, which is the second largest district after Bogota, and has missing court identifiers in all of their records. I keep only crimes committed after 2005, which results in 193,520 records. Next, I drop all records from court levels for which there was only one judge (5,963 cases dropped), and also in cases in which the number of records per judge in a year is fewer than 15 (44,806). I also only keep courtrooms for which I have judge/year conviction rates from the Attorney General’s Office database. This leaves me with 128,792 criminal records from 105,133 adults. I retain only the first conviction in my sample, and collect data on the crime, courtroom identifier, and decisions regarding sentence and incarceration. I merge the criminal records back into the SISBEN data and keep only the first parental conviction in the household. My final data set consists of 91,032 convicted parents. These parents are linked to 67,770 children who were born between 1990 and 2007 and who experienced parental incarceration between ages 0 and 14.

I link these data to two outcome variables for these children: educational attainment and criminal records. I find school records for 77% of them, similar to the share of children between ages 12 and 17 who attend school (76%, 2005 Census). Table B2 in the Appendix shows evidence that having a missing education record is mostly due to actually not being in school, as reported in SISBEN, and is not a problem in the match; it is also not related to parental incarceration. Missing values are also more prevalent for boys, as well as for households with lower income and lower education levels for the head of household, both of which are predictors for a child not attending school. I also search for criminal records for all children of convicted parents who were 18 years of age by 2017. My final data set consists of 52,419 children born between 1990 and 2007 who have a convicted parent. In the following section, I characterize the population of convicted and incarcerated individuals, as well as their households and children.

---

14 In 2005, there was a reform in the judicial system that renders the two periods incomparable. In the previous system, a judge served as both prosecutor and judge at the same time, and he or she was anonymous to the defendant. Additionally, at the time of this reform, there were other changes put in place regarding sentencing guidelines.
3.3 Summary statistics

The population in my sample is negatively selected along three margins: education, income and criminal activity. In Table 1, I present socioeconomic characteristics for adults in the overall population, for parents in SISBEN with and without a conviction, and for parents with a conviction, by incarceration status. By comparing column 1 and columns 2 and 3, we see that parents in the SISBEN have fewer years of education, are less likely to have a high school degree, live in larger households, and are more likely to be single than all adults. Among parents in the SISBEN, individuals with a conviction are also negatively selected across a host of variables (column 3 relative to column 2). Convicted adults have fewer years of schooling, are less likely to have a high school degree or more (23% vs. 31%), and have lower income scores. They also live in larger households and are more likely to be single (41% vs. 35%, respectively). Adults with criminal records are disproportionally male (84%), they are more likely to work and to be the head of the household than those without a criminal record.\textsuperscript{15}

Among convicted parents, incarcerated parents have lower education and lower income levels (columns 4 and 5). Gender differences in the probability of incarceration conditional on conviction are far smaller than those in conviction. Incarceration is associated with lower probabilities of working, as well as being the head of the household. Table 2 splits the sample by gender. On average, convicted women have lower levels of education relative to convicted men, and they tend to come from poorer households. Compared to men, women are less likely to be the head of the household; yet they are still much more likely to be the heads of their respective households than in the country’s overall female population (36% vs. 29%, respectively). Convicted women are also more likely to be single.

Property crimes are the most common type of offense (25%), followed closely by drug-trafficking crimes (24%). Violent crimes account for 20% of the records, followed by gun-trafficking and misdemeanor offenses at 18% and 12%, respectively. Incarceration rates vary substantially by crime. Figure 2 ranks crimes by their incarceration rates for selected crimes. Serious crimes, such as kidnapping or rape, have the highest incarceration rates, whereas failure to pay child support, simple assault, and property damage have the lowest. In the middle of the distribution, we find crimes such as drug trafficking, domestic violence, counterfeit currency trafficking, theft, and smuggling, among others.

\textsuperscript{15}In the US context, for example, 29% of parents in state prisons have a high school degree or more, 48% are single, 92% are male, and the median age is 32 (Mumola, 2000).
4 Identification

Children from households with incarcerated parents are disadvantaged along many dimensions. As a result, simple comparisons of outcomes of children with and without incarcerated parents would lead to negatively biased estimates of the effects of parental incarceration. A common way to address this endogeneity concern is to exploit the random assignment of defendants to judges who differ in their leniency to incarcerate.\footnote{See Kling (2006); Aizer and Doyle 2013; Di Tella and Schargrodsky (2013); Mueller-Smith 2015; and Bhuller et al. 2016, among others.} The intuition of this identification approach is that for a group of defendants on the margin, their incarceration decision will only be determined by whether they were assigned to a harsh or lenient judge.

In this literature, the authors have data on the pool of cases randomly assigned across judges and use this to construct their incarceration instrument, as the share incarcerated by each judge—the leave-out mean. They compare incarcerated defendants with non-incarcerated defendants, which includes those who were not convicted as well as those who were convicted but did not receive a prison sentence. I cannot follow this strategy because in my data I only observe defendants who are convicted. Conviction is determined after random assignment to a judge, so the observed sample of convicted defendants is not balanced across judges. To address this challenge, I provide a new identification result for a setting in which sample selection invalidates the exogeneity of an instrument. Using the technology in Lee and Salanie (2018), my result extends the insight in Ahn and Powell to a setting with heterogenous treatment effects, where I also relax the assumptions on scalar unobservables, linearity in the choice equation, and separability in the outcomes equation.

In the model, treatment can take three values (not convicted, convicted but not incarcerated, and convicted and incarcerated), and is decided upon crossing two thresholds along distinct margins of selection. The first threshold determines whether the defendant is guilty and conditional on the crossing of the this threshold, the second determines the severity of punishment. I show that given an instrument for each decision margin, conditional treatment effects related to the crossing of the second threshold are identified. This approach has the additional advantage of estimating a more policy relevant treatment effect of incarceration, than the one previously in the literature; instead of comparing incarceration to conviction without incarceration and to those who were found innocent, I only compare incarcerated to the former. In the following section I provide intuition for the identification, after which I formalize this result.
4.1 A simplified framework

To fix ideas, let us consider the following framework: Judges are randomly assigned to defendants to make conviction and incarceration decisions by evaluating two distinct attributes of the defendant. When deciding on conviction $C$, a judge assesses the strength of the evidence of the case at hand. Without loss of generality, the distribution of the strength of the evidence across defendants $U^c$ is uniform $[0,1]$, where zero is the smoking gun and one is no evidence against the defendant. The judge can be one of two types in conviction: harsh ($P_c(H)$) or lenient ($P_c(L)$). Harsh judges do not require much evidence to convict a defendant. They have a threshold of 0.8, and thus they convict 80% of defendants; this corresponds to all defendants with a level of evidence below 0.8. Lenient judges require more evidence to convict a defendant, choosing a threshold of 0.2, such that they convict only 20%.

Next, if a defendant is convicted, the judge decides on incarceration $I$. The judge makes this decision based on an assessment of how harmful the convicted defendant may be to society, and how much punishment the defendant deserves. This trait, which I denote $U^I$, is also distributed uniformly $[0,1]$. Very harmful defendants have low values of $U^I$, and non-harmful defendants have values close to 1. A harsh judge ($P_I(H)$) would send 70% of convicted defendants to prison, whereas a lenient one ($P_I(L)$) would only incarcerate 30%. It is the same judge making both decisions, but a judge can be of different types on each decision. Figure 3 illustrates this situation. The x-axis traces the strength of the evidence the conviction decision is based on. That is, we can order defendants along one relevant dimension—namely, the strength of the evidence in the $[0,1]$ interval. A judge splits the space into two when she or he sets her or his conviction rate: Defendants to the right are free, and defendants to the left are convicted. Similarly, the y-axis traces the defendant’s punishment level, which is related to the assessment of predicted future criminal activity; unobserved—to the econometrician, not the judge—crime severity; and any mitigating/aggravating factors or family ties. I refer to this dimension as a measure of the defendants’ overall quality. For a fixed level of evidence required for conviction, a judge’s incarceration level splits the space of convicted individuals into two: A defendant below the threshold will go to prison, and a defendant above will not.

Due to randomization, all judges start with a statistically identical pool of defendants. However, after the conviction decision is made, the pool of convicted

---

17 As mentioned above, sentencing laws guide the judge’s incarceration decisions; however, there is large scope for discretion, even within a specific crime. What this dimension tries to capture are the factors that cause a judge to make different incarceration decisions for criminals who have the same charges.
Defendants is no longer comparable across judges with different conviction thresholds. Defendants convicted under a judge who requires solid evidence to convict will have, on average, a stronger case against them than those convicted under a judge who convicts even under weak evidence of guilt.

Defendants convicted under a harsh judge can face two types of judges \([P_c(H), P_I(H)]\) or \([P_c(H), P_I(L)]\), where the first term refers to the judge’s conviction stringency, and the second refers to the incarceration stringency. Similarly, those convicted under lenient judges can also have judges of types \([P_c(L), P_I(H)]\) and \([P_c(L), P_I(L)]\). Within these partitions, defendants are balanced across judges: first, because they were randomly assigned to their judge, and second, because they were selected into conviction under the same threshold. As a result, within partitions, there is exogenous variation in the probability of going to prison. For example, convicted defendants who were assigned to a \([P_c(H), P_I(L)]\) judge face a 30% chance of incarceration, whereas those assigned to a \([P_c(H), P_I(H)]\) judge face a 70% probability. Figure 4 illustrates this argument. This means that for 40% of defendants whose harmfulness assessment is located above the worst 30% of the population, but still in the bottom 70%, incarceration is only a function of judge assignment. Thus, I will be able to estimate LATE-type parameters for defendants who fall into this range.

Specifically, for this example I estimate the following two LATE parameters:

\[
LATE_{P_c(H)} = E[Y(t_I) - Y(t_c)|U^c < 0.8, 0.3 < U^I < 0.7]
\]

and

\[
LATE_{P_c(L)} = E[Y(t_I) - Y(t_c)|U^c < 0.2, 0.3 < U^I < 0.7]
\]

Where \(LATE_{P_c(H)}\) is the causal effect of incarceration relative to conviction for those convicted under a harsh judge \((U^c < 0.8)\), and \(LATE_{P_c(L)}\) is the one for conviction under a lenient judge. \(Y(t_I)\) and \(Y(t_c)\) represent counterfactual outcomes (years of education of the child) for incarceration \((I)\) and conviction \((C)\), and \(U^c\) traces the selection on the conviction stage.

\[
LATE_{P_c(H)} = \frac{E[Y|P_c(H), P_I(H)] - E[Y|P_c(H), P_I(L)]}{E[T = I|P_c(H), P_I(H)] - E[T = I|P_c(H), P_I(L)]}
\]

Where \(T = I\) in the denominator represents treatment assignment equal to incarceration. Similarly, we can have the analogous expression for \(LATE_{P_c(L)}\).

\[\text{18See Appendix D for an illustration of the failure of the simple IV estimator in this context.}\]
4.2 Model

In this section, I formalize the previous intuition and extend it to the case of continuous instruments to deliver a new identification result.

The model is described by the standard IV model, which consists of five main random variables: \( T, Z, Y, V, X \). Those variables lie in the probability space \( (\Omega, F, P) \), where individuals are represented by elements \( i \in \Omega \) of the sample space \( \Omega \). The variables are defined below:

- \( T_i \) denotes the assigned treatment of individual \( i \), and takes values in \( \text{supp}(T) = \{t_f, t_c, t_I\} \). \( t_f \) stands for not convicted, \( t_c \) for convicted but not incarcerated, and \( t_I \) for convicted and incarcerated.

- \( Z_i \) is the instrumental variable in this analysis and takes values in \( \text{supp}(Z) \), and represents judge assignment.

- \( Y_i \) denotes the outcome of interest for individual \( i \), —e.g., years of education of the child.

- \( X_i \) represents the exogenous characteristics of individual \( i \).

- \( V_i \) stands for the random vector of unobserved characteristics of individual \( i \), and takes values in \( \text{supp}(V) \).

The random vector \( V \) is the source of selection bias in this model. It causes both the treatment \( T \) and outcome \( Y \). The standard IV model is defined by two functions and an independence condition, as follows:

\[
\text{Outcome Equation: } Y = f_Y(T, X, V, \epsilon_Y) \tag{1}
\]
\[
\text{Treatment Equation: } T = f_T(Z, X, V) \tag{2}
\]
\[
\text{Independence: } Z \perp V, \epsilon_Y | X \tag{3}
\]

where \( \epsilon_Y \) is an unobserved zero-mean error term associated with the outcome equation.

In this notation, a counterfactual outcome is defined by fixing \( T \) to a value \( t \in \text{supp}(T) \) in the outcome equation. That is, \( Y(t) = f_Y(t, V, X, \epsilon_Y) \). The observed outcome for individual \( i \) is given by:

\[
Y = Y(T) = \sum_{t \in \{t_f, t_c, t_I\}} Y(t) \cdot 1[T = t]. \tag{4}
\]
The independence condition (3) implies the following exclusion restriction:

\[
\text{Exclusion Restriction : } Z \perp Y(t) | X \quad \text{for all } t \in \text{supp}(T).
\]

For the sake of notational simplicity, I suppress exogenous variables \(X\) henceforth. All of the analysis can be understood as conditional on pre-treatment variables.

I assume that the treatment equation is governed by a combination of two threshold-crossing inequalities. First, there is a conviction stage:

\[
\begin{cases}
\text{Free} & \text{if } 1[\phi_c(V) > \xi_c(Z)] \\
\text{Convicted} & \text{if } 1[\phi_c(V) \leq \xi_c(Z)].
\end{cases}
\]

where \(1[\cdot] \) denotes a binary indicator and \(\phi_c(\cdot), \xi_c(\cdot)\) are real-valued functions. Function \(\phi_c(\cdot)\) measures the degree of culpability assessed by the judicial system. This function looks at variables and information that are not observed by the econometrician but that are observed by the judge, such as the evidence, crime intensity, effort of the defense and prosecutor lawyers, as well as unobserved characteristics of the defendant such as aggression, antisocial behavior, etc. The function \(\xi_c(\cdot)\) assesses judge leniency on conviction. This function can be understood as a threshold of reasonable doubt beyond which the defendant is convicted by the judge. Judges differ in their leniency and may set different thresholds for evidence. The judge convicts defendant \(i\) whenever \(\phi_c(V) \leq \xi_c(Z)\). If that is the case, a second stage is held and the judge makes a decision regarding incarceration:

\[
\begin{cases}
\text{Not incarcerated} & \text{if } 1[\phi_I(V) > \xi_I(Z)] \\
\text{Incarcerated} & \text{if } 1[\phi_I(V) \leq \xi_I(Z)].
\end{cases}
\]

Similarly, \(\phi_I(V)\) is a function whose arguments are the case and defendant’s characteristics relevant for assessment of the punishment level. As before, the judge compares \(\phi_I(V)\) to her/his threshold to incarcerate \(\xi_I(Z)\).

Treatment assignment can be summarized as follows:

\[
T = f_T(Z, V) = \begin{cases}
t_f & \text{if } 1[\phi_c(V) > \xi_c(Z)] \\
t_c & \text{if } 1[\phi_c(V) \leq \xi_c(Z)] \cdot 1[\phi_I(V) > \xi_I(Z)] \\
t_I & \text{if } 1[\phi_c(V) \leq \xi_c(Z)] \cdot 1[\phi_I(V) \leq \xi_I(Z)].
\end{cases}
\]
This model relies on two separable threshold functions that play the role of the monotonicity condition.\footnote{Consider two judges, $j$ and $j'$, who see defendants $i$ and $i'$, who differ in their level of culpability. Say $i'$ has more evidence against him than $i$; namely $\phi_c(i') < \phi_c(i)$. Suppose that judge $j$ convicts defendant $i'$ but not $i$. Then the threshold function implies that it cannot be the case that judge $j'$ convicts defendant $i$, but not $i'$. More generally, let $D_i(j) = 1[T_i(j) = t_c]$ denote the binary indicator that judge $j$ convicts defendant $i$. Thus if judge $j$ convicts $i'$ but not $i$, it implies: $D_i(j) > D_{i'}(j)$ Then it cannot be the case that judge $j'$ convicts defendant $i$, but not $i'$. Which means: $D_i(j) > D_{i'}(j) \rightarrow D_i(j') > D_{i'}(j')$, which is equivalent to stating that: $D_i(j) > D_i(j') \rightarrow D_{i'}(j) > D_{i'}(j')$. We can generalize this to all individuals to arrive at the standard monotonicity assumption of Imbens and Angrist (1994).}

Without loss of generality, it is useful to express treatment assignment using the following variable transformation:

\[
U_c = F_{\phi^c(V)}(\phi^c(V)) \sim \text{Unif}[0,1], \quad (6)
\]

\[
U_I = F_{\phi^I(V)}(\phi^I(V)) \sim \text{Unif}[0,1], \quad (7)
\]

\[
P_c = F_{\phi^c(V)}(\xi^c(Z)); z \in \text{supp}(Z), \quad (8)
\]

\[
P_I = F_{\phi^I(V)}(\xi^I(Z)); z \in \text{supp}(Z), \quad (9)
\]

where $F_K(\cdot)$ denotes the cumulative distribution function of a random variable $K$. $U_c, U_I$ are uniformly distributed random variables in $[0,1]$ due to assumption (iii). Let $P_c(z)$ denote the conditional random variable $P_c(Z = z)$ which is simply the probability of conviction when $Z = z$. Moreover, independence condition (3) implies $P_c, P_I \perp U_c, U_I$. In this notation, the model can be expressed as:

\[
T \equiv f_t(Z,V) = g_T(U^c, U^I, P_c, P_I) = \begin{cases} 
t_I & \text{if } 1[U^c > P_c(z)] 
t_c & \text{if } 1[U^c \leq P_c(z)] \cdot 1[U^I > P_I(z)] 
t_I & \text{if } 1[U^c \leq P_c(z)] \cdot 1[U^I \leq P_I(z)] \end{cases} \quad (10)
\]

In the model, $U^c$ and $U^I$ have the same interpretation as in the previous section, and $P_c$ is interpreted as the share convicted by judge $z$. Without the assumption of independence of $U^c$ and $U^I$, variation in incarceration leniency is only identified once I fix the conviction threshold. Thus, the counterfactual of interest are $Y(t_I)$ and...
Y\((t_c)\) for those who were convicted under \(P_c = p_c\). This means that the objective is to identify causal effects of the form: \(E(Y(t_I) - Y(t_c)|U^c < p_c)\), which is the same exercise explained in Section 4.1. Let:

\[
P^*_I(z) = Pr[U^I < P_I(z)|U^c < P_c(z)]
\]

(11)

\(P^*_I\) is the judge’s incarceration probability conditional on conviction.

**Proposition:** The difference in counterfactual outcomes \(E(Y(t_I) - Y(t_c)|U^c < p_c)\) is identified from the data as follows:

\[
E(Y(t_I) - Y(t_c)|U^c < p_c) = \int_0^1 \frac{\partial E(Y \cdot 1[T \in \{t_c, t_I\}]|P_c(Z) = p_c, P^*_I(Z) = p^*_I, U^c < p_c)}{\partial p^*_I} dp^*_I
\]

(12)

See the appendix for the proof.

What this result says is that we can trace the treatment effect of incarceration relative to conviction once we fix a threshold for conviction. We do this by evaluating changes in the outcome variable when we change \(P^*_I\). This delivers the MTE along the unobservable dimension \(U^I|U^c < p_c\). The integral over the support of the instrument gives the LATE, or the ATE when the instrument has full support. In the next section I use this identification approach to estimate the effects of parental incarceration in my data.

## 5 Estimation

To apply the identification result of the previous section, I start by estimating the sample analogs of \(P_c(Z)\) and \(P^*_I(Z)\) in the model. The interpretation of these variables is the probability of being convicted/incarcerated, given the assignment to a specific judge. Following the literature, these are estimated as judge fixed effects from regressions after parsing out variation at the unit at which the randomization of judges occurred and specific case characteristics That is, the conviction/incarceration decision can be decomposed into a portion that is related to the individual, the judge, the crime, and the randomization unit/year. I do this as follows:

\[
D_{itor} = \gamma_{rt} + \gamma_o + \epsilon_{itor}
\]

Where \(D_{itor}\) corresponds to a conviction or incarceration dummy, \(i\) indexes indi-
individuals, $t$ year, $o$ offense, and $r$ court-level/judicial district. $\gamma_{rt}$ corresponds to randomization-level fixed effects, which is a court-level/judicial-district and year-level fixed effect. $\gamma_o$ is an offense-level fixed effect (161 different crimes); and $\epsilon_{itor}$ is a zero mean term. Following the literature, I estimate the judge instrument $\hat{p}_{z-i}$ for defendant $i$ to be the following leave-out estimator:

$$\hat{p}_{zi} = \frac{1}{n_z - 1} \sum_{k \neq i} \hat{res}_{z,k}$$

where $n_z$ is the number of cases of judge $z$, and $\hat{res}_{z,k}$ is the residual from a regression of the conviction/incarceration dummy on $\gamma_{rt}$ and $\gamma_{c}$.

Figure 5 shows the distribution of $D_{itor}$ at the judge level, and $\hat{p}_z$ for both conviction and incarceration. From the graph, we can see that although court-level/year and crime-level fixed effects explain most of the variation, judge’s fixed effects still represent a sizable share of the variance in conviction and incarceration.

### 5.1 Instrument validity

Next, I examine how much judge fixed effects predict individual-level decisions by estimating a first-stage regression for defendants, as follows:

$$D_{iztor} = \beta_0 + \hat{p}_{zi} + \beta_1 X_{iztor} + \epsilon_{iztor}$$

As before, $D_{iztor}$ corresponds to the conviction or incarceration dummy, and $p_z$ is the leave-out mean of judge $z$ assigned to person $i$. I run this regression with and without controls $X_i$. In the conviction regression, where I use anonymized data from the Attorney General’s Office, I can only control for age, gender, and number of crimes charged. In the incarceration regression, I control for schooling, income, occupation, gender, year of birth, and year in the survey. According to the results in Table 3, judges have a strong influence on conviction and incarceration decisions. The estimates are highly significant and suggest that being assigned to a judge with a 10 percentage point higher conviction/incarceration rate increases the defendant’s probability of conviction and incarceration by seven and eight percentage points, respectively. This relationship is robust to the inclusion of controls, as expected by random assignment. Figure 6 depicts this first-stage relationship for conviction (left panel) and incarceration (right panel). These graphs show a strong positive relationship between the instrument and individual trial decisions. The F-stats on the first stage correspond to regressions on judge dummies to account for the true dimension of the instruments. These F-stats are above the critical value for the
leave-out mean instrument for weak instruments (see Figure 4 in Stock et al., 2002).

Recall from the previous section that, the variation in incarceration stringency conditional on a level of conviction stringency is what identifies treatment effects in this context. Figure 7 shows a scatter plot of both conviction and incarceration fixed effects. From the graph we can see that there is substantial variation along the incarceration axis for each conviction rate.

For the instrument to be valid, the judge’s fixed effects must be orthogonal to the defendant’s characteristics. I test this in the anonymized data from the Attorney General’s Office, where the universe of cases the judge has heard is available. Table 4 checks the balance across defendants for my judge-stringency measures for conviction and incarceration. Across gender, age, and type of crime—which are the only variables available in these data—I find no individual or joint statistical significance. In addition, the identification result is supported by the observation that once $P_c$ is fixed, the pool of convicted defendants is balanced across judges. I test whether covariates are associated with incarceration stringency for the convicted sample, once I split the sample by conviction group (low, medium, or high) or control for the conviction level with a polynomial of $P_c$. In Table 5 I test the individual and joint significance of variables associated with education, income, and occupation status, and find no evidence of a relationship with judge stringency.

To interpret the results of the IV as the causal effect of incarceration, judge stringency must only affect child’s outcomes through incarceration. This may not be the case if the judge fixed effects capture other dimensions of trial decisions, such as fines or guilt (Mueller-Smith, 2017). In my setting, this is less of a concern because in the case of Colombia, fines are rare and only associated with large property crimes; and because I model the conviction decision directly.

Finally, the instrument must satisfy the monotonicity assumption: Conviction or incarceration decisions made by a lenient judge would also have been made by a stricter judge; this is called the monotonicity assumption. One testable implication is that first-stage estimates should be non-negative for all sub-samples. That is, if a judge is lenient, he or she is going to be lenient for both women and men, and for both violent crimes and nonviolent crimes. To test this assumption, I construct judge fixed effects for just one group in the population, for example, for men and use this fixed effect in a first-stage regression to predict individual conviction and incarceration for women. I do this for gender, type of crime, and age group. Table C1 in the Appendix shows these first-stage tests, in which I find positive first-stage estimates across all slices of the data, which supports the monotonicity assumption.

The other side of monotonicity is separability (Vytlacil, 2002). In terms of the
former, the model assumes that judges weigh the same characteristics of defendants and value them in a similar fashion. Specifically, in my model, one testable implication of such an assumption is that I can write the conviction and incarceration decisions as functions $\xi^c(\cdot)$ and $\xi^I(\cdot)$, and that these functions are not judge specific. This is a reasonable modeling assumption, given that all judges go through the same training and ultimately should have the same objective function. To evaluate whether the data support this assumption, for a handful of covariates I estimate a random coefficient model, with different coefficients for each judge, and test whether this model provides a better fit than a model with fixed judge coefficients. Table C2 in the appendix shows that overall, the data fail to reject the model with fixed coefficients per judge.

5.2 Results

I now turn to the estimation of the effect of parental incarceration on children’s educational attainment. I restrict attention to children who were ages 0 to 14 at the time of parental incarceration, and born between 1990 and 2007; this ensures that children appear in the educational attainment data, which I observe from 2005 to 2016. I consider only incarceration cases in households in which the person incarcerated was the parent and not any other household member.\(^{20}\)

Following the identification result, I need to account for the different levels of conviction stringency at which defendants were found guilty. I do this in two ways: First, I sort my data by stringency in the conviction stage ($P_c$) and split the sample into terciles: low ($0.7 < P_c < 0.88$), medium ($0.88 < P_c < 0.9$), and high ($0.9 < P_c < 1$) conviction levels. Second, I pool the data and add a second-degree polynomial on $P_c$ with interaction terms. This last estimate can be interpreted as an average effect across the different conviction thresholds. The first three columns of Tables 6 and 7 show regressions for the split sample, and the fourth one has the pooled regression.

I begin by showing the OLS estimate of this design. Table 6 shows a regression of parental incarceration on years of education. Following Abadie et al. (2017), I cluster standard errors at the judge level. Without controls, a child whose parent went to prison has 0.4 to 0.3 fewer years of schooling than a child whose parents did not. Once I add controls, this difference reduces drastically to less than 0.1 years. Still, we expect that incarcerated parents are negatively selected on unobservables that cannot be accounted for, so -0.1 years is a lower bound on the causal effect.

Next, Figure 8 shows a graphical representation of the reduced-form regression. This graph plots the distribution of judges’ incarceration fixed effects against the

\(^{20}\)The number of cases in which this is the case is not large enough to study this population.
predicted years of education from a local polynomial regression. From the graph, we can see that there is a strong positive relationship between judge stringency in incarceration and years of education. That is, as we move to the right, where the probability of having a parent in prison increases exogenously, I estimate that the years of education also increase. The top panel of Table 7 shows the regression results for this reduced form: I estimate large increases in years of education for all specifications, and for all but the second column, the increase in years of education is statistically significant. Finally, the bottom panel of Table 7 shows results from the IV; I estimate that having an incarcerated parent increases years of schooling by 0.7 to 0.9 years. These estimates are statistically different from zero for the first and third terciles, as well as for the pooled regression. I find that the increase in years of education is mostly accrued through higher graduation rate from middle school. Figure C2 in Appendix E plots the treatment effect of parental incarceration on grade completed from 6th grade to 11th grade. There are positive treatment effects for all grades, but the effect is the larger for 9th grade which corresponds to the last grade of middle school.

I also study how parental incarceration affects the chance that the child is later convicted of a crime. For this exercise, I restrict the data to children who were 18 years old by 2017, so that their criminal records would be public. Figure C3 graphically depicts reduced-form estimates of judge stringency on conviction probability; the effect is close to zero. However, the analysis is under-powered to detect to estimate reasonably sized treatment effects. This is not surprising, since conviction is a low incidence event; only 1.6% of children had a criminal record, and the difference in the OLS is only 0.1 pp.

### 5.3 Heterogeneity

In this section I examine the heterogeneity of the results along observables and unobservables. In my context, marginal treatment effects (MTE) are particularly interesting, because they trace the causal effect of incarceration along parents’ unobserved characteristics ($U^I$) that matter for incarceration and that are correlated with defendants’ quality, broadly defined. What this exercise does is to evaluate the possibility of different effects of parental incarceration given the type of defendant that is going to prison, which is characterized by his or her location along the y-axis of Figure C3. The intuition is as follows: Parents who are incarcerated under the most lenient judges have worse characteristics than those incarcerated under strict judges. In other words, a strict judge incarcerates almost everyone, but a lenient judge incarcerates only the worst defendants, so that those incarcerated under rela-
tively lenient judges are more negatively selected. I follow Heckman and Vitlacyl (2005) to estimate this MTE. I find that at the 5% level, there are heterogeneous treatment effects along parental quality (Figure 9). Specifically, I find that the positive effects of incarceration on schooling accrue when the worst defendants go to prison.

The magnitude of the effect of parental incarceration on children’s education is a function of the relationship between the parent and the child prior to the incarceration episode, the type or quality of this parent, and the role of the child in the household. To document this heterogeneity, I estimate the IV regression for different subgroups in the data. Following previous literature in economics, as well as that in psychology and sociology, I estimate different regressions by gender of the child, gender of the parent, child’s age at the time of the incarceration episode, birth order, and the nature of the offense—violent, property, drug- or gun-related, and misdemeanor. In Table 8, I show IV results for the pooled model for these different groups in the data.

According to the estimates, the benefits of parental incarceration are larger for boys than girls, and this difference is statistically significant. Specifically, I find that boys’ schooling increases by one year, whereas girls’ schooling increases by 0.4 years, but the latter is not statistically significant. This result is consistent with previous research in psychology and economics, which documents that boys are more vulnerable than girls to negative experiences in the household (Bertrand & Pan (2013); Autor et al. (2016); Parke & Clarke-Stewart, (2002); Hetherington et al., 1998). Specifically, Autor et al. find that boys, relative to their sisters, have higher rates of disciplinary problems, lower achievement scores, and fewer high school completions when growing up in disadvantaged environments.

I split the sample by gender of the parent and find that incarceration is more beneficial in cases in which the mother is the one going to prison. This result might be surprising at first glance. However, it is important to bear in mind that children’s well-being is more closely affected by their mothers’ behavior because of their main role as primary caregivers, and that criminal women are more negatively selected than criminal men (Table 2). This result is consistent with the findings of previous research in the US, where Billings (2018) and Turanovic et al. (2012) estimate larger positive effects from maternal incarceration. It is also the case that in the US, incarcerated women have worse socioeconomic backgrounds than incarcerated women.
men (Harrison & Beck, 2006). In addition, Glaze and Maruschak (2008) survey incarcerated parents and find that 60% of imprisoned mothers, compared to 16% of fathers, have histories of being physically or sexually abused.

A source of heterogeneity associated with the quality of the parent going to prison is the type of crime they committed. Thus, in the lower panel of Table 8 I split the sample by crime categories: violent, property, drug-related, gun-related, and misdemeanors. The largest benefits are observed for defendants convicted for violent crimes, whereas the smaller benefits are for misdemeanors. These differences, however, are not statistically significant. Nonetheless, this is in line with the previous result on unobserved heterogeneity, in which the positive effects are a function of how good the defendant is as a parent.

Lastly, I look at heterogeneous effects depending on the age of the child at the time of parental conviction. I split the sample into three groups: children who were 0 to 5 years, 6 to 10 years, and 11 to 15 years at the time of parental conviction. I find a U pattern in the effects on schooling. Studies in developmental psychology conclude that children in the first age group are the most vulnerable, as they do not yet have the abilities and skills to process trauma on their own (Johnston, 1995). These skills and abilities develop over time, and help children cope with distress. On the other hand, the increase in the positive effect in the later years may be the result of how salient the decision is to continue in school or drop out at older ages.

5.4 Robustness

In this section I go over various exercises that evaluate the robustness of the results in the paper along different dimensions.

In Table 3 I report the first-stage regression on incarceration, and in the bottom of the table I report the F-test on the excluded instruments. This F-test corrects for the fact that the dimensionality of the instrument is the number of judges and not one (my measure of judge leniency). With this correction, the F-stats are low, but above the critical values for weak instruments. The consequence of weak instruments is that 2SLS-IV estimates will be biased toward the OLS (Stock et al., 2002). In my context, given that the OLS estimates are negative, the bias of the OLS is also negative, and the 2SLS IV estimates are positive, this means that we could expect even larger positive effects. To assess the size of this residual bias, I estimate the IV using the LIML estimator, which is less sensitive to weak instruments—the bias does not increase with the number of instruments (Rothenberg, 1993; Stock et al., 2002). Table C3 in the Appendix shows the estimates for the LIML estimator. I
find that the 2SLS and LIML estimator are very close, with both around a point estimate of 0.8 years.

In the Results section, I show my preferred specifications for the estimates of the effect of parental incarceration on educational attainment. This decision to split the sample into three groups of $P_c$ was arbitrary. To assess the robustness of the results, in Figure C4 I instead order observations along $P_c$, and run multiple regressions on a rolling window of 20,000 observations over $P_c$, moving the window 500 observations each time. Figure C4 in the Appendix shows that for each sample, I find a positive effect of incarceration on education.

Lastly, as a placebo check, I evaluate whether there are differences in schooling for children of incarcerated versus non-incarcerated parents before the date of the sentence. Table C6 in the Appendix shows that there is no supporting evidence that the positive effects I estimate are the result of preexisting differences in educational attainment.

6 Mechanisms

6.1 What explains the positive effect?

The results presented here suggest that living with a convicted parent has negative consequences. There are many reasons to believe that this is plausible. First, criminals are more likely to exert psychological and physical violence at home, and this can often be detrimental to a child’s well-being. In the US context, Western et al. (2004) find that incarcerated men engage in domestic violence at a rate about four times higher than the rest of the population. Further, psychology research documents that spending time with parents who engage in high levels of antisocial behavior is associated with more conduct problems for their children (Jaffee et al., 2003). This literature concludes that the salutary effects of being raised by married biological parents depend on the quality of care the parents provide.

Second, Chimeli and Soares (2017) document the causal effect of trading illegal commodities on violence. In light of their work, we can expect that households that take part in illegal businesses face constant violence or threats of violence related to guaranteeing property rights or resolving disputes within the business, all of which affect the quality of life in a household. There is also literature on the intergenerational transmission of violence, substance abuse, and crime. Specifically, in the role-model theory, in which children directly observe and model their parents’ behavior, incarcerating parents could be beneficial, as it removes bad role models from the house and forces children to update their beliefs about the consequences
of criminal behavior (Hjalmarsson and Lindquist, 2012). Beyond intergenerational transmission, childhood exposure to negative behaviors is documented to have direct adverse effects on outcomes in both childhood and adulthood (Balsa, 2008; Chatterji and Markowitz, 2000).

6.2 How does the environment of the child change?

To characterize the changes that households and children experience after an episode of incarceration, I analyze households for which I have two observations in the SISBEN (44% of cases), in which the parent was convicted of a crime between observations. Appearing in both waves of the SISBEN is not random; on the contrary, leaving the sample is associated with an improvement in living standards. This is particularly relevant for children who might be moving to a household outside of SISBEN after the episode of parental incarceration. With this caveat, Table 9 shows that incarceration is associated with an increase in labor force participation (LFP) of the spouse, a worsening of the income score of the household, and a decrease in the probability of a male as the head of the household. I also find that the probability of living with grandparents increases and the probability of being in the second wave of SISBEN decreases, suggesting that incarceration induces children to move in with relatives who are better off financially.

6.3 Parents at the margin

To derive policy implications, it is important to acknowledge the local feature of my estimates. This paper estimates the effects of parental incarceration for a particular sub-population: children of convicted poor parents at the margin of incarceration. A large share of the convicted—for example, those guilty of murder or rape—would be incarcerated regardless of judge assignment, and this paper cannot provide any insights into the effects on educational attainment of the children of those individuals. At the other end of the distribution, defendants convicted of minor crimes will also avoid prison, regardless of judge assignment. Defendants convicted of drug- or gun-trafficking, domestic violence, and medium-sized property crimes compose the complier group in my estimation, and they are the group my estimates apply to. This marginal population, however, is particularly relevant because it is the population that is more likely to be affected by policy interventions to the criminal justice system. Following Dahl et al. (2014), I find that compliers make up approximately
29.8% of the sample.

6.4 External validity and policy implications

To assess the external validity of my results, I provide a framework motivated by my heterogeneity analysis, which links parental quality to parenting treatment effects and the probability of incarceration. Figure 10 summarizes this framework. The x-axis traces parental quality; as we move to the right, parental quality increases. The y-axis measures the treatment effect of parenting: Having better parents is better for children. Most importantly, however, there is a segment in the support of parental quality for which parents are detrimental for children. The secondary y-axis measures incarceration probability: In the model, the probability of being incarcerated decreases when parental quality increases. Each society chooses a level of incarceration, which is characterized by a threshold in the support of parental quality. This threshold determines the average effect of incarcerating parents (the gray area in Figure 10). To determine how much the results in this paper apply to other settings, we need to think about the location of the incarceration threshold along the parental quality axis and the shape of the treatment effects’ function in each country. Countries with higher incarceration rates will incarcerate, on average, better parents than those with lower rates, and as a result we should expect lower benefits or even costs from parental incarceration. We can also expect a much flatter function of treatment effects of parenting in generous welfare states, such as the Nordic countries, in which children’s education and health vary less with parental characteristics. As a consequence, we would find smaller treatment effects of parental incarceration (both positive and negative). This framework reconciles contemporaneous papers with opposing results, and provides a clear framework to think about external validity.

In terms of policy implications, these results call for greater support from the government in assisting children from fragile households. There is a strong body of

Parental compliers are defendants who would have received a different incarceration decision had their case been assigned to the most lenient judge instead of the strictest judge. We can define the size of this group ($\pi_c$) as follows:

$$\pi_c = \text{Prob}(\text{Incarceration} = 1|z_j = \bar{z}) - \text{Prob}(\text{Incarceration} = 1|z_j = \bar{z})$$

where $\bar{z}$ and $\bar{z}$ correspond to the incarceration rates of a judge at the 99th and 1st percentiles, respectively. Because of monotonicity, the share of parents who would go to prison regardless of the judge assigned to their case—always takers—is given by the incarceration rate for the most lenient judge and is equal to 22.5%. On the other hand, 47.7% of the sample are children of never takers who would not go to prison no matter which judge was assigned to their case. I estimate that children of compliers make up approximately 29.8% of the sample.
experimental evidence on the powerful role of parenting and parenting supplements in shaping skills, but also on the lack of parenting knowledge among disadvantaged parents (see Cunha et al., 2013). Early childhood interventions have been remarkably successful in complementing parental care with positive economic, psychological, behavioral, and health benefits (Heckman et al., 2010). The Perry Preschool Program, which targeted very disadvantaged children from backgrounds in which incarceration was a common feature, is an example of this. These programs have been successful in providing early supplements to parenting and can be a starting point to complement and improve parenting among this population.

7 Conclusions

The rise in incarceration in the US has led to an equivalent increase in the number of children growing up with a parent in prison. Children of incarcerated parents fare worse than those without one on a wide range of outcomes. Yet separating the causal effects of parental incarceration from preexisting risk factors has been a significant challenge. In this paper, I estimate the causal effects of parental incarceration on educational attainment in Colombia. My results suggest that children benefit when their convicted parents are incarcerated. Specifically, I estimate that parental incarceration increases schooling by 0.8 years on average.

I conclude with a discussion of three important limitations of this paper. First, I consider only the short-term effects of parental incarceration. This is important, as these parents eventually leave prison and will perhaps return to live with their children. Further, if incarceration decreases one’s human capital and social and emotional skills, the type of parent who returns after incarceration can be much worse than the one who left. In that case, the long-term effects may be very different from what I estimate here. Another significant limitation of this paper is that, effectively, I can only study one outcome variable. As shown by Dobbie et al. (2018), parental incarceration can have sizable effects on other variables such as earnings and teen pregnancy. These are important results that help characterize the complex shock of having an incarcerated parent, but that due to data limitations I cannot explore here. Finally, given my sample selection, my analysis is restricted to cases in which the convicted parents are living with their children; which is not the majority of the cases, and to poor households. There are significant reasons to believe that my results do not extend to different groups of children living in other situations.
References


Billings, Stephen (2017) Parental Arrest and Incarceration: How Does it Impact the Children? (Preliminary draft)


Hjalmarsson, Randi, and Matthew J. Lindquist. 2011. “The Origins of Inter-


### Tables

#### Table 1: Population by conviction and incarceration

<table>
<thead>
<tr>
<th>Sample:</th>
<th>Census: Adult population</th>
<th>SISBEN Criminal record</th>
<th>SISBEN w/ conviction By incarceration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Years of education</td>
<td>7.36</td>
<td>6.82</td>
<td>6.68</td>
</tr>
<tr>
<td>Finished High School D=1</td>
<td>44.0%</td>
<td>31.2%</td>
<td>22.8%</td>
</tr>
<tr>
<td>Income score</td>
<td>34.01</td>
<td>30.90</td>
<td>31.72</td>
</tr>
<tr>
<td>Gender (Male=1)</td>
<td>49.0%</td>
<td>47.6%</td>
<td>83.3%</td>
</tr>
<tr>
<td># HH members</td>
<td>3.90</td>
<td>4.28</td>
<td>4.47</td>
</tr>
<tr>
<td>Occupation: Working D=1</td>
<td>48.0%</td>
<td>47.3%</td>
<td>65.4%</td>
</tr>
<tr>
<td>Head of the household D=1</td>
<td>41.2%</td>
<td>47.1%</td>
<td>46.9%</td>
</tr>
<tr>
<td>Year of birth</td>
<td>1965</td>
<td>1966.9</td>
<td>1974.8</td>
</tr>
<tr>
<td>Marital status: Single D.</td>
<td>45.0%</td>
<td>34.7%</td>
<td>40.7%</td>
</tr>
<tr>
<td>Obs</td>
<td>26,757,687</td>
<td>16,195,178</td>
<td>89,257</td>
</tr>
<tr>
<td>Years of education for children</td>
<td>8.41</td>
<td>7.20</td>
<td>6.71</td>
</tr>
</tbody>
</table>

Notes: Columns 1-5 are group means. HHH: Head of the household, HS: High School. D: Dummy. Income Score: Score from 0 to 100, calculated using variables on income and education of the members of the household, size and characteristics of the house. Source: 2005 Census, SISBEN and criminal records.
Table 2: Convicted parents by incarceration and gender

<table>
<thead>
<tr>
<th>Convicted sample: by gender and incarceration status</th>
<th>Women</th>
<th>Men</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Years of education</td>
<td>6.50</td>
<td>6.06</td>
</tr>
<tr>
<td>Dummy Has HS degree =1</td>
<td>20%</td>
<td>16%</td>
</tr>
<tr>
<td>Income Score</td>
<td>17.2</td>
<td>16.1</td>
</tr>
<tr>
<td>Occupation: Dummy Working=1</td>
<td>45%</td>
<td>40%</td>
</tr>
<tr>
<td>Dummy head of the household=1</td>
<td>36.2%</td>
<td>37.1%</td>
</tr>
<tr>
<td>Age at sentence</td>
<td>35.5</td>
<td>36.2</td>
</tr>
<tr>
<td>Marital status: Dummy Single=1</td>
<td>47.8%</td>
<td>45.1%</td>
</tr>
<tr>
<td>Obs</td>
<td>9,375</td>
<td>6,028</td>
</tr>
</tbody>
</table>

Notes: Columns 1-4 are group means. HHH: Head of the household, HS: High School. D: Dummy. Income Score: Score from 0 to 100, calculated using variables on income and education of the members of the household, size and characteristics of the house. Source: SISBEN and criminal records.
Table 3: First stage - Parents

<table>
<thead>
<tr>
<th>Dep var: Decision Dummy</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conviction</td>
<td>Conviction</td>
<td>Incarceration</td>
<td>Incarceration</td>
</tr>
<tr>
<td>Judge Stringency</td>
<td>0.697***</td>
<td>0.697***</td>
<td>0.792***</td>
<td>0.786***</td>
</tr>
<tr>
<td></td>
<td>[0.0368]</td>
<td>[0.0368]</td>
<td>[0.0416]</td>
<td>[0.0430]</td>
</tr>
<tr>
<td>Controls</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>F stat*</td>
<td>4.4</td>
<td>4.0</td>
<td>3.9</td>
<td>4.2</td>
</tr>
<tr>
<td>Obs</td>
<td>233,050</td>
<td>116,062</td>
<td>91,854</td>
<td>90,774</td>
</tr>
<tr>
<td>Judges</td>
<td>392</td>
<td>392</td>
<td>262</td>
<td>262</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.124</td>
<td>0.124</td>
<td>0.243</td>
<td>0.242</td>
</tr>
<tr>
<td>adj. R-sq</td>
<td>0.118</td>
<td>0.118</td>
<td>0.238</td>
<td>0.237</td>
</tr>
</tbody>
</table>

Controls column 2: Gender, age, number of crimes, and crime category. Controls column 3: Years of education, gender, income score, year of birth, occupation, year of survey. Standard errors clustered at the judge level. Sources: Attorney General’s Office, criminal records and poverty census. Fstat is calculated from a regression on judge dummies.
Table 4: Balance test-Trial sample

<table>
<thead>
<tr>
<th></th>
<th>Judge: Conviction stringency</th>
<th>Judge: Incarceration stringency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dep. Var:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conviction / Incarceration stringency</td>
<td>0.0000024 [0.0000208]</td>
<td>0.00000914 [0.0000354]</td>
</tr>
<tr>
<td>Age</td>
<td>0.000324 [0.000509]</td>
<td>-0.000291 [0.000753]</td>
</tr>
<tr>
<td>Number of charges</td>
<td>0.000867 [0.000835]</td>
<td>0.000718 [0.00157]</td>
</tr>
<tr>
<td>Violent crime</td>
<td>-0.000293 [0.000805]</td>
<td>0.0014 [0.00129]</td>
</tr>
<tr>
<td>Property crime</td>
<td>0.00203 [0.00224]</td>
<td>0.00117 [0.00360]</td>
</tr>
<tr>
<td>Drugs related crime</td>
<td>-0.000927 [0.00157]</td>
<td>-0.00189 [0.00271]</td>
</tr>
<tr>
<td>Guns related crime</td>
<td>-0.000666 [0.00142]</td>
<td>-0.00101 [0.00213]</td>
</tr>
<tr>
<td>Misdeminour</td>
<td>-0.000867 [0.00112]</td>
<td>0.00139 [0.00183]</td>
</tr>
<tr>
<td><strong>Obs</strong></td>
<td>187,231</td>
<td>162,960</td>
</tr>
<tr>
<td><strong>Judges</strong></td>
<td>1,272</td>
<td>683</td>
</tr>
<tr>
<td><strong>F test</strong></td>
<td>0.52</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Standard errors clustered at the randomization unit/year level. Each rows corresponds to a different regression of judge leniency and defendant characteristics. When testing balance across crime categories I construct an alternative measure of conviction stringency that doesn't parse-out crime level conviction rates. The F-test corresponds to a regression where I include all the variables at the same time. Source Attorney General’s office and criminal records.
Table 5: Balance test II-Incarcerated sample

<table>
<thead>
<tr>
<th>Dep var: Incarceration FE</th>
<th>(1) 0.74&lt;Pc&lt;0.88</th>
<th>(2) 0.88&lt;Pc&lt;0.9</th>
<th>(3) 0.9&lt;Pc&lt;1</th>
<th>(4) Pooled Pc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of education</td>
<td>-0.0000292</td>
<td>-0.0000215</td>
<td>0.000274</td>
<td>0.000111</td>
</tr>
<tr>
<td></td>
<td>[0.000119]</td>
<td>[0.000136]</td>
<td>[0.000169]</td>
<td>[0.0000873]</td>
</tr>
<tr>
<td>Income score</td>
<td>-0.0000174</td>
<td>0.00000267</td>
<td>0.000013</td>
<td>0.0000106</td>
</tr>
<tr>
<td></td>
<td>[0.0000283]</td>
<td>[0.0000292]</td>
<td>[0.0000364]</td>
<td>[0.0000175]</td>
</tr>
<tr>
<td>Age at sentence</td>
<td>0.0000218</td>
<td>-2.08E-08</td>
<td>0.0000107</td>
<td>0.0000197</td>
</tr>
<tr>
<td></td>
<td>[0.0000338]</td>
<td>[0.0000320]</td>
<td>[0.0000435]</td>
<td>[0.0000266]</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.00142</td>
<td>0.001</td>
<td>-0.00212**</td>
<td>-0.00104</td>
</tr>
<tr>
<td></td>
<td>[0.00127]</td>
<td>[0.000793]</td>
<td>[0.00100]</td>
<td>[0.000633]</td>
</tr>
<tr>
<td>Years of education HH</td>
<td>-0.0000463</td>
<td>0.000106</td>
<td>-0.000153</td>
<td>-0.0000165</td>
</tr>
<tr>
<td></td>
<td>[0.000157]</td>
<td>[0.000136]</td>
<td>[0.000162]</td>
<td>[0.0000996]</td>
</tr>
<tr>
<td>D: Working</td>
<td>-0.0000919</td>
<td>-0.000981</td>
<td>0.000137</td>
<td>-0.000126</td>
</tr>
<tr>
<td></td>
<td>[0.000672]</td>
<td>[0.000763]</td>
<td>[0.00108]</td>
<td>[0.000493]</td>
</tr>
<tr>
<td>D: Studying</td>
<td>-0.0022</td>
<td>-0.000602</td>
<td>0.00103</td>
<td>0.00108</td>
</tr>
<tr>
<td></td>
<td>[0.00316]</td>
<td>[0.00278]</td>
<td>[0.00364]</td>
<td>[0.00199]</td>
</tr>
<tr>
<td>D: Both census surveys</td>
<td>-0.000844</td>
<td>-0.000942</td>
<td>0.000587</td>
<td>-0.000305</td>
</tr>
<tr>
<td></td>
<td>[0.000897]</td>
<td>[0.000634]</td>
<td>[0.000857]</td>
<td>[0.000488]</td>
</tr>
<tr>
<td>D: First survey</td>
<td>0.000355</td>
<td>0.000691</td>
<td>0.000648</td>
<td>0.000511</td>
</tr>
<tr>
<td></td>
<td>[0.00124]</td>
<td>[0.00123]</td>
<td>[0.00162]</td>
<td>[0.000800]</td>
</tr>
<tr>
<td>Constant</td>
<td>0.178*</td>
<td>-3.04E-01</td>
<td>6.64E-02</td>
<td>0.360***</td>
</tr>
<tr>
<td></td>
<td>[0.107]</td>
<td>[0.226]</td>
<td>[0.124]</td>
<td>[0.00594]</td>
</tr>
</tbody>
</table>

F Test                     | 0.8494            | 0.5001           | 0.564        | 0.5763       |
Obs                        | 16,684            | 17,416           | 15,845       | 49,945       |
R-sq                       | 0.128             | 0.149            | 0.137        | 0.03         |

Additional controls: Pc, Municipality FE, sentence year FE. Standard errors clustered at the randomization unit year level.
Table 6: OLS Regression

**Children with a convicted parent by age 14**

<table>
<thead>
<tr>
<th>OLS: no controls</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep var: Years of education</td>
<td>0.7&lt;Pc&lt;0.88</td>
<td>0.88&lt;Pc&lt;0.9</td>
<td>0.9&lt;Pc&lt;1</td>
<td>Pooled Pc</td>
</tr>
<tr>
<td>Parental Incarceration Dummy</td>
<td><strong>-0.400</strong>*</td>
<td><strong>-0.270</strong>*</td>
<td><strong>-0.408</strong>*</td>
<td><strong>-0.388</strong>*</td>
</tr>
<tr>
<td></td>
<td>[0.0776]</td>
<td>[0.0716]</td>
<td>[0.0727]</td>
<td>[0.0423]</td>
</tr>
<tr>
<td></td>
<td>[0.0695]</td>
<td>[0.0695]</td>
<td>[0.0718]</td>
<td>[0.0421]</td>
</tr>
<tr>
<td>Obs</td>
<td>17,347</td>
<td>18,672</td>
<td>17,045</td>
<td>53,718</td>
</tr>
<tr>
<td>Clusters</td>
<td>264</td>
<td>197</td>
<td>303</td>
<td>780</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.004</td>
<td>0.002</td>
<td>0.005</td>
<td>0.004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>OLS: Adding controls</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental Incarceration Dummy</td>
<td>-0.0675</td>
<td>-0.0938**</td>
<td>0.0134</td>
<td>-0.0572**</td>
</tr>
<tr>
<td></td>
<td>[0.0456]</td>
<td>[0.0398]</td>
<td>[0.0394]</td>
<td>[0.0242]</td>
</tr>
<tr>
<td>Constant</td>
<td>6.128***</td>
<td>8.911***</td>
<td>6.033***</td>
<td>8.668***</td>
</tr>
<tr>
<td></td>
<td>[1.263]</td>
<td>[3.244]</td>
<td>[0.748]</td>
<td>[2.635]</td>
</tr>
<tr>
<td>Obs</td>
<td>17,347</td>
<td>18,672</td>
<td>17,045</td>
<td>53,064</td>
</tr>
<tr>
<td>Clusters</td>
<td>264</td>
<td>197</td>
<td>303</td>
<td>764</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.408</td>
<td>0.386</td>
<td>0.391</td>
<td>0.387</td>
</tr>
</tbody>
</table>

Controls: Municipality FE, gender, YOB FE, SISBEN score, years of education of HH head, years of education incarcerated parent, gender of incarcerated parent, pc, year of sentence, birth order and year of survey. Column 4 in the second panel adds a second order polynomial on Pc with interactions. Sample: Children between 1990 and 2007 who had a convicted parent between ages 0 and 14. SE in brackets clustered at the judge level.
Table 7: Results: Reduced form and IV

<table>
<thead>
<tr>
<th>Reduced form</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep var: Years of education</td>
<td>0.7&lt;Pc&lt;0.88</td>
<td>0.88&lt;Pc&lt;0.9</td>
<td>0.9&lt;Pc&lt;1</td>
<td>Pooled Pc</td>
</tr>
<tr>
<td>Judge stringency</td>
<td>0.918**</td>
<td>0.593</td>
<td>0.558**</td>
<td>0.664***</td>
</tr>
<tr>
<td></td>
<td>[0.421]</td>
<td>[0.510]</td>
<td>[0.258]</td>
<td>[0.205]</td>
</tr>
<tr>
<td>Obs</td>
<td>17347</td>
<td>18672</td>
<td>17045</td>
<td>53064</td>
</tr>
<tr>
<td>Clusters</td>
<td>415</td>
<td>386</td>
<td>404</td>
<td>610</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.409</td>
<td>0.386</td>
<td>0.392</td>
<td>0.387</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IV Dep var: Years of education</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental Incarceration Dummy</td>
<td>0.770**</td>
<td>0.904</td>
<td>0.753*</td>
<td>0.842***</td>
</tr>
<tr>
<td></td>
<td>[0.356]</td>
<td>[0.761]</td>
<td>[0.399]</td>
<td>[0.282]</td>
</tr>
<tr>
<td>Obs</td>
<td>17,347</td>
<td>18,672</td>
<td>17,045</td>
<td>53,064</td>
</tr>
<tr>
<td>Clusters</td>
<td>264</td>
<td>197</td>
<td>303</td>
<td>764</td>
</tr>
</tbody>
</table>

Controls: Municipality FE, gender, YOB FE, Sisben score, years of education HH head, years of education of incarcerated parent, gender of incarcerated parent, pc, year of sentence, birth order and year of survey. Column 4 controls add a second order polynomial on Pc. Sample: Children between 1990 and 2007 who had a convicted parent between ages 0 and 14. SE in brackets clustered at the judge level. AR confidence interval result in the same significance levels.
Table 8: Heterogenous effects

<table>
<thead>
<tr>
<th>IV</th>
<th>Girls</th>
<th>Boys</th>
<th>Mother</th>
<th>Father</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Dep var: Years of education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parental Inc.</td>
<td>0.484*</td>
<td>1.278***</td>
<td>1.223**</td>
<td>0.745**</td>
</tr>
<tr>
<td></td>
<td>[0.258]</td>
<td>[0.452]</td>
<td>[0.592]</td>
<td>[0.307]</td>
</tr>
<tr>
<td>Obs</td>
<td>26148</td>
<td>26916</td>
<td>12019</td>
<td>41045</td>
</tr>
<tr>
<td>Age of the child</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-5 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parental Inc.</td>
<td>0.798**</td>
<td>0.57</td>
<td>1.384**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.297]</td>
<td>[0.337]</td>
<td>[0.615]</td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>18,630</td>
<td>23,505</td>
<td>13,019</td>
<td></td>
</tr>
<tr>
<td>5-10 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-15 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of crime</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Violent</td>
<td>1.341</td>
<td>0.855*</td>
<td>0.883</td>
<td>0.803</td>
</tr>
<tr>
<td></td>
<td>[1.178]</td>
<td>[0.505]</td>
<td>[0.663]</td>
<td>[0.506]</td>
</tr>
<tr>
<td>Property</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drug-related</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gun-related</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Misdemeanour</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parental Inc.</td>
<td>0.803</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.506]</td>
<td>[1.693]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs</td>
<td>10259</td>
<td>13385</td>
<td>12846</td>
<td>9937</td>
</tr>
<tr>
<td>Pooled Pc</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Controls: Municipality FE, gender, YOB FE, Sisben score, years of education head, years of education incarcerated parent, gender of incarcerated parent, pc, year of sentence, birth order and year of survey. Column 4 controls add a second order polynomial on Pc. Sample: Children between 1990 and 2007 who had a convicted parent between ages 0 and 14. SE in brackets clustered at the randomization unit and year level.
Table 9: Changes after incarceration

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dep var</td>
<td>LFP spouse</td>
<td>Income score</td>
<td>Years of educ. HHH</td>
<td>D: Male HHH</td>
<td># of people in HH</td>
<td>D: Lives w/ Grandparents</td>
<td>D: In 2nd SISBEN</td>
</tr>
<tr>
<td>Parental Inc.</td>
<td>0.0680***</td>
<td>-2.365***</td>
<td>0.103***</td>
<td>-0.0786***</td>
<td>-0.0996***</td>
<td>0.0196*</td>
<td>-0.0303***</td>
</tr>
<tr>
<td></td>
<td>[0.0187]</td>
<td>[0.193]</td>
<td>[0.0300]</td>
<td>[0.00604]</td>
<td>[0.0303]</td>
<td>[0.0110]</td>
<td>[0.00492]</td>
</tr>
<tr>
<td>Obs</td>
<td>9,673</td>
<td>82,779</td>
<td>82,779</td>
<td>82,779</td>
<td>81,615</td>
<td>16,372</td>
<td>32,881</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.22</td>
<td>0.75</td>
<td>0.20</td>
<td>0.19</td>
<td>0.33</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Mean dep var</td>
<td>0.399</td>
<td>26.41</td>
<td>5.1</td>
<td>0.595</td>
<td>4.659</td>
<td>0.215</td>
<td>0.242</td>
</tr>
<tr>
<td>St dev dep var</td>
<td>0.49</td>
<td>20.13</td>
<td>2.911</td>
<td>0.491</td>
<td>2.42</td>
<td>0.411</td>
<td>0.428</td>
</tr>
</tbody>
</table>

Panel regressions. Controls: Poverty score, years of education of HHH, Municipality FE and year of survey FE. Dummy for living with grandparents also includes uncles and cousins. Households with data on both cross-sections of the poverty census and who had an conviction episode in between surveys. Source: SISBEN and criminal records.
Figures

Figure 1: Prosecution and trial stages

Source: Colombian Penal proceedings code, Informe de la Comision Asesora de Politica Criminal (2012), SPOA and Criminal records.
Source: Criminal records. Selected crimes. I restrict to crimes with at least 100 cases.
A defendant is characterized by a point in the unitary square. A judge is defined by a pair of thresholds along each axis which determine treatment assignments. Defendants to the left of the conviction threshold are convicted, and those to the right are freed. Among the convicted, defendants below the incarceration threshold go to prison, and those above do not.
Figure 4: Identification under 4 types of judges

The left panel features harsh judges on the conviction margin ($P_c(High)$). This judges can be harsh ($P_l(High)$) or lenient ($P_l(Low)$) on the incarceration margin. We can identify the causal effect of incarceration for defendants in the shaded area. Those whose incarceration decision is only a function of judge assignment. The right panel is analogous and it features lenient judges on the conviction margin ($P_c(Low)$).
Figure 5: Judges’ fixed effects

Source: Attorney General’s office and criminal records. Raw rates are conviction/incarceration averages by judge. To construct the judge’s fixed effect I take the residual at the judge level after regressing conviction/incarceration on (demeaned) randomization unit/year dummies, (demeaned) crime-level conviction/incarceration rates, without a constant.
Figure 6: First stage

Source: Attorney General’s office and criminal records. Raw rates are conviction/incarceration averages by judge. To construct the judge’s fixed effect I take the residual at the judge level after regressing conviction/incarceration on (demeaned) randomization unit/year dummies, (demeaned) crime-level conviction/incarceration rates, without a constant.
Figure 7: Scatter plot: Judges’ fixed effects

Source: Attorney General’s office and criminal records. To construct the judge’s fixed effect I take the residual at the judge level after regressing conviction/incarceration on (demeaned) randomization unit/year dummies, (demeaned) crime-level conviction/incarceration rates, without a constant.
Notes: Histograms of parental incarceration judge stringency and the fitted value of local polynomial regressions of children’s educational attainment on judge stringency. Pooled regression I control for $p_c$. 

Figure 8: Reduced form
Notes: Following the LIV approach in Heckman and Vytlacil (2005) I regress \( Y_{educ} = \alpha + \beta_1 P_i + \beta_2 P_i^2 + \beta_3 X \). This graphs plots: \( \beta_1 P_i + 2\beta_2 P_i \) for the pooled regression.
Figure 10: Model of parenting and incarceration
A Appendix: Model and proofs

The model is described by the standard IV model that consists of five main random variables: $T, Z, Y, V, X$. Those variables lie in the probability space $(\Omega, F, P)$, where individuals are represented by elements $i \in \Omega$ of the sample space $\Omega$. The variables are defined below:

- $T_i$ denotes the assigned treatment of individual $i$, and takes values in $\text{supp}(T) = \{t_f, t_c, t_I\}$. $t_f$ stands for not convicted, $t_c$ for convicted but not incarcerated, and $t_I$ for convicted and incarcerated.
- $Z_i$ is the instrumental variable in this analysis and takes values in $\text{supp}(Z)$, and represents judge assignment.
- $Y_i$ denotes the outcome of interest for individual $i$, e.g. years of education of the child.
- $X_i$ represents the exogenous characteristics of individual $i$.
- $V_i$ stands for the random vector of unobserved characteristics of individual $i$, and takes values in $\text{supp}(V)$.

The random vector $V$ is the source of selection bias in this model. It causes both the treatment $T$ and outcome $Y$. The standard IV model is defined by two functions and an independence condition as follows:

Outcome Equation: $Y = f_Y(T, X, V, \epsilon_Y)$ \hspace{1cm} (13)

Treatment Equation: $T = f_T(Z, X, V)$ \hspace{1cm} (14)

Independence: $Z \perp V, \epsilon_Y | X$ \hspace{1cm} (15)

where $\epsilon_Y$ is an unobserved zero-mean error term associated with the outcome equation.

In this notation, a counterfactual outcome is defined by fixing $T$ to a value $t \in \text{supp}(T)$ in the outcome equation. That is, $Y(t) = f_Y(t, V, X, \epsilon_Y)$. The observed outcome for individual $i$ is given by:

$$Y = Y(T) = \sum_{t \in \{t_f, t_c, t_I\}} Y(t) \cdot 1[T = t].$$ \hspace{1cm} (16)

The independence condition (3) implies the following exclusion restriction:

$$Z \perp Y(t) | X \text{ for all } t \in \text{supp}(T).$$ \hspace{1cm} (17)

For the sake of notational simplicity, I suppress exogenous variables $X$ henceforth. All of the analysis can be understood as conditional on pre-treatment variables.

I assume that the treatment equation is governed by a combination of two threshold crossing inequalities. First, there is a conviction stage:

$$\begin{cases} 
\text{Free} & \text{if } 1[\phi_c(V) > \xi_c(Z)] \\
\text{Convicted} & \text{if } 1[\phi_c(V) \leq \xi_c(Z)] 
\end{cases}$$
where \( 1[\cdot] \) denotes a binary indicator and \( \phi_c(\cdot), \xi_c(\cdot) \) are real-valued functions. Function \( \phi_c(\cdot) \) measures the degree of culpability assessed by the judicial system. This function looks at variables and information that are not observed by the econometrician but that are observed by the judge, such as the evidence, crime intensity, the effort of the defense and prosecutor lawyers, as well as unobserved characteristics of the defendant such as aggression, anti-social behavior, etc. The function \( \xi_c(\cdot) \) assesses the judge leniency on conviction. This function can be understood as a threshold of reasonable doubt beyond which the defendant is convicted by the judge. Judges differ in their leniency and may set different threshold of evidence. The judge convicts defendant \( i \) whenever: \( \phi_c(V) \leq \xi_c(Z) \). If that is the case, a second stage is held and the judge makes a decision regarding incarceration:

\[
\begin{cases}
\text{Not incarcerated} & \text{if } 1[\phi_I(V) > \xi_I(Z)] \\
\text{Incarcerated} & \text{if } 1[\phi_I(V) \leq \xi_I(Z)]
\end{cases}
\]

Similarly, \( \phi_I(V) \) is a function whose arguments are case and defendant’s characteristics that are relevant for the assessment of the punishment level. Same as before, the judge compares \( \phi_I(V) \) to her/his threshold to incarcerate \( \xi_I(Z) \).

Treatment assignment can be summarized as follows:\(^{23}\)

\[
T = f_T(Z, V) = \begin{cases}
t_I & \text{if } 1[\phi_c(V) > \xi_c(Z)] \\
t_c & \text{if } 1[\phi_c(V) \leq \xi_c(Z)] \cdot 1[\phi_I(V) > \xi_I(Z)] \\
t_I & \text{if } 1[\phi_c(V) \leq \xi_c(Z)] \cdot 1[\phi_I(V) \leq \xi_I(Z)]
\end{cases}
\]

This model relies on two separable threshold functions that play the role of the monotonicity condition:\(^{24}\)

I assume the following standard regularity conditions: i) \( E(|Y(t)|) < \infty \) for all \( t \in supp(T) \), ii) \( P(T = t|Z = z) > 0 \) for all \( t \in supp(T) \) and all \( z \in supp(Z) \) and, iii) \((\phi_c(V), \phi_I(V))\) are absolutely continuous with respect to Lebesgue measure in \( \mathbb{R}^2 \). The first assumption guarantees the existence

\(^{23}\)See example 4 in Lee and Salanie (2017).

\(^{24}\)Consider two judges \( j \) and \( j' \), that see defendants \( i \) and \( i' \) who differ in their level of culpability. Say \( i' \) has more evidence against him than \( i \), namely \( \phi_c(i') < \phi_c(i) \). Supposed that judge \( j \) convicts defendant \( i' \) but not \( i \). Then the threshold function implies that it cannot be the case that judge \( j' \) convicts defendant \( i \), but not \( i' \). More generally, let \( D_i(j) = 1[T_i(j) = t_c] \) denote the binary indicator that judge \( j \) convicts defendant \( i \). Thus if judge \( j \) convicts \( i' \) but not \( i \), it implies:

\[
D_i(j) > D_{i'}(j)
\]

Then, it cannot be the case that judge \( j' \) convicts defendant \( i \), but not \( i' \). Which means:

\[
D_i(j) > D_{i'}(j) \implies D_i(j') > D_{i'}(j')
\]

which is equivalent to state that:

\[
D_i(j) > D_{i'}(j') \implies D_{i'}(j) > D_{i'}(j')
\]

We can generalize this to all individuals to arrive at the standard monotonicity assumption of Imbens and Angrist (1994).
of the expectation, the second one assures that there is a share of the population assigned to each
treatment group for every judge, and the third one allows me to apply the Lebesgue differentiation
theorem.

Without loss of generality, it is useful to express treatment assignment using the following
variable transformation:

\[ U^c = F_{\phi^c(V)}(\phi^c(V)) \sim Unif[0, 1], \]
\[ U^I = F_{\phi^I(V)}(\phi^I(V)) \sim Unif[0, 1], \]
\[ P_c = F_{\phi^c(V)}(\xi^c(Z)); z \in supp(Z), \]
\[ P_I = F_{\phi^I(V)}(\xi^I(Z)); z \in supp(Z), \]

where \( F_K(\cdot) \) denotes the cumulative distribution function of a random variable \( K \). \( U^c, U^I, P_c, P_I \)
are uniformly distributed random variables in \([0, 1]\) due to assumption (iii). Let \( P_c(z) \) denote the
conditional random variable \( P_c(Z = z) \) which is simply. Moreover, independence condition (3)
implies \( P_c, P_I \perp (U^c, U^I) \). In this notation, the model can be expressed as:

\[
T \equiv f_t(Z, V) = g_T(U^c, U^I, P_c, P_I) = \begin{cases} 
   t_f & \text{if } 1[U^c > P_c(z)] \\
   t_c & \text{if } 1[U^c \leq P_c(z)] \cdot 1[U^I > P_I(z)] \\
   t_I & \text{if } 1[U^c \leq P_c(z)] \cdot 1[U^I \leq P_I(z)]
\end{cases}
\]

In the model, \( U^c \) and \( U^I \) have the same interpretation as in the previous section, and \( P_c \) is
interpreted as the share convicted for judge \( z \). Moreover, under the assumption that \( U^c \perp U^I \),
we can identify \( P_I(z) \) from the data, that is:

\[ P(U^I < P_I(z)|U^c \leq P_c(z)) = P(U^I < P_I(z)) = P_I(Z) \]

The left hand side is observed from the data, the first equality follows directly from the in-
dependence assumption and the last one the uniform distribution of \( U^I \). \( P_I \) is interpreted as the
share incarcerated. For ease of exposition, I will first explore identification under this assumption
(see also Lee & Salanie, 2017) and then I will go over the results without it.

The goal is to identify and evaluate the treatment effect: \( E(Y(t_I) - Y(t_c)) \) which is a function
of counterfactual variables \( Y(t_I) \) and \( Y(t_c) \). To achieve this goal, it is useful to express the observed
expectations in terms of the variables that define the model:

\[
E(Y \cdot 1[T = t_c]|P_c(Z) = p_c, P_I(Z) = p_I) = \\
= E(Y(t_c) \cdot 1[T = t_c]|P_c(Z) = p_c, P_I(Z) = p_I) \\
= E(Y(t_c) \cdot 1[U^c \leq p_c] \cdot 1[U^I > p_I]|P_c(Z) = p_c, P_I(Z) = p_I) \\
= E(Y(t_c) \cdot 1[U^c \leq p_c] \cdot 1[U^I > p_I]) \\
= \int_0^{p_c} \int_{p_I}^1 E(Y(t_c)|U^c = u^c, U^I = u^I) f_{u^c u^I}(u^c, u^I) \, du^c \, du^I
\]

(28)
The main idea is that changes in \( P \) of Heckman and Vytlacil (1999). In Appendix B I explain graphically the intuition of this result. Looking at the derivative of the outcome variables with respect to square of \( u^c, u^I \) in Equation (30) traces the MTE of incarceration relative to conviction throughout the unitary and punishment assessments, \( U \) causal effect of incarceration versus conviction only, for the share of defendants whose culpability (MTE) of outcome \( Y \) factual outcome (RHS Eq. 29). We can use the same steps applied to counterfactual result gives me is a connection between the observed outcomes (Eq. 23) and the targeted counterfactual \( Y \) obtain the counterfactual for \( Y(t_c) \) to obtain the counterfactual for \( Y(t_I) \). Combining these two I obtain:

\[
\frac{\partial^2 E(Y \cdot 1[T = t_c]|P_c(Z) = p_c, P_I(Z) = p_I)}{\partial p_c \partial p_I} = -E(Y(t_c)|U^c = p_c, U^I = p_I) = E(Y(t_I) - Y(t_c)|U^c = p_c, U^I = p_I) \tag{29}
\]

Equality (29) arises as a direct application of the Lebesgue differentiation theorem. What this result gives me is a connection between the observed outcomes (Eq. 23) and the targeted counterfactual outcome (RHS Eq. 29). We can use the same steps applied to counterfactual \( Y(t_c) \) to obtain the counterfactual for \( Y(t_I) \). Combining these two I obtain:

\[
\frac{\partial^2 E(Y \cdot 1[T \in \{t_c, t_I\}]|P_c(Z) = p_c, P_I(Z) = p_I)}{\partial p_c \partial p_I} = E(Y(t_I) - Y(t_c)|U^c = p_c, U^I = p_I) \tag{30}
\]

In the language of Heckman and Vytlacil (2005), Eq.30 defines the marginal treatment effect (MTE) of outcome \( Y \) with respect to treatment assignment \( t_c \) and \( t_I \). It is interpreted as the causal effect of incarceration versus conviction only, for the share of defendants whose culpability and punishment assessments, \( U^c \) and \( U^I \) respectively, is set at quantiles \( p_c \) and \( p_I \). The derivative in Equation (30) traces the MTE of incarceration relative to conviction throughout the unitary square of \( U^c, U^I \). This result is an application of Lee and Salanie (2017) and extends the result of Heckman and Vytlacil (1999). In Appendix B I explain graphically the intuition of this result. The main idea is that changes in \( P_c \) and \( P_I \) affect exogenously treatment assignment. Then, by looking at the derivative of the outcome variables with respect to \( P_c \) and \( P_I \), we capture how the outcome variable changes when treatment changes at each point in the space of the unobservable confounding variables.

The average treatment effect (ATE) is the causal effect of \( t_c \) and \( t_I \) on \( Y \) in the population, and it corresponds to the integral of the MTE over the support of \( U^c \) and \( U^I \).

\[
E(Y(t_I) - Y(t_c)) = \int_0^1 \int_0^1 \frac{\partial^2 E(Y \cdot 1[T \in \{t_c, t_I\}]|P_c(Z) = p_c, P_I(Z) = p_I)}{\partial p_c \partial p_I} dp_c dp_I \tag{31}
\]

Without the assumption of independence of \( U^c \) and \( U^I \), variation in \( P_I \) is only identified once I fix the conviction threshold. Thus, the counterfactual of interest is now: \( Y(t_I) \) and \( Y(t_c) \) for those who were convicted under \( P_c = p_c \). This means the objective is to identify causal effects of
the form: $E(Y(t_I) - Y(t_c)|U^c < p_c)$, which is the same exercise explained in Section 4.1. Let:

$$E(Y \cdot 1[T = t_c]|P_c(Z) = p_c, P_t(Z) = p_t, U^c < p_c) =$$

$$= E(Y(t_c) \cdot 1[T = t_c]|P_c(Z) = p_c, P_t(Z) = p_t, U^c < p_c)$$

$$= E(Y(t_c) \cdot 1[U^I > p_t]|P_c(Z) = p_c, P_t(Z) = p_t, U^c < p_c)$$

$$= E(Y(t_c) \cdot 1[U^I > p_t]|U^c < p_c)$$

Where I followed the same steps as before. Let:

$$P^*_I = Pr[U^I < P_I|U^c < P_c] = G(P_I)$$

$P^*_I$ is the object I observe so I will define the observed expectations in terms of this variable:

$$E(Y(t_c) \cdot 1[U^I > G^{-1}(p^*_I|U^c < p_c)|U^c < p_c)$$

$$\int_{P^*_I} E(Y(t_c)|U^I = u^I, U^c < p_c)f_{u^I|U^c < p_c}(p^*_I)du^I$$

And applying the Lebesgue differentiation theorem this results in:

$$\frac{\partial E(Y \cdot 1[T \in \{t_c\}][p_c, P_t, U^c < p_c)}{\partial p_t} = -E(Y(t_c)|U^I = p_I, U^c < p_c)f_{u^I|U^c < p_c}(p^*_I)$$

And ultimately:

$$E(Y(t_I) - Y(t_c)|U^c < p_c) = \int_0^1 \frac{\partial E(Y \cdot 1[T \in \{t_c, t_I\}][P_c(Z) = p_c, P^*_I(Z) = p^*_I, U^c < p_c)dp^*_I}{\partial P^*_I}$$

What this result says is that we can trace the treatment effect of incarceration relative to conviction once we fix a threshold for conviction. We do this by evaluating the changes on the outcome variable when we change $P^*_I$. This delivers the MTE along the unobservable dimension $U^I|U^c < P_c$. The integral over the support of the instrument gives the LATE, or the ATE when the instrument has full support. In the next section I use this identification approach to estimate the effects of parental incarceration in my data.

**B Appendix: Intuition for the 2 dimension LATE**

In this section I go over the intuition of the results in eq. 18 and eq.19. This result extends the intuition behind LATE to a two-dimensional space. To make this point clear, let us think in discrete terms and use an example with 4 judges with threshold levels $\{P^1_c, P^1_I\}$, $\{P^2_c, P^2_I\}$, and $\{P^2_c, P^2_I\}$.

---

25Where $f_{u^I|U^c < p_c}(p^*_I)$ in eq. (39) corresponds to: $f_{u^I|U^c < p_c}(p^*_I)\frac{\partial P^*_I(p^*_I)}{\partial P^*_I}$. 
26Equivalent to $\{HL\}$, $\{HH\}$, $\{LH\}$, and $\{LL\}$ in Section 4.
For notation purposes, let:

\[ f(p_c, p_I) = E(Y \mathbb{1}[T \in \{ t_c \}]| P_c(Z) = p_c, P_I(Z) = p_I) \]  

and

\[ g(p_c, p_I) = E(Y \mathbb{1}[T \in \{ t_I \}]| P_c(Z) = p_c, P_I(Z) = p_I) \]  

Next, I can rewrite, in discrete terms, the identification result in equation 5 as:

\[ \Delta f(p_c, p_I) \Delta p_c + \Delta f(p_c, p_I) \Delta p_I = \left[ f(p_c^2, p_I^2) - f(p_c^1, p_I^2) \right] - \left[ f(p_c^2, p_I^1) - f(p_c^1, p_I^1) \right] + \left[ g(p_c^2, p_I^2) - g(p_c^1, p_I^2) \right] - \left[ g(p_c^2, p_I^1) - g(p_c^1, p_I^1) \right] = E(Y(t_I) - Y(t_c)| u_c = p_c, u_I = p_I) \]  

Now, let us go over each term in (31). First, \( f(p_c^2, p_I^2) \) represents the outcomes of convicted but not incarcerated individuals who had a judge with thresholds \( \{ P_c^2, P_I^2 \} \). Panel a in Figure C.3 shades the area in the \( u_c, u_I \) square that identifies these individuals. The next panels in Figure C.4 highlight the following terms in equation 8 and their differences. Ultimately, what equation (31) is doing is identifying the complier range in a two-dimensional space, which instead of an interval is a rectangle.

I estimate (18) by fitting a polynomial on \( p_I \) and \( p_c \) and evaluating the cross-derivative on the support of the instruments. Figure C5 shows the MTE in the relevant segment of the \( u_c, u_I \) square. There are some interesting features of these results; first, as before, as we increase \( u_I \) (defendants’ quality), the effect on years of schooling decreases, confirming that this positive effect is accrued when incarceration removes a bad parent from the household. What is new in Figure C.5 is that now we can also move along the \( u_c \) margin, or the "strength of the evidence" margin. The data also show that as evidence becomes weaker, the positive effects also decrease. Ultimately, what this exercise shows is that the effect on children is very sensitive to the type of case a judge is deciding on. In the case of Colombia, marginal incarcerations are of defendants still very negatively selected and with sufficient evidence against them, so that their children are better off without that parent. How this result extends to other settings is a function of the location of the marginal cases in the \( u_c, u_I \) square.

C Appendix : Data construction

In this appendix, I explain in detail the construction of the sample and variables I use throughout the paper. The starting point for my data construction are the two SISBEN surveys. These data are collected by the government to target social programs for the poor. The survey is conducted at the household level, and consists of two modules. In the first, it asks about the characteristics of the house (flooring material, number of bedrooms, etc), access to utilities, and assets in the households (TV, refrigerator, car, etc.). In the second part, all members of the household are listed with names and national identification numbers, and their relationship to the head of the household is specified. The questionnaire then asks about gender, age, education level, marital status, disability status,
and occupation. This survey is applied to everyone living in a municipality with a population of 30,000 or less, and in larger municipalities local authorities target households who could be potential beneficiaries of welfare programs. If a household is not targeted by local authorities and wishes to be surveyed, it can easily request to be included. The government uses this information to create a formula that measures the household’s ability to provide resources for its members, and computes a score for each household that determines eligibility for different social programs. These data provide me with i) identification numbers with municipality location to web-scrape criminal records and, ii) parent-to-child links.

I select the population of adults who lived in the 17 out of 33 municipalities that have criminal records online. These districts represent 67% of the population, and 69% of homicide and 83% of property crimes. I then web-scrape criminal records (from http://procesos.ramajudicial.gov.co/consultaprocessos/) by selecting the district and then searching individually for records with the ID numbers. From a 5% sample in which I look for criminal records in all 17 districts I estimate that I will miss 8.6% of the sample due to crimes committed in districts different from the one in the SISBEN.

I find 328,937 criminal records that belong to 256,366 individuals. I start by dropping observations that have missing values in year of sentence, crime or courtroom identifier (81,049 observations deleted). Next, I drop all records before 2005 (59,872 observations deleted), and all cases in which there is only one judge per district (4,635 observations deleted). I keep only the courtrooms for which there is data on convictions (14,786 observations deleted). Finally, I drop all observations where there are less than 15 cases in a year/judge cell (56,268 observations deleted). After this, I end up with 112,696 criminal records which correspond to 93,676 individuals. Table B.1 shows differences between the characteristics of individuals in the final data-set and those who were dropped. For the set of observations that have sentence data, I find that there is no evidence of differential incarceration rates across samples.

To assess how representative my sample is of the prison population, I compare counts of individuals sentenced by year from my data with counts of new inmates from official records of the Prison Authority (INPEC). I only have information available for 2015; according to INPEC, there were 27,287 new inmates that year, from my data, I find that 5,932 defendants were sent to prison, which would suggest that I have data on 22% of the prison population. This number, however, should be taken with caution, because INPEC data include flows of inmates across prisons, and I don't have data on the size of these flows.

I then link these convicts to the 436,309 individuals living in their households, of whom 179,699 are in the relevant cohort years (1991-2007), and 106,465 are the child of a convict. Of this, 67,770 experienced the sentencing episode between ages 0 and 14. Finally, I have education data for 52,419 (77%) of these children. This rate is close to the share of children between ages 12 and 17 who attend school, according to the census (76%). Table B.2 in the appendix shows evidence that a missing education record is not related to parental incarceration, but to the child’s not being at school or being working. Missing values are also more prevalent for boy, and for household with lower income and lower education of the head of the household.

27Judicial districts with online data: Armenia, Barranquilla, Bogota, Bucaramanga, Buga, Cali, Ibagué, Florencia, Manizales, Medellín, Neiva, Palmira, Pasto, Pereira, Popayán, Tunja, and Villavicencio.
Table B1: Sample selection—Defendants

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incarceration</td>
<td>0.00141</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.00204]</td>
<td></td>
</tr>
<tr>
<td>Years edu.</td>
<td>0.0018</td>
<td>0.00118</td>
</tr>
<tr>
<td></td>
<td>[0.00150]</td>
<td>[0.00157]</td>
</tr>
<tr>
<td>Income score</td>
<td>0.00118***</td>
<td>0.000837***</td>
</tr>
<tr>
<td></td>
<td>[0.0000822]</td>
<td>[0.0000879]</td>
</tr>
<tr>
<td>Male D.</td>
<td>-0.0400***</td>
<td>-0.0209***</td>
</tr>
<tr>
<td></td>
<td>[0.00279]</td>
<td>[0.00290]</td>
</tr>
<tr>
<td>Head HH D.</td>
<td>0.00877**</td>
<td>0.00771**</td>
</tr>
<tr>
<td></td>
<td>[0.00370]</td>
<td>[0.00389]</td>
</tr>
<tr>
<td>Single</td>
<td>-0.0298***</td>
<td>-0.0213***</td>
</tr>
<tr>
<td></td>
<td>[0.00222]</td>
<td>[0.00239]</td>
</tr>
<tr>
<td>Years edu. HHH</td>
<td>0.0004</td>
<td>0.000919</td>
</tr>
<tr>
<td></td>
<td>[0.00150]</td>
<td>[0.00157]</td>
</tr>
<tr>
<td>D: Studying</td>
<td>0.0264***</td>
<td>-0.00653</td>
</tr>
<tr>
<td></td>
<td>[0.00490]</td>
<td>[0.00486]</td>
</tr>
<tr>
<td>D: Working</td>
<td>0.0177***</td>
<td>0.0154***</td>
</tr>
<tr>
<td></td>
<td>[0.00209]</td>
<td>[0.00226]</td>
</tr>
<tr>
<td>Yob</td>
<td>-0.00708***</td>
<td>-0.00312***</td>
</tr>
<tr>
<td></td>
<td>[0.0000877]</td>
<td>[0.0000956]</td>
</tr>
<tr>
<td>Constant</td>
<td>14.55***</td>
<td>6.55E+00</td>
</tr>
<tr>
<td></td>
<td>[0.173]</td>
<td>[3279.3]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Obs</th>
<th>R-sq</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>260,968</td>
<td>196,314</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.306</td>
</tr>
</tbody>
</table>

Additional controls: Municipality FE and survey year FE. The first column includes all criminal records and the second restricts to the ones that have data on sentence length.
Table B2: Sample selection

<table>
<thead>
<tr>
<th>Dep var: Missing Educ.</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental incarceration</td>
<td>-0.00245</td>
<td>-0.00314</td>
<td>-0.00335</td>
</tr>
<tr>
<td></td>
<td>[0.00309]</td>
<td>[0.00309]</td>
<td>[0.00308]</td>
</tr>
<tr>
<td>Gender</td>
<td>0.00851***</td>
<td>0.00853***</td>
<td>0.00758***</td>
</tr>
<tr>
<td></td>
<td>[0.00282]</td>
<td>[0.00282]</td>
<td>[0.00280]</td>
</tr>
<tr>
<td>Yob</td>
<td>0.0205***</td>
<td>0.0200***</td>
<td>0.0125***</td>
</tr>
<tr>
<td></td>
<td>[0.000478]</td>
<td>[0.000479]</td>
<td>[0.000582]</td>
</tr>
<tr>
<td>Gender of the parent</td>
<td>-0.00366</td>
<td>0.00251</td>
<td>0.00186</td>
</tr>
<tr>
<td></td>
<td>[0.00344]</td>
<td>[0.00347]</td>
<td>[0.00346]</td>
</tr>
<tr>
<td>Income score</td>
<td>-0.000886***</td>
<td>-0.000582***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.000136]</td>
<td>[0.000136]</td>
<td></td>
</tr>
<tr>
<td>Years edu. HHH</td>
<td>-0.00493***</td>
<td>-0.00469***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.000620]</td>
<td>[0.000617]</td>
<td></td>
</tr>
<tr>
<td>D: Studying</td>
<td>-0.0872***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.00384]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D: Working</td>
<td></td>
<td>-0.0633</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.0583]</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-40.43***</td>
<td>-39.54***</td>
<td>-24.40***</td>
</tr>
<tr>
<td></td>
<td>[0.958]</td>
<td>[0.959]</td>
<td>[1.167]</td>
</tr>
<tr>
<td>Obs</td>
<td>65,125</td>
<td>65,125</td>
<td>65,125</td>
</tr>
<tr>
<td>R-sq</td>
<td>0.279</td>
<td>0.281</td>
<td>0.286</td>
</tr>
</tbody>
</table>

Additional controls: Municipality FE, survey year FE and birth order.
Appendix: Failure of the IV

Following the notation of Section 4.1, to illustrate the failure of the simple IV, I need to compute the share incarcerated for every judge as in the previous papers in this literature. Recall that those papers define only two treatment assignments: incarceration vs. everything else — which includes those convicted who receive probation, and those not convicted. For a judge type \((h^c, l^I)\), the probability of incarceration corresponds to: \(0.8 \times 0.2 = 0.16\) which is the same as the one for \((l^c, h^I)\). For judges type: \((h^c, h^I)\) is \(0.8 \times 0.8 = 0.64\), and for \((l^c, l^I)\) equals \(0.2 \times 0.2 = 0.04\). At first glance, it looks like we have exogenous variation in incarceration, which can serve as an instrument. However, what this exercise ignores is that the pool of defendants is not being held constant across judges, and as a result, differences will reflect not only the effect of incarceration but also the differences in the samples. Figure B1 plots a situation in which I use the variation in incarceration rates from judges \((h^c, h^I)\) and \((l^c, l^I)\). From the graph it is clear that there are not well defined groups for a valid comparison. This is because we are not observing the same group of people across judges. Specifically, defendants with \(U^c > 0.2\) are only observed for judge \((h^c, h^I)\), and as a result the IV estimates cannot deliver valid causal effects.
Figure B1: IV estimator

\begin{figure}
\centering
\includegraphics[width=\textwidth]{iv_estimator}
\caption{IV estimator}
\end{figure}

\section*{E Appendix: Extra tables and figures}

Table C1: Monotonicity

\begin{table}[ht]
\centering
\begin{tabular}{lcccccc}
\hline
& Males & Females & Violent & Not violent & Young & Old \\
\hline
Conviction-Judge FE & 0.789*** & 0.194*** & 0.164*** & 0.376*** & 0.334*** & 0.310*** \\
Out of sample & [0.0520] & [0.0102] & [0.00870] & [0.0208] & [0.0278] & [0.0198] \\
Obs & 20,665 & 147,066 & 143,567 & 75,345 & 50,267 & 70,042 \\
\hline
Incarceration-Judge FE & 0.587*** & 0.163*** & 0.0517*** & 0.189*** & 0.360*** & 0.451*** \\
Out of sample & [0.0565] & [0.0148] & [0.0163] & [0.0275] & [0.0237] & [0.0336] \\
Obs & 23,345 & 104,672 & 78,652 & 48,582 & 75,710 & 50,387 \\
\hline
\end{tabular}
\caption{Monotonicity test: Out of sample First stage}
\end{table}

I compute out of sample judge stringency measures and estimate first stage regressions.
Table C2: Random coefficients test

<table>
<thead>
<tr>
<th>Random coefficients for:</th>
<th>LR test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years of education</td>
<td>2.81</td>
<td>0.2452</td>
</tr>
<tr>
<td>Income score</td>
<td>1.82</td>
<td>0.4031</td>
</tr>
<tr>
<td>Head of the HH Dummy</td>
<td>0.00</td>
<td>0.9999</td>
</tr>
<tr>
<td>Single Dummy</td>
<td>5.49</td>
<td>0.0641</td>
</tr>
<tr>
<td>Working Dummy</td>
<td>-4.94</td>
<td>0.9999</td>
</tr>
<tr>
<td>Male is head of HH Dummy</td>
<td>5.81</td>
<td>0.0548</td>
</tr>
<tr>
<td>Sex Dummy</td>
<td>34.22</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Due to computational constraints I run this mixed effects logistic regression only for judges in Bogota which is the largest district.

Table C3: Sentencing guidelines

<table>
<thead>
<tr>
<th>Sentencing guidelines</th>
<th>Prison time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Crime</strong></td>
<td><strong>Colombia</strong></td>
</tr>
<tr>
<td>Possession of cocaine:</td>
<td>14</td>
</tr>
<tr>
<td>grams -100 grams</td>
<td>5 to 9 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assault</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple/third degree</td>
<td>1 to 3 years</td>
</tr>
<tr>
<td>2nd degree</td>
<td>2 to 7 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Theft</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>2 to 9 years</td>
</tr>
<tr>
<td>Aggravated theft</td>
<td>6 to 14 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Domestic violence</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic violence</td>
<td>4 to 8 years</td>
</tr>
</tbody>
</table>

Source: Colombia articles 376, 112 239, 240 of the penal code, respectively. For New York: 220.16, 120.00, 120.00, 155.25 or 165.40, 155.30 and 120.00 to 120.12 sections of New York penal law code, respectively.
Table C4: Placebo check

**Placebo test**

<table>
<thead>
<tr>
<th>Dep var: Years of education</th>
<th>OLS</th>
<th>RF</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental inc.</td>
<td>-0.0182***</td>
<td>0.0609</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.00705]</td>
<td>[0.187]</td>
<td></td>
</tr>
<tr>
<td>Judge leniency</td>
<td>0.0533</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.143]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>4.075</td>
<td>3.908</td>
<td>4.152</td>
</tr>
<tr>
<td></td>
<td>[4.106]</td>
<td>[4.103]</td>
<td>[4.085]</td>
</tr>
<tr>
<td>Obs</td>
<td>46,257</td>
<td>46,257</td>
<td>46,257</td>
</tr>
</tbody>
</table>

Controls: Municipality FE, gender, YOB FE, Sisben score, years of education head, years of education incarcerated parent, gender of incarcerated parent, pc, year of sentence, birth order and year of survey. Sample: Children between 1990 and 2007 who had a convicted parent between ages 0 and 14. SE in brackets clustered at the randomization unit and year level.

Table C5: LIML estimates

**IV LIML**

<table>
<thead>
<tr>
<th>Dep var: Years of education</th>
<th>(1) 0.7&lt;Pc&lt;0.88</th>
<th>(2) 0.88&lt;Pc&lt;0.9</th>
<th>(3) 0.9&lt;Pc&lt;1</th>
<th>(4) Pooled Pc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parental Incarceration</td>
<td><strong>0.741</strong></td>
<td><strong>0.89</strong></td>
<td><strong>0.748</strong></td>
<td><strong>0.827</strong></td>
</tr>
<tr>
<td></td>
<td>[0.371]</td>
<td>[0.834]</td>
<td>[0.356]</td>
<td>[0.280]</td>
</tr>
<tr>
<td>Obs</td>
<td>17,347</td>
<td>18,672</td>
<td>17,045</td>
<td>53,064</td>
</tr>
</tbody>
</table>

Controls: Municipality FE, gender, YOB FE, Sisben score, years of education head, years of education incarcerated parent, gender of incarcerated parent, pc, year of sentence, birth order and year of survey. Column 4 controls add a second order polynomial on Pc. Sample: Children between 1990 and 2007 who had a convicted parent between ages 0 and 14. SE in brackets clustered at the randomization unit and year level.
Notes: Histograms of parental incarceration judge stringency and the fitted value of local polynomial regressions of parental incarceration on judge stringency. I divide the sample by terciles of judge stringency in the conviction stage, and in the pooled regression I control for $p_c$. 
Figure C2: Treatment Effects by grade
Figure C3: Reduced form

Notes: Histograms of parental incarceration judge stringency and the fitted value of local polynomial regressions of children’s criminal records on judge stringency.
Figure C4: Rolling reduced form

Notes: Reduced form estimates of a sample size of 18,000, with a rolling window of 500 on $P_c$. Grey lines represent 90% confidence intervals.
Figure C5: Identification in 2 dimensions

\[ a. [f(p_1^2, p_1^1) - f(p_1^2, p_2^1)] - [f(p_2^1, p_1^1) - f(p_2^1, p_2^1)] \]

\[ b. [f(p_1^2, p_2^2) - f(p_2^1, p_2^1)] - [f(p_2^1, p_1^1) - f(p_1^2, p_1^1)] \]

\[ c. [f(p_1^2, p_2^1) - f(p_1^1, p_2^1)] - [f(p_2^1, p_2^1) - f(p_2^1, p_2^1)] \]

\[ d. [f(p_1^1, p_2^2) - f(p_1^2, p_1^1)] - [f(p_1^1, p_1^1) - f(p_1^2, p_1^1)] \]

\[ e. [f(p_1^1, p_2^2) - f(p_1^1, p_2^1)] - [f(p_1^2, p_2^1) - f(p_1^2, p_2^1)] \]

\[ f. [f(p_1^1, p_2^2) - f(p_1^1, p_2^1)] - [f(p_1^1, p_1^1) - f(p_1^2, p_1^1)] \]

\[ g. [f(p_1^2, p_2^2) - f(p_2^2, p_2^2)] - [f(p_2^2, p_2^1) - f(p_2^2, p_2^2)] \]
Figure C6: Compliers rectangle

\[
\frac{\Delta E[Y|U=U^c, T = t_c | p_c, p_I]}{\Delta p_c, \Delta p_I} - \frac{\Delta E[Y|U=U^c, T = t_c | p_I, p_c]}{\Delta p_c, \Delta p_I}
\]

Figure C7: Unconditional MTE

\[
\frac{\Delta E[Y|U=U^c, T = t_c | p_c, p_I]}{\Delta p_c, \Delta p_I} - \frac{\Delta E[Y|U=U^c, T = t_c | p_I, p_c]}{\Delta p_c, \Delta p_I}
\]