

Identification with External Instruments in Structural VARs under Partial Invertibility

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Abstract

This paper discusses the conditions for identification with external instruments in Structural VARs under partial invertibility. We observe that in this case the shocks of interest and their effects can be recovered using an external instrument, provided that a condition of limited lag exogeneity holds. This condition is weaker than that required for LP-IV, and allows for recoverability of impact effects also under VAR misspecification. We assess our claims in a simulated environment, and provide an empirical application to the relevant case of identification of monetary policy shocks.

Keywords: Identification with External Instruments; Structural VAR; Invertibility; Monetary Policy Shocks.

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1 Introduction

A central endeavour in empirical macroeconomics is the study of the dynamic causal effects connecting macroeconomic variables. Since Sims (1980), this has been addressed by first fitting a reduced-form vector autoregression (VAR) to the data of interest, and then using a set of identifying assumptions in order to select a Structural VAR (SVAR) among the set of all models that can generate the variance covariance matrix of the reduced-form innovations. The structural shocks thus identified are thought of as the disturbances to the system of stochastic equations by which data are generated. The structural moving average, obtained by inverting the identified SVAR, allows inference on the dynamic causal effects, represented in the form of structural impulse response functions (IRFs).

An almost always maintained assumption in the Structural VAR literature is that of ‘fundamentalness’, or ‘invertibility’ of the structural shocks given the chosen model. This assumption implies that all the structural shocks can be accurately recovered from current and lagged values of the observed data included in the model. If this assumption is satisfied, VAR residuals are a linear transformation of the structural shocks, and given the variance-covariance matrix of the residuals, the causal relationships are identified up to an orthogonal matrix.¹ Much of creativity in the SVAR literature has been devoted to the formulation of appropriate assumption to inform the choice of this ‘rotation’ matrix.

In contrast with the standard statistical identifications, an important innovation in the more recent practice has seen the adoption of external instruments – that can be thought of as noisy observations of the shocks of interest –, for the identification of the dynamic causal effects. These instruments can be used either in conjunction with Structural VARs (SVAR-IV), or with direct regression methods, such as Jordà (2005)’s Local Projections (LP-IV).

¹A matrix Q is an orthogonal matrix if its transpose is equal to its inverse, i.e. $Q'Q = QQ' = \mathbb{I}_n$. The group $O(n)$ of the $n \times n$ orthogonal matrices is spanned by $n(n-1)/2$ unrestricted parameters.

As observed in Stock and Watson (2018), the assumption of invertibility is a crucial one in order to fully identify the system, both for the standard statistical identification, and for the modern external instrument approach.² However, often the researcher is only interested in ‘partially’ identifying the system, that is, in retrieving the dynamic effects of one or a subset of the structural shocks. In such a case, weaker conditions may suffice to identify a shock with an external instrument. Stock and Watson (2018) consider the case in which the shock of interest is observed – i.e. the case of a perfectly exogenous instrument –, and observe that in such a case the assumption of invertibility can be dispensed with for the validity of SVAR-IV. The intuition for this result is that, being the instrument strictly exogenous, the dynamic responses can be consistently estimated by a distributed lag regression.

This paper discusses the conditions for identification with external instruments in SVARs under the looser assumption of partial invertibility, and the more general case in which the shocks of interest are not perfectly observable. We show that, in general, fairly weak conditions are required. This allows to generalise the application of this method to many empirical cases in which, while some of the structural disturbances may be non-fundamental, the shock of interest is yet partially invertible.

First, we show that under partial invertibility a covariance-stationary stochastic vector process admits a ‘semi-structural’ representation that is the sum of two terms. The first one depends on the invertible shocks only. The second one is instead a function of linear combinations of the Wold innovations, and is orthogonal to the shock of interest. This result implies that if the VAR lag order correctly captures the autocorrelation structure of the Wold representation, the partially identified structural moving average produces impulse response functions that reveal the dynamic causal effects to the identified shock of interest.

²Stock and Watson (2018) observe that direct methods, such as local projections, do not need to explicitly assume invertibility of the system under strict exogeneity of the instrument. However, if lagged observables are required as control variables for an instrument that violates the lagged exogeneity condition, then, in general, the same invertibility conditions of a structural VAR are required.

Second, we observe that the use of VARs allows for identification under much weaker conditions on the instruments than those required with LP-IV methods. In fact, under partial invertibility, it is enough to assume that the instrument correlates with the VAR residuals only via the shock of interest. This weaker limited lag exogeneity condition allows for several sources of contamination of the instruments. In fact, the external instrument can be correlated with other shocks at different lags and leads (but not contemporaneously), as long as these do not enter the Wold innovations.

Third, we discuss the case of misspecification of the VAR model along several dimensions – lag order, missing moving average components, missing variables, missing higher order terms. We observe that while in this case the dynamic responses will in general be biased, the impact effects of the shock of interest are still correctly identified. In fact, given a valid instrument and a misspecified VAR, we should expect correct impact responses, but potentially biased dynamics. Conversely, an instrument contaminated by lagged shocks which are not ‘filtered out’ by the VAR would also yield biased impact responses.

Finally, we reckon that these latter observations give rise to an important empirical trade-off between efficiency and accuracy in SVAR-IV methods. As such, they also provide intuition for simple checks that can be used to assess the sources of lack of robustness in SVAR-IV systems. In fact, an instrument that fails the lagged exogeneity condition would produce unstable impact responses under the inclusion of additional variables in the system. Conversely, a valid instrument in conjunction with a misspecified system, such as one that omits variables relevant to the transmission of the shock, would result into unstable shapes of the dynamic responses, while still correctly retrieving the impact effects.

To assess our claims, we generate artificial data from a small standard New-Keynesian DSGE model with price stickiness and three shocks – monetary policy, government spending, and technology. Due to the introduction of a learning-by-doing

element in the law of motion for the technology process (Lippi and Reichlin, 1993), the system fails the ‘poor man’s invertibility condition’ of Fernandez-Villaverde et al. (2007) for a VAR in the observables, namely, output growth, inflation, and the interest rate. Hence, the structural shocks cannot all be recovered from the VAR residuals. Nonetheless, the system is partially invertible in the monetary policy shock. We show that an external instrument constructed as a noisy measure of the monetary policy shock can accurately recover impact responses in the simulated data. Conversely, if the external instrument for monetary policy is also contaminated by lagged technology shocks, whose space is not spanned by the VAR innovations by construction, then both the impact responses and the IRFs are severely biased.

Finally, in an empirical application, we test the hypothesis that contamination of the instrument can be revealed by the dependence of the impact responses on different information sets. We assess three variants of the high-frequency instruments popularised by Gürkaynak et al. (2005) to identify monetary policy shocks. Two of these, we show, are potentially contaminated by other past structural shocks. In fact, in these cases we show that impact responses depend on the model of choice. This can be thought of as an indication of the fact that the limited lag exogeneity condition is not satisfied by these instruments. The third instrument, defined as in Miranda-Agrippino and Ricco (2017) with a pre-whitening step to remove correlation with past shocks, recovers impact responses that are invariant to the VAR specification. This lends support to its exogeneity.

This paper builds on the recent literature on the use of external instruments in macroeconomics. This rapidly expanding research programme, surveyed in Ramey (2016), has produced, among other applications, a number of instruments for the identification of monetary policy (e.g. Romer and Romer, 2004; Gürkaynak et al., 2005; Gertler and Karadi, 2015; Miranda-Agrippino and Ricco, 2017; Paul, 2017), fiscal spending (e.g. Ramey, 2011; Ramey and Zubairy, 2014), tax (e.g. Romer and

Romer, 2010; Leeper et al., 2013; Mertens and Ravn, 2012), and oil shocks (e.g. Hamilton, 2003; Kilian, 2008).

The econometric technique supporting the use of SVAR-IV was first introduced by Stock (2008), and then explored in Stock and Watson (2012) and Mertens and Ravn (2013). As an alternative, IV can be used in direct regressions, with or without controls. This approach, that goes under the name of Local Projections-IV, has been proposed independently by Jordà et al. (2015) and Ramey and Zubairy (2014). The econometric conditions for instruments validity in the direct regression without control variables have first appeared in lecture notes by Mertens (2014), while conditions for instruments validity with control variables are discussed in Stock and Watson (2018). Stock and Watson (2018) have recently provided a thoughtful and unified discussion of the econometric theory supporting the use of external instruments in macroeconomics, and explored the connections between the SVAR-IV and LP-IV methods.

This paper is close in spirit to Forni et al. (2018) – that expands on the approach proposed by Giannone and Reichlin (2006) and results in Forni and Gambetti (2014) –, and studies the conditions under which a SVAR is informative enough to estimate the dynamic effects of a shock. While the two papers share the emphasis on partial invertibility (referred to in Forni et al. 2018 as informational sufficiency), this paper focuses on the recent debate on the use of IV in empirical macro, and on its interaction with misspecifications in the modelling choices.

The paper is organised as follows. In Section 2 we review the concepts of invertibility and fundamentalness and some other useful results in the literature. Section 3 discusses partial identification, and how this allows for semi-structural representations of covariance-stationary vector processes, while Section 4 proposes conditions for the identification of structural shocks in SVAR-IV under partial invertibility of the shock of interest. In Section 5 we analyse the case of misspecified systems. We

apply the concept discussed in this paper in Sections 6 and 7, where we study the identification of monetary policy shocks in a simulated environment with artificial data, and in an empirical application using US data, respectively. Finally, Section 8 concludes.

2 Non-Fundamental Representations

To introduce this concept, let us consider a covariance-stationary $n \times 1$ vector process Y_t , for which we assume a linear data generating process.³ A process of this form is a VARMA(p,q), i.e. a stationary solution of the stochastic difference equation

$$\Phi(L)Y_t = \Psi(L)u_t \quad u_t \sim \mathcal{WN}(0, \Sigma_u), \quad (1)$$

where $\Phi(L)$ and $\Psi(L)$ are generic autoregressive (AR) and moving average (MA) filters of order p and q respectively

$$\Phi(L) = \sum_{i=0}^p \Phi_i L^i, \quad \Psi(L) = \sum_{i=0}^q \Psi_i L^i, \quad (2)$$

and u_t are the ‘true’ innovations of the data generating process (i.e. the ‘structural’ innovations in the economic jargon), generally assumed to be orthogonal or orthonormal processes. If the process is causal – i.e., $\det(\Phi(L))$ has all roots outside the unit circle, $\det(\Phi(z)) \neq 0 \forall z = \zeta_i$ such that $|\zeta_i| < 1$ –, then it can be written as a (possibly infinite) MA

$$Y_t = \Theta(L)u_t, \quad u_t \sim \mathcal{WN}(0, \Sigma_u). \quad (3)$$

³In the economic literature, the issue of non-fundamentality (see Rozanov, 1967; Hannan, 1970) was first pointed out by Hansen and Sargent (1980, 1991) in a purely theoretical setting, while Lippi and Reichlin (1993, 1994) provided the first empirical application. Other more recent contributions on fundamentality in macro models are in Chari et al. (2004), Christiano et al. (2007) and Fernandez-Villaverde et al. (2007). A useful review is in Alessi et al. (2011).

If $\det(\Psi(z))$ – and hence $\det(\Theta(z))$ – have all roots outside the unit circle, i.e.

$$\det(\Theta(z)) \neq 0, \quad \forall z = \zeta_i \text{ s.t. } |\zeta_i| < 1, \quad (4)$$

then the process in Eq. (1) is ‘invertible’, and can be written as a VAR process

$$A(L)Y_t = \Theta_0 u_t . \quad (5)$$

If, instead, $\det(\Theta(z))$ has at least one root inside the unit circle, then the process in Eq. (1) is ‘non-invertible’, and u_t is said to be Y_t -non-fundamental.⁴

The Wold Representation Theorem guarantees that Y_t always admits a Wold decomposition of the form

$$Y_t = \eta_t + C(L)\nu_t \quad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu), \quad (6)$$

where $C(L)$ is a causal (no terms with $C_j \neq 0$ for $j < 0$), time-independent, square summable filter with $C_0 = \mathbb{I}_n$ and η_t is a deterministic term (that we will disregard in the following to focus on purely non-deterministic processes). ν_t is the innovation process – an uncorrelated sequence – to Y_t

$$\nu_t = Y_t - Proj(Y_t | Y_{t-1}, Y_{t-2}, \dots) , \quad (7)$$

that, by definition, belongs to the space generated by present and past values of Y_t . Given the invertibility of $C(L)$, we can rewrite Eq. (6) in a VAR form

$$A(L)Y_t = \nu_t \quad A_0 = \mathbb{I}_n . \quad (8)$$

If the Wold representation has absolute summable coefficients, then it admits a VAR representation with coefficient matrices that decay to zero rapidly; hence, it can be

⁴A borderline case is for $\det(\Theta(L))$ with at least one root on the unit circle. This case implies non-invertibility but not non-fundamentality.

well approximated by a finite order VAR process. This is always the case for causal finite-order ARMA processes.

If the structural shocks u_t are Y_t -fundamental, then u_t and ν_t generate the same space ($\mathcal{H}_t^u \equiv \mathcal{H}_t^\nu \forall t$). This implies that

$$\nu_t = \Theta_0 u_t , \quad (9)$$

where Θ_0 is non-singular. Hence, the structural disturbances u_t can be determined from current and lagged values of Y_t

$$u_t = Proj(u_t | Y_t, Y_{t-1}, \dots) . \quad (10)$$

If, however, the process is not invertible, and u_t is not Y_t -fundamental, the space generated by the VAR innovations does not coincide with that spanned by the structural shocks, i.e. $\mathcal{H}^\nu \neq \mathcal{H}^u$. The following result guarantees that the Wold and the structural MA representations (Eq. 3) are connected by a class of transformations generated by means of Blaschke matrices.

Theorem 1. *Let Y_t be a covariance-stationary vector process with rational spectral density, i.e. an ARMA process. Let $Y_t = C(L)\nu_t$ be a fundamental representation of Y_t , i.e.*

(i) ν_t is a white noise vector;

(ii) $C(L)$ is a matrix of rational functions in L with no poles of modulus smaller or equal to unity (Causality);

(iii) $\det(C(L))$ has no roots of modulus smaller than unity (Invertibility).

Let $Y_t = \Theta(L)u_t$ be any other MA representation, i.e. one which fulfils (i), and (ii), but not necessarily (iii). Then

$$C(L) = \Theta(L)B(L) ,$$

where $B(L)$ is a Blaschke matrix.

Blaschke matrices are filters capable to flip the roots of a fundamental representation inside the unit circle (see Lippi and Reichlin, 1994). A complex-valued matrix $B(z)$ is a Blaschke matrix if: (i) It has no poles inside the unit circle; (ii) $B(z)^{-1} = B^*(z^{-1})$, where $*$ indicates the complex conjugation.⁵ The following result guarantees that any Blaschke matrix can be written as the product of orthogonal matrices, and matrices with a Blaschke factor as one of their entries.

Theorem 2. *Let $B(z)$ be an $n \times n$ Blaschke matrix, then $\exists m \in \mathbb{N}$ and $\exists \zeta_i \in \mathbb{C}$ for $i = 1, \dots, m$ such that*

$$B(z) = \prod_{i=1}^m K(\zeta_i, L) R_i, \quad (11)$$

where R_i are orthogonal matrices, i.e. $R_i R_i' = \mathbb{I}_n$, and

$$K(\zeta_i, L) = \begin{pmatrix} \mathbb{I}_{n-1} & 0 \\ 0 & \frac{z - \zeta_i}{1 - \zeta_i^* z} \end{pmatrix}, \quad (12)$$

are matrices with a Blaschke factor as one of the entries.

The above results indicate that in general we can connect the structural and the Wold representation using a Blaschke matrix $B(L)$, that is

$$Y_t = \Theta(L)u_t = \Theta(L)B(L)^{-1}B(L)u_t = C(L)\nu_t, \quad (13)$$

where $B(L)$ flips the roots of the Wold fundamental representation inside the unit circle to obtain the structural MA. Hence,

$$\nu_t = \Theta_0 B(L)u_t. \quad (14)$$

In the case in which the structural representation is invertible, $B(L)$ is just the

⁵See Lippi and Reichlin (1994) for a proof of Theorems 1 and 2.

product of the orthogonal matrices R_i .

It is important to observe that, as it is clear from Eqs. (11-12), Blaschke factors may be acting only on a subset of the shocks. The remaining shocks can be recovered from current and past realisations of the variables are said to be partially invertible. We discuss this relevant case in the next section.

3 Partial Invertibility

The property of invertibility guarantees the identifiability of all the structural disturbances of a correctly specified VAR. Under invertibility, the problem of identification amounts to finding the correct matrix Θ_0 that connects the VAR residuals to the structural shocks as in Eq. (9). However, phenomena such as anticipation and foresight of economic shocks, which are often a feature of rational expectation models, can generate non-invertible representations (see e.g. Leeper et al., 2013). In such cases, the search for the correct Blaschke matrix can be a daunting problem (see Lippi and Reichlin, 1994).

In most empirical applications, however, often only a subset of the ‘structural’ innovations is of interest. For example, one may want to identify monetary policy shocks while not being interested in fully identifying the system. Let us consider the case in which one structural shock – u_t^1 – is partially invertible, and hence Y_t –fundamental

$$u_t^1 = Proj(u_t^1 | Y_t, Y_{t-1}, \dots) . \quad (15)$$

This implies that u_t^1 is a linear combination of the innovations ν_t

$$\kappa u_t^1 = \lambda' \nu_t , \quad (16)$$

where λ is an n -dimensional unit norm vector, i.e. $\lambda\lambda' = 1$, and κ is a constant of proportionality. Following from the discussion in the previous section, in this case

Eq. (14) would read

$$\nu_t = \Theta_0 B(L)u_t = \tilde{B}(L)u_t = [\tilde{b}_1 \ \tilde{b}_2(L)]u_t, \quad (17)$$

where \tilde{b}_1 is $n \times 1$, and $\tilde{b}_2(L)$ is a matrix of dimensions $n \times (n - 1)$ obtained as a combination of Blaschke factors and orthogonal transformations.

Let us consider a non singular matrix

$$\Lambda' = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \quad (18)$$

such that $\tilde{\lambda}'\lambda = 0_{(n-1) \times 1}$, and $\tilde{\lambda}'\tilde{\lambda} = \mathbb{I}_{n-1}$.⁶ Λ is an orthogonal matrix, $\Lambda'\Lambda = \Lambda\Lambda' = \mathbb{I}_n$.⁷ Also,

$$\Lambda'\nu_t = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} \nu_t = \begin{pmatrix} \kappa u_t^1 \\ \xi_t \end{pmatrix}. \quad (19)$$

$\xi_t \equiv \tilde{\lambda}'\nu_t$ is a combination of structural shocks, and is orthogonal to u_t^1 at different lags and leads, i.e. $\xi_t' u_t^1 = (\tilde{\lambda}'\nu_t)' u_t^1 \propto (\tilde{\lambda}'\nu_t)' \lambda'\nu_t = \nu_t' \tilde{\lambda} \lambda'\nu_t = 0$. It is worth noticing that while the requirement that ξ_t and u_t^1 are orthogonal is important, we do not require ξ_t to span the space of all the shocks orthogonal to u_t^1 .

Let us consider the representation obtained by acting with Λ on the reduced form

⁶It is possible to constructively obtain a non-singular matrix Λ by observing that since λ is normalised to be of unitary norm, it can be thought of as the first column of an orthogonal matrix. $\tilde{\lambda}$ has to live in the orthogonal complement subspace of \mathbb{R}^n of the space defined by λ . This space is spanned by a generic basis of $n - 1$ independent vectors of norm one, orthogonal to λ . Any such a base can be used as column vectors of $\tilde{\lambda}$. Λ is then non-singular, and an orthogonal matrix.

⁷This follows trivially from the assumptions on the sub-matrices λ and $\tilde{\lambda}$ and the choice of a non-singular Λ . First, observe that

$$\Lambda'\Lambda = \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} (\lambda \ \tilde{\lambda}) = \begin{pmatrix} \lambda'\lambda & \lambda'\tilde{\lambda} \\ \tilde{\lambda}'\lambda & \tilde{\lambda}'\tilde{\lambda} \end{pmatrix} = \begin{pmatrix} 1 & 0_{1 \times (n-1)} \\ 0_{(n-1) \times 1} & \mathbb{I}_{n-1} \end{pmatrix} = \mathbb{I}_n.$$

This also implies

$$\mathbb{I}_n = (\lambda \ \tilde{\lambda}) (\lambda \ \tilde{\lambda})^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix}^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = (\lambda \ \tilde{\lambda}) \left(\begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} (\lambda \ \tilde{\lambda}) \right)^{-1} \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = (\lambda \ \tilde{\lambda}) \begin{pmatrix} \lambda' \\ \tilde{\lambda}' \end{pmatrix} = \Lambda\Lambda'.$$

VAR representation in Eq. (8)

$$\Lambda' A(L)Y_t = \Lambda' \nu_t . \quad (20)$$

Eq. (20) is a ‘partially’ identified SVAR of the form

$$\Lambda' Y_t = \sum_{i=1}^k \Lambda' A_i Y_{t-i} + \begin{pmatrix} \kappa u_t^1 \\ \xi_t \end{pmatrix} . \quad (21)$$

A partially-identified MA is obtained by pre-multiplying Eq. (20) for $A(L)^{-1} \Lambda^{-1}$, where $\Lambda^{-1} = \Lambda = \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix}$, to get

$$Y_t = C(L) \begin{pmatrix} \lambda & \tilde{\lambda} \end{pmatrix} \begin{pmatrix} \kappa u_t^1 \\ \xi_t \end{pmatrix} = \kappa C(L) \lambda u_t^1 + C(L) \tilde{\lambda} \xi_t . \quad (22)$$

Eq. (22) implies that the Wold moving average can be factorised into two terms. The first one depends on the invertible shock u_t^1 , and the second one is a function of the $n - 1$ linear combinations of the Wold innovations orthogonal to u_t^1 . We summarise the above discussion in the following proposition.

Proposition 1. *Let the covariance stationary vector process Y_t be a solution to*

$$\Phi(L)Y_t = \Psi(L)u_t \quad u_t \sim \mathcal{WN}(0, \Sigma_u) , \quad (23)$$

and let $\Psi(L)$ be a non-invertible moving average filter, i.e. $\det(\Psi(z)) = 0$ for some ζ_i such that $|\zeta_i| < 1$. Let the Wold representation of Y_t be equal to

$$Y_t = C(L)\nu_t \quad \nu_t \sim \mathcal{WN}(0, \Sigma_\nu). \quad (24)$$

If the system is partially invertible in a shock u_t^i for some $i \in n$, viz. exists a unit-norm vector λ such that $\lambda' \nu = \kappa u_t^i$, then Y_t admits a semi-structural moving average

representation of the form

$$Y_t = \kappa C(L)\lambda u_t^i + C(L)\tilde{\lambda}\xi_t, \quad (25)$$

with $\mathbb{E}(u_t^i \xi_t') = 0$.

The above result implies that if the VAR has a correctly specified lag order, under partial invertibility the ‘partially’ identified SVAR impulse response functions reveal the dynamic causal effects to the identified shock u_t^i . The argument above can be readily extended to λ of dimension $n \times m$ for $m < n$.

4 Identification with External Instruments under Partial Invertibility

Let us consider a partially invertible VAR with reduced form representation as in Eq. (8), repeated below for convenience

$$A(L)Y_t = \nu_t \quad A_0 = \mathbb{I}_n. \quad (8)$$

Given an external instrument z_t , it is possible to identify u_t^1 and its effects on Y_{t+h} , $h = 0, \dots, H$, under the set of conditions in the following proposition.

Proposition 2. (Identification in SVAR-IV under Partial Invertibility)

Let z_t be an instrument for the shock u_t^1 that satisfies the following conditions:

- (i) $\mathbb{E}[u_t^1 z_t'] = \alpha$ (Relevance)
- (ii) $\mathbb{E}[u_t^{2:n} z_t'] = 0$ (Contemporaneous Exogeneity)
- (iii) $\mathbb{E}[u_{t+j}^k z_t'] = 0$ for all $\{j, k\}$ such that $E[u_{t+j}^k \nu_t] = 0$ and $\{j, k\} \neq \{0, 1\}$. (Limited Lag Exogeneity)

The impact effect λ of u_t^1 onto Y_t is identified (up to a scale) as

$$\lambda \propto \mathbb{E}[\nu_t z_t'].$$

The above conditions (i) and (ii) are the conventional relevance and exogeneity conditions of IV that are standard in the micro and macro literatures (see Stock and Watson, 2018). Condition (iii) arises because of the dynamics. If the system is invertible and the VAR correctly captures the data generating process of Y_t , then the third condition is trivially satisfied, since ν_t is in this case a linear combination of the elements of u_t only at time t . However, in the case of partial invertibility, ν_t are linear combinations of (some of the) past and future shocks as well. Hence, identification with an external instrument is possible only as long as the instrument is contaminated only by past and future shocks that do not appear in ν_t . In other words, if the instrument only depends on other shocks that are already filtered out by the VAR – i.e. $\mathbb{E}[\xi_t z_t'] = 0$ where ξ_t is defined as in Eq. (25). This is a relatively stronger condition than that required for a well specified and invertible SVAR (where lag exogeneity is not required), but still much weaker than standard LP-IV conditions. In fact, these require a strong lag exogeneity condition (iii') whereby $\mathbb{E}[u_{t+j}^k z_t'] = 0$ for all $j \neq 0$ and for all $k \neq 1$. Importantly, under partial invertibility, the impact effects of the shock of interest will be correctly recovered also in a misspecified VAR, as long as Condition (iii) holds. We discuss this case in the next section.

5 An Observation on VAR Misspecifications

Let us consider a purely nondeterministic, stationary VARMA(p,q) process $Y_t = (y_{1,t}' \ y_{2,t}')'$

$$\begin{pmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} \Psi_1(L) \\ \Psi_2(L) \end{pmatrix} u_t. \quad (26)$$

For the m -dimensional subprocess $y_{1,t} = JY_t$, where $J_t = (\mathbb{I}_m \ 0_{n-m})$ is a selector matrix, we can write

$$\Phi_{11}(L)y_{1,t} = -\Phi_{12}(L)y_{2,t} + \Psi_1(L)u_t . \quad (27)$$

The Wold Representation Theorem implies that also $y_{1,t}$ has an invertible MA representation. In fact, if Y_t is covariance-stationary, $y_{1,t}$ is also covariance stationary, with first and second moments respectively equal to $\mathbb{E}(y_{1,t}) = J\mathbb{E}(Y_t)$, and $\Gamma_{y_1}(h) = J\Gamma_Y(h)J'$, where $\Gamma(h)$ is the autocovariance of Y_t at lag h . The Wold Representation Theorem also guarantees the existence of an ARMA representation of the form

$$\tilde{\Phi}_1(L)y_{1,t} = \tilde{\Psi}_1(L)\nu_t . \quad (28)$$

The true innovations u_t are trivially non-invertible in $y_{1,t}$. In fact, the n innovations u_t are compounded and reduced to the $m < n$ innovations ν_t which do not have a meaningful structural interpretation. However, the presence of a Wold representation guarantees that if the system is partially invertible in a shock u_t^i , i.e. exists λ such that $\lambda'\nu = \kappa u_t^i$, then it is possible to retrieve impact effects of the shock u_t^i onto $y_{1,t}$ as discussed in the previous section. Due to the misspecification and the resulting bias in the estimated VAR coefficients, the dynamic responses estimated from the VAR are going to be biased, as discussed in Braun and Mittnik (1993). Direct methods à la Jordà (2005) can in such cases be used to improve over VAR estimates.

6 Responses From A Simulated System

We simulate data from a simple New Keynesian model that features (i) a representative infinitely-lived household that chooses between consumption and leisure; (ii) firms that produce a continuum of goods using a Cobb-Douglas technology to aggregate capital and labour; (iii) a government that uses a share of output for wasteful public spending; and (iv) a central bank that sets the interest rate using a Taylor rule

with smoothing. There are three stochastic disturbances that generate fluctuations in the observables, namely, a monetary policy shock u_t^r , a government spending shock u_t^g , and a technology shock u_t^a . The processes for technology and the policy rate are defined as follows. Log technology a_t evolves with a learning-by-doing term as

$$a_t = \rho_a a_{t-1} + u_t^a + \omega u_{t-1}^a , \quad (29)$$

where u_t^a is an i.i.d normally distributed technology shock, and $\omega > 1$ is the learning-by-doing parameter that produces a non-invertible moving average. The monetary authority sets the nominal interest rate by a Taylor rule with smoothing

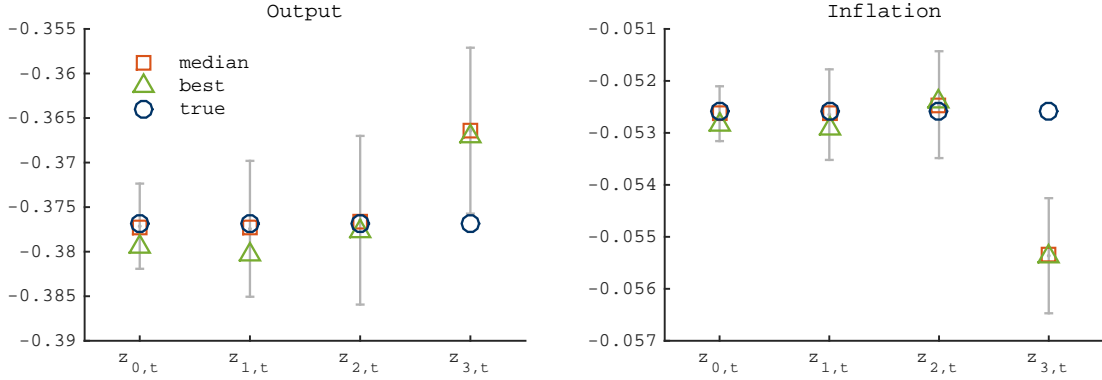
$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\phi_\pi \bar{\pi}_t + \phi_y \overline{\Delta y}_t) + u_t^r , \quad (30)$$

where $\bar{\pi}_t$ is the average inflation over the last four periods, $\overline{\Delta y}_t$ is the average output growth, and u_t^r is an white noise i.i.d normally distributed monetary policy shock. The monetary policy innovation is partially invertible, and can be recovered from current and past values of the policy rate, inflation and output. All the model details, including the calibrated parameters, are reported in Appendix A.

We consider a VAR in output growth, inflation, and the policy interest rate. Under the chosen set of parameters, the model fails the ‘poor man’s invertibility condition’ of Fernandez-Villaverde et al. (2007); hence, the three structural shocks cannot all be recovered from a VAR in the observables. However, as observed, the system is invertible in the monetary policy shock.

From the model, we simulate 1000 economies each of sample size $T = 300$ periods. For each set of simulated data, we then estimate a VAR(p) in the three observables, and identify the monetary policy shock using the following four different external instruments:

FIGURE 1: IMPACT RESPONSES TO MONETARY POLICY SHOCK



Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(2) in three observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bars from the distribution of impact responses across 1000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).

$$z_{0,t} = u_t^r, \quad (31)$$

$$z_{1,t} = \frac{4}{5}u_t^r + \varsigma_t, \quad (32)$$

$$z_{2,t} = \frac{4}{5}u_t^r + \frac{3}{5}(u_{t-1}^g + u_{t-2}^g + u_{t-3}^g) + \varsigma_t, \quad (33)$$

$$z_{3,t} = \frac{4}{5}u_t^r + \frac{3}{5}(u_{t-1}^a + u_{t-2}^a + u_{t-3}^a) + \varsigma_t. \quad (34)$$

In Eq. (31) the shock is perfectly observable. This is the case discussed in Stock and Watson (2018). The instrument in Eq. (32) is an instrument contaminated by classic white noise measurement error. The instruments in Eqs. (33-34) both fail the lag exogeneity condition. In fact, while $z_{2,t}$ is contaminated by lagged spending shocks, $z_{3,t}$ correlates with lagged technology shocks. In all cases, ς_t is a normally distributed random measurement error with zero mean and variance equal to that of the structural shocks. A VAR(2) captures the model's dynamics sufficiently well. Hence, we use $p = 2$ as the benchmark case, but discuss also the cases $p = 4$ and $p = 1$.

Impact responses for output and inflation recovered from the 4 instruments and a VAR(2) are in Figure 1.⁸ In each subplot, we use blue circles for the model’s responses (true), orange squares for the median across simulations, and green triangles for the simulation which is the closest to the median (best).⁹ The error bars are two standard deviations intervals constructed from the distribution across simulations. A few elements are worth highlighting. As also noted in Stock and Watson (2018), when the shock is observable ($z_{0,t}$), the assumption of full invertibility can be dispensed with for the validity of SVAR-IV. However, the shock is correctly recovered also under the milder conditions introduced in Section 4. In fact, correct impact responses are recovered also with $z_{1,t}$. The introduction of a measurement error in $z_{1,t}$ widens the distribution of impact responses across simulations, but recovers the correct impact effects. The picture changes substantially when we consider the case of $z_{3,t}$. In this case, the instrument correlates with lagged non-invertible technology shocks which the data in the VAR cannot provide sufficient information for by construction. This results in severely biased impact responses.

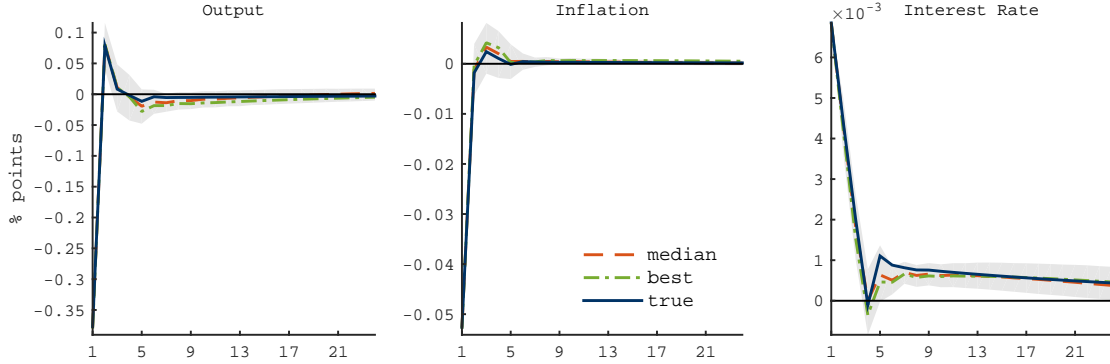
Finally, an interesting case arises in the case in which the instrument also correlates with lagged spending shocks ($z_{2,t}$). The spending shock is not invertible in the system, however, it is responsible for a negligible share of the variance of the simulated variables. Hence, in this case the impact responses are virtually correctly recovered.

The discussion extends in an equivalent way for responses at farther away horizons. Figure 2 reports responses estimated over 24 periods using $z_{1,t}$ (Panel A, top), $z_{2,t}$ (Panel B, centre), and $z_{3,t}$ (Panel C, bottom). In the first two cases the model’s responses lie comfortably within the bands generated across the simulations. On the contrary, the response of both output and the policy rate are well outside the

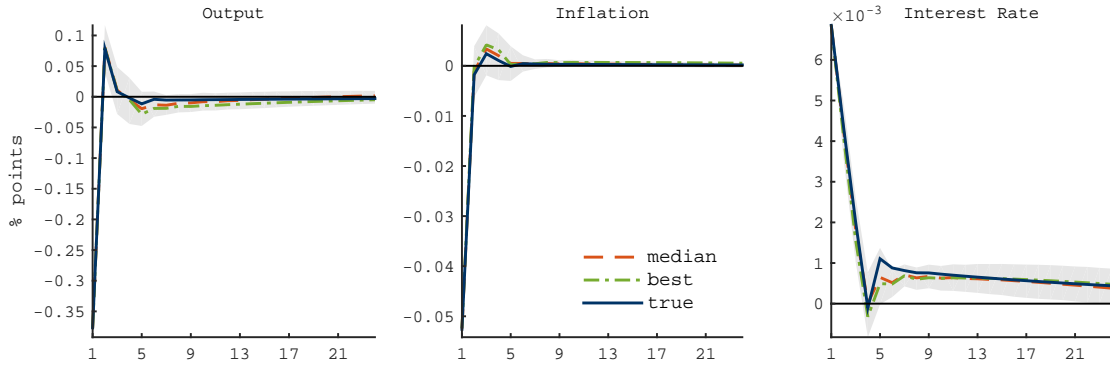
⁸IRFs are normalised such that the impact response of the policy rate to a monetary policy shock equals that of the model.

⁹We select the simulation whose IRFs minimise the sum of square deviations from median IRFs over the first 4 periods. The choice is to avoid overweighting longer horizon responses which are essentially zero, and put more weight at shorter ones, where responses display richer dynamics. Changing the truncation horizon to either 6 or 12 periods yields qualitatively similar results.

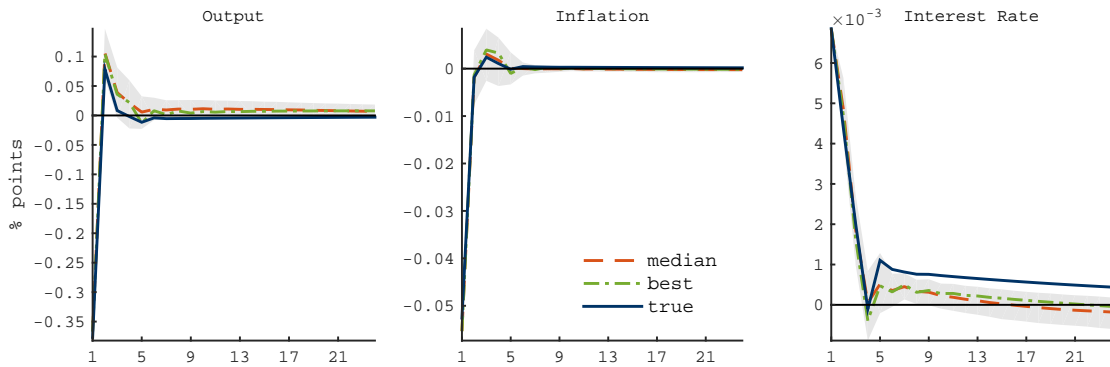
FIGURE 2: RESPONSES TO MONETARY POLICY SHOCK – SIMULATION



(A) $z_{1,t}$: external instrument correlates with monetary policy shock only



(B) $z_{2,t}$: external instrument also correlates with lagged spending shocks



(C) $z_{3,t}$: external instrument also correlates with lagged technology shocks

Notes: Impulse responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(2) in three observables. Instrument correlates with monetary policy shock only (Panel A). Instrument correlates with monetary policy shocks and lagged spending shocks (Panel B). Instrument correlates with monetary policy shocks and lagged technology shocks (Panel C). Grey shaded areas denote 90th quantiles of the distribution of IRFs across 1000 simulated economies of sample size $T = 300$ periods. Model responses (true, blue solid), median across simulations (orange dashed), minimum distance from median (best) simulation (green dash-dotted).

simulation confidence region when the impact responses are estimated using $z_{3,t}$.

When the VAR is estimated with one lag only, the difference among the impact responses estimated using the different instruments become starker, as the bias introduced by $z_{3,t}$ is more pronounced (Figure B.1 in the Appendix, top panel). In this case the use of one lag only introduces another degree of misspecification of the type discussed in Section 5. In fact, we note that while impact responses are still correctly recovered using both $z_{1,t}$ and $z_{2,t}$, the VAR dynamic places model responses outside the confidence region at short horizons for both output growth and the policy rate (Figure B.1, bottom panel). Not surprisingly, the inclusion of more lags can help mitigating the effect of misspecifications. In particular, including more lags can help producing ‘whiter’ VAR innovations, which make the limited lag exogeneity condition somewhat easier to achieve. This can be seen in Figure B.2 where impact responses are now computed using a VAR(4).¹⁰ The inclusion of more lags removes the bias in the impact response of inflation to a monetary policy shock. However, enriching the VAR specification is not per se sufficient to remove all the distortions altogether: even with 4 lags, the model response of output still lies at the edge of the simulated distribution of impact responses.

7 IV Identification of Monetary Policy Shocks

In this section we use an empirical application to test the hypothesis that contamination of the instrument can be revealed by the dependence of the impact responses on different information sets. We compare responses to monetary policy shocks identified with different external instruments in an informationally sufficient VAR, and in a misspecified VAR that (*i*) omits variables that are relevant for the transmission of monetary policy, and (*ii*) severely understates the lag order.

We consider three external instruments, all of which are constructed as a variant of

¹⁰Impulse response functions are not reported due to space considerations, but are available upon request.

the high-frequency surprises in Gürkaynak et al. (2005), and measure monetary policy shocks using the surprise reaction of federal funds futures markets around FOMC announcements, following the insight of Kuttner (2001). The first such instrument is constructed by summing up high-frequency surprises around FOMC meetings within each month from 1990 to 2012. This is equivalent to the instrument used in e.g. Stock and Watson (2018) and Caldara and Herbst (2018), and we denote it by $z_{A,t}$. The second instrument is a monthly moving average of high-frequency surprises around FOMC announcements from 1990 to 2012. This is the instrument in Gertler and Karadi (2015), denoted $z_{B,t}$. The third external instrument – $z_{C,t}$ – is the residual of a projection of high-frequency surprises (constructed as $z_{A,t}$) onto their lags and Fed Greenbook forecasts from 1990 to 2009 (see discussion in Miranda-Agrippino and Ricco, 2017). This projection can be seen as a pre-whitening step that removes contamination with past and contemporaneous shocks.

Table 1 reports Granger causality tests for the three instruments on macroeconomic and financial factors estimated from the monthly dataset in McCracken and Ng (2015), that include a constant and one lag of the selected instrument (see Miranda-Agrippino and Ricco, 2017). Results point to possible contamination of the $z_{A,t}$ and $z_{B,t}$ instruments by lagged macroeconomic shocks. This serves as motivation for our next exercise.

We evaluate the effect of the instruments' contamination on the estimation of the IRFs in the empirical setup of Gertler and Karadi (2015), where the effects of monetary policy shocks are estimated in a monthly VAR(12) from 1979:7 to 2012:6, and consisting of the one-year government bond rate as the policy variable, an index of industrial production, the consumer price index, a commodity price index, and the excess bond premium (EBP) of Gilchrist and Zakrajšek (2012).¹¹ Stock and Watson (2018) show that in this system there is no statistically significant evidence against

¹¹Data for bond yields, industrial production, and the consumer price index are from the St Louis FRED Database, the commodity price index is from the Commodity Research Bureau, the EBP data are from the Federal Reserve Board.

TABLE 1: CONTAMINATION OF MONETARY POLICY INSTRUMENTS

	$z_{A,t}$		$z_{B,t}$		$z_{C,t}$	
$f_{1,t-1}$	-0.012	[-1.97]*	-0.011	[-2.74]***	0.006	[0.98]
$f_{2,t-1}$	0.001	[0.38]	0.004	[1.79]*	0.005	[1.56]
$f_{3,t-1}$	0.002	[0.41]	-0.001	[-0.23]	0.001	[0.29]
$f_{4,t-1}$	0.015	[2.09]**	0.008	[1.92]*	0.005	[0.70]
$f_{5,t-1}$	0.002	[0.26]	0.001	[0.12]	0.008	[1.18]
$f_{6,t-1}$	-0.011	[-2.19]**	-0.007	[-2.58]**	-0.008	[-1.63]
$f_{7,t-1}$	-0.010	[-1.69]*	-0.006	[-1.40]	-0.004	[-0.54]
$f_{8,t-1}$	-0.001	[-0.35]	0.001	[0.32]	-0.001	[-0.15]
$f_{9,t-1}$	-0.002	[-0.59]	-0.002	[-0.53]	0.000	[0.07]
$f_{10,t-1}$	0.004	[0.75]	0.000	[-0.03]	-0.003	[-0.70]
R^2	0.073		0.140		0.033	
F	2.230		3.572		2.225	
p	0.014		0.000		0.014	
N	236		236		224	

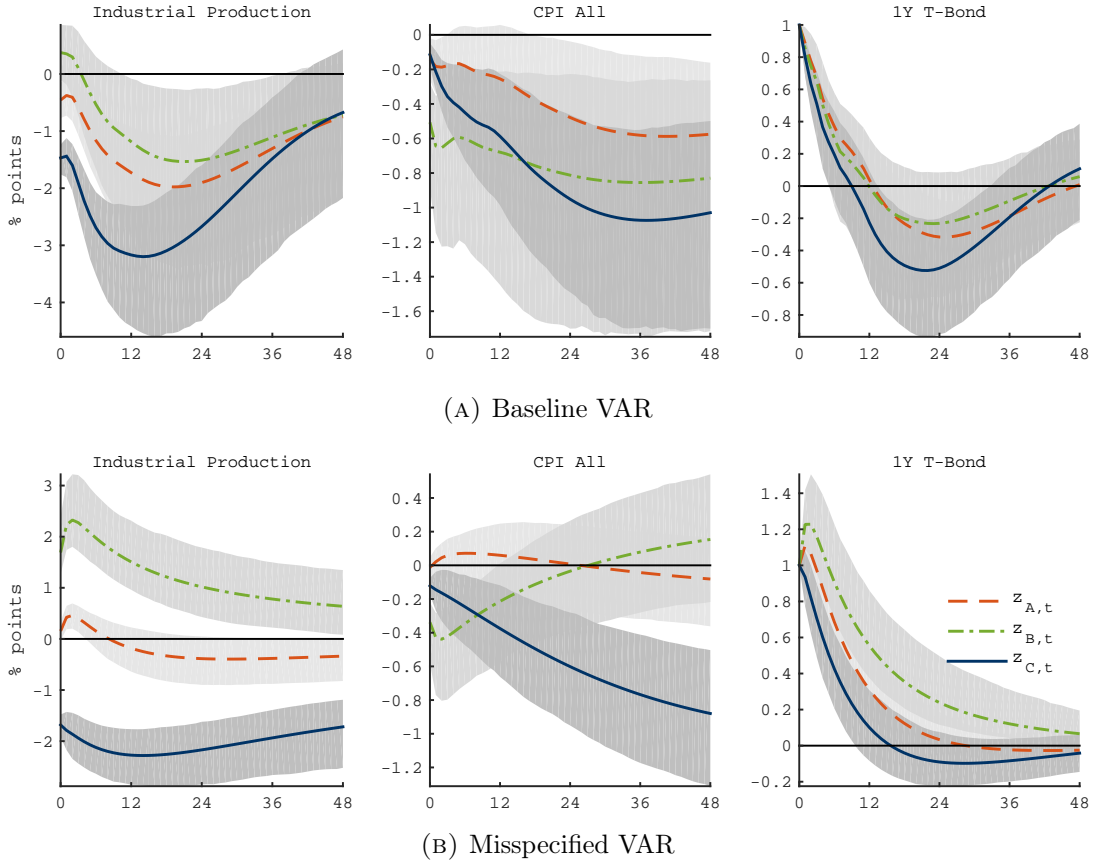
Note: Regressions include a constant and 1 lag of the dependent variable. t-statistics are reported in square brackets, * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, robust standard errors.

the null hypothesis of invertibility.

We also consider a misspecified VAR which omits both the EBP variable and the commodity price index, and includes 2 lags only. In both cases, we estimate the impact responses from a regression of the VAR innovations onto one of the above instruments, while IRFs are retrieved from the coefficients of the VAR.

Results are reported in Figure 3. In the baseline VAR, all instruments identify a monetary policy shock that triggers an economic recession, accompanied by a significant contraction in prices. While qualitatively coherent, these responses come with substantially diverse impact effects. However, the picture changes quite materially as we move to the misspecified VAR (bottom row of Figure 3). Modal impact responses of production to a contractionary monetary policy shock are now strongly positive at almost 2% under $z_{B,t}$, and small, yet positive and significant under $z_{A,t}$. The impact response under $z_{C,t}$ is largely unchanged. Similarly for prices, the impact response turns from negative and significant to positive and non-significant under $z_{B,t}$, while

FIGURE 3: RESPONSES TO MONETARY POLICY SHOCKS – 1979:2012



Notes: Baseline VAR(12) in all variables, top panel (A). Misspecified VAR(2) in three variables, bottom panel (B). VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{A,t}$: sum of high-frequency surprises within the month; $z_{B,t}$: moving average of high-frequency surprises within the month; $z_{C,t}$: residuals of $z_{A,t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.

it is essentially unaltered in the $z_{C,t}$ case. This finding suggests that neither $z_{A,t}$ nor $z_{B,t}$ satisfy the limited lag exogeneity condition, i.e. they correlate with other shocks, likely related to developments in financial markets and the real economy, that have not been filtered out by the VAR.^{12,13}

¹²The first factor in Table 1 is typically regarded as a synthetic measure of real activity, see e.g. McCracken and Ng (2015). Other than a barometer for financial markets' health levels, the EBP has strong predictive powers for an array of measures of economic activity, and its inclusion is likely to account for other omitted variables too (see e.g. Gilchrist and Zakrajšek, 2012; Gertler and Karadi, 2015).

¹³These results are invariant to a number of robustness tests, including on the estimation sample and the use of scheduled FOMC meetings only, as discussed extensively in Miranda-Agrippino and

8 Conclusions

This paper discusses conditions for identification with external instruments in Structural VARs under partial invertibility. We show that SVAR-IV methods allow for identification of the dynamic causal effects of interest under much weaker conditions than those required by LP-IV. Under partial invertibility, identification is achieved provided that the external instrument satisfies a limited lag exogeneity condition. This allows recoverability of the correct impact responses also when the instrument is correlated with other future or past shocks, so long as these sources of contamination are filtered out by the VAR dynamics. Hence, identification can be attained without resorting to full invertibility. Lastly, we show that identification is possible even in the presence of misspecification. This leads to the emergence of an empirical trade-off between efficiency and accuracy for IRFs analysis with SVAR-IV models.

Ricco (2017). We report IRFs estimated on a sample starting in 1990:01, selected to coincide with the start date of all the instruments used here, in Figure B.3 in the Appendix. Results are qualitatively the same.

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A Model

The economy is populated by a representative infinitely-lived household seeking to maximise

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t) , \quad (\text{A.1})$$

with a period utility

$$U(C_t, H_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{H_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} , \quad (\text{A.2})$$

where σ is the risk aversion parameter, φ is the Frisch elasticity, and H_t are hours worked. C_t is a consumption bundle defined as

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon}{1-\varepsilon}} , \quad (\text{A.3})$$

where $C_t(i)$ is the quantity of a good i consumed by the household in period t . A continuum of goods $i \in [0, 1]$ exists. The log-linearised households optimality conditions are given by

$$c_t = \mathbb{E}[c_{t+1}] - \frac{1}{\sigma} (r_t - \mathbb{E}[\pi_{t+1}]) , \quad (\text{A.4})$$

and by the labour supply schedule

$$w_t = \frac{1}{\varphi} h_t + \sigma c_t , \quad (\text{A.5})$$

where w_t is the labour wage on a competitive labour market. Agents maximise their intertemporal utility subject to a flow budget constraint. Agents can hold bonds or firms capital, and a no arbitrage condition between bonds and capital holds

$$\frac{1}{\beta} (r_t - \mathbb{E}[\pi_{t+1}]) = \frac{1}{\beta - (1 - \delta)} \mathbb{E}[z_{t+1}] . \quad (\text{A.6})$$

Firms produce differentiated goods $j \in [0, 1]$ by using a Cobb-Douglas technology to aggregate capital and labour

$$Y_t(j) = A_t K_{t-1}(j)^\alpha H_t(j)^{1-\alpha} \quad (\text{A.7})$$

where, importantly, log technology $a_t \equiv \log(A_t)$ evolves with a learning-by-doing term

$$a_t = \rho_a a_{t-1} + u_t^a + \omega u_{t-1}^a, \quad (\text{A.8})$$

where u_t^a is an i.i.d normally distributed technology shock, and $\omega > 1$ is a parameter that generates learning-by-doing effects. The static optimality condition on the production inputs delivers the linearised relation

$$w_t + h_t = k_{t-1} + z_t. \quad (\text{A.9})$$

The log-linearised production function of the firms is

$$y_t = a_t + k_{t-1} \alpha + h_t (1 - \alpha). \quad (\text{A.10})$$

Firms set prices in a staggered way à la Calvo (1983) with an indexation mechanisms of the type proposed by Galì and Gertler (1999). Thus, each period, a measure $1 - \theta$ of firms reset their prices, while prices for a fraction θ of the firms are $P_t(j) = P_{t-1} \pi_{t-1}^\gamma$. θ is an index of price stickiness. The firms that can reset their prices maximise the expect sum of profits

$$\max_{P_t^*(j)} \sum_{\tau=0}^{\infty} (\beta\theta)^\tau \left(P_t^*(j) \left(\frac{P_t - 1 + \tau}{P_{t-1}} \right)^\gamma - MC_{t+\tau} \right) Y_{t+\tau}(j), \quad (\text{A.11})$$

where MC_t are the real marginal costs in period t . The first order conditions from this problem, combined with the aggregate price equation, form a hybrid New Keynesian

Phillips Curve

$$\pi_t = \gamma \pi_{t-1} + \beta \mathbb{E}[\pi_{t+1}] + \lambda mc_t, \quad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \quad (\text{A.12})$$

where marginal costs evolve as

$$mc_t = \alpha z_t + w_t (1 - \alpha) - a_t. \quad (\text{A.13})$$

The linearised law of motion for firms capital is

$$I_t = K_{t+1} - (1 - \delta)K_t, \quad (\text{A.14})$$

where K_t is the physical capital and I_t is the investment. The log-linearisation of this equation yields

$$\delta i_t = k_t - (1 - \delta) k_{t-1}. \quad (\text{A.15})$$

A fiscal authority absorbs a share of output into wasteful government spending

$$G_t = (1 - \rho_g)G + \rho_g G_{t-1} e^{u_t^g} \quad (\text{A.16})$$

and the log-linearised equation for government spending is

$$g_t = \rho_g g_{t-1} + u_t^g, \quad (\text{A.17})$$

where u_t^g is an i.i.d normally distributed government demand shock. At the steady state $G = gY$. A monetary authority sets the nominal interest rate using a monetary rule with a smoothing term

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) (\phi_\pi \bar{\pi}_t + \phi_y \overline{\Delta y_t}) + u_t^r, \quad (\text{A.18})$$

TABLE A.1: CALIBRATED PARAMETERS

Parameter	Value	Description
α	0.4	share of capital in output
β	0.99	discount factor
δ	0.025	depreciation of capital
σ	1	risk aversion consumption
φ	2	labor disutility
g	0.2	share of public spending in output
θ	0.75	price stickiness
γ	0.3	indexation parameter (NK Phillips curve backward term)
ϵ	10	substitutability goods
ρ_r	0.9	monetary policy smoothing
ϕ_y	0.125	monetary policy output growth
ϕ_r	1.5	monetary policy inflation
ρ_a	0.95	productivity autocorrelation
ρ_g	0.95	public spending autocorrelation
ω	3	learning by doing

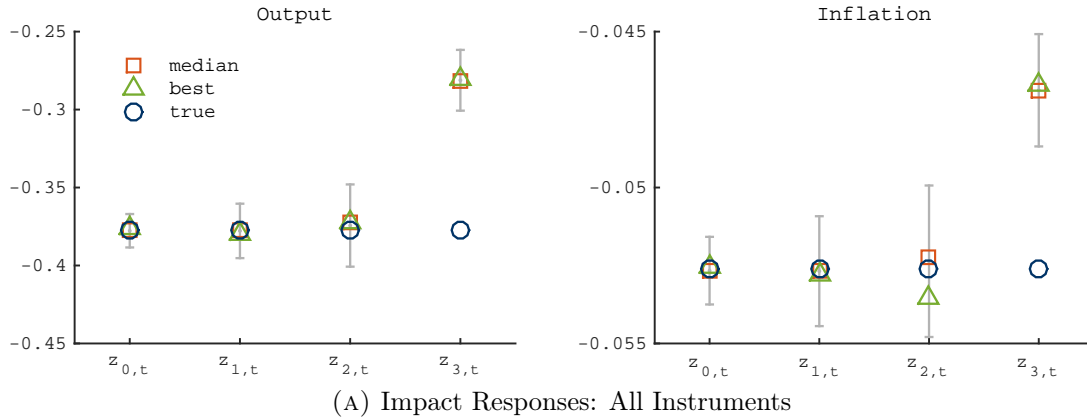
where $\overline{\pi}_t$ and $\overline{\Delta y}_t$ are respectively the average inflation and the average output growth over the last four periods, and u_t^r is a white noise i.i.d normally distributed monetary policy shock. Importantly, the monetary policy innovation can be recovered from current and past values of the policy rate, inflation and output. Finally the aggregate economy clears

$$Y y_t = C c_t + I i_t + G g_t . \quad (\text{A.19})$$

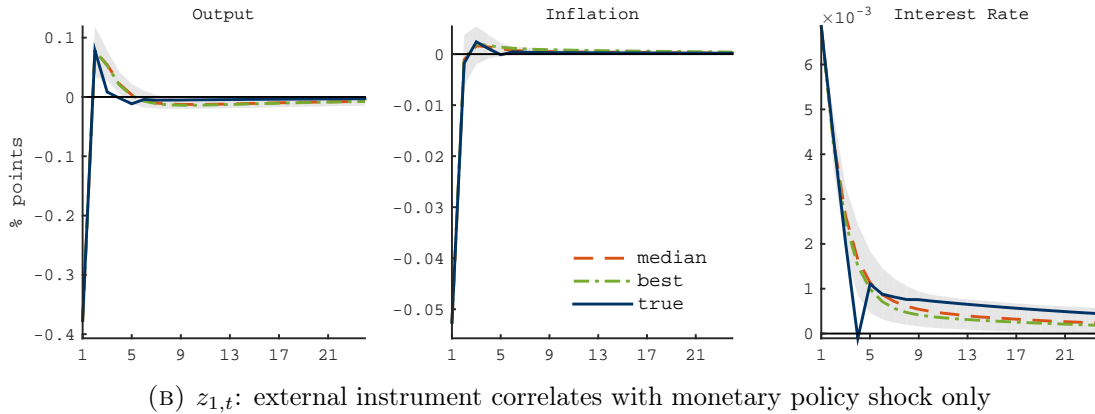
Table A.1 reports the calibration for this benchmark NK model. For this set of parameters the model fails the ‘poor man’s invertibility condition’ of Fernandez-Villaverde et al. (2007).

B Additional Charts

FIGURE B.1: RESPONSES TO MP SHOCK – SIMULATION & VAR(1)

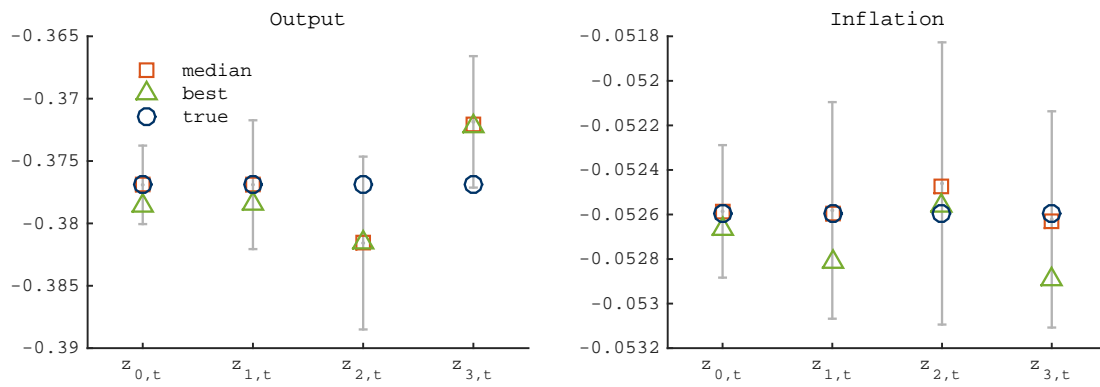


Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(1) in three observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bars from the distribution of impact responses across 1000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).



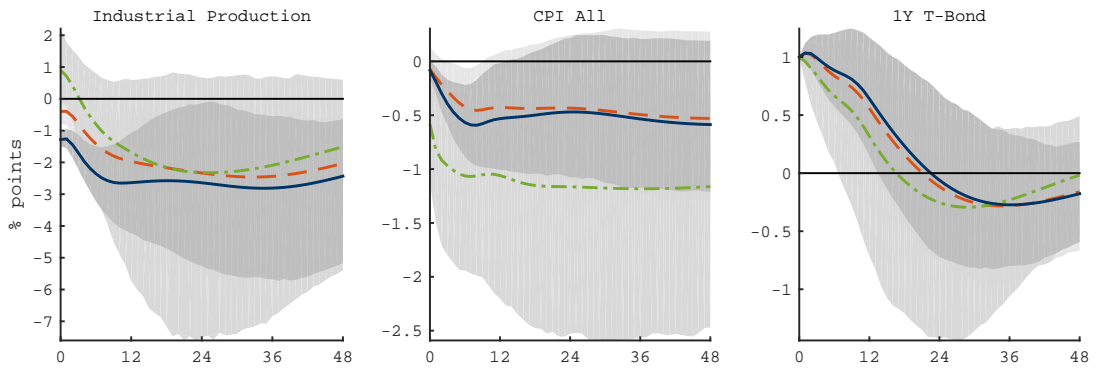
Notes: Impulse responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(1) in three observables. Instrument correlates with monetary policy shocks only. Grey shaded areas denote 90th quantiles of the distribution of IRFs across 1000 simulated economies of sample size $T = 300$ periods. Model responses (true, blue solid), median across simulations (orange dashed), minimum distance from median (best) simulation (green dash-dotted).

FIGURE B.2: IMPACT RESPONSES TO MP SHOCK – SIMULATION & VAR(4)

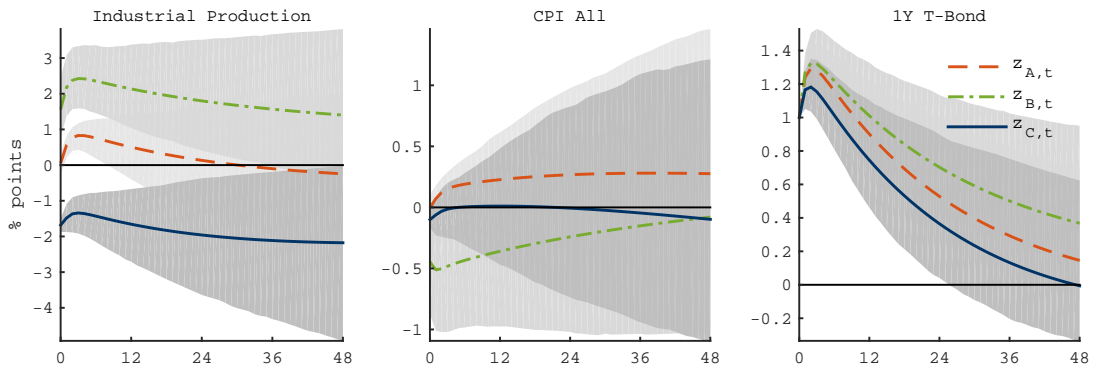


Note: Impact responses to monetary policy shock from partially-invertible DSGE identified with external instruments and estimated with a VAR(4) in three observables. $z_{0,t}$: observed shock case; $z_{1,t}$: instrument correlates with monetary policy shock only; $z_{2,t}$: instrument also correlates with past spending shocks; $z_{3,t}$ instrument correlates also with past technology shocks. Grey vertical lines are 2 standard deviations error bars from the distribution of impact responses across 1000 simulated economies of sample size $T = 300$ periods. True impact (blue circle), median across simulations (orange square), minimum distance from median (best) simulation (green triangle).

FIGURE B.3: RESPONSES TO MONETARY POLICY SHOCKS – 1990:2012



(A) Baseline VAR



(B) Misspecified VAR

Notes: Baseline: VAR(12) in all variables. Misspecified: VAR(2) in three variables. VARs estimated with standard macroeconomic priors. Identification in all cases uses the full length of the instruments. $z_{A,t}$: sum of high-frequency surprises within the month; $z_{B,t}$: moving average of high-frequency surprises within the month; $z_{C,t}$: residuals of $z_{A,t}$ on Fed Greenbook forecasts. Shaded areas denote 90% posterior coverage bands.