A Critique of Cross-Lagged Correlation

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Cross-lagged correlation is not a useful procedure for the analysis of longitudinal panel data. In particular, the difference between the cross-lagged correlations is not a sound basis for causal inference. Demonstrations of the failure of cross-lagged correlation are based mainly on results for the two-wave, two-variable longitudinal panel design. Extensions of these results to panels with multiple waves and multiple measures reveal additional problems.

The topic of this article is the analysis of reciprocal causal effects. Often, questions about reciprocal causal effects have been phrased, Does X cause Y or does Y cause X? More formally, many have spoken of a determination of causal predominance or of a preponderant causal effect. Examples of such research questions in developmental psychology include the reciprocal influences in mother-child interaction (Clarke-Stewart, 1973) and relationships between infant intelligence and infant behavior (Crano, 1977). Examples from educational research include the relationship between teacher expectation and student achievement (Crano & Mellon, 1978; Humphreys & Stubbs, 1977; West & Anderson, 1976) and the relationship between self-concept and achievement (Bachman & O'Malley, 1977; Calsyn & Kenny, 1977; Purkey, 1970). Empirical research on topics such as these has resulted in the collection and analysis of large amounts of longitudinal panel data.

Data from a longitudinal panel consist of observations on n cases at T (t = 1, . . . , T) time points or waves. At each time point, observations on one or more variables are obtained. Much attention is given to the simplest relevant panel design: the two-wave, two-variable (2W2V) longitudinal panel. For 2W2V panels the two variables are labeled X and Y. These variables are subscripted to indicate the time of measurement. Thus for each individual case, measures $X_t, Y_t, X_{t+1},$ and $Y_{t+1}$ are available.

Cross-lagged correlation (CLC) is currently the most popular procedure in many areas of psychological and educational research for identifying causal effects from longitudinal panel data. Most often CLC is used to determine a predominant causal influence—the causal winner.

Users of CLC often make enthusiastic claims. For example, Crano and Mellon (1978) asserted,

With the introduction of the cross-lagged panel correlational method. . ., causal inferences based on correla-
tional data obtained in longitudinal panel studies can be made and enjoy the same logical status as those derived in the more standard experimental settings. (p. 41)

Although technical deficiencies in CLC have been noted (Bohrnstedt, 1969; Duncan, 1969; Goldberger, 1971; Heise, 1970), CLC is widely recommended (e.g., Calsyn, 1976; Calsyn & Kenny, 1977; Clarke-Stewart, 1973; Crano, 1974, 1977; Humphreys & Stubbs, 1977; Kenny, 1975).

I find that CLC does not provide sound information about casual effects. CLC may indicate the absence of direct causal influence when important causal influences, balanced or unbalanced, are present. Also, CLC may indicate a causal predominance when no causal effects are present. Moreover, CLC may indicate a causal predominance opposite to that of the actual structure of the data; that is, CLC may indicate that X causes Y when the reverse is true.

A basic deficiency in CLC is the lack of an explicit definition of a causal effect. Without a clearly defined quantity to be estimated, it is not surprising that CLC fails to provide sound information about causal effects. The assessment of causal effects should be based on a model for the data in which causal effects are identified. In the next section definitions of causal effects derived from consideration of structural regression models, continuous-time models, and multiple time-series models are identical.

Structural Regression Models

A structural regression model for longitudinal panel data is one plausible model from which the longitudinal panel data could have been generated. For two variables, X and Y, the causal influences are represented by the regression parameters of the path from a prior X to a later Y and from a prior Y to a later X. This representation can be formulated for two-wave or multiwave panel data. (When the regression model is formulated in terms of latent variables having multiple indicators at each time point, the causal effects are represented by the regression parameters for the structural regression equations that relate the latent variables.)

Previous formulations of regression models for panel data with reciprocal causal effects have focused on models for 2W2V data (Duncan, 1969, 1972, 1975; Goldberger, 1971; Heise, 1970). For this simple configuration, the structural regression model is equivalent to a path analysis model. Figure 1 is a representation of a specific structural regression model for 2W2V data. This configuration can also be represented by the structural regression equations:

\[ X_2 = \beta_0 + \beta_1 X_1 + \gamma_1 Y_1 + \alpha, \]
\[ Y_2 = \gamma_0 + \beta_2 X_1 + \gamma_1 Y_1 + \beta. \] (1)

The parameters \( \beta_1 \) and \( \gamma_1 \) represent the influence of a variable on itself over time. The parameters \( \beta_2 \) and \( \gamma_2 \) represent the lagged, reciprocal causal effects between X and Y. Thus \( \beta_2 \) and \( \gamma_2 \) are key quantities in the investigation of reciprocal causal effects in 2W2V panels.

Restrictions on the nature of the causal influences between X and Y are built into Figure 1 and Equation 1. Most important is the assumption that all causal influences are lagged;
simultaneous causal influences between \( X_2 \) and \( Y_2 \) are not included. Also, assumptions of linearity and additivity of causal influences are built into this model.

The absence of a causal effect between variables is represented by a zero value of the relevant model parameter. In particular, in Equation 1 the absence of any direct causal effects between \( X \) and \( Y \) is represented by \( \beta_2 = \gamma_2 = 0 \). (Definitions of the absence of any causal effects between \( X \) and \( Y \) will prove to be important in the examination of "spuriousness" in CLC.) Also, a causal predominance of \( X \) over \( Y \) would be represented by a zero (or negligible) value of \( \gamma_2 \) and a large value of \( \beta_2 \). A causal predominance of \( Y \) over \( X \) is represented in the same manner.

Standardized versions of the structural parameters in Equation 1 also may be used to define causal effects. Standardized parameters are denoted by an asterisk. Explicitly,

\[
\beta_1^* = \beta_1 \left( \frac{\sigma_{X_1}}{\sigma_{X_2}} \right), \quad \beta_2^* = \beta_2 \left( \frac{\sigma_{X_1}}{\sigma_{Y_2}} \right),
\]
\[
\gamma_1^* = \gamma_1 \left( \frac{\sigma_{Y_1}}{\sigma_{Y_2}} \right), \quad \gamma_2^* = \gamma_2 \left( \frac{\sigma_{Y_1}}{\sigma_{X_2}} \right).
\]

Similar representations of reciprocal causal effects can be made in structural regression models for data from longitudinal panel designs that are far more complex than a 2W2V design. For example, multiple indicators of latent variables \( \xi \) and \( \eta \) may be available at each time of measurement. In the structural regression model for these data, the role of \( \beta_2 \) and \( \gamma_2 \) is unchanged; these parameters represent the lagged reciprocal causal influences between the latent variables \( \xi \) and \( \eta \).

**Continuous-Time Models**

Another approach to modeling panel data is to formulate equations for the rate of change of variables over time (Coleman, 1968; Hannan & Tuma, 1979). A simple two-variable model for rates of change that incorporates reciprocal influences between \( X \) and \( Y \) is

\[
\frac{dX(t)}{dt} = b_0 + b_1X(t) + c_2Y(t),
\]
\[
\frac{dY(t)}{dt} = c_0 + c_1Y(t) + b_2X(t). \quad (2)
\]

Equation 2 is a system of coupled differential equations which stipulates that the rates of change of \( X \) and \( Y \) at any time depend linearly on the levels of \( X \) and \( Y \). The parameters \( b_2 \) and \( c_2 \) represent the cross-effects or couplings between \( X \) and \( Y \). Note that Equation 2 is deterministic; for my purposes this limitation is not crucial.

Although rates of change are not directly observable, the solution of the system of differential equations in Equation 2 yields equations in terms of the observable variables of the same form as in Equation 1. The parameters \( \beta_2 \) and \( \gamma_2 \) are monotone increasing functions of \( b_2 \) and \( c_2 \) and depend on the time between waves and the parameters of Equation 2 (Kaufman, 1976). Thus the representation of reciprocal effects in the structural regression model in Equation 1 is consistent with that of the model for rates of change in Equation 2. Also, instead of only corresponding to the experimental lag between waves, regression models for panel data can be thought of as reflecting a process in which causal influences and resulting adjustments are continuous in time (Coleman, 1968; Hannan & Tuma, 1979).

**Multiple Time-Series Models**

In econometrics the detection of reciprocal causal effects from time-series data has attracted considerable interest. The analysis by Sims (1972) of the reciprocal causal influences of money stock and income is the best known example of this work. Definitions of reciprocal causal effects in multiple time-series data were formulated by Granger (1969). Subsequent work (Pierce, 1977; Pierce & Haugh, 1977) has interpreted and extended these definitions.

The definitions of causality are based on predictability criteria. Loosely speaking, one time series, for example, \( X(t) \), causes another
Definitions of causal effects in panel data also can be formulated using the predictability parameters of multiple time-series models (see Pierce & Haugh, 1977).

The conditions are particularly simple for 2W2V data. For example, the condition for X causes Y for the 2W2V model in Figure 1 can be stated in terms of linear prediction with no simultaneous causation as a relation between the population multiple correlations:

$$\hat{R}_{y_1 x_1}^2 > \hat{R}_{y_1 y_1}^2.$$  

This inequality is satisfied when

$$\rho_{y_1 y_1}^2 > 0.$$  

That condition is satisfied if and only if $$\beta_2 \neq 0.$$ Also, the condition for Y causes X is that $$\gamma_2 \neq 0.$$ And the condition for feedback is that both $$\beta_2$$ and $$\gamma_2$$ are nonzero. Although much of the sophistication of the time-series formulation is lost when these definitions of causality are adapted to longitudinal panel data, it is useful to show that these definitions are consistent with the other representations of causal effects.

Method of Cross-Lagged Correlation

In this section the procedures and assumptions of CLC are described and discussed. Procedures for both two-wave and multiwave panel data are considered. This material provides the groundwork for the results in the next two sections. The discussion in this section is restricted to variables measured without error; complications that result from fallible measurement are not crucial in this analysis.

Procedures for Two-Wave Panels

Figure 2 is the diagram that accompanies expositions of CLC. Figure 2 presents the population correlations among the variables in a 2W2V panel. The population cross-lagged correlations are $$\rho_{x_1 y_1}$$ and $$\rho_{y_1 x_1}.$$ The within-time-period, between-variables correlations, $$\rho_{x_1 y_1}$$ and $$\rho_{y_1 x_1},$$ are the synchronous correlations. The between-time-periods correlations of the same variable, $$\rho_{x_1 x_1}$$ and $$\rho_{y_1 y_1},$$ are the stabilities of the variables.

The attribution of causal predominance\(^1\) in CLC is based on the difference between the cross-lagged correlations, $$\rho_{x_1 y_1} - \rho_{y_1 x_1}.$$ If the data indicate that $$\rho_{x_1 y_1} - \rho_{y_1 x_1}$$ is positive, the causal predominance is concluded to be that of X causing Y. If the data indicate that $$\rho_{x_1 y_1} - \rho_{y_1 x_1}$$ is negative, the causal predominance is concluded to be that of Y causing X (Campbell, 1963). Usually, attributions of causal predominance are made only when the null hypothesis of equal cross-lagged correlations (Ho: $$\rho_{x_1 y_1} = \rho_{y_1 x_1}$$) is rejected.

CLC discards much information. A statistically significant difference between the sample cross-lagged correlations is given the same in-

\(^1\) In the literature on CLC, the recognition of both a source and a direction of a predominant causal influence has generated much discussion (Rozelle & Campbell, 1969; Yee & Gage, 1968). Source is the variable that is causally predominant, and direction is whether the causally predominant variable causes an increase or a decrease in the other variable. In terms of the model in Equation 1, source is related to the relative magnitudes of $$\beta_2$$ and $$\gamma_2,$$ and direction is determined by the sign of the parameter. Causal predominance in this article refers to an identification of both source and direction.

\(^2\) Pelz and Andrews (1964) proposed that causal attributions be based on the partial cross-lagged correlations, $$\rho_{y_1 x_1}$$ and $$\rho_{x_1 y_1}.$$ I do not consider this proposal further for two reasons. First, this proposal has been ignored in applications and technical development of CLC. Second, and more important, the partial correlation strategy fundamentally differs from CLC because the partial correlation strategy is not in any way based on the model in Figure 3 that is the basis for the use of zero-order correlations. In fact the partial correlation strategy is best thought of as an incomplete structural regression strategy based on Figure 1 (see, also, Heise, 1970).
interpretation regardless of the magnitude of the difference and regardless of the magnitudes of the individual correlations. Also, no distinction is made between large and equal cross-lagged correlations and small and equal cross-lagged correlations.

Kenny (1975) interpreted the null hypothesis of equal cross-lagged correlations as a null hypothesis that the relationship of \( X \) and \( F \) is a result of the common influence of an unmeasured third variable and not a result of direct influences between \( X \) and \( Y \). Kenny termed this a null hypothesis of spuriousness. Rejection of this null hypothesis (under certain assumptions) leads to a conclusion of direct causal effects between \( X \) and \( F \). Although Kenny emphasized this test for spuriousness over an attribution of causal predominance, all applications of CLC seek an attribution of causal predominance.

Kenny (1975) posited a very specific model for his null hypothesis of spuriousness, which is depicted in Figure 3. This 2W2V model with one evolving common factor (\( F \)) is also analyzed from the standpoint of path analysis by Duncan (1972, especially pp. 63-74). Duncan showed that this model is underidentified; that is, only the parameter \( h \) can be estimated from the 2W2V data.\(^3\)

The strategy of investigating the tenability of a null hypothesis of a spurious association between \( X \) and \( Y \) is not without merit. However, Kenny's (1975) model of spurious association is just one of several models in which direct causal influences between \( X \) and \( Y \) are absent. The term spuriousness is better considered as a generic term for the absence of direct causal influences between \( X \) and \( Y \).

Many alternative models for 2W2V data can be constructed that are true to this broadened notion of spuriousness. Two examples are the common factor model with lagged effects considered in Duncan (1972, Figure 12) and a restricted version of the 2W2V model in Figure 1 in which \( \beta_2 = 0 \) and \( \gamma_2 = 0 \). As \( \beta_2 \) and \( \gamma_2 \) represent the magnitudes of the lagged causal effects between \( X \) and \( Y \), it follows from the definition of causal effects that the absence of direct causal effects between \( X \) and \( Y \) be represented by the model in Figure 1, with \( \beta_2 = \gamma_2 = 0 \).

Rarely is it recognized that the objectives of CLC are modest and limited. In its most complete form, CLC purports to distinguish only between spuriousness and a causal predominance for one of the variables. Even if CLC were valid for its stated objectives, it falls short of providing an adequate description of causal influence in panel data. The failure of CLC to achieve even these limited objectives makes the status of CLC as the primary analysis method for panel data in education and psychology very unfortunate.

\[^3\] From examination of the models in Figures 1 and 3, it might appear that the model in Figure 1 contains the restriction that the autocorrelations among the residuals in Figure 3 (denoted by the parameters \( s \) and \( t \)) are zero. This is not the case, as nonzero values of these parameters are incorporated into the parameters \( \beta_1 \) and \( \gamma_1 \). That the parameters of the model in Figure 1 do reflect nonzero values of \( s \) and \( t \) can be seen from the result that the restriction \( s = t = 0 \) requires that \( \beta_1 \gamma_1 = \beta_2 \gamma_2 \). Also, the model in Figure 3 does not require that \( u \) and \( v \) in Equation 1 be uncorrelated.

The restrictions that \( h \) (a correlation) not be greater than one in magnitude requires that \( r_{X_2Y_2} \leq r_{X_1Y_1} \leq r_{X_1Y_2} \). Kenny (1975) stated that a violation of this restriction in sample data is indicative of a causal effect. Also, if \( s \) and \( t \) are both positive, then the model requires that \( r_{X_1Y_2} \leq r_{X_1Y_1} \leq r_{X_2Y_2} \).
Assumptions in CLC

Articles on CLC often contain complex non-mathematical discussions of the assumptions deemed necessary for the interpretation of the difference between the cross-lagged correlations. Taken together, these assumptions are very restrictive. Not all expositions of CLC include all assumptions, and most empirical applications of CLC ignore the assumptions.

Kenny (1973, 1975) formulated the assumptions of CLC with regard to his model for spuriousness (Figure 3). The major assumption of CLC is stationarity, that is, that the causal structures for X and Y do not change over time. For the parameters in Figure 3, stationarity requires that $a_1 = a_2$ and $b_1 = b_2$. (Weaker assumptions, called proportional stationarity and quasi stationarity are also considered in Kenny, 1975.) Because the synchronous correlations, $\rho_{X_1,Y_1}$ and $\rho_{X_2,Y_2}$, are equal if $a_1b_1 = a_2b_2$, stationarity implies equality of the synchronous correlations. Equality of the synchronous correlations is a necessary but not sufficient condition for stationarity.

The assumption of stationarity is far from innocent. The assumption that the causal parameters do not change over time (or, less restrictively, that the synchronous correlations do not change over time) is closely linked to an assumption that the system (composed of the relations between X and Y) is in equilibrium (see Coleman, 1968). This assumption of equilibrium is often invoked to justify the use of cross-sectional data as a proxy for longitudinal data. The analysis of longitudinal panel data should not depend on a restrictive assumption closely linked to cross-sectional research.

Restrictive assumptions are necessary in CLC because the model in Figure 3 is under-identified. Dependence on the stationarity assumption appears to limit applications of CLC. However, in many well-known applications, substantial violations of stationarity are present (e.g., Clarke-Stewart, 1973; Eron, Huesmann, Lefkowitz, & Walder, 1972).

Given stationarity, unequal cross-lagged correlations are inconsistent with the model in Figure 3. Consequently, in CLC unequal cross-lagged correlations indicate rejection of spuriousness. Because equal synchronous correlations are not a sufficient condition for stationarity, the cross-lagged correlations may be unequal when the synchronous correlations are equal and the representation of spuriousness in Figure 3 is valid. \(^3\) Kenny had previously noted problems with verifying stationarity in two-wave data and had indicated ways (Kenny, 1973) to verify stationarity in multiwave data.

A second key assumption is synchronicity, that the measures at each wave are obtained at the same time. This assumption is implicit in our description of the longitudinal panel data and will not be considered further.

Kenny (1975) introduced an assumption termed homogeneous stability that is invoked to help distinguish between the alternative hypotheses that X causes an increase in Y and that Y causes a decrease in X (or vice versa) once the null hypothesis of spuriousness has been rejected. Homogeneous stability is not used to determine whether spuriousness should be rejected. Nor is homogeneous stability used to decide between X causes an increase in Y and Y causes an increase in X. The assumption is not stated explicitly in terms of parameters in Figure 3, but Kenny did state that equal stabilities for X and Y (measured without error) are consistent with this assumption. In recent applications of CLC (Calsyn & Kenny, 1977; Crano, 1977; Crano & Mellon, 1978; Humphreys & Stubbs, 1977), this assumption has not been used or discussed.

Extension of CLC to Multiwave Panels

The extension of CLC to multiwave panel data consists of comparisons of cross-lagged

\[^1\] The restriction that the synchronous correlations be equal has no special interpretation in terms of the parameters of the model in Figure 1. The restriction in terms of the standardized versions of the structural parameters is

$$\rho_{X_1Y_1} = \frac{\beta_1^* \beta_2^* + \gamma_1^* \gamma_2^* + (\sigma_{XY}/\sigma_{XX} \sigma_{YY})}{1 - \beta_1^* \gamma_1 - \beta_2^* \gamma_2}.$$  

\[^1\] From Figure 3

$$\rho_{X_1Y_2} - \rho_{Y_1X_2} = b(a_1b_2 - a_2b_1).$$

Equality of the synchronous correlations requires that $a_1b_1 = a_2b_2$. Substituting this into the difference between the cross-lagged correlations yields

$$\rho_{X_1Y_2} - \rho_{Y_1X_2} = b \left( \frac{a_1}{b_1} \right) (b_2^2 - b_1^2).$$
correlations from all possible two-wave combinations (see Calsyn, 1976; Calsyn & Kenny, 1977; Crano, 1977). Multiple waves of data are used to replicate 2W2V patterns. For example, for three-wave, two-variable (3W2V) panel data, three pairs of cross-lagged correlations would be examined: cross-lagged correlations between Waves 1 and 2, between Waves 2 and 3, and between Waves 1 and 3.

The analysis of \( T \) waves of data as \( \binom{T}{2} \) two-wave pieces represents a short-sighted view of the value of multiwave data. Many patterns of causal influence cannot be detected from a series of two-wave snapshots. Using multiple waves to replicate two-wave patterns depends on a static pattern of causal influence over time. This strategy rules out investigation of change over time in patterns of causal influence and is not an effective use of multiple waves of data.

When \( T \) is large or when many indicators of each variable are available, the number of comparisons of cross-lagged correlations becomes large. (With \( p_x \) measures of \( \xi \) and \( p_y \) measures of \( \eta \) at each of the \( T \) time points, \( p_x p_y \binom{T}{2} \) comparisons of cross-lagged correlations can be made.) The usual practice in CLC analyses of such data (Calsyn, 1976; Crano, 1977) is to tally the statistically significant differences between the cross-lagged correlations in both directions; the causal winner is the variable with the greater number of significant differences in the appropriate direction.

Results for Two-Wave Panels

In this section I demonstrate that the difference between the cross-lagged correlations is not a sound basis for causal inference. Neither determinations of causal predominance nor spuriousness are defensible. Complications resulting from measurement error, specification error, and multiple indicators are also discussed. All results are presented in terms of population parameters.

**Difference Between the Cross-Lagged Correlations**

The difference between the population cross-lagged correlations can be written in terms of the parameters of Equation 1:

\[
\rho_{X_t,Y_t} - \rho_{Y_t,X_t} = (1 - \rho^2_{X_t,Y_t}) \left[ \beta_2 \left( \frac{\sigma_{X_t}}{\sigma_{Y_t}} \right) - \gamma_2 \left( \frac{\sigma_{X_t}}{\sigma_{Y_t}} \right) \right] + \rho_{X_t,Y_t} (\rho_{Y_t,X_t} - \rho_{X_t,X_t}).
\]

The difference between the cross-lagged correlations can also be expressed in terms of the standardized versions of the structural parameters:

\[
\rho_{X_t,Y_t} - \rho_{X_t,X_t} = (1 - \rho^2_{X_t,Y_t}) (\beta_2^* - \gamma_2^*) + \rho_{X_t,Y_t} (\rho_{Y_t,X_t} - \rho_{X_t,X_t}),
\]

\[
\rho_{X_t,Y_t} - \rho_{Y_t,X_t} = (\beta_2^* - \gamma_2^*) + \rho_{X_t,Y_t} (\gamma_1^* - \beta_1^*).
\]

The difference between the cross-lagged correlations does not have a direct correspondence to measures of causal effects.

The expressions above are used to demonstrate that the difference between the cross-lagged correlations does not provide a sound basis for a determination of spuriousness or causal predominance, even when the assumptions of CLC are satisfied. Equal cross-lagged correlations do not support a conclusion of the absence of direct causal effects, and unequal cross-lagged correlations do not support a conclusion of causal predominance. The examples and generalizations based on Equations 3, 4, and 5 almost always incorporate the assumption that the structural parameters and the correlations are nonnegative. This assumption is not limiting, but in some cases negative values would require changes in wording.

**Equal cross-lagged correlations.** In CLC, equal cross-lagged correlations indicate a conclusion of a spurious (or nonexistent) pattern of causal influence between \( X \) and \( Y \). This conclusion is unsound because equal cross-lagged correlations are consistent with many patterns of direct causal influence. Equal cross-

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6 The assumption of stationarity can be partially represented in Equations 3, 4, and 5 by incorporating the restriction of equal synchronous correlations, using the expression given in Footnote 4. Substitution of this equality has no effect on the interpretations and on conclusions that are based on Equations 3, 4, and 5. Consequently, this complication is not considered in the text.
Figure 4. Numerical examples of misleading cross-lagged correlations in 2W2V panels. (Standardized structural parameters are listed above each of the diagrams that display the correlations.)

Lagged correlations require only that

$$\beta_2^* - \gamma_2^* = \frac{\rho_{X_1Y_1}}{1 - \rho_{X_1Y_1}^2} (\rho_{X_1X_2} - \rho_{Y_1Y_2}),$$

or

$$\beta_2^* - \gamma_2^* = \rho_{X_1Y_1}(\beta_1^* - \gamma_1^*).$$

In particular, equal cross-lagged correlations are consistent with (a) large and equal causal effects, (b) large and unequal causal effects, and (c) the absence of causal effects between $X$ and $Y$. These cases are considered below, and numerical examples are displayed in Figure 4.\(^7\)

As can be seen from Equation 4, if $\beta_2^*$ and $\gamma_2^*$ are large and equal and if the stabilities of $X$ and $Y$ are equal, the cross-lagged correlations are equal. Panel a in Figure 4 is one of many possible illustrations that a zero difference between the cross-lagged correlations is consistent with nonnegligible and equal direct causal influences. Equal cross-lagged correlations are

\(^7\)In the numerical examples displayed in Figures 4 and 5, stationarity is not grossly violated. In each of these examples, the correlations were generated under the assumption that $\sigma_{uv} = 0$. A nonzero value of $\sigma_{uv}$ could have been specified in each of these examples that would make the synchronous correlations identical; however, it was felt that the slight discrepancies among synchronous correlations did not justify the complication of an additional specification.
also consistent with unequal causal effects, the difference between the stabilities offsetting the difference between $\beta_1^* = \gamma_2^*$. Panel b in Figure 4 presents an example in which $\beta_1^*$ is twice as large as $\gamma_2^*$ and in which the cross-lagged correlations are equal. Equal cross-lagged correlations are consistent with the absence of causal effects—$\beta_1^* = \gamma_2^* = 0$—only when the stabilities of $X$ and $Y$ are equal.

All of the difficulties with the interpretation of equal cross-lagged correlations persist when causal effects are defined in terms of unstandardized structural parameters. Because the variance changes when the cross-lagged correlations increase, the variances change when the cross-lagged correlations become equal. Cross-lagged correlations are consistent with an even wider variety of configurations of unstandardized causal parameters than were indicated previously for standardized parameters.

The evidence used in CLC to reach a determination of spuriousness is equal cross-lagged correlations (and equal synchronous correlations to satisfy stability). Because this evidence is consistent with many patterns of direct causal influence, the determination of spuriousness in CLC is unsound.

Unequal cross-lagged correlations. Unequal cross-lagged correlations indicate a causal predominance in CLC. However, a nonzero difference between the cross-lagged correlations is no more dependable than a zero difference for determining the pattern of causal influence. Unequal cross-lagged correlations are consistent with (a) the absence of causal effects between the variables, (b) equal causal effects, and (c) unequal causal effects. Moreover, the predominant cause indicated by the unequal cross-lagged correlations may be opposite to that defined by the structural parameters; for example, the cross-lagged correlations indicate that $X$ is causally predominant, when the reverse is true.

As before, the shortcomings of CLC are seen from Equations 3, 4, and 5. First, in the absence of causal effects ($\beta_1 = \gamma_2 = 0$), $\rho_{X_1X_2} - \rho_{Y_1X_2} = \rho_{X_1Y_1}(\rho_{Y_1X_2} - \rho_{Y_1Y_2})$. Unequal cross-lagged correlations result whenever $\rho_{X_1X_2} \neq \rho_{Y_1X_2}$. CLC will assign causal predominance to the variable with the lower stability, even though causal effects are not present.

Similarly, a causal predominance will be indicated by CLC when $\beta_1^* = \gamma_2^* = 0$ and the stabilities are unequal. Causal predominance is attributed to the variable with the lower stability because, as in the previously mentioned expression, $\rho_{X_1Y_2} - \rho_{Y_1X_2} = \rho_{X_1Y_1}(\rho_{Y_1Y_2} - \rho_{X_1X_2})$. Panel c in Figure 4 presents a numerical example in which $\beta_1^* = \gamma_2^*$, and the cross-lagged correlations indicate a causal predominance for $X$, the variable with the lower stability.

In addition to indicating a causal predominance when there is none, CLC may indicate a causal predominance opposite to that indicated by the structural parameters. Whenever $\rho_{X_1Y_2} - \rho_{Y_1X_2}$ and $\beta_2 - \gamma_1$ (or $\beta_1^* - \gamma_2^*$) differ in sign, the direction of causal predominance indicated by the cross-lagged correlations is antipodal to that determined by the structural parameters. In Panels d and e in Figure 4, the stronger causal effect, defined by the standardized structural parameters, is from $Y$ to $X$. However, in each example the cross-lagged correlations indicate that the predominant causal influence is from $X$ to $Y$. In each example CLC picks the wrong causal winner.

When causal effects are expressed in terms of the unstandardized structural parameters, changes in variances over time further complicate the interpretation of unequal cross-lagged correlations. As noted previously, the difference between the cross-lagged correlations increases as $\sigma_{XY}\sigma_{X_1}/\sigma_{Y_2}\sigma_{Y_1}$ increases. As a result, CLC (other parameters constant) favors the attribution of causal predominance to the variable with increasing variance over time. One way of highlighting the role of changing variances over time is to rewrite Equation 3 as

$$\rho_{X_1Y_2} - \rho_{Y_1X_2} = \frac{1}{\sigma_{Y_2}} (\sigma_{X_1}\beta_2 + \sigma_{Y_1}\rho_{X_1Y_1}\gamma_1)$$

$$- \frac{1}{\sigma_{X_1}} (\sigma_{Y_1}\gamma_2 + \sigma_{X_1}\rho_{X_1Y_1}\beta_1).$$

For example a relatively large $\sigma_{X_1}$ tends to make $\rho_{X_1Y_2} - \rho_{Y_1X_2}$ positive, thus favoring the attribution of causal predominance to $X$.

Measurement Error

In the preceding discussion $X$ and $Y$ were assumed to be measured without error. A number of techniques, both standard psychometric
methods and special methods peculiar to CLC, have been used in CLC analyses to deal with measurement error. It is not vital to consider these correction techniques explicitly because any scheme for disattenuation can at best reproduce a situation with perfect measurement. But CLC is unsound even when measurement is perfect, and better performance cannot be expected when measurement error is present. Equations 3, 4, and 5 are valid both for fallible and for perfectly measured variables.

The presence of measurement error is cited in discussions of CLC as invalidating alternative regression-based analyses (see Kenny, 1975). If this claim were sound, it would justify some despair but would not justify the use of CLC. The argument in support of CLC is that partial regression slopes are more severely affected by measurement error than are zero-order correlations. With only one fallible measure of each variable at each time point, it is not possible in general to obtain acceptable estimates of the parameters of a regression model for the panel data. However, additional measures of each variable can be used to consistently estimate the regression parameters. The problem of measurement error can be viewed as a problem of inadequate research design; a research design that incorporates multiple indicators of the underlying variables is not as easily invalidated by fallible measurement.

**Specification Error**

Sometimes claims are made that CLC is not affected by omitted causal variables or by other forms of specification error, whereas regression analyses are destroyed by specification error. A fervent argument of this type was made by Humphreys and Stubbs (1977), who asserted that

> the comparison of correlations among the same set of measures obtained at two or more intervals of time (cross-lagged correlations) either avoids completely or minimizes the difficulties involved in the interpretation of regression weights. (p. 262)

The claim is specious. CLC is not immune to specification error. The effects of specification error on CLC are opaque because a zero-order correlation does not depend on additional variables, whether they be measured or omitted. The problems arise in the interpretation of the zero-order correlations in the determination of causal effects, specifically from the effects of specification error on the definitions of causal effects and on the model underlying CLC. When the model in Equation 1 is misspecified because of the omission of important causal variables, the structural parameters in Equation 1 are not valid representations of the causal influences. The cross-lagged correlations are functions of these structural parameters, and thus their interpretation is also affected. For example, when \( \rho_{x_1x_2} = \rho_{y_1'y_1'} \)

\[
\rho_{x_1'y_2} - \rho_{x_1'x_2} = (1 - \rho_{x_1'y_1'}^2) (\beta^x - \gamma^y).
\]

The causal interpretation of the zero-order correlations is based on specific assumptions about causal structures. The model (Figure 3) underlying CLC includes assumptions about unmeasured variables that are unlikely to be satisfied when important causal variables are omitted. For example, the requirements that \( U_z \) and \( V_1 \) be uncorrelated with each other and that \( U_1 \) and \( V_1 \) be similarly uncorrelated are susceptible to violations resulting from specification error. Figure 15 in Duncan (1972) contains some forms of specification errors for Figure 3.

**Multiple Indicators**

A CLC analysis of panel data with multiple indicators of each variable consists of the comparison of all possible 2W2V cross-lagged correlations. In a two-wave panel with \( p_x \) measures of the latent variable \( \xi_i \) and with \( p_y \) measures of the latent variable \( \eta_i \) at each time, \( p_x \times p_y \) differences between cross-lagged correlations are computed. Multiple indicators are considered to be an opportunity for replications across different operationalizations of the same construct (Kenny, 1975, p. 894).

The \( p_x \times p_y \) differences between the cross-lagged correlations obtained from a 2W2V design with multiple indicators can be written as

\[
\rho_{x_i'x_j'} - \rho_{y_i'y_j'} = \left( \frac{\delta_{ij}}{\sigma_{x_i'}\sigma_{y_j'}} \right) \rho_{x_i'y_{j'}} = \left( \frac{\lambda_{ij} \delta_{ij}}{\sigma_{y_j'}\sigma_{x_i'}} \right) \rho_{y_i'x_j'},
\]

\[i = 1, \ldots, p_x; \ j = 1, \ldots, p_y. \]
The latent variables $\xi$ and $\eta$ are related by the structural equations,
\[\xi_t = \beta_0 + \beta_1 \xi_{t-1} + \gamma_1 \eta_{t-1} + u_t,
\]
\[\eta_t = \gamma_0 + \beta_2 \xi_{t-1} + \gamma_2 \eta_{t-1} + \nu_t.\]
The observed indicators are related to the latent variables by the measurement model,
\[X_{it} = \delta_{i0} \xi_t + \epsilon_{it},
\]
\[Y_{jt} = \lambda_{j0} \eta_t + \nu_{jt},\]
\[i = 1, \ldots, p_x; \quad j = 1, \ldots, p_y; \quad t = 1, 2.\]

Equation 6 shows the additional problems with CLC resulting from the use of multiple indicators for replication. The relations in Equations 3, 4, and 5 can be thought of as relations for the latent variables $\xi$ and $\eta$. Even if measures of $\xi$ and $\eta$ were available, the difference between their cross-lagged correlations is not a sound basis for causal inference. Equation 6 shows that these shortcomings are likely to be compounded by the CLC strategy for using multiple indicators. The difference between cross-lagged correlations of the multiple indicators depends on the specific relations (the loadings $\delta$ and $\lambda$) of each indicator to the latent variable it represents. This dependence is likely to produce inconsistency and confusion, not replication.

Results for Multiwave Panels

The analysis of CLC for multiwave data is based on a 3W2V panel design. The representation of causal effects for the 3W2V design is a simple extension of Equation 1 (intercepts omitted):
\[X_2 = \beta_1 X_1 + \gamma_2 Y_1 + u_2,
\]
\[Y_2 = \beta_2 X_1 + \gamma_1 Y_1 + v_2,
\]
\[X_3 = \beta_3 X_2 + \gamma_3 Y_2 + \beta_4 X_1 + \gamma_6 Y_1 + u_3,
\]
\[Y_3 = \beta_1 X_2 + \gamma_3 Y_2 + \beta_6 X_1 + \gamma_4 Y_1 + v_3.\]

In this model, causal effects between $X$ and $Y$ may span two waves; both between-variables and within-variable effects are so represented in Equation 7. Three separate comparisons of cross-lagged correlations are made in CLC for the 3W2V data: comparisons between Waves 1 and 2, between Waves 2 and 3, and between Waves 1 and 3. In terms of the standardized parameters of Equation 7, the differences between the cross-lagged correlations are
\[p_{x_1 y_1} - p_{x_1 y_2} = \beta_6^* - \gamma_6^* + \rho_{x_1 y_1} (\gamma_6^* - \beta_6^*),
\]
\[p_{x_2 y_2} - p_{x_2 y_3} = \beta_4^* + \gamma_6^* + \rho_{x_2 y_2} (\gamma_6^* - \beta_6^*)
+ \rho_{x_1 y_1} (\gamma_6^* - \beta_6^*) + \rho_{x_2 y_1} (\gamma_6^* - \beta_6^*)\]
\[p_{x_3 y_3} - p_{x_3 y_1} = \beta_6^* - \gamma_6^* + \rho_{x_3 y_3} (\gamma_6^* - \beta_6^*)
+ \rho_{x_2 y_2} (\gamma_6^* - \beta_6^*) + \rho_{x_3 y_2} (\gamma_6^* - \beta_6^*)
- \beta_6^* + \gamma_6^* - \beta_6^* \gamma_6^* - \beta_6^* \gamma_6^* \gamma_6^*. \]

Equation 8 indicates that the differences between the cross-lagged correlations may be inconsistent in causal attribution, even when causal influences are pronounced and consistent across waves. Figure 5 displays one of many possible patterns of inconsistency in the cross-lagged correlations. In Figure 5 the stronger causal effect, as defined by the standardized structural parameters, is from $Y$ to $X$ consistently across the three waves. The cross-lagged correlations do not indicate this causal pattern; Waves 1 and 2 indicate a causal predominance for $X$; Waves 2 and 3 indicate a causal predominance for $Y$, and Waves 1 and 3 indicate spuriousness.

Unfortunately, inconsistencies in the cross-lagged correlations over waves generate clever substantive interpretations. For example, Clarke-Stewart (1973) conducted a CLC analysis on the relationship between maternal attention and infant attachment using three waves of data. Waves 1 and 2 indicated a causal predominance for maternal attention, and Waves 2 and 3 indicated a causal predominance for infant attachment (Wave 1–Wave 3 comparison not reported). Clarke-Stewart suggested that “as mother and child search for harmonious, balanced interaction over the course of development, first one then the other assumes the ‘causal role’ ” (p. 91).
Statistical Inference in CLC

In the preceding sections relations among population parameters were used to demonstrate the failure of CLC. In this section additional problems with the application and interpretation of statistical inference procedures in CLC are noted.

Rejection of the null hypothesis of equal cross-lagged correlations \( H_0: \rho_{x_1y_2} = \rho_{y_1x_2} \) often is interpreted with little regard for the power of the statistical test. Users of CLC are advised to use large samples; Kenny (1975) advises that "cross-lagged analysis is a low-power test" (p. 887) and that even with moderate sample sizes (defined as 75 to 300), statistically significant differences are difficult to obtain. With large enough samples, trivial deviations from the null hypothesis lead to rejection. For example, Crano, Kenny, and Campbell (1972) found significant differences between cross-lagged correlations of .65 and .67 because the sample size was 5,495. Other authors ignore sampling variation and with small sample sizes interpret differences between the sample cross-lagged correlations as if these sample estimates were population values. The use of interval estimates for \( \rho_{x_1y_2} - \rho_{y_1x_2} \) would
be an improvement over current practice in CLC; the construction of the interval estimate can be found in Olkin (1966).

Also, significant differences between the cross-lagged correlations are interpreted without regard for the assumptions of CLC. Rarely is any assessment of stationarity made in applications of CLC. In many sets of longitudinal data, the difference between the synchronous correlations is about as large as the difference between the cross-lagged correlations (e.g., Humphreys & Stubbs, 1977, Table 4; Kenny, 1975, Tables 3 and 5). In a few applications a test of the null hypothesis—\( H_0: \rho_{X_1Y_1} = \rho_{X_2Y_2} \)—is performed. However, nonrejection does not prove the null hypothesis true, and in addition this null hypothesis is only a necessary condition for stationarity. The statistical test of equal cross-lagged correlations ignores the assumptions and the use of a preliminary test for stationarity. The complete null hypothesis should be that the cross-lagged correlations are equal, conditional on stationarity. That is, the conditional null hypothesis of interest is

\[ H_0: \rho_{X_1Y_1} = \rho_{X_2Y_2} \mid \rho_{X_1X_2} = \rho_{X_1X_2} \]

No exact statistical test is available for this conditional null hypothesis, although methods such as covariance structure analysis (Jöreskog & Sörbom, 1979) could be used to form a large-sample, normal-theory test. Of course, no improvement in the use of statistical inference procedures can offset the basic deficiencies of CLC.

**Summary and Discussion**

No justification was found for the use of CLC. In CLC both determinations of spuriousness and attributions of causal predominance are unsound. The results for 2W2V panels demonstrate that when reciprocal causal effects are absent, the difference between the cross-lagged correlations may be either small or large, and when reciprocal causal effects are present, the difference between the cross-lagged correlations may be either small or large. Also, the practice in CLC of reducing the analysis of data with multiple waves and multiple measures to a collection of 2W2V analyses produces additional problems. CLC is best forgotten.

It should be stressed that this article is not devoted to identifying perverse situations in which CLC might break down. Rather, a straightforward and explicit formulation of causal effects in panel data showed that CLC is not a sound basis for causal inference over a wide range of plausible situations. Possibly, CLC could be patched up, primarily through additional restrictive assumptions, in response to the deficiencies demonstrated in this article. This article is not intended to stimulate such activities. CLC should be set aside as a dead end.

In some sense this article is a flight of fantasy. The fantasy is the notion of a closed two-variable causal system on which the exposition and results are based. This simple formulation serves well for investigating the worth of CLC. Also, almost all applications and technical development of CLC have been limited to two-variable causal systems. CLC fails even in this idealized situation, and no grounds for optimism exist for better performance in more complex causal systems.

In his articles on longitudinal panel data, Duncan (1969, 1972, 1975) stressed that the analysis of panel data cannot be reduced to a mechanical procedure that yields trustworthy inferences about causal structures. A minimal requirement for success is the careful formulation of explicit (and often specialized) models for the substantive processes. An intent of this article is to emphasize this message. Trying to answer a causal question from a set of (longitudinal) data is asking a lot from those data. Minimal requirements are that the right variables be measured well. Often the state of theoretical and empirical knowledge in a substantive area is not sufficiently advanced that the relevant variables have been identified or that sufficient measurement techniques have been developed.

Alternative methods for analyzing panel data were not endorsed or discussed in detail. This omission reflects the fact that methods for detecting patterns of causal influence from panel data are far from fully developed. The contribution of this article is to demonstrate that CLC certainly is not the method to rely on for the analysis of panel data. It is hoped that this article will direct efforts away from further development and application of CLC and toward the development and evaluation of productive approaches for the analysis of panel data.
References


Calsyn, R. J., & Kenny, D. A. Self-concept of ability and perceived evaluation of others: Cause or effect of academic achievement? *Journal of Educational Psychology, 1977,* 69, 136–145.


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