Exploring Arithmetic Strategy Instruction Interventions on Strategy Sophistication Growth and Fact Fluency

Amanda Cole

Executive Summary

The purpose of this study was to determine the effectiveness of an intervention tailored to individual students' addition strategy use would increase the sophistication of their strategy usage and their fact fluency in addition. The study was aimed to fill a gap in literature around assessing students on strategies and using the data to inform explicit strategy instruction (defined here as teaching arithmetic strategies through modeling and practice). Existing literature around the Mathematics Recovery Programme has examined its effects as it was rolled out in its entirety across the math curriculum, and required a large portion of the instructional day (Holliday, 2005; Holliday, 2006; Wiley, Holliday & Martland, 2007). There was a lack of literature on quick, specific interventions which could be put into place focusing only on strategy sophistication and the relationship between strategy usage and addition fact fluency. This study used primarily quantitative methods to track student's growth in sophisticated strategy usage and fact fluency across a six week intervention. The study showed that this intervention was effective in raising student's use of mathematical strategies, but the author was left with questions and points for further study in how the intervention related to fact fluency growth.

Problem

Within the specific classroom, students showed a heavy reliance on very simple arithmetic strategies, such as using fingers and drawing pictures. Higher level strategies had been introduced whole group, but the strategies students were exposed to did not take into account their current levels of strategy usage. In fact fluency assessments, students were continuing to rely on these simple strategies and were unable to efficiently and quickly complete addition facts.

Within the wider scope of current mathematical research, the problem of early mathematical difficulty takes on new meaning and makes the need for intervention much greater in the primary classroom. Researchers have found that early mathematical understandings have been shown as critical for later math performance. For example, the seminal Cockcroft Report (1982) found students who have low number knowledge in the primary school years tend to remain low throughout their schooling, with the knowledge gap between these students and average students only increasing. Shockingly, a three year gap between these groups of students early in school became a seven year difference after ten years of schooling. However, Dowker's (2004) research showed some hope for the state of mathematics education. Her research has shown that children's difficulty in early numeracy is highly influenced by intervention. This intervention is most effective when it focuses on specifically what students need rather than using a set curriculum or program. It is also very important that the intervention takes place as early as possible.

Hypothesis

It was hypothesized if students were grouped into homogeneous groups based on their current level of strategy sophistication and were given frequent small group intervention specifically on the next strategy level, they would be able to grow at least 1 strategy level in 6 weeks. It was also hypothesized that this intervention would lead to students' increase in the number of addition facts completed correctly in a minute (fact fluency). This hypothesis was formed using the data on strategy levels that already exists. This data shows a clear progression in strategy usage, and has been well researched.

Students initially use strategies that researchers refer to as direct modeling or counting all (Carpenter et al, 1999; Gervasoni & Sullivan, 2007). Children using this strategy have to put two sets of counters together and count from one to answer an addition problem. According to Carpenter et al (1999), "over time direct modeling strategies give way to more efficient counting strategies, which are generally more abstract ways of modeling a problem". Gervasoni & Sullivan's (2007) research identified counting all as a "on the way growth point" for first grade students. They maintain that first grade students who have not yet reached the counting all strategy are vulnerable, meaning they have not learned the skills that "underpin the initial mathematics curriculum in a particular domain and grade level". For the purpose of this study, students who had not yet reached this initial stage were given the strategy label "Pre Counting All" (SL 0). Carpenter et al (1999) refers to the next group of strategies as "counting strategies". Developmentally, children tend to develop the "counting on" strategy next. In this strategy, students count up from one of the addends the number represented by the other addend (Stock et al, 2007).

The most efficient strategy is utilizing mental math. Students "eventually come to rely on number facts, but the learning of number facts is not necessarily a rote skill" (Carpenter et al, 1999). While this strategy refers to students recalling the fact from memory, these facts build on a child's understanding of the relationship between numbers. Derived facts, using a known fact to help one solve an unknown number fact, are very important to eventually being able to answer number facts by memory (Carpenter et al, 1999). Examples of derived facts are using doubles (ie 8+9 is one more than 8+8) and decomposition of ten (ie 9+6 is the same thing as 10+5) (Carpenter et al, 1999; Verschaffel et al, 2007).

Methods

The participants in this study were 20 students in a general education classroom in New Orleans, LA. These students ranged in ability levels. Two of the students had been diagnosed with math delays and had specific IEP goals related to math. One student had been diagnosed as having a mild mental retardation, and also had specific IEP goals. Two other students were diagnosed with ADHD. There were other IEPs within the classroom related to speech and language, but those children did not have any specific math goals. These diagnoses demonstrate the inclusion nature of the classroom, and contributed to the wide range of ability levels.

This study was largely quantitative, and used The Mathematics Recovery Programme as the primary source of assessment. The portion of the assessment that was used in this study assessed the student's Stage of Early Arithmetical Learning (SEAL). The tasks in the assessment check the student's ability to complete addition problems using different levels of strategies. The first task asks the student to add two single digit numbers using mental math. A number card is placed over each corresponding number of cubes. If the student is unable to complete this level of the task, the assessor moves to an easier task, where the first number of cubes is still covered, but the second number of cubes is exposed. The student can use the extra cubes to "count on" from the first number. If this is also too difficult, the student moves to the easiest of the three tasks. In this task, the number cards are moved so that the student can see all the cubes (Wright, Martland, & Stafford, 2006).

Each task level corresponds with a strategy. These strategies are leveled from least sophisticated to most sophisticated based on previously cited research: Pre-Counting all, Counting all, Counting on, Counting on Over 10, and Mental Math. For students who were able to complete the mental math portion, a missing addend assessment was given also from The Mathematics Recovery Program to assess their ability to find the missing number in an addition sentence. Each student in the class was given the assessment as baseline data. Based on what strategy they used, they were placed in one of four leveled groups.

Students were also given a baseline timed mental math assessment. This assessment was 18 single digit addition problems with sums of no more than 12. In each assessment, each number (0-12) was used roughly the same number of times, with no more than 3 doubles problems. Students had one minute to complete as many problems as possible. These assessments were based off of the Math Learning Center's fact fluency assessment. The Math Learning Center's assessment recommends a goal of 10 math facts in a minute (Math Learning Center, 2011). This was the teacher's goal for each student, but the students were given 18 problems to challenge some of the students who were more advanced. Every other week after the initial assessment, students were given a similar, but not identical, math fluency assessment. A total of 3 fact fluency assessments were given. The teacher tracked the number of problems the students answered correctly.

Every day, the teacher met with one group, so that each group was met with at least once a week. However, the lower groups were met with twice a week. These group meetings were 15 minutes long. This time was in addition to the math block. According to Dowker (2004), the amount of time spent on individualized instruction does not have to be very long to also be effective. Moreover, while each group was met with no more than 30 minutes a week, this time was used to meet each child on his/her individual level, and thus was perceived to be enough time to reach the goal of the experiment. The objective of these group meetings was to give explicit instruction on how to get to the next strategy level. Every meeting with the students followed this timeline:

1-2 minutes: Teacher and students discuss what strategy they have been using. Teacher and students brainstorm why using the next strategy level will help the student in math. This was the time the teacher and students discussed how using a more sophisticated strategy can help the students do math problems more quickly, and thus excel on their biweekly mental math challenges and in more difficult mathematics in the next grade. The purpose of this lesson introduction is grounded in motivation research, which states the critical link to success is that the "why" is at the center (Sinek, 2009).

10 minutes: Teacher begins instruction on the student's SL goal, always working to move students from a SL 0 to 1, 1 to 2, etc. Teacher focused on scaffolding within a student's Zone of Proximal Development. Vygotsky (1978) originally coined this term and refers to it as "the distance between the actual

developmental level, as determined by independent problem solving, and the level of potential development, as determined through problem-solving under adult guidance, or in collaboration with more capable peers" (p. 86). Wiley, Holliday & Martland (2007) explain that the teacher's role is to: "select appropriate problems for the child, present them in a suitable setting, support the child successfully to find their own solution to the problem and help the child to reflect on what they are doing" (p. 112). The teacher would observe student behaviors as they attempted to solve addition problems, and use those responses and behaviors to select a teaching step that would push the child's thinking and prompt them towards a deeper understanding of the strategy. It is important to note that because this intervention focused on explicit strategy instruction, the teacher did directly model the strategy for the students. This is a difference between the MR teaching model, which rarely models how to get to a solution.

This 10 minutes was divided between explicit modeling and guided practice using manipulatives, white boards, number lines, and hundreds charts to encourage student participation and critical thinking. Students who had already reached SL 4 or 5 received instruction on more advanced strategies, primarily using derived facts.

Remaining minutes: Each student was given a problem from the teacher to do on their own using the next strategy level. These problems were set up in a similar way to the assessment.

At the end of the 6 weeks, students completed the Mathematics Recovery Programme assessment once more. This information was documented and analyzed to see what strategy level the student was doing at the end of the intervention. Because of unexpected extended absences for three students, only 17 students were assessments in the Mathematics Recovery Programme at the end of the intervention. The students were also given their final timed mental math challenge.

Data analysis was performed in Excel and Origin Pro. Students who had no room for growth within the assessments were partially excluded from data analysis. Students who were not assessed using the Mathematics Recovery Programme at the end of the intervention were not analyzed on that variable. Analysis of significance was found using a paired student's t test and a student's t test assuming unequal variances.

Results

The mean strategy level growth for students was 1.21 strategies, excluding the students who already were able to use mental math and complete the missing addend. A significant difference was found between baseline and post-intervention data with $p<.05$ ($p=.0002$). Student data was also grouped by baseline strategy level to determine if significant difference existed among the groups. The Pre-Count All group (strategy level (SL) 0) was uniformly distributed , each student grew 1 level. The Count All group (SL 1) showed the largest mean growth, 1.75 strategy levels. The Count On group (SL 2) averaged 1.5 SLs of growth. The Count On Over Ten group (SL 3) had a mean SL growth of 1. The two highest groups, Mental Math and Mental Math+Missing Addend (SLs 4-5) both had a mean of 0 growth. It is important to note that SL 5 is the highest assessed herein, and thus any growth was not assessed via Mathematics Recovery.

Baseline SL	Average Growth (in number of strategy levels)
SL 0: Pre Count All	1
SL 1: Count All	1.75
SL 2: Count On	1.5
SL 3: Count On Over 10	1
SL 4: Mental Math	0
SL 5: Mental Math + Missing Addend	0

Table 1: This table shows the average strategy growth (in number of levels) based on the intervention groups, which were grouped by baseline strategy levels.

The data showed the intervention was most effective for students of initial SL 1: count all and SL 2: count on. Visual inspection shows an obvious difference between student of initial SL 0-3 and SL 4-5. The difference in mean strategy growth was found to be significant between SLs 0-3 and 4-5 at $p<.05$ ($p=.0004$), tested with a student's t test.

Within the Fact Fluency assessment, the class, on average, improved in the number of problems students could complete in a minute from 9.75 problems per minute baseline to 12.53 post intervention.. Thus, on average students grew to meet the goal of 10 problems in a minute set out prior to the intervention. A significant difference was found between the baseline and post intervention fact fluency assessments with $p<.05$ ($p=.01$). For Further analysis, students who had no significant room for growth, defined as able to complete 17-18 problems per minute, were excluded from the fact fluency data. When those students were excluded, the data showed that at baseline, 14.29% of students were able to meet the goal of 10 addition problems in a minute. Post intervention, 50% met the goal. It should be noted no connection was found between the SL growth and the fact fluency growth.

The inconsistency between individual growth in SL and fact fluency presents more questions. When data was grouped based on initial strategy level and fact fluency growth, no significance was found. There also was no significant difference between fact fluency of growth among students who grew 0, 1 or 2 SL. Lastly, there was no significant difference in students with ending SLs 0-3 and 4-5 in their fact fluency growth.

To look deeper at the patterns in student growth, the timed mental math data was divided based on the students' ending strategy use. In other words, students who were counting all at the end of the intervention were placed in one data group to track that group's growth on the mental math challenges. Once the data was divided, some trends emerged. Some children's growth was flat across the mental math challenges. This was especially true of the Mental Math strategy group. Only one student in this growth saw any significant growth, while most other students were able to complete almost all 18 problems on the mental math assessments prior to the intervention. Other groups showed more inconsistency in their growth. This was especially true of the Count on Over 10 group. Within this group of 7 students, 3 showed no growth on the mental math assessments, while the other four showed growth ranging from 5 more problems in a minute to 10 more problems. This wide range was typical of the other ending strategy levels.

Conclusions

This study showed that an intervention on addition strategy levels was effective in increasing the average child's strategy level. The results exceeded the hypothesized mean growth of 1 strategy level. It also demonstrated that students with the middle strategy levels (2-3) were more impacted by the intervention. However, even strategy level 0 and strategy level 1 saw the hypothesized level of growth. Thus, this study supports the use of targeted instruction on strategy levels to increase students' use of sophisticated strategies.

Because no statistical significance was found for the fact fluency assessment, this data did not show that the intervention consistently impacted students' growth in fact fluency. This study also demonstrated that growth in sophistication of addition strategy levels and fact fluency do not have a linear relationship. The inconsistent growth in fact fluency between students in the same group raised the question of why students who use the same level of sophistication in math strategies would have such a wide range of ability in fact fluency. Dowker (2005) found that children have marked discrepancies between their ability levels in different math sub-components, and that these discrepancies can result from a variety of factors, including neurological, social, educational and cultural. Carpenter (1999) offers one specific conclusion: while students may be capable of sophisticated strategies, they may not use them in all situations. Therefore, students who use sophisticated strategies in un-timed settings may not be able to use them in timed assessments. Other possible reasons could be that deficits in working memory hinder students from excelling in fact fluency, which could particularly impact some of the students with identified developmental delays and mental retardation in this study (DeStefano & LeFevre, 2004). However, further study would need to be conducted on the specific relationship between strategies and fact fluency to draw any firm conclusions.

This study's results are limited by its small sample size. It is also limited because it only addresses strategy instruction for addition; subtraction strategies were not assessed. It is recommended to complete further study on how interventions on addition and subtraction strategies build and support each other. It is also recommended to complete a year long study where students can be tracked across more than one strategy development.

This study focused primarily on the way that small group intervention contributed to the students' mathematical growth. However, this study does not take into account the many ways in which students were engaging in problem solving throughout the rest of the math block. A full math curriculum can never be compartmentalized into 15 minutes. However, because these small group interventions focused on explicit, in depth instruction on specific strategies, the author contributes much of the growth to this specific block.

Citations

Carpenter, T. et al. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth: Heinemann.

Cockcroft, W.H. (1982). *Mathematics Counts*. London, HMSO.

DeStefano, D & LeFevre, J. (2004). The role of working memory in mental arithmetic. Journal of Cognitive Psychology, 16 (3), 353 - 386.

Dowker, A. (2004). What works for children with mathematical difficulties. Oxford, England: University of Oxford. Retrieved from: http://www.catchup.org.uk/LinkClick.aspx?fileticket=59GXj0uNY1A%3D&tabid=105

Dowker, A. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education.* Abingdon: Psychology Press.

Gervasoni, A & Sullivan, P. (2007). Assessing and teaching children who have difficulty learning arithmetic. *Educational and Child Psychology*, 24 (2), 40-53.

Holliday, A. (2005). Mathematics recovery evaluation of funding April 2004 to March 2005. Unpublished report of Cumbria Children's Services.

Holliday, A. (2006). Mathematics recovery evaluation April 2005 to March 2006. Unpublished report of Cumbria Children's Services.

Math Learning Center (2011). Retrieved from: http://www.mathlearningcenter.org/

Sinek, S. (2009). *Start with why: How great leaders inspire everyone to take action.* Richmond: Portfolio Hardcover.

Stock, P., Desoete, A., & Roeyers, H. (2007). Early markers for arithmetical difficulties. *Educational and Child Psychology*, 24 (2), 28-39.

Willey, R., Holliday, A., & Martland, J. (2007). Achieving new heights in Cumbria: Raising standards in early numeracy through mathematics recovery. *Educational and Child Psychology*, 24 (2), 108-118.

Wright, R., Martland, J. & Stafford, A. (2006a). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publishing/Sage.

Verschaffel, L. et al. (2007) Strategy flexibility in children with low achievement in mathematics. *Educational and Child Psychology*, 24(2), 16-27.

Vygotsky, L.S. (1978). *Mind in society: The development of higher mental processes.* Cambridge, MA: Harvard University Press.