Competition in Two-part Tariffs between Asymmetric Firms∗

Jorge Tamayo† and Guofu Tan‡

August 3, 2019

Abstract

We study competitive two-part tariffs in a model of asymmetric duopoly firms offering (vertically and horizontally) differentiated products. We provide necessary and sufficient conditions for marginal-cost pricing to be in equilibrium, in both the Hotelling and general discrete choice approaches to horizontal differentiation. When firms face symmetric demands but have asymmetric marginal costs, we show that in equilibrium the less-efficient firm sets its marginal price below its own marginal cost and compensates for this loss with the fixed fee, while the more-efficient firm sets its marginal price above its own marginal cost but below its rival’s price. A similar pattern holds in a setting where firms have identical marginal costs but asymmetric demands, that is, the inferior firm “cross-subsidizes” between fixed fee and marginal price. When the market shares are determined by Logit with outside option we show that, even in the symmetric model, the equilibrium outcome with two-part tariffs is not efficient.

Keywords: Competition, two-part tariffs, marginal-cost pricing, cross-subsidies, product differentiation

JEL Codes: L11, L13, L15

∗We are grateful to Mark Armstrong, Odilon Camara, Yongmin Chen, Kenneth Chuk, Michele Fioretti, Yılmaz Kocer, Anthony Marino, Steven Puller, Nicolas Schutz, Alex White, Simon Wilkie, Junjie Zhou, and seminar participants at the the 14th Annual International Industrial Organization Conference, the 91st Annual Conference–Western Economic Association International, the 2016 Asian Meeting of the Econometric Society, LACEA-LAMES Annual Meeting 2016, Asia-Pacific Industrial Organization Conference 2016, Workshop on Industrial Organization and Competition Policy (University of International Business and Economics), University of Southern California, The Hong Kong University of Science and Technology, East China Normal University, Shanghai University of Finance and Economics, and Lingnan University for their valuable comments.

†Harvard University, Harvard Business School; jtamayo@hbs.edu.

‡University of Southern California, Department of Economics; guofutan@usc.edu.
1 Introduction

In this paper, we study competitive two-part tariffs (2PTs) offered by duopolistic firms when consumers have variable demands and private information regarding horizontal brand preferences and quality preferences for the products, and when firms have asymmetric marginal costs and face asymmetric vertically differentiated demands. Firms that use 2PTs usually charge a membership fee that allows customers to buy products and services at a unit price (or usage price). We show that when the necessary and sufficient condition for marginal cost pricing is not satisfied, the equilibrium strategy involves “cross-subsidization” between the unit price and the fixed fee for the “disadvantaged” firm, while the “advantaged” firm always offers a unit price above its marginal cost.

2PTs are prevalent and widely practiced in different industries. Examples include credit cards, membership retail stores (e.g., Costco and Sam’s Club), and TV and wireless carrier subscriptions, among others.\(^1\) The liberalization of the British electricity market at the end of the 1990s is perhaps one of the best examples of competition in 2PTs between asymmetric firms. The retail sector had been separated into 14 monopolized regional markets before it was opened to competition. In the years before and after liberalization, firms predominantly offered single 2PTs. An important characteristic of these markets is the considerable variability of the tariffs offered in each region at each point in time, after the liberalization. According to Davies et al. (2014), in two-thirds of the cases, the entrant offered a lower unit price with a higher fixed fee than the incumbent.\(^2\) Moreover, they show that this tariff asymmetry was persistent.

Further, the largest membership (or subscription) business model, Amazon Prime—Amazon’s loyalty program—has largely driven Amazon’s share of US e-commerce sales to up to 44% of the market in 2017 (One Click Retail, 2018).\(^3\)

More recently, Internet-enabled subscription services have been growing exponentially: business-to-consumer subscription services have been growing at 200% annually since 2011 (McCarthy and Fader, 2017); the on-demand economy is attracting more than 22.4 million

\(^1\)Other examples include telephone services, car rentals, club memberships, equipment leasing, and amusement parks.

\(^2\)In this market, there are other types of asymmetries not considered in our paper. In particular, most of the electricity suppliers were also active in the gas market. Some of the firms were vertically integrated into electric generation; National Grid provides transmission and there is a monopoly distributor in each of these regions. For a complete description of the British electricity market, see Davies et al. (2014).

\(^3\)According to Morgan Stanley Research (2017), Amazon Prime penetration is expected to increase to 51% of US households by the end of 2018. Amazon has expanded Amazon Prime, offering various benefits: including access to Amazon Instant Video, free cloud storage through Amazon Web Services, and access to special deals (Lightning Deals) on Prime Days.

\(^4\)Business-to-consumer subscription businesses sell a wide variety of products, including food (Hello Fresh and Blue Apron), grooming products (Dollar Shave Club), and clothes (Stitch Fix and Trunk Club). For
consumers annually (Colby and Bell, 2016). Amazon has expanded its membership services beyond Amazon Prime, offering shipping on everyday essentials (Prime Pantry) and groceries (Amazon Fresh). During 2016 and 2017, Uber tested Uber Plus, which allows consumers to purchase fixed-price rides for a monthly fee (Kominers, 2017). In all of the above examples, consumers pay a positive fixed fee that allows them to buy products and services at a given (positive or zero) unit price, that is, from firms charging 2PT.

Most of the firms described above share three main features. First, firms offer multiple products and consumers may purchase multiple units of each product. Examples of this (variable demand) are online grocery delivery services (e.g., Amazon Fresh and Instacart), membership-only retail stores (e.g., Costco and Sam’s Club), health clubs (e.g., tennis and country clubs), credit cards, and the retail electricity market (e.g., British electricity market). Second, consumers are often heterogeneous and differentiated by unobservable tastes for quality, as well as by their brand (horizontal) preferences. For example, for the British electricity market, Davies et al. (2014) show that consumption varies significantly across households. Third, firms are asymmetric. That is, they usually have asymmetric costs (e.g., marginal costs) and offer asymmetrically differentiated products. In fact, some of the industries in which 2PTs are widely practiced have evolved from natural monopolies before the recent worldwide liberalization of their sectors–e.g., energy and communication–and have experienced competition from more efficient firms (those with lower marginal costs) with new products and differentiated demands.

Most of the previous literature on competitive price discrimination assumes horizontally differentiated consumers with homogeneous tastes for quality (or homogeneous demand) and symmetrically competing firms; that is, firms with symmetrically marginal costs and product demands. However, these assumptions are restrictive for many applications of interest and do not match the main characteristics of the industries that use 2PTs, as described above.

The recent literature on competitive price discrimination shows that when the market is fully covered and symmetric firms offer nonlinear pricing schedules, the optimal strategy for firms in equilibrium is to offer marginal-cost-based 2PTs (Armstrong and Vickers, 2001; Rochet and Stole, 2002), implying that the equilibrium outcome is efficient. Since the analysis of general nonlinear pricing with asymmetric firms and general consumer preferences is details, see McCarthy and Fader (2017).

5Prime members pay a membership of $4.99 per month for Prime Pantry and $14.99 for Amazon Fresh.
6In fact, implementing 2PTs is often subject to uncertainty regarding consumers’ preferences, which may explain why so little price discrimination is observed compared to what the theory suggests (Armstrong, 2006).
72PTs, were traditionally viewed as a monopoly price discrimination tool. In fact, traditional economic theories viewed them as “price discrimination devices, employed exclusively by firms with market power” (i.e., Hayes, 1987). A seminal contribution is the study of Oi (1971).
complicated, we may want to understand whether and when the equilibrium outcome is still efficient when firms use 2PTs (a smaller strategy space) and infer about the general case with nonlinear pricing schedules.\textsuperscript{8}

In this paper, we construct a general model with two asymmetric firms both offering 2PTs, and both competing for horizontally differentiated consumers with heterogeneous tastes for product quality and variable demands.\textsuperscript{9} We consider two different assumptions regarding the horizontal differentiation. In the next four sections, we study a model in which consumers have horizontal brand preferences, \textit{à la} Hotelling, uniformly distributed. In Section 6, we consider a general discrete choice model of random utility maximization.\textsuperscript{10}

In Section 2, we introduce our framework and discuss the basic assumptions of the model. In Section 3, we assume that firms have asymmetric marginal costs and asymmetric demands (differentiated products). We start by considering a model in which consumers have private information about their horizontal-brand preferences and \textit{homogeneous} tastes for quality. We show that there exists a unique equilibrium in which firms set marginal prices equal to marginal costs.\textsuperscript{11} Further, we provide necessary and sufficient conditions such that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT when consumers have \textit{heterogeneous} tastes for quality. We show that if the marginal costs are asymmetric and the products of the two firms are symmetric—or if the marginal costs are symmetric, but firms offer vertically differentiated products—then marginal-cost-based 2PT is not a Nash equilibrium.

In Section 4, we assume that both firms have symmetric demands but asymmetric marginal costs (\textit{Asymmetric Costs Model}). From our results in Section 3, if consumers have heterogeneous tastes for quality, marginal-cost pricing is not a Nash equilibrium.\textsuperscript{12} We show that the optimal strategy for the less-efficient firm (the firm with a higher marginal cost) is to set its marginal price below its own marginal cost and compensate for this loss with the fixed fee. On the other hand, the optimal strategy for the efficient firm is to set its marginal price above its own marginal cost but below that of its rival. This result contrasts with the one in a model in which both firms use linear pricing (LP): as the number of tools available

\textsuperscript{8}We explain in detail the relationship between our paper and the literature on competitive price discrimination on page 6.

\textsuperscript{9}We assume that consumers types are described by a \textit{horizontal} brand preference parameter and a taste parameter for product quality, which are independently distributed.

\textsuperscript{10}For most of the paper, we assume that consumers are single-homing and assume full market coverage; that is, all consumers buy from one firm and both firms sell strictly positive quantities. In section 6, we provide an example of a logit demand model with an outside option.

\textsuperscript{11}This result is intimately related to the proposition in Mathewson and Winter (1997) for a multi-product firm selling goods that are strongly complementary in demand.

\textsuperscript{12}Whereas, if consumers are homogeneous in their tastes for quality, under the assumption of full market coverage, marginal-cost pricing is an equilibrium.
to firms increases (from one to two), they have incentives to establish “cross-subsidies” across the tariff instruments (fixed fee and marginal price), which is not possible in the LP model.

Section 5 presents the Asymmetric Demands Model, in which both firms have symmetric marginal costs but asymmetric demands. As in our model in Section 3, we know that if consumers are heterogeneous in their tastes for quality, marginal-cost pricing is not a Nash equilibrium. In fact, the optimal strategy for the firm whose products are vertically inferior is to set its own price below the marginal cost, whereas the optimal strategy for the firm with superior vertical goods is to set its marginal price above its rival’s price (and above its own marginal cost). This result contrasts with that in our previous setting, where the firm with the higher marginal cost (the disadvantaged firm in the Asymmetric Costs Model) sets its price below its own marginal cost but above its rival’s marginal price. This is because in the Asymmetric Costs Model, the efficient firm sets a marginal price below its rival’s price but above its own marginal cost, while in the Asymmetric Demands model, by setting a price below the common marginal cost, the firm with vertically superior goods (the advantaged firm in the Asymmetric Demands Model) would have to compensate for this loss by increasing the fixed fee, thus decreasing its market share. Hence, in both models, the disadvantaged firm uses cross-subsidies between the tariffs (e.g., sets its marginal price below its marginal cost and positive fixed fee). However, the equilibrium pricing strategy of the efficient firm depends on the nature of each model.

Section 6 extends our analysis by considering a discrete choice model of random utility maximization. We propose a model in which consumers have private information about their horizontal brand preferences and we consider homogeneous as well as heterogeneous tastes for quality. In the first case (homogeneous tastes for quality), we show that there exists an equilibrium in which firms set marginal prices equal to marginal costs, and we provide comparative static properties of the equilibrium. In the second case, with heterogeneous tastes preferences, we provide a necessary and sufficient such that any pure-strategy Nash equilibrium involves marginal-cost pricing. We show that when the market shares are determined by Logit, marginal-cost pricing is not an equilibrium, even in the symmetric model. We further show that the results of Section 4 hold in a simplified discrete-type model.

In Section 7, we study the conditions under which marginal-cost-based 2PT is an equilibrium if both firms use nonlinear tariffs instead of 2PTs, under Hotelling horizontal differentiation. Armstrong and Vickers (2001) and Rochet and Stole (2002) showed that if the two firms are symmetric and under full market coverage, in equilibrium each firm offers a marginal-cost-based 2PT. We extend this result, consider asymmetric firms, and derive necessary and sufficient conditions for a marginal-cost-based 2PT to be an equilibrium.
Related literature. A seminal contribution to the literature on competitive price discrimination is Armstrong and Vickers (2001), who studied competitive nonlinear pricing when consumers are differentiated à la Hotelling, have private information about their tastes for quality, and purchase all products from a single firm (one-stop shopping). They show that when the market is fully covered and firms are symmetric, each firm offers a simple 2PT contract with a marginal price equal to the marginal cost in equilibrium. Rochet and Stole (2002) interpreted the quantity in Armstrong and Vickers (2001) as quality (so consumers choose a price-quality pair) and show that if firms are symmetric and transportation cost is low enough to guarantee full coverage, firms offer a cost-plus-fee pricing schedule in equilibrium. However, this surprisingly simple yet elegant result strongly depends on the assumption of symmetry of the firms, excluding cases in which firms may have different marginal costs or may offer asymmetrically differentiated products.

Our analysis extends the findings of Armstrong and Vickers (2001) and Rochet and Stole (2002) in two ways. First, we provide necessary and sufficient conditions under which marginal-cost-based 2PT is an equilibrium under horizontally differentiated consumers with heterogeneous quality preferences and asymmetric firms. This condition allows us to identify environments in which marginal-cost-based 2PTs is not an equilibrium when firms have smaller pricing spaces like 2PTs and hence are also not an equilibrium when firms are allowed to use larger pricing spaces. Similarly, we show that even if firms are symmetric, marginal-cost-based 2PTs may not be an equilibrium, as in the Logit Model with outside option. Second, we characterize the equilibrium outcome of the model when marginal-cost-based 2PT is not an equilibrium and show that for the asymmetric marginal costs and asymmetric demands model, the optimal solution involves cross-subsidization between the marginal price and the fixed fee for the disadvantaged firm.

Yang and Ye (2008) consider a model similar to Armstrong and Vickers (2001) and Rochet and Stole (2002) but study the case in which consumer types on the vertical dimension are not fully covered. That is, they consider a model in which the lowest type of consumer covered (in the market) is endogenously determined. They show that when the market structure moves from monopoly to duopoly, more types of consumers are served and quality

---

[^13]: Note that if firms are symmetric and the market is competitive (all consumers buy from at least one firm), the results of Armstrong and Vickers (2001) and Rochet and Stole (2002) imply that there would be an efficient quantity (or quality) provision supported by the marginal-cost-based 2PTs.

[^14]: Armstrong and Vickers (2010) generalize the model in Armstrong and Vickers (2001) by assuming that consumers are allowed to multi-shop (buy from both firms or from just one) and find that in equilibrium, firms offer marginal-cost-based 2PTs. Hoernig and Valletti (2011) consider a simple version of the model in Armstrong and Vickers (2010) in which vertical and horizontal taste parameters are correlated. They show that neither 2PTs nor full exclusivity can arise in equilibrium. For a review of this literature see Armstrong (2016).
distortions decrease. Similarly, Shen et al. (2016) consider a model similar to Yang and Ye (2008) and provide conditions under which entry prompts an incumbent to expand or contract its low end of the product line. Note that in our model, we assume that all consumer types on the vertical dimension are covered; that is, firms do not exclude the low/mid-end of the market.

Yin (2004) considers a model of 2PT competition with general horizontal preferences in which the transportation cost interacts with the quantity (transportation cost is a “shipping” cost) and consumers have homogeneous tastes for quality. He shows that marginal prices are equal to marginal cost if and only if the demand of the marginal consumer (who is indifferent between buying the $i$-good and the $j$-good for $i \neq j$ in the full competition equilibrium) is equal to the average demand.\footnote{Schmalense (1981) studies 2PT monopoly equilibria with vertically differentiated consumers and shows that when the marginal consumer has the same demand as the average consumer, the optimal 2PT involves cost-based 2PT (see Varian, 1989, for a summary). However, if the marginal consumer demands more than the average consumer, the optimal price in the 2PT would be less than the marginal cost. Note that in our model, consumers are horizontal and vertically differentiated, and we assume that the market is fully covered, which provides a different intuition for “cross-subsidization.”} For instance, if the horizontal taste parameter is additively separable from the price (transportation cost is a “shopping” cost), marginal price is equal to the marginal cost in equilibrium.\footnote{Note that in our model, all consumers with the same taste preference purchase the same quantity of the goods, independently of their location. Thus, by construction, the demand of the marginal consumer is equal to the average demand.} We show that this result does not hold if consumers have heterogeneous taste preferences and firms have asymmetric marginal costs or asymmetric demands. In this case, the disadvantaged firm (the one with the higher marginal cost) sets its prices below its own marginal cost.

Hoernig and Valletti (2007) consider a model where consumers are horizontally differentiated, à la Hotelling, and mix goods offered by two firms, and show that the tariff structure affects location decision, consumers, and profits. The authors show that when both firms use 2PTs, marginal prices are equal to the marginal cost if and only if both firms are located at the same spot. Griva and Vettas (2015) consider a duopoly model in which firms use 2PTs and offer homogeneous goods to a population of vertically differentiated consumers (heterogeneous usage rate). The authors show that when one price of the components is fixed for both firms, the market is segmented, that is, low-usage consumers choose the low-fee firm and high-usage consumers choose the low-rate firm. Our analysis does not consider any of these cases; for example, interaction of the transportation cost with the quantity or location decisions. Thus, the reasons for marginal-cost-based 2PT are different from those in the previous models.

Also related to our study is the literature on cross-subsidization, which is commonly ob-
served in multiproduct firms, which often price some products below marginal cost and sub sidize the resulting loss of the profits from other products. The literature provides different explanations for competitive cross-subsidization. DeGraba (2006) shows that pricing below cost could serve as a strategy to screen the most profitable consumers in a setting in which firms face heterogeneous consumers. Chen and Rey (2012) show that pricing the products on which the large firm competes with the smaller rival below marginal cost and increasing the price on other products, allows the large firm to discriminate between multi-shoppers and one-stop shoppers. Note that in this context, loss leading serves as an exploitative device rather than as an exclusionary instrument. 17 Chen and Rey (2019) study multi-product firms with different comparative advantages, competing for customers with heterogeneous transaction costs. They show that firms price strong products (on which they have a comparative advantage) above cost and weak products below cost. Our paper provides a different rationale for “cross-subsidization.” Here, the disadvantaged firm is the one that has incentives to use cross-subsidies between the tariffs (fixed fee and marginal price) as an optimal strategy to extract consumer surplus.

2 Model

Two firms, A and B, offer differentiated products to a population of heterogeneous consumers. We assume that both firms can produce their products at constant marginal costs, denoted by \(c_A\) and \(c_B\), respectively. There is a mass of consumers with types \((x, \theta)\), where \(x\) is uniformly distributed on the unit interval independently of the distribution of \(\theta \equiv (\theta_1, \ldots, \theta_n) \in \Theta \equiv [\bar{\theta}, \tilde{\theta}]^n\), which is continuously distributed with cumulative distribution \(G(\cdot)\). 18 We adopt a one-stop shopping Hotelling model with heterogeneous consumers with different tastes for quality; that is, consumers buy all products from one or the other firm or else consume their outside option. The consumer’s preferences for the two differentiated products can be represented by the utility function \(u_A(q_A, \theta) - tx\) if she buys from A and \(u_B(q_B, \theta) - (1-x) t\) if she buys from B, where \(x\) is the distance to firm A (and \(1-x\) the distance to firm B), \(t>0\)

17 A related literature on loss leader in multi-good setting is Lal and Matutes (1994). The authors show that firms advertise a loss-leader product in order to attract consumers, in a single stop shopping model where consumers are initially unaware of prices. Rhodes (2014) develops a multi-product search model where competing firms randomly advertise one product at a low price, and may even set its advertised price below cost. Ellison (2005) considers an “add-on pricing” game in which add-on prices are unobserved, and firms advertise a base good in hopes of selling add-ons at high unadvertised prices. In equilibrium, firms may price the base product below cost to subsidize the loss with the profit from add-on prices. A driving force behind the result in Ellison (2005), is the correlation between the vertical taste and the horizontal preferences. Verboven (1999) gets a similar result—high prices for add-ons—assuming independent vertical and horizontal consumer heterogeneity.

18 In Section 6, we consider a general discrete choice model of random utility maximization.
is the consumer transportation cost per unit of distance, and $\theta$ represents the preference for quality.

The next assumption characterizes the set of utility functions.

**Assumption 1.** The utility function $u_i(q_i, \theta) : \mathbb{R}^+_+ \times \Theta \to \mathbb{R}^+_+$ is twice continuously differentiable and satisfies $\frac{\partial u_i(q, \theta)}{\partial q_i} \bigg|_{q_i=0} > c_i$, $\frac{\partial^2 u_i(q, \theta)}{\partial q_i^2} < 0 \ \forall \theta \in \Theta$ and $\frac{\partial^2 u_i(q, \theta)}{\partial q_i \partial \theta_j} > 0$, $\forall j \in \{1, 2, \ldots, n\}$.

The firms use 2PTs, which include a marginal (unit) price, $p_i$, and a lump-sum fee, $F_i$, for $i \in \{A, B\}$. To avoid expositional complications, we define the set of feasible unit prices of both firms as $\mathcal{P}$. Given $(p_i, F_i)$, a consumer with the “vertical” taste parameter $\theta \in \Theta$ decides to buy $q_i : \mathcal{P} \times \Theta \to \mathbb{R}^+_+$ units from firm $i \in \{A, B\}$, where

$$q_i(p_i, \theta) = \arg \max_{q_i \in \mathbb{R}^+_+} \{u_i(q_i, \theta) - p_i q_i\}.$$

So the net utility $U_i(p_i, F_i, \theta)$ is

$$U_i(p_i, F_i, \theta) \equiv v_i(p_i, \theta) - F_i,$$

where $v_i(p_i, \theta)$ is the indirect utility “offered” by firm $i$, defined by

$$v_i(p_i, \theta) \equiv \max_{q_i \in \mathbb{R}^+_+} \{u_i(q_i, \theta) - p_i q_i\}.$$

We will focus on the case with $E[v_i(c_i, \theta)] > 0$, where $v_i(c_i, \theta)$ is the maximum surplus offering a good at the marginal cost, $c_i$, by firm $i \in \{A, B\}$ for any $\theta \in \Theta$.

Note that the indirect utility function, $v_i(p_i, \theta)$, satisfies $q_i(p_i, \theta) = -\partial v_i(\cdot) / \partial p_i$ by Roy’s identity with $\frac{\partial^2 v_i(\cdot)}{\partial p_i \partial \theta_j} < 0$ for all $i \in \{A, B\}$ and $\forall j \in \{1, 2, \ldots, n\}$. Moreover, by continuity of the first and second derivatives of $v_i(p_i, \theta)$ and by Roy’s identity, we know that $v_i(p_i, \theta)$ is submodular in $(p_i, \theta)$.

From the properties of supermodular (submodular) functions, we know that $-v_i(p_i, \theta)$ satisfies the increasing differences property. That is, $v_i(p_i, \theta) - v_i(p'_i, \theta)$ must be monotone nondecreasing in $\theta$ for all $p_i, p'_i \in \mathcal{P}$ and $p_i \leq p'_i \ \forall i \in \{A, B\}$.

In order to simplify our analysis, we assume full market coverage in which all consumers buy from at least one firm $i \in \{A, B\}$, and both firms sell strictly positive quantities. This assumption implies a lower and an upper bound for $t$, which will depend on the model
considered in each section.

Note that (A1) implies that the buyer’s demand function and the monopoly profit function—\( q_i(p_i, \theta) \) and \( \pi_i(p_i, \theta) \), respectively—are continuously differentiable and that \( q_i(p_i, \theta) \) is strictly decreasing in \( p_i, \forall i \in \{A, B\} \).

**Assumption 2.** \( \frac{\partial \mu_i(p_i)}{\partial p_i} < 1, \forall i \in \{A, B\} \), where \( \mu_i(p_i) \equiv -\frac{E[q_i(p_i, \theta)]}{E[q_i'(p_i, \theta)]} \) and \( q_i'(p_i, \theta) \equiv \frac{\partial q_i(p_i, \theta)}{\partial p_i} \).

Under (A2), there is a unique optimal monopoly price \( p^*_i \in \mathcal{P} \). Furthermore, the expected value of the monopoly profit function,

\[
E[\pi_i(p_i, \theta)] = E[q_i(p_i, \theta)] (p_i - c_i)
\]

is single-peaked in \( p_i \) under (A2).

Due to our full market coverage assumption, the share of \( \theta \)-consumers who decide to buy from firm \( i \in \{A, B\} \) is\(^{23} \)

\[
s_i(p_i, F_i, p_j, F_j; \theta) \equiv \frac{1}{2} + \frac{v_i(p_i, \theta) - v_j(p_j, \theta) - F_i + F_j}{2t},
\]

and the share of firm \( j \neq i \) is \( s_j(p_j, F_j, p_i, F_i; \theta) = 1 - s_i(p_i, F_i, p_j, F_j; \theta) \). The problem of each firm \( i \in \{A, B\} \) is

\[
\max_{p_i, F_i} E\{s_i(p_i, F_i, p_j, F_j; \theta) [\pi_i(p_i, \theta) + F_i]\}
\]

for \( j \neq i \).

We present conditions for marginal-cost-based 2PT under the assumption of homogeneous and heterogeneous taste preferences in consumers.\(^{24} \) We restrict our general model to studying the equilibrium pricing strategy of each asymmetry separately. First, we assume that

\(^{22}\)Carrillo and Tan (2015) use an identical assumption in a model of platform competition. Likewise, Armstrong and Vickers (2001) have a similar assumption for a model with consumers with homogeneous tastes for quality and symmetric firms with a common marginal cost, \( c \). They assume \( \sigma'(u) \leq 0 \), where \( \sigma(p) = -\frac{2v(p)}{E[v]} (p - c) \) for \( u = v(p) \). The function \( \sigma(p) \) represents the elasticity of demand expressed in terms of the markup \( p - c \) rather than in terms of the price \( p \). It is straightforward to show that \( \rho'(p) < 1 \) implies that \( \sigma'(u) \leq 0 \).

\(^{23}\)The full market coverage assumption requires a lower bound for \( t \) that guarantees that both firms sell strictly positive quantities and an upper bound such that all consumers buy from at least one firm. Note that for each model, we need different bounds. In Section 3 we define the lower and upper bound for \( t \). For the rest of the models, the bounds are similar, so we exclude them from the analysis.

\(^{24}\)Note that the terms “homogeneous” and “heterogeneous” refer to the taste parameter \( \theta \). We will denote “homogeneous preferences” when \( \theta \) is constant in the model and “heterogeneous preferences” when \( \theta \) follows a distribution \( G(\cdot) \) independent of \( x \). Note that in both cases, consumers are horizontally differentiated.
the indirect utilities provided by both firms are equal—that is, \( v_i(p, \theta) = v_j(p, \theta) = v(p, \theta) \) for all \( p \in \mathcal{P} \) and \( \theta \in \Theta \) where \( v(p, \theta) \) satisfies (A1)—but that the marginal cost for firm \( A \), the efficient firm, is lower than the marginal cost for firm \( B \), the less-efficient firm, that is, \( c_A < c_B \). The second model assumes that both firms have symmetric marginal costs but offer differentiated goods. In particular, we assume that the products offered by firm \( A \) are vertically superior to the products offered by firm \( B \); that is, \( v_A(p, \theta) > v_B(p, \theta) \) for all \( p \in \mathcal{P} \) and \( \theta \in \Theta \).

3 Marginal-cost Pricing

In this section, we assume that firms offer differentiated products and have different marginal costs. We start by considering a model in which consumers are homogeneous in their taste for quality, whereas their horizontal brand preferences remain unknown to the firm. We show that there exists a unique equilibrium in which firms set their marginal prices equal to their marginal costs. Next, we consider a model in which consumers have heterogeneous taste preferences and provide necessary and sufficient conditions for marginal-cost-based 2PT Nash equilibrium. We show that under certain conditions this equilibrium is unique.

**Homogeneous preferences.** The set of feasible unit prices is

\[ \mathcal{P} = [c, \bar{p}] , \]

where \( c \equiv \min \{c_A, c_B\} \) and \( \bar{p} = \max \{p^m_A, p^m_B\} \).

Now consider the choice of prices and fixed fees by each firm. Due to our full market coverage assumption, the market share of consumers, \( s_i(p_i, F_i, p_j, F_j) \), who decide to buy from firm \( i \in \{A, B\} \) is defined by the analogue of (1) for \( \theta \) identical for all consumers.\(^{25}\)

The problem of firm \( i \in \{A, B\} \) is

\[ \max_{p_i, F_i} \Pi^i = \max_{p_i, F_i} s_i(p_i, F_i, p_j, F_j) [\pi_i(p_i) + F_i] \]

for \( j \neq i \).

**Proposition 1.** Suppose the analogues of (A1) and (A2) for \( \theta \) identical for all consumers are satisfied. Then, marginal-cost-based 2PT is a unique equilibrium where \( F^*_i = \]

\[^{25}\text{The full market coverage assumption requires } t \in \{ t \in \mathbb{R}_+; \frac{v_A(c_A) - v_B(c_B)}{3} < t < \frac{v_A(c_A) + v_B(c_B)}{3} \}. \text{ For the rest of the paper, we omit the conditions for } t.\]
\[ t + \frac{v_i(c_i) - v_j(c_j)}{3} \] for \( i \in \{A, B\} \) and \( j \neq i \).  \(^{26}\)

Proposition 1 shows that if consumers are homogeneous in their tastes for quality, under the assumption of full market coverage, the optimal strategy for each firm is to set its prices equal to its marginal costs and extract surplus through the fixed fee. Note that in this model, the marginal costs of the two firms may be different, which implies that the marginal prices (and fixed fees) may also be different.  \(^{27}\)

Proposition 1 is close to the result in Mathewson and Winter (1997) for goods that are strongly complementary in demand. In our model of one-stop shopping and homogeneous taste preferences, consider firm \( i \)'s choices for \( i \in \{A, B\} \): we can interpret the permission to allow consumers to enter the shop as the first product (product 1) and its price to be equal to the fixed fee \( F_i \), and treat the real product offered by firm \( i \) as product 2, with a price equal to \( p_i \). The demand for product 1 is the market share of firm \( i \)'s product, \( s_i(p_i, F_i, p_j, F_j) \), and the demand for product 2 is the market share multiplied by the individual demand for that product, \( s_i(p_i, F_i, p_j, F_j)q_i(p_i) \). Note that the ratio is independent of the fixed fee, \( F_i \). Hence the two “products” are strong complements. Using Proposition 2 in Mathewson and Winter, we could conclude that the profits are maximized for firm \( i \) at \( p_i = c_i \). Hence, independently of firm \( j \)'s actions, firm \( i \neq j \) always charges the marginal cost of the second product, \( c_i \).  \(^{28}\)

Note that if we modify our model to make it compatible with Yin (2004), our Proposition 1 would be able to be derived from his Proposition 1. When the location parameter does not interact with quantity, the demand of the marginal consumer is equal to the average demand, satisfying the condition for marginal-cost pricing. However, there are three important remarks: First, Yin assumes a general distribution for the consumers while we assume they are uniformly distributed on \([0, 1]\). Second, although Yin considers the particular case in which consumers are uniformly distributed, firms in this case have symmetric costs and

\(^{26}\)If \( t < \frac{v_A(c_A) - v_B(c_B)}{3} \), then there exists a corner equilibrium in which firm B sets \( p_B = c_B \) and \( F_B = 0 \) while firm A sets \( p_A = c_A \) and \( F_A = \frac{t}{2} + \frac{v_A(c_A) - v_B(c_B)}{2} \). For the rest of the paper we consider only interior equilibria.

\(^{27}\)Note that we exclude from the analysis cases in which the fixed fees offered by the two firms are equal to zero; otherwise, we will end up considering an LP game. If the full market coverage assumption is satisfied, firms will have incentives to deviate and offer a 2PT scheme with positive fixed fees.

\(^{28}\)Note that this game (and, in general, the set of games presented here) satisfies strategic complementarity on rivals’ strategies, as in Bulow et al. (1985). However, neither is a game with strategic complementarities as in Vives (1990) nor a supermodular game as in Milgrom and Roberts (1994). The reason is that the product under consideration and access by each firm are complements to consumers, not substitutes. These two “products” are therefore substitutes across the firms but complements within each firm. Schmalense (1981) and Varian (1989) pointed out that 2PTs are a pricing problem with complementary goods. Thus we cannot use the results derived from this set of games (e.g., the existence of Nash and the order structure of any equilibria).
product demand. Third, we cannot show uniqueness (or the conditions needed) from his result.

From Proposition 1, we know that in equilibrium both firms set their prices equal to their marginal costs and in equilibrium the firm that provides the highest surplus (at its own marginal cost) has the highest fixed fee, market share, and total profits. Similarly, in the asymmetric marginal costs model (indirect utilities are symmetric, that is, \( v_i(p) = v_j(p) \) for all \( p \in \mathcal{P} \) and \( i \neq j \), if \( c_i < c_j \), then in equilibrium \( F_i > F_j \), \( s_i > s_j \) and \( \Pi_i > \Pi_j \). Likewise, note that for the asymmetric demands model (marginal costs are symmetric, that is, \( c_i = c_j \) for \( j \neq i \)), if \( v_i(p) > v_j(p) \) for all \( p \in \mathcal{P} \), then in equilibrium \( F_i > F_j \), \( s_i > s_j \) and \( \Pi_i > \Pi_j \). Finally, note that if both the marginal cost and the indirect utility are symmetric, we get the standard 2PTs result.

**Heterogeneous preferences.** We shift our attention to the case in which consumers differ both in their brand preferences (horizontal differentiation) and in their taste preferences (vertical quality parameter) and we provide necessary and sufficient conditions under which any equilibrium involves marginal-cost pricing. Due to our full market coverage assumption, the market share and the problem of firm \( i \in \{A, B\} \) are defined by (1) and (2), respectively. First-order conditions for firm \( i \in \{A, B\} \) are

\[
[p_i] : -E[q_i(p_i, \theta) \pi_i(p_i, \theta)] - E[q_i(p_i, \theta)] \cdot F_i + 2t \cdot E[\pi'_i(p_i, \theta) s_i(p_i, F_i, p_j, F_j; \theta)] = 0 \tag{3}
\]

and

\[
[F_i] : 2t \cdot E[s_i(p_i, F_i, p_j, F_j; \theta)] - E[\pi_i(p_i, \theta)] - F_i = 0. \tag{4}
\]

We can establish general conditions under which any pure-strategy Nash equilibrium involves marginal-cost pricing. From (3) and (4), we get the condition:

\[
\text{Cov} (v_i(p_i, \theta) - v_j(p_j, \theta), q_i(p_i, \theta)) = 0 \tag{5}
\]

for \( p_i = c_i, i \in \{A, B\} \), and \( j \neq i \). We summarize this result in the following proposition.

**Proposition 2.**

(i) For a given \( c_i, c_j \in \mathcal{P} \), a pure-strategy Nash equilibrium involves marginal-cost-based 2PT if and only if (5) holds for \( p_i = c_i \) for \( i, j \in \{A, B\} \) and \( i \neq j \).

(ii) If for any \( p_i, p_j \in \mathcal{P} \), (5) holds for \( i, j \in \{A, B\} \) and \( i \neq j \), marginal-cost-based 2PT is a unique equilibrium.

Note that if (5) holds for \( p_i = c_i \), the covariance of the demand, \( q_i(p_i, \theta) \), and the market
share, \( s_i(p_i, F_i, p_j, F_j; \theta) \), is also zero. The reason is that the market share is linear with respect to the difference between the two indirect utilities, \( v_i(p_i, \theta) - v_j(p_j, \theta) \), for the uniformly distributed model, à la Hotelling. Thus, any pure-strategy Nash equilibrium involves marginal-cost-based 2PT if and only if the covariance of the demand and the market share for both firms, evaluated at the marginal cost, is zero. Now, if the demand is independent of the market share for firm \( i \in \{ A, B \} \) for all feasible prices marginal-cost-based 2PT is a unique equilibrium.

We can use our previous example to explain the conditions under which Mathewson and Winter’s result holds in our model when consumers have heterogeneous tastes for quality. Remember that we can interpret the permission to allow consumers to enter the shop as the first product and treat the real product offered by firm \( i \) as product 2 with prices \( F_i \) and \( p_i \), respectively. In this case the demand for product 1 is the expected market share for firm \( i \)’s product, \( E[s_i(p_i, F_i, p_j, F_j, \theta)] \), and the demand for product 2 is the expected value of the market share multiplied by the individual demand for that product, \( E[s_i(p_i, F_i, p_j, F_j, \theta) q_i(p_i, \theta)] \).

Proposition 2(i) shows indirectly that if, for \( i, j \in \{ A, B \} \) and \( i \neq j \),
\[
\frac{E[s_i(c_i, F_i, c_j, F_j, \theta) q_i(c_i, \theta)]}{E[s_i(c_i, F_i, c_j, F_j, \theta)]} = E[q_i(c_i, \theta)],
\]
then any equilibrium involves marginal-cost pricing. That is, if (5) holds, the ratio of the demands for the two products is independent of the fixed fee, \( F_i \), for \( p_i = c_i \). Hence, from Mathewson and Winter’s result, we know that marginal-cost pricing is part of the equilibrium.\(^{29}\) Note that this condition is always satisfied for the symmetric case, as we discuss in the following corollary.

**Corollary 1.** If \( c_i = c_j = c \) and \( v_i(p, \theta) = v_j(p, \theta) \) for all \( p \in \mathcal{P} \), \( \theta \in \Theta \) and \( j \neq i \), any pure-strategy Nash equilibrium involves marginal-cost-based 2PT.

Corollary 1 is related to the standard result of 2PTs (e.g., Armstrong and Vickers, 2001; Rochet and Stole, 2002); that is, marginal-cost pricing is an equilibrium for the symmetric case.

**Assumption 3.** \( \theta \) is associated.

A vector \( \theta \) of random variables is associated if \( \text{Cov}[f(\theta), g(\theta)] \geq 0 \) for all nondecreasing functions \( f \) and \( g \) for which \( E[f(\theta)], E[g(\theta)] \), and \( E[f(\theta) g(\theta)] \) exist.\(^{30}\)

\(^{29}\)Note that the fact that (5) implies (6) for \( p_i = c_i \) is a special feature of Hotelling’s market share. That is, this result would not be true for a model like that in Section 6, with a general market share.

\(^{30}\)We use a strict version of association for the rest of the paper. For a complete reference on association of random variables and its properties, see Esary et al. (1967). See also Holmstrom and Milgrom (1994) and
Proposition 2 implies that if $\theta$ is associated, since $q_i(c_i, \theta)$ is monotonic increasing, and if $v_i(c_i, \theta) - v_j(c_j, \theta)$ is monotonic increasing or decreasing (depending on marginal costs and the functional form of $v_i(\cdot)$ for $i \in \{A, B\}$) with respect to $\theta$, marginal-cost pricing is not an equilibrium; that is (5) is violated. Moreover, note that Proposition 2 is a general result in the following sense.

**Corollary 2.**

(i) If $c_i \neq c_j$ and $v_i(p, \theta) = v_j(p, \theta) = v(p, \theta)$ for all $p \in \mathcal{P}$ and $\theta \in \Theta$, then marginal-cost-based 2PT is not a Nash equilibrium.

(ii) If $c_i = c_j$ and $v_i(p, \theta) - v_j(p, \theta)$ is monotonic with respect to $\theta$, then marginal-cost-based 2PT is not a Nash equilibrium.

Corollary 2(i) shows that if marginal costs are asymmetric and the products of the two firms are symmetric, then (5) does not hold. Likewise, if marginal costs are symmetric but $v_i(p, \theta) - v_j(p, \theta)$ is monotonic with respect to $\theta$, then (5) does not hold from Corollary 2(ii).

An illustrative example of the last case with symmetric marginal cost is the following: Suppose for instance that for any $p \in \mathcal{P}$ and $\theta \in \Theta$, the indirect utility offered by firm $i$ is $v(p, \theta)$ (satisfies A1) and the indirect utility offer by firm $j$, $j \neq i$, is $\alpha \cdot v(p, \theta)$ for any $\alpha \in (0, 1)$. Then, marginal-cost-pricing is not an equilibrium. However, if marginal costs are asymmetric and $v(p, \theta)$, $c_i$ for $i \in \{A, B\}$ and $\alpha$ are such that $v(c_i, \theta) - \alpha v(c_j, \theta) = 0$ for all $\theta \in \Theta$, marginal-cost-based 2PT is an equilibrium.

The reason for this difference between Corollaries 1 and 2 is related to the dependence of the fixed fees and marginal prices on $\theta$. If both marginal costs and indirect utilities (demand for the two goods) are symmetric, the most profitable way for both firms to attract consumers and extract consumer surplus is to set the marginal price equal to the marginal cost and set the fixed fee equal to $t$. Note that because the marginal price and the fixed fee do not depend on $\theta$, this tariff remains an equilibrium even when $\theta$ is unknown for both firms (Armstrong, 2006; Armstrong and Vickers, 2001). However, if marginal costs or the products offered by the two firms are asymmetric, the marginal price and the fixed fee would depend on $\theta$. In the next section, we show that as the dispersion of $\theta$ increases, the quasi-best response function in terms of $(p_i, p_j)$ rotates clockwise for firm $i$ and counterclockwise

---

By “monotonic with respect to $\theta$,” we refer to the following example: Let $\theta, \theta' \in \Theta$ such that $\theta > \theta'$ ("high" and "low" type). Then if $v_i(p, \theta) - v_j(p, \theta)$ is monotonic with respect to $\theta$, the sum of the indirect utilities offered by firm $i$ and $j$ to the high and low type, respectively, is higher than the sum of indirect utilities offered by firm $i$ and $j$ to the low and high type, respectively, that is, $v_A(p, \theta) + v_B(p, \theta') > v_A(p, \theta') + v_B(p, \theta)$. Thus, product A is "vertically" superior to product B.

Marginal-cost-based 2PT is also an equilibrium if, for example, the indirect utilities offered by the two firms are such that $v_i(p, \theta) - v_j(p, \theta) = k$, where $k$ is a constant for all $p \in \mathcal{P}$ and $\theta \in \Theta$. 

Milgrom and Weber (1982) for economic applications.
for firm \( j \), for \( i \neq j \). This result implies that firms would have incentives to deviate from marginal-cost-based 2PT.

In sum, when firms have asymmetric marginal costs or asymmetric demands, information about vertical taste preferences has a substantial effect on the equilibrium pricing strategy. That is, vertical uncertainty affects the slope of the implicit best-response functions regarding the marginal prices. We will investigate this issue further in the next sections.

4 Asymmetric Costs

In this section, we suppose that the indirect utilities offered by the two firms are symmetric but that marginal costs differ. Without loss of generality, we assume that the marginal cost of firm \( A \), the efficient firm, is lower than the marginal cost of firm \( B \), the less-efficient firm; that is, \( c_B > c_A \geq 0 \). We assume that consumers are heterogeneous in their taste for quality; thus, from Corollary 2 we know that marginal-cost-based 2PT is not a Nash equilibrium.

We first show in Proposition 3 that no equilibrium exists for \( p_B \geq c_B \). Indeed, we show that in any pure-strategy Nash equilibrium, \((p_A, p_B) \in (c_A, c_B)^2\). Next, we show that the quasi-best response functions of the two firms are increasing and that the equilibrium is unique.\(^{33}\) Finally, we provide comparative statics with respect to \( \theta \).

To make the notation compatible with previous sections, we need to redefine the set of feasible unit prices for both firms:

\[
\hat{P} = [c_A, p^m_B],
\]

where \( p^m_B \) corresponds to the monopoly price of firm \( B \). We restrict the set of feasible unit prices of the efficient firm to always be above its marginal cost, \( c_A \), while prices for firm \( B \) are allowed to be lower than its marginal cost, \( c_B \).\(^{34}\) The problem of each firm \( i \in \{A, B\} \) is

\[
\max_{p_i, F_i} E \left\{ \left( \frac{1}{2} + \frac{v(p_i, \theta) - v(p_j, \theta) - F_i + F_j}{2t} \right) [\pi_i(p_i, \theta) + F_i] \right\}.
\]

From the first-order conditions with respect to \( p_i \), for firm \( i \),

\[
(p_i - c_i) E \left[ 2t \cdot q'(p_i, \theta) s_i - q(p_i, \theta)^2 \right] + E \left[ q(p_i, \theta) (t + v(p_i, \theta) - v(p_j, \theta)) \right] + E \left[ q(p_i, \theta) (F_j - 2F_i) \right] = 0,
\]

\(^{33}\)By quasi-best response functions, we refer to the best response functions only in terms of \( p_A \) and \( p_B \). We show later that we can find an explicit solution for \((F_i, F_j)\) as a function of \((p_i, p_j)\). This fact, allows to simplified the first-order conditions as a function (only) of \((p_i, p_j)\).

\(^{34}\)We discuss in the Appendix that this assumption is without loss of generality.
where \( s_i \) is the market share of firm \( i \in \{A, B\} \). When both firms use LP, the first and the second term on the left-hand side of (7) characterize the best response functions of each firm \( i \).

Thus, there are two main differences when firms use 2PT compared to LP: First, there is a direct effect that moves the curve of the quasi-best response function of firm \( i \) to the left, in the \( p_i, p_j \) plane for \( j \neq i \).

This implies that firm \( i \) reacts more aggressively with its marginal price for each value of \( p_j \). Second, there is an indirect effect, since as \( F_i \) increases, firm \( j \) reacts more aggressively, decreasing \( p_j \) for each value of \( p_i \), increasing \( F_j \).

The intuition for these two effects is the following: when firms are allowed to use 2PTs, they can also extract consumer surplus with the fixed fee, which does not depend directly on the curvature of the demand. Firms react aggressively with their marginal prices by setting a low price, allowing them to attract consumers and to extract surplus more efficiently with the fixed fee. We next show that in equilibrium, both firms decrease their marginal prices compared to the case when both firms use LP.

From the first-order conditions with respect to \( F_i \) for each firm \( i \), the fixed fee in equilibrium is defined by

\[
F_i^* + E[\pi_i(p_i, \theta)] = t + TS_i(p_i) - TS_j(p_j),
\]

where \( TS_i(p_i) \equiv (E[v(p_i, \theta)] + E[\pi_i(p_i, \theta)]) / 3 \) is proportional to the expected total surplus for firm \( i \in \{A, B\} \) for \( j \neq i \). From (8), we know that the total profit per consumer depends on the transportation cost and the difference between the surplus offered by both firms. Similarly, from (7) and (8), \( p_i \) in equilibrium is implicitly defined by

\[
(p_i - c_i) \{2t \cdot E[q'(p_i, \theta)s_i] - \text{Var}(q(p_i, \theta)) + \text{Cov}(v(p_i, \theta) - v(p_j, \theta), q(p_i, \theta)) = 0,
\]

where \( s_i \equiv \frac{1}{2} + \frac{\Delta v_i - \Delta v_j + TS_i - TS_j}{2t} \) and \( \Delta v_i \equiv v(p_i, \theta) - E[v(p_i, \theta)] \) for \( i \in \{A, B\} \) and \( j \neq i \). Note that if the second term on the left-hand side is zero, any equilibrium involves marginal-cost-based 2PT (Proposition 2); that is, the first term on the left-hand side is zero if and only if \( p_i = c_i \).

If \( p_i > c_i \), the first term on the left-hand side of (9) is negative. This implies that the second term must be positive; that is, \( p_i < p_j \). If \( p_i < c_i \), the first term is positive, which implies that, in this case, \( p_i > p_j \). We therefore conclude that no equilibrium exists for \( p_B > c_B \). We formalize this idea in the following proposition.

**Proposition 3.** In any pure-strategy Nash equilibrium, the following hold:

(i) \( c_A < p_A^* < p_B^* < c_B \) and

35If both firms use LP, the problem of firm \( i \) is: \( \max_{p_i} E \left\{ \left( \frac{1}{2} + \frac{v(p_i, \theta) - v(p_j, \theta)}{2t} \right) \pi_i(p_i, \theta) \right\} \).

36As illustrated in Figure 1.

37Remember that we assume that \( \theta \) is associated and that from (A1) it follows that \( -v_i(p_i, \theta) \) has an increasing differences property.
(ii) the expected market share, per-customer profits and total revenue are greater for firm $A$ than those for firm $B$.

We first show that no equilibrium exists for $p_B \geq c_B$, as we mentioned before. Thus we show that the two curves defined by (9) for $i \in \{A,B\}$ cross each other at least once in the set $(c_A, c_B)^2$, as shown in Figure 1.\textsuperscript{38} Note that (ii) follows from (i) and (8), and from the fact that the expected market share is a linear function of the difference of the expected surplus. In any equilibrium $p_A^* < p_B^*$, then, the expected total surplus and the market share are greater for firm $A$ than for firm $B$. Finally, note that in any pure-strategy Nash equilibrium in which $c_A < p_A^* < c_B$, we have $E[\pi_B (p_B^*, \theta)] < 0 < E[\pi_A (p_A^*, \theta)]$, so that the expected revenue per consumer is greater for firm $A$ than for firm $B$.

From Proposition 3, it follows that if the difference between $c_B$ and $c_A$ is small, $F_A > F_B$.\textsuperscript{39} From the Implicit Function Theorem, it follows that given $c_A$, as $c_B$ increases, $F_A$ increases and $F_B$ decreases.\textsuperscript{40} In summary, in any equilibrium, the efficient firm charges a lower marginal price and a higher fixed fee than those of the less-efficient firm.

The intuition for this result is the following: Suppose that initially both firms offer their products at the marginal cost and charge a positive fixed fee. In order for firm $B$ to increase the share of revenues extracted with the fixed fee (compared to firm $A$), it needs to decrease the marginal price even below its own marginal cost, keeping the market share for its products constant. Thus, firm $B$ has incentives to deviate, offering a marginal price below its own marginal cost and compensating for the loss by increasing the fixed fee. On the other hand, firm $A$ increases its marginal price, but keeps it below its rival’s price, slightly decreasing its fixed fee. That is, firm $A$ does not need to decrease its marginal price below its own marginal cost to get a higher market share than firm $B$, due to its advantage over firm $B$.

In sum, in any equilibrium, the expected market share, profits, total revenue per consumer, and total revenue are greater for firm $A$ than for firm $B$. In particular, note that the marginal price is lower and the fixed fee is higher for the efficient firm than for the less-efficient one. These results may explain the empirical regularities observed in the British electricity market and highlighted by Davies et al. (2014); if the entrant firms are more efficient than the incumbent, we should expect lower marginal prices and higher fixed fees for the entrant.\textsuperscript{41}

\textsuperscript{38}Particularly, note that as $p_A \to c_A$ in (9) for $i = A$, we have $p_B \to c_A$ and, as $p_A \to c_B$ is not true, $p_B \to c_B$. In fact, $p_B \to \alpha_A > c_B$. Similarly, from (9) for $i = B$, as $p_B \to c_B$, we have $p_A \to c_B$ while as $p_A \to c_A$, $p_B \to \alpha_B \in (c_A, c_B)$.

\textsuperscript{39}A formal definition of the upper bound of $c_B$, as a function of $c_A$, is provided in the proof of Proposition 4.

\textsuperscript{40}If the heterogeneity of consumers’ vertical preferences is small Proposition 1 offers a similar conclusion (i.e., $F_A > F_B$).

\textsuperscript{41}As we mentioned before, there are other types of asymmetry that are important in the British electricity
To determine the sufficient condition and uniqueness of equilibrium, we need to analyze the slope of the quasi-best response functions and introduce a new assumption that helps us to characterize it.\textsuperscript{42} First we introduce the following definition:

**Definition 1.** \( v(p, \theta) : \hat{P} \times \Theta \rightarrow \mathbb{R}_+ \) is separable if there exist functions \( v : \hat{P} \rightarrow \mathbb{R}_+ \), \( h : \Theta \rightarrow \mathbb{R} \) and \( l : \Theta \rightarrow \mathbb{R} \), where \( v(\cdot) \) is strictly decreasing and \( h(\cdot) \) as well as \( l(\cdot) \) are strictly increasing, such that for all \( (p, \theta) \in \hat{P} \times \Theta \),

\[
v(p, \theta) = v(p) \cdot h(\theta) + l(\theta).
\]

**Assumption 4.** \( v(p, \theta) : \hat{P} \times \Theta \rightarrow \mathbb{R}_+ \) is separable.

Examples of the classes of indirect utilities that satisfy (A4) are: (i) the power functions (or constant elasticity demand) e.g., suppose that \( u(q, \theta) = \theta \sqrt{q} \) then \( v(p, \theta) = \frac{\theta^2}{4p} \); (ii) the log function, e.g., \( u(q, \theta) = \theta \log q \), then \( v(p, \theta) = \theta (\log \theta - 1) - \theta \log p; \) and (iii) the linear demand-type function, e.g., \( u(q, \theta) = \alpha q - \frac{\theta q^2}{2} \), then \( v(p, \theta) = \frac{(\alpha - p)^2}{2\theta} \).

(A4) allows us to simplify (9) and to show in Lemma 1 that the slope of the implicit functions defined by (9) for each firm \( i, R^i(p_A) : \hat{P} \rightarrow \hat{P} \), is positive, where \( R^i(p_A) = p_B \) is such that \( \tilde{p}^i_A \) and \( \tilde{p}^i_B \) satisfy (9) for each \( i \in \{A, B\} \).

**Lemma 1.** Suppose (A4) is satisfied. Then the slope of the implicit functions defined by (9), \( \frac{\partial R^i(p_A)}{\partial p_A} \) for \( i \in \{A, B\} \), is positive for \((p_A, p_B) \in [c_A, c_B]^2\).

Although both firms are using fixed fees to extract surplus, both quasi-best response functions in terms of the unit prices are increasing, as in the standard LP game. Note that if consumers have homogeneous taste parameters for quality, the quasi-best response function is vertical for firm A and horizontal for firm B, which contrasts with Lemma 1.

Note that (A4) allows us to express (9) as a function of \( \bar{\theta} \equiv E[h(\theta)] \) and \( \sigma \equiv \text{Var}[h(\theta)] \).

Using Lemma 1, we show the uniqueness for the game in the following proposition.

**Proposition 4.** Suppose (A4) is satisfied, \( 3\sigma > \bar{\theta}^2 \), and \( c_B < \rho(c_A) \).\textsuperscript{43} Then there exists a unique equilibrium in 2PTs in which \( p^*_i \in \hat{P} \) is determined by (9) and \( F^*_i \) satisfies (8) for market. Some of the firms were integrated upstream into generation and some were active in the gas market. Although Davies et al. (2014) suggest small costs asymmetries between firms, we need to assume that these other type of asymmetry can be projected into the firms’ marginal costs. This may result in firms with asymmetric marginal costs.

\textsuperscript{42}Note that (9) implicitly defines a quasi-best response function for each \( i \in \{A, B\} \) as a function of \((p_A, p_B) \).

\textsuperscript{43}The condition, \( c_B < \rho(c_A) \), establishes an upper bound for \( c_B \), given that \( \rho(\cdot) \) is such that \( \rho(c_A) > c_A \). A formal definition of \( \rho(\cdot) \) is provided in the Proof of Proposition 4 in the Appendix. These two assumptions (\( 3\sigma > \bar{\theta}^2 \) and \( c_B < \rho(c_A) \)) allow us to simplify our proof of uniqueness. A more complex proof, in which these two assumptions are not used is available upon request.
From Proposition 3, we know that the two implicit functions, $R^A(p_A)$ and $R^B(p_A)$ derived from (9) for $i \in \{A, B\}$ cross at least once (see Figure 1). Next, from Lemma 1 we know that the slope of the implicit functions $R^A(p_A)$ and $R^B(p_A)$ is positive. To prove uniqueness, we show that in equilibrium the slope of $R^A(p_A)$ is greater than the slope of $R^B(p_A)$.

Corollary 3. In equilibrium, as both $c_B$ and $c_A$ go to $c$, $p_i^*$ converges to $c$ and $F_i^*$ to $t$ for $i \in \{A, B\}$.

Note that Corollary 3 follows from Proposition 2 and Proposition 4. As the marginal costs for both firms converge to a common value $c$, both marginal prices tend to the marginal cost and both fixed fees to the transportation cost, $t$ (e.g., Armstrong and Vickers, 2001).

Corollary 4. In equilibrium,

(i) as $\sigma \to 0$, $p_A \to c_A$ and $p_B \to c_B$ and

(ii) as $\sigma \to \infty$, $p_A \to \bar{p}_A$ and $p_B \to \bar{p}_B$ where $c_A < \bar{p}_A < \bar{p}_B < c_B$.

Corollary 4(i) follows from Proposition 1 and the monotonicity of the quasi-best response functions with respect to the unit prices for both firms. Note that when $\sigma = 0$, the quasi-best response function for firm $A$ is a vertical line at $p_A = c_A$ in the $p_A, p_B$ plane. For firm $B$ it is a horizontal line at $p_B = c_B$. From numerical simulations, we find that as $\sigma$ increases, $p_A$ increases and $p_B$ decreases. That is, as $\sigma$ increases, the quasi-best response function
rotates to the right around \((c_A, c_A)\). Similarly, for firm \(B\), as \(\sigma\) increases, the quasi-best response function rotates to the left (counterclockwise) around \((c_B, c_B)\). Thus, for \(p_i > c_i\), as \(\sigma\) increases, firm \(i\) reacts less aggressively (sets a higher price) for each \(p_j\), for \(j \neq i\). However, for \(p_i < c_i\), as \(\sigma\) increases, firm \(i\) reacts more aggressively (sets a lower price) for each \(p_j\), for \(j \neq i\). This explains why when consumers are heterogeneous in their tastes, marginal-cost-based 2PT is not a Nash equilibrium. In particular, it explains why the optimal strategy for the less-efficient firm is to set its marginal price below its own marginal cost and to compensate for this loss with the fixed fee. On the other hand, the optimal strategy for the efficient firm is to set its marginal price above its own marginal cost but below that of its rival. Finally, from Corollary 4(ii), note that as \(\sigma\) increases, the marginal change of \(p_A\) and \(p_B\) decreases.

5 Asymmetric Demands

This section introduces the second type of asymmetry related to the goods offered (or equivalently to the demand) by the two firms. We assumed that both firms have the same marginal cost, \(c\), but offer differentiated products. Without loss of generality, we assume that for any \(p \in \tilde{P}\) and \(\theta \in \Theta\), the indirect utility offered by firm \(A\) is higher than the one offered by firm \(B\); that is, \(v_A(p, \theta) - v_B(p, \theta) > 0\) for all \(\theta \in \Theta\).\(^{44}\) To simplify the analysis, we introduce the following assumption:

**Assumption 5.** Let \(v_A(p, \theta) = v(p, \theta)\) and \(v_B(p, \theta) = \alpha v(p, \theta)\) for \(\alpha \in (0, 1)\), where \(v(p, \theta) = h(\theta) v(p)\), \(v(\cdot)\) is strictly decreasing and \(h(\cdot)\) is strictly increasing.

Intuitively, (A5) implies that for any \(p \in \tilde{P}\) and for \(\theta, \theta' \in \Theta\) such that \(\theta > \theta'\) (e.g., high and low type, respectively), the sum of the indirect utilities offered by firms \(A\) and \(B\) to the high and low type, respectively, is higher than the sum of the indirect utilities offered by firm \(A\) and \(B\) to the low and high type, respectively, so that, \(v_A(p, \theta) + v_B(p, \theta') > v_A(p, \theta') + v_B(p, \theta)\). Thus, product \(A\) is “vertically” superior to product \(B\).\(^{45}\) Suppose, for example, that \(u_A(q, \theta) = \theta \sqrt{q}\) and \(u_B(q, \theta) = \theta' \sqrt{aq}\), then \(v_A(p, \theta) = \frac{p^2}{4p}\) and \(v_B(p, \theta) = \alpha \frac{p^2}{4p}\), which satisfies (A5).

The set of feasible unit prices for both firms is

\[
\tilde{P} = [\tilde{\gamma}_B, \tilde{p}_A],
\]

\(^{44}\)The formal definition of \(\tilde{P}\) is presented below.

\(^{45}\)Note that (A5) excludes functions such that the two indirect utilities offered by both firms differ by an additive constant.
where $\tilde{\gamma}_B$ is such that

$$v(c) = \alpha v(\tilde{\gamma}_B).$$

(10)

Note that from (10), it follows that $\tilde{\gamma}_B$ is strictly less than $c$. We restrict our analysis to
the set of indirect utilities that satisfy (A1) such that $\tilde{\gamma}_B$ is strictly positive. This condition
implies that the difference between the two indirect utilities is bounded.\(^{(46)}\)

We proceed to characterize the equilibrium of the game following a strategy similar to
that in the previous section. From Proposition 2, we know that marginal-cost pricing is not
a Nash equilibrium. We first show that no solution exists for $p_B > c$. Next, we show that
there exists an equilibrium in 2PTs and that it is uniquely defined.

From the first-order conditions, $p_i$ in equilibrium is implicitly defined by\(^{(47)}\)

$$\text{Cov} (v_i(p_i, \theta) - v_j(p_j, \theta), q_i(p_i, \theta)) + (p_i - c_i) \{2t \cdot E [q'_i(p_i, \theta) \cdot \tilde{s}_i] - \text{Var} [q_i(p_i, \theta)]\} = 0,$$

(11)

where $\tilde{s}_i \equiv \frac{1}{2} + \frac{\Delta v_i - \Delta v_j + T \tilde{S}_i - T \tilde{S}_j}{2t}, \Delta v_i \equiv v_i(p_i, \theta) - E[v_i(p_i, \theta)]$ and $T \tilde{S}_i = (E[v_i(p_i, \theta)] + E[\pi_i(p_i, \theta)])/3$ for $i \in \{A, B\}$ and $j \neq i$.

**Proposition 5.** Suppose (A5) is satisfied and $\alpha \in O$.\(^{(48)}\) Then there exists a unique
equilibrium in 2PTs in which $p^*_A, p^*_B \in \tilde{P}$ are determined by (11) and $F^*_A, F^*_B$ by (8). Moreover,
$p^*_B < c < p^*_A$.

For the proof of Proposition 5, we follow a strategy similar to that used in the previous
section. We first show that no equilibrium exists for $p_B > c$, which implies that if
any equilibrium of the game exists, it must be in the set $\tilde{\Omega}(p_A, p_B)$, where $\tilde{\Omega}(p_A, p_B) \equiv \{p_A, p_B \in \tilde{P} \mid (p_A, p_B) \in [c, \tilde{\alpha}_A] \times [\tilde{\gamma}_B, c]\}$, $\tilde{\alpha}_A$ is such that $(\tilde{\alpha}_A, c)$ satisfy (11) for $i = B$, and
$\tilde{\gamma}_B$ is as defined in (10). Next, we show that for $(p_A, p_B) \in \tilde{\Omega}(p_A, p_B)$, the slope of the implicit
functions defined by (11) for $i \in \{A, B\}$, $\tilde{R}^i(p) : \tilde{P} \to \tilde{P}$, where $\tilde{R}^i(p^*_A) = \tilde{p}^*_B$, are such that
$\tilde{p}^*_A$ and $\tilde{p}^*_B$ satisfy (11) for $i \in \{A, B\}$, is positive.

Finally, we show that there exists at least one Nash equilibrium, that is, the two implicit
curves defined by (11) for $i \in \{A, B\}$ always cross each other in the region $\tilde{\Omega}(p_A, p_B)$.\(^{(49)}\) To

\(^{(46)}\)Equivalently, we can say that the difference between the demands of the two products offered by the firms is bounded.

\(^{(47)}\)Note that the model is similar to that in the previous section. Thus we omit the profit function and first-order conditions.

\(^{(48)}\)We present the formal definition of the set $O$ in the Appendix (Proof of Proposition 5).

\(^{(49)}\)Particularly, note that as $p_A \to c$ in (11) for $i = A$, we have that $p_B \to \tilde{\gamma}_B > c$, and as $p_A \to \tilde{\gamma}_A$, $p_B \to c$
where $\tilde{\gamma}_A$ is such that $(\tilde{\gamma}_A, c)$ satisfy (11) for $i = A$. Similarly, from (11) for $i = B$, as $p_A \to c$, we have that
$p_A \to \tilde{\alpha}_B$, while as $p_A \to \tilde{\alpha}_A$, $p_B \to c$, where $\tilde{\alpha}_B$ is such that $(c, \tilde{\alpha}_B)$ satisfy (11) for $i = B$ and $\tilde{\alpha}_B > \tilde{\gamma}_B$.
Similarly, we show that as $p_A \to \tilde{\gamma}_A$ in (11) for $i = B$, $p_B \to \omega_B < c$. Thus, both curves cross each other at
least once in the set $\tilde{\Omega}(p_A, p_B)$ (see Figure 2).
prove uniqueness, we show that in equilibrium the slope of the implicit function $\tilde{R}^A(p_A)$ is greater than the slope of $\tilde{R}^B(p_A)$.

Figure 2: Equilibrium with Asymmetric Demand

Note: Figure 2 shows the inverse of the quasi-best response function of firm A as a function of $p_A$ and the quasi-best response function of firm B as a function of $p_A$.

Note that as the difference between the indirect utilities offered by the two firms tends to 0 (i.e., $\alpha$ tends to 1) for any $p_i \in \tilde{P}$, $i \in \{A, B\}$, and $\theta \in \Theta$, both marginal prices tend to the marginal cost, $c$, and the fixed fees become independent of $\theta$, equal to $t$ (the standard result of 2PT).

Moreover, from the Implicit Function Theorem, it follows that $p_A^*$ and $F_A^*$ decrease and $p_B^*$ and $F_B^*$ increases as $\alpha$ increases, for $\alpha$ close to 1, which implies that $F_A^* > F_B^*$.

In summary, in equilibrium, the disadvantaged firm (vertically inferior product demand) sets its marginal price below its rival’s price and below its marginal cost, while the advantaged firm (vertically superior product demand) offers a unit price above its marginal cost and its rival’s unit price. The disadvantaged firm compensates for the loss of subsidizing the unit price below the marginal cost with a positive fixed fee that is below that of its rival’s fixed fee.

This result contrasts with Proposition 4, in which the disadvantaged firm sets a higher unit price (but below its own marginal cost) and a lower fixed fee than those of its rival. In the Asymmetric Costs Model, the efficient firm sets a marginal price below its rival’s price but above its own marginal cost, while in the Asymmetric Demands Model, if the advantaged firm offers a price below the common marginal cost, it would have to compensate for this
loss by increasing the fixed fee and decreasing its market share. Hence, firm $A$ has incentives to deviate and offer a higher price than firm $B$, due to its advantage in product demand.

We may reconcile Proposition 5 with the empirical regularities observed in the British electricity market (described by Davies et al., 2014, i.e., the entrant firm offers a lower marginal price and a higher fixed fee than the marginal price and fixed fee, respectively, offered by the incumbent) in the following way: Suppose that the marginal cost of firm $A$ is lower than that of firm $B$ (i.e., $c_A < c_B$). Likewise, suppose that $v_A(p, \theta) = v(p, \theta)$ and $v_B(p, \theta) = \alpha v(p, \theta)$ for $\alpha \in (0, \bar{\alpha}]$, where $\bar{\alpha} > 1$ and is such that an interior equilibrium exists, $v(p, \theta) = h(\theta) u(p)$, $v(\cdot)$ is strictly decreasing, and $h(\cdot)$ is strictly increasing. Then, from Propositions 4 and 5 and the Implicit Function Theorem, it follows that there exist $\alpha^1 < 1$ and $\alpha^2 \in (1, \bar{\alpha})$ such that for $\alpha \in (\alpha^1, \alpha^2)$, $c_A < p^*_A < p^*_B < c_B$, and $F^*_A > F^*_B$. That is, if $\alpha \in (\alpha^1, \alpha^2)$, firm $A$ offers a lower marginal price and a higher fixed fee than firm $B$, as in the British electricity market for the entrants and the incumbent firm, respectively.

## 6 General Market Share Functions

In this section, we extend our analysis to allow for general market share functions. We assume that there is a mass of consumers with types $(\xi, \theta)$, where $\xi \equiv (\xi_A, \xi_B, \xi_0)$ is distributed independently of the distribution of $\theta$. Following Armstrong and Vickers (2001), we consider a discrete-choice model in which the consumer’s preference for the two differentiated products can be represented by the utility function $u_A(q_A, \theta) + \xi_A$ if she buys from $A$, $u_B(q_B, \theta) + \xi_B$ if she buys from $B$, and $u_0 + \xi_0$ if no purchase is made. Consumers buy all products from one or the other firm, or else take their outside option.  

Given $(p_i, F_i)$, the share of $\theta$-consumers who choose to buy from firm $A$ is $s(v_A(p_A, \theta) - F_A, v_B(p_B, \theta) - F_B)$ and the share of consumers who choose firm $B$ is $s(v_B(p_B, \theta) - F_B, v_A(p_A, \theta) - F_A)$, where $v_i(p_i, \theta)$ is the indirect utility offered by firm $i$, defined as before, for $i \in \{A, B\}$. Finally, both firms can produce their products at constant marginal costs, $c_i$.

We impose the following regularity assumptions: First, $s(u_A, u_B)$ is increasing with respect to $u_A$ and decreasing with respect to $u_B$.  

Second, $s(u_A, u_B) / s_1(u_A, u_B)$ is weakly increasing with respect to $u_A$ and weakly decreasing with respect to $u_B$.  

---

50 For the $\theta$-consumer, the aggregate consumer utility is

$$V(u_A(\theta), u_B(\theta)) = E[\max\{u_A(\theta) + \xi_A, u_B(\theta) + \xi_B, u_0 + \xi_0\}] .$$

By the envelope theorem, $V_1(u_A(\theta), u_B(\theta)) \equiv s(v_A(p_A, \theta) - F_A, v_B(p_B, \theta) - F_B)$ is the share of $\theta$-consumers who choose to buy from firm $A$. We assume that consumers’ tastes for the two firms’ products are symmetrically distributed; i.e., $V(u_A(\theta), u_B(\theta)) = V(u_B(\theta), u_A(\theta))$.

51 Note that this assumption follows from consumers’ utility maximization.

52 Armstrong and Vickers (2001) make a similar assumption. Quint (2014) shows that we get a similar
The problem of each firm is
\[
\max_{p_i, F_i} E \left[ s \left( v_i (p_i, \theta) - F_i, v_j (p_j, \theta) - F_j \right) \left( \pi_i (p_i, \theta) + F_i \right) \right]
\]
for \(i, j \in \{A, B\}\) and \(j \neq i\). We first provide necessary and sufficient conditions such that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT. We show that a model in which consumers have homogeneous taste preferences trivially satisfies these conditions. Next, we show that for Logit market shares with an outside option, these conditions may be violated even when firms are symmetric (i.e., identical marginal cost and product demand), which contrasts with the Hotelling model of Section 3 in which any equilibrium involves marginal-cost pricing. We further consider a general market share and asymmetric costs setting with discrete types and show that the qualitative results of Section 4 hold.

### 6.1 Marginal-cost Pricing

To study necessary and sufficient conditions under which marginal-cost-based 2PT is an equilibrium, let
\[
\phi_i (\theta) \equiv \frac{s \left( v_i^* (\theta), v_j^* (\theta) \right) - s_1 \left( v_i^* (\theta), v_j^* (\theta) \right)}{E \left[ s \left( v_i^* (\theta), v_j^* (\theta) \right) \right] - E \left[ s_1 \left( v_i^* (\theta), v_j^* (\theta) \right) \right]} \tag{12}
\]
where \(v_i^* (\theta) \equiv v_i (c_i, \theta) - F_i^*\) and \(F_i^*\) is implicitly defined by
\[
F_i^* = \frac{E \left[ s \left( v_i^* (\theta), v_j^* (\theta) \right) \right]}{E \left[ s_1 \left( v_i^* (\theta), v_j^* (\theta) \right) \right]} \tag{12}
\]
for \(j \neq i\) and \(i, j \in \{A, B\}\). Necessary and sufficient conditions such that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT are given by the equalities:
\[
Cov \left( \phi_i (\theta), q_i (c_i, \theta) \right) = 0 \tag{13}
\]
for \(j \neq i\) and \(i, j \in \{A, B\}\), which we formally summarize these in the next proposition.

**Proposition 6.** For a given \(c_i, c_j \in \mathcal{P}\), a pure-strategy Nash equilibrium involves marginal-cost-based 2PT if and only if (13) holds for \(i, j \in \{A, B\}\) and \(j \neq i\).

The result of Proposition 6 is quite general. For the specific case in which \(\xi_i\) is distributed uniformly \((\text{à la Hotelling})\), conditions in (13) trivially depend on the covariance between each firm’s efficient quantity demand, \(q_i (c_i, \theta)\), and the difference between the efficient consumer condition if \(\xi\) follows a log-concave distribution.
surpluses offered by the two firms, \( v_i(c_i, \theta) - v_j(c_j, \theta) \), as we showed in Section 3. For
the general model presented here, the market share may not be linear with respect to the
difference between the efficient consumer surplus offered by the two firms. This may have two
implications: First, we may need stronger necessary and sufficient conditions for marginal-
cost-based 2PT as a Nash equilibrium, compared to the Hotelling model. That is, if (13) is
satisfied, then (5) is also satisfied, but not vice versa. Second, condition (13) depends on the
market share and not on the difference between the efficient consumer surpluses offered by
the two firms (as in the Hotelling model).

These two conditions imply that even if firms are symmetric, that is, \( c_i = c_j \) and \( v_i(p, \theta) = \)
\( v_j(p, \theta) \) for all \( p \in P \) and \( \theta \in \Theta \) and for \( i \neq j \), marginal-cost-based 2PT may not be an
equilibrium. In the Hotelling setting, if firms are symmetric, marginal-cost pricing is an
equilibrium. For the model presented here, even if the market share is constant with respect
to \( \theta \) (e.g., symmetric firms and no outside option), the left side of (13) may be different from
zero (e.g., if \( \frac{E_1(c_i)}{E_1(c_j)} \) is monotonic with respect to \( \theta \)). Thus, necessary and sufficient conditions
such that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT depend on
both \( s(\cdot) \) and \( s_1(\cdot) \).

If consumers are homogeneous, (13) is trivially satisfied for each firm \( i \in \{A, B\} \). We
show in the next corollary that for this case, marginal-cost-based 2PT is an equilibrium.

**Corollary 5.** Suppose consumers have homogeneous vertical tastes and \( v_i(c_i) > \frac{s(0,0)}{s_1(0,0)} \)
for \( i \in \{A, B\} \). Then, marginal-cost-based 2PT is an equilibrium, where \( F_i^* = v_i(c_i) - v_i^* \)
and \( v_i^* \) are implicitly defined by \( v_i(c_i) = v_i^* + \frac{s(v_i^*, v_j^*)}{s_1(v_i^*, v_j^*)} \) for \( i, j \in \{A, B\} \) and \( j \neq i \).\(^{53}\)

If consumers are homogeneous, under the assumption of full market coverage, Corollary 5
shows that the optimal strategy for each firm is to set the unit price equal to its marginal cost
and extract surplus with the fixed fee. Note that \( v_i(c_i) \) is the efficient (maximum) surplus
offered by firm \( i \) to the consumers and \( v_i^* \) is the net surplus when competing firms engage
in efficient surplus extraction. In the case of a monopoly, the firm would set \( F_i^* = v_i(c_i) \)
and hence the net consumer surplus would be \( v_i^* = 0 \) (full extraction). In the presence of
competition, full extraction is not possible and hence \( v_i^* > 0 \). Under full market coverage,
the equilibrium net surpluses \((v_i^*, v_j^*)\) are determined by the above equations, which imply
that \( 0 < v_i^* < v_i(c_i) \) for each \( i \in \{A, B\} \). The ratio \( \frac{s(v_i^*, v_j^*)}{s_1(v_i^*, v_j^*)} \) represents the competitive effect
that prevents firms from full extraction. Note that in equilibrium, the firm that provides the

\(^{53}\)Note that any equilibrium involves marginal-cost-based 2Pt, where \( F_i^* = v_i(c_i) - v_i^* \) and \( v_i^* \) are implicitly
defined by \( v_i(c_i) = v_i^* + \frac{s(v_i^*, v_j^*)}{s_1(v_i^*, v_j^*)} \). Sufficient condition can be imposed to guarantee uniqueness of \( F_i^* \) and,
in consequence, uniqueness of the Nash equilibrium. This condition is satisfied by the Logit model with and
without outside option.
highest surplus (at its own marginal cost) has the highest fixed fee, market share and total profits, similar to the model in Section 3; that is, if \( v_i(c_i) > v_j(c_j) \), then \( F^*_i > F^*_j \), \( s^*_i > s^*_j \) and \( \Pi^*_i > \Pi^*_j \) for \( i \neq j \) and \( i, j \in \{A, B\} \).

Corollary 5 extends Armstrong and Vicker’s Proposition 1 to allow for asymmetric firms. Here, firms set their marginal prices equal to the corresponding marginal costs. This result allows us to show that a Nash equilibrium in utility space exists.

### 6.2 Logit Market Shares with an Outside Option.

In order to illustrate Proposition 6 for symmetric firms, we assume that \( \xi_i \) follows a type-I extremum distribution and we allow for an outside option. The market share of firm \( i \) is

\[
s(u_i, u_j) \equiv \frac{e^{u_i}}{e^{u_i} + e^{u_j} + e^{u_0}},
\]

where \( u_i \equiv v(p_i, \theta) - F_i \) for \( i, j \in \{A, B\}, j \neq i \), and \( u_0 \) is the value of the outside option. Note that in a symmetric equilibrium, the covariance between \( \phi_i(\theta) \) and firms’ efficient quantity would be positive, since \( s(v^*(\theta), v^*(\theta)) \), and \( s_1(v^*(\theta), v^*(\theta)) \) are increasing with respect to \( \theta \), but the rate of increase is higher for \( s(v^*(\theta), v^*(\theta)) \) than for \( s_1(v^*(\theta), v^*(\theta)) \), and thus if \( \theta \) is associated, (13) is not satisfied.

**Proposition 7.** In a symmetric model with Logit shares and an outside option, any pure-strategy Nash equilibrium involves \( p^* > c \).

Proposition 7 contrasts with Corollary 5, which shows that marginal-cost pricing is an equilibrium for a general market share setting (including Logit) when consumers have homogeneous taste preferences.\(^{54}\) When consumers are homogeneous, a 2PT game is formally equivalent to a linear pricing game with Logit market shares—as was pointed out by Nocke and Schutz (2018),—which has a unique equilibrium.

Likewise, Proposition 7 contrasts with Corollary 1, in which we show that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT when consumers are differentiated \( \text{à la} \) Hotelling and have heterogeneous tastes for quality and firms have identical marginal costs and product demand (i.e., firms are symmetric, as in Proposition 7). Remember that in the Hotelling model, the market share is linear with respect to the difference in the consumer surplus offered by the two firms, which simplifies the necessary and sufficient conditions for marginal-cost pricing (Proposition 2).

For the Logit model without an outside option, \( s(v^*(\theta), v^*(\theta)) \) and \( s_1(v^*(\theta), v^*(\theta)) \) are constant, thus (13) is satisfied and marginal-cost-based 2PT is an equilibrium.

\(^{54}\)That is, (13) is trivially satisfied for each firm.
Note, however, that this result holds only if firms are symmetric; if firms have asymmetric marginal costs or offer differentiated products, marginal-cost pricing is not an equilibrium. Since the analysis with asymmetric firms and Logit market shares is complicated, here we present a numerical example. We consider constant elasticity demand and two types of vertical taste. In particular we assume the following functional form: 
\[ u(q, \theta) = \theta \eta \eta^{1 - \frac{1}{\epsilon}} \epsilon q^{1 - \frac{1}{\epsilon}} - 1 \eta^{1 - \frac{1}{\epsilon}} \epsilon p^{\epsilon}, \]
and hence \[ q(p) = \eta \left( \frac{\theta}{p} \right)^{\epsilon}, \]
v(p) = \frac{1}{(\epsilon - 1) \eta^{\epsilon}} p^{\epsilon}, and \[ q'(p) = -\eta \epsilon \theta \eta^{\epsilon - 1} p^{\epsilon - 1}. \]
We use the following parameters: \( c_A = 0.2, c_B = 0.25, \eta = 0.2, \epsilon = 2, \theta_L = 0.3, \) and \( \theta_H = 0.5. \) Let \( \lambda_H = \lambda \) and \( \lambda_L = 1 - \lambda \) for \( \lambda \in (0, 1). \) Figures 3(i) and 3(ii) show the changes in marginal prices and fixed fees, respectively, when \( \lambda \) (x axis) varies from 0 to 1. Note that marginal prices are always above marginal costs for both firms, and lower for firm A than for firm B, while fixed fees are always higher for firm A than for firm B.

Figure 3(i): Marginal Prices

Figure 3(ii): Fixed Fees

Note: Figure 3(i) shows the changes in marginal prices for firm A (left y-axis, blue line) and firm B (right y-axis, red line) when \( \lambda \) (x axis) varies from 0 to 1. Similarly, Figure 3(ii) shows the changes in fixed fees for firm A (blue line) and firm B (red line), when \( \lambda \) (x axis) varies from 0 to 1.

In the Logit model with an outside option, firms also compete with the outside option, but the outside option does not react to the changes in marginal prices (or fixed fees) of any of the firms. Hence, firms react less aggressively to their competitor’s pricing strategy than they do in the Logit model without an outside option, allowing them to set prices above the marginal costs.

From previous results, we may expect that as the value of the outside option \( u_0 \) varies from 0 to \( -\infty, \) the less efficient firm again sets prices below the marginal cost. Figures 4(i) and 4(ii) show equilibrium values of \( p_A \) and \( p_B, \) respectively, when the outside option (\( u_0 \))
varies from $-\infty$ to 0 ($x$ axis is equal to $\exp(u_0)$).

Figures 4(i) and 4(ii) show that there is a non-monotonic relationship (inverse u-shape) between the outside option and $p_A$ and $p_B$, respectively. Note that as $u_0 \rightarrow -\infty$, $p_A \rightarrow \hat{p}_A > c_A$ and $p_B \rightarrow \hat{p}_B < c_B$.

In the next section we explore whether the equilibrium strategy for the less-efficient firm involves cross-subsidization between the unit price and the fixed fee in a general market share setting with asymmetric marginal costs.

6.3 General Market Shares and Asymmetric Marginal Costs

In this section we suppose that firms have identical product demand but asymmetric marginal costs. To illustrate, we suppose $\theta$ is drawn from the distribution on $\Theta = \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$ (low and high type), with probabilities $\lambda$ and $1 - \lambda$, respectively. The next proposition generalizes Proposition 3, allowing for a general market share without outside option.

**Proposition 8.** In any pure-strategy Nash equilibrium $c_A < p_A^* < p_B^* < c_B$.

Note first that from Proposition 6, it follows that marginal-cost-based 2PT is not an equilibrium. The result in Proposition 8 (and the strategy used for its proof) is similar to

---

55The results presented here can be generalized to the case of $n$ types.
that of Proposition 3. That is, when consumers are heterogeneous in their taste for quality, firms have incentives to deviate from marginal-cost pricing. The efficient firm increases its marginal price,—keeping it below its rival’s price—and slightly decreases its fixed fee. On the other hand, the less-efficient firm has incentives to decrease the marginal price below its own marginal cost, so that the revenue losses arising from this strategy are more than offset by the revenue gains obtained from the fixed fee.

7 Extensions

In this section, we extend our previous model and study under what conditions marginal-cost-based 2PT is an equilibrium if both firms use nonlinear tariffs. Armstrong and Vickers (2001) and Rochet and Stole (2002) showed that if the two firms are symmetric and under full market coverage, each firm offering marginal-cost-based 2PT is an equilibrium.

Here we keep all the assumptions of Section 2: we assume that there are two firms competing on either end of the market, offering differentiated products to a population of heterogeneous consumers, and with constant but differentiated marginal costs (i.e., $c_A \neq c_B$). We suppose that, instead of 2PTs, firm $i \in \{A, B\}$ offers a nonlinear tariff $T_i(q_i)$.

The utility of a consumer with taste parameter $\theta \in \Theta$, excluding transportation cost, is

$$u_i(q_i, \theta) - T_i(q_i),$$

if she buys $q_i$ units from firm $i$ in return for a payment $T_i(q_i)$, where $u_i(q_i, \theta)$ satisfies (A1) for $i \in \{A, B\}$. Given $T_i(q_i)$, a type $\theta \in \Theta$ consumer chooses the price-quantity pair \{\(q_i(\theta), T_i(q_i)\}\} that maximizes her utility,

$$\hat{u}_i(\theta) \equiv \max_{q_i} \{u_i(q_i, \theta) - T_i(q_i)\},$$

if she buys from firm $i$. Finally let

$$v_i(\theta) \equiv \max_{q_i} \{u_i(q_i, \theta) - c_i q_i\}$$

be the type-$\theta$ consumer’s utility when firm $i$ sets its prices at its own marginal cost.

Using the dual approach, we can write the market share and the total expected profit of firm $i$ as a function of consumers’ maximum utility $u_i(\theta)$. Due to our full market coverage
assumption, the problem of each firm \( i \in \{ A, B \} \) is
\[
\max_{u_i,q_i} E \left\{ \left( \frac{1}{2} + \frac{u_i(\theta) - u_j(\theta)}{2t} \right) \left( S_i(q_i(\theta) \cdot \theta) - u_i(\theta) \right) \right\},
\]
where \( S_i(q_i(\theta) \cdot \theta) = u_i(q_i, \theta) - c_i q_i(\theta) \) is the surplus from trade with firm \( i \).

**Proposition 9.** Suppose both firms use nonlinear tariffs and consumers are horizontally differentiated à la Hotelling. Then, any equilibrium involves marginal-cost-based 2PT if and only if for \( i,j \in \{ A, B \} \) and \( j \neq i \), \( v_i(\theta) - v_j(\theta) \) is constant over \( \theta \in \Theta \).

Proposition 9 extends Proposition 5 in Armstrong and Vickers (2001) and Proposition 6 in Rochet and Stole (2002) to allow for general asymmetric demands and costs. The proof of Proposition 9 can be constructed, by adapting the strategies used by those authors. It states that marginal-cost-based 2PT is an equilibrium if and only if the difference between the utilities offered by the two firms at their marginal costs (or the efficient utilities) is constant (with respect to consumer heterogeneity \( \theta \)). Two examples that satisfy this condition are:

1. \( v_A(p,\theta) = v(p,\theta) \) and \( v_B(p,\theta) = v(p,\theta) + k \), where \( k \in \mathbb{R} \) and \( v(p,\theta) \) is derived from a utility function that satisfies (A1).

2. \( v_A(p,\theta) = v(p,\theta) \) and \( v_B(p,\theta) = \alpha v(p,\theta) \), where \( \alpha \in (0,1) \), \( v(p,\theta) = h(\theta) v(p) \), \( v(\cdot) \) is strictly decreasing and \( h(\cdot) \) is strictly increasing. Note that in this example, there exists \( (c_A,c_B) \) such that the difference between the efficient utilities is zero at \( (c_A,c_B) \).

Finally, note that if \( q_i(p,\theta) \) is increasing with respect to \( \theta \) for \( i \in \{ A, B \} \), the condition in Proposition 9 is the same as that in Proposition 3. That is, the covariance of the demand and the difference of the utilities evaluated at the marginal costs is equal to zero.

**8 Concluding Remarks**

In this paper, we study competition in 2PTs between asymmetric firms in a general model of consumer preferences in which consumers have elastic demands and private information regarding their horizontal brand preferences and tastes for product quality. We first provide necessary and sufficient conditions for marginal-cost pricing as a Nash equilibrium in the model with consumer horizontal preferences described à la Hotelling. We show that when

---

\(^{56}\)Note that if both firms offer symmetric goods and have symmetric marginal costs (as in Armstrong and Vickers, 2001; Rochet and Stole, 2002), this condition is trivially satisfied.
these conditions are not satisfied and when firms have asymmetric marginal costs but the same product demand, the equilibrium strategy involves cross-subsidization between the unit price and fixed fee for the less-efficient firm, while the efficient firm always offers a unit price above its marginal cost. We find a similar result when both firms have identical marginal costs but asymmetric product demands.

We extend our analysis to allow for a general discrete choice model of random utility maximization and provide necessary and sufficient conditions for marginal-cost pricing as a Nash equilibrium. We show that some of our previous results in the Hotelling setting hold for this general market share specification, particularly when interior equilibrium without an outside option is considered. However, other results do not hold when we allow for an outside option. For instance, we show that when the market shares are determined by Logit with outside option, even in the symmetric model, marginal-cost pricing is not an equilibrium.

A direct extension of our analysis would be to consider a multi-product setting in which firms offer more than one product. Furthermore, we could consider more than two firms, using our general discrete choice model of random utility maximization presented in Section 6. It would also be interesting to understand the impact of mergers on equilibrium 2PTs and consumer welfare. Since firms compete aggressively with their marginal prices, and set them close to their marginal costs, mergers between efficient and less-efficient firms may have a substantial effect on the rest of the competing firms that also offer 2PTs and hence on consumer welfare.
Appendix

Proof of Proposition 1. We solve the two-variable optimization problem of firm $i$ sequentially. First we show that for any $p_i \in \mathcal{P}$, firm $i$ chooses $F_i$ to maximize its profits. The first order condition with respect to $F_i$ yields

$$s_i (p_i, F_i, p_j, F_j) - \frac{1}{2 \ell} [\pi_i (p_i) + F_i] = 0. \quad (14)$$

The profit is quadratic and strictly concave in $F_i$, and the unique solution is given by

$$2F_i^* = t + v_i (p_i) - v_j (p_j) + F_j - \pi_i (p_i). \quad (15)$$

Next, firm $i$ chooses $p_i$ to maximize its maximum profits (we substitute $F_i^* (p_i)$ in the objective function of firm $i$)

$$s_i (p_i, F_i^* (p_i), p_j, F_j) [\pi_i (p_i) + F_i^* (p_i)]. \quad (16)$$

The derivative of (16) with respect to $p_i$, after using the Envelope Theorem, is

$$q' (p_i) (p_i - c_i) s_i (p_i, F_i^* (p_i), p_j, F_j) = 0. \quad (17)$$

Given $p_j \in \mathcal{P}$ and $F_j \geq 0$, $s (p_i, F_i^* (p_i), p_j, F_j) = 1/\ell \{ t + v_i (p_i) - v_j (p_j) + F_j + \pi_i (p_i) \}$, is strictly decreasing with respect to $p_i$ for any $p_i > c_i$. Thus, if there exists a $\tilde{p}_i (p_j, F_j) \in \mathcal{P}$, such that $s (\tilde{p}_i, F_i^* (\tilde{p}_i), p_j, F_j) = 0$, then, for any $p_i \geq \tilde{p}_i$ the profit is zero. Then, for any $p_j \in \mathcal{P}$ and $F_j \geq 0$, note that (17) is positive for $p_i < c_i$ and it is negative for any $p_i \in (c_i, \tilde{p}_i (p_j, F_j))$. Thus, (16) is single-peaked in $p_i$ and reaches a unique maximum at $p_i = c_i$. Analogously, the profit function for firm $j \neq i$ in terms of $p_j$ is single-peaked in $p_j$ and reaches a unique maximum at $p_j = c_j$. Thus the equilibrium is unique, and it follows that $F_i^* = t + \frac{v_i (c_i) - v_j (c_j)}{3}$ for $j \neq i$ and $i, j \in \{A, B\}$.

Proof of Proposition 2.

(i) Let’s show first the “only if” part. Assume that any pure-strategy Nash equilibrium involves marginal-cost-based 2PT. Then, from (3) and (4) for firm $i \in \{A, B\}$, marginal-cost pricing is an equilibrium if

$$-E [q_i (c_i, \theta)] E [\varphi_i (c_i, c_j, \theta)] + E [q_i (c_i, \theta) \varphi_i (c_i, c_j, \theta)] = 0,$$

where $\varphi_i (p_i, p_j, \theta) \equiv v_i (p_i, \theta) - v_j (p_j, \theta)$ for $j \neq i$, which implies that (5) holds for $p_i = c_i$. 

33
Let’s prove the other direction. Suppose that (5) holds in equilibrium. Note that, if firm $j \neq i$ uses marginal-cost pricing, then from (3), (4) and (5) it follows that marginal-cost pricing is also an equilibrium for firm $i$ for $i, j \in \{A, B\}$. Then it follows that any pure strategy Nash equilibrium involves, marginal-cost pricing.

(ii) We split the proof in two steps: In (ii,a) and using the result in (ii,b), we show that the objective function of firm $i$ is single-peaked so that it has a global maximum if for any $p_i, p_j \in \mathcal{P}$, (5) holds. (ii,b) For $p^*_i = c_i$ and $p^*_j = c_j$, we show that there do not exist $p_i, p_j \in \mathcal{P}$ with $p_i \neq p^*_i$ and $p_j \neq p^*_j$, such that the first order conditions are satisfied.

(ii,a) We show that the objective function of firm $i$ is single-peaked and then has a global maximum. We solve the two-variable optimization problem of firm $i$ sequentially. First we show that for any $p_i \in \mathcal{P}$, firm $i$ chooses $F_i$ to maximize its profits. The first order condition with respect to $F_i$ yield

$$2F^*_i = t + E [v_i (p_i, \theta) - v_j (p_j, \theta)] + F_j - E [\pi_i (p_i, \theta)].$$ (18)

The profit is quadratic and strictly concave in $F_i$ and the unique solution is given by (18). Next, firm $i$ chooses $p_i$ to maximize its profits (we substitute $F^*_i (p_i)$ in the objective function of firm $i$)

$$E [s (p_i, F^*_i (p_i), p_j, F_j; \theta) (\pi_i (p_i, \theta) + F^*_i (p_i))],$$ (19)

where $4t \cdot E [s (p_i, F^*_i (p_i), p_j, F_j; \theta)] = t + E [v_i (p_i, \theta) - v_j (p_j, \theta)] + F_j + E [\pi_i (p_i, \theta)]$. The derivative of (19) with respect to $p_i$, after using the envelope theorem, (18) and the fact that for any $p_i, p_j \in \mathcal{P}$, (5) holds, is

$$-\frac{1}{2t} \left[ E \left[ q_i (p_i, \theta)^2 \right] - E \left[ q_i (p_i, \theta) \right]^2 \right] (p_i - c_i)$$

$$+ E \left[ q_i^* (p_i, \theta) (p_i - c_i) s_i (p_i, F^*_i (p_i), p_j, F_j; \theta) \right].$$ (20)

Given $p_j \in \mathcal{P}$ and $F_j \geq 0$, for any $\theta \in \Theta$, $s (p_i, F^*_i (p_i), p_j, F_j; \theta)$, is strictly decreasing with respect to $p_i$ for any $p_i > c_i$. Note that that there exists a $\tilde{p}_i \in \mathcal{P}$ and a $\theta \in \Theta$, such that $s (\tilde{p}_i, F^*_i (\tilde{p}_i), c_j, F^*_j; \theta) = 0$, then for any $p_i \geq \tilde{p}_i (p_j, F_j) \equiv \sup_{\theta \in \Theta} \tilde{p}_i (p_j, F_j; \theta)$ the profit is zero. Then, for any $p_j \in \mathcal{P}$ and $F_j \geq 0$, note that (20) is positive for $p_i < c_i$ and it is negative for any $p_i \in (c_i, \tilde{p}_i (p_j, F_j))$. Thus, (19) is single-peaked in $p_i$ and reaches a unique maximum at $p_i = c_i$. Analogously, the profit function for firm $j \neq i$ in terms of $p_j$ is single-peaked in $p_j$ and reaches a unique maximum at $p_j = c_j$. Thus, it follows that, in
equilibrium, \( F_i^* = t + \frac{E[v_i(c_i, \theta) - v_j(c_j, \theta)]}{3} \) for \( j \neq i \) and \( i, j \in \{A, B\} \).

(ii.b) We now show that for \( p_i^* = c_i \) and \( p_j^* = c_j \), there do not exist \( p_i, p_j \in \mathcal{P} \) with \( p_i \neq p_i^* \) and \( p_j \neq p_j^* \), such that the first order conditions are satisfied. From (3) and (4) for firm \( i \in \{A, B\} \),

\[
F_i = t + \frac{E[v_i(p_i, \theta) - v_j(p_j, \theta)]}{3} - \frac{E[\pi_j(p_i, \theta)]}{3} - \frac{2E[\pi_i(p_i, \theta)]}{3} \quad (21)
\]

and

\[
F_i - F_j = \frac{2E[v_i(p_i, \theta) - v_j(p_j, \theta)]}{3} + \frac{E[\pi_j(p_j, \theta)]}{3} - \frac{E[\pi_i(p_i, \theta)]}{3} \quad (22)
\]

for \( i \neq j \). Using (21) and (22) in (3) and the fact that (5) holds for any \( p_i, p_j \in \mathcal{P} \), we get

\[
2t \cdot E[q'(p_i, \theta)s_i](p_i - c_i) - (p_i - c_i) \text{ Var } [q(p_i, \theta)] = 0, \quad (23)
\]

where \( s_i \equiv \frac{1}{2} + \frac{\Delta v_i - \Delta v_j + TS_i - TS_j}{2t}, \quad TS_i(p_i) \equiv \left(E[v_i(p_i, \theta)] + E[\pi_i(p_i, \theta)]\right)/3, \) and \( \Delta v_i \equiv v_i(p_i, \theta) - E[v_i(p_i, \theta)] \) for \( i \in \{A, B\} \) and \( j \neq i \). Now, note that if (5) holds for any \( p_i, p_j \in \mathcal{P} \), then

\[
-E[q_i(p_i, \theta)]E[v_i(p_i, \theta) - v_j(p_j, \theta)] + E[q_i(p_i, \theta)(v_i(p_i, \theta) - v_j(p_j, \theta))] = 0. \quad (24)
\]

If we take the derivative to both sides of (24) with respect to \( p_i \), multiply by \( p_i - c_i \) and substitute in (23) we get

\[
E[q_i'(p_i, \theta)(p_i - c_i)]\{t + TS_i(p_i) - TS_j(p_j)\} = 0. \quad (25)
\]

Note that if \( p_i \neq c_i \) the left-hand side of (25) implies that \( E[\pi_i(p_i, \theta)] + F_i = 0 \) for \( i \in \{A, B\} \). However, both firms have a profitable deviation by setting marginal prices equal to marginal costs and the fixed fee equal to \( F_i^* = t + \frac{E[v_i(c_i, \theta) - v_j(c_j, \theta)]}{3} \). If both firms have strictly positive profits, the second term of the left side of (25) is strictly positive, then \( p_i = c_i \) for \( i \in \{A, B\} \). Thus, marginal-cost pricing is a unique equilibrium.

**Proof of Corollary 1.** It follows directly from Proposition 2, that is, \( \varphi(c, c, \theta) = 0, \forall \theta \), where \( \varphi(p_i, p_j, \theta) \equiv v(p_i, \theta) - v(p_j, \theta) \).

**Proof of Corollary 2.** It follows directly from Proposition 2.

**Proof of Proposition 3.** (i) We first show that for every \( p_A, p_B \in \hat{\mathcal{P}} \) that satisfies (9) for \( i = A \), it has to be true that \( p_A < p_B \). Second we show that for \( p_B \geq c_B \) and for every
\(p_A, p_B \in \hat{P}\) that satisfy (9) for \(i = B\), we have that \(p_B < p_A\). Thus we conclude that no equilibrium exists for values of \(p_B \geq c_B\). Finally we show that the two curves defined by (9) for \(i \in \{A, B\}\) cross each other at least once in the set \([c_A, c_B]^2\).

Let us first show that for every \(p_A, p_B \in \hat{P}\) that satisfy (9) for \(i = A\), it has to be true that \(p_A < p_B\). Suppose is not true, i.e., \(p_A \geq p_B\), then note that from (9),

\[
\frac{E[q(p_A, \theta)(\Delta v_A - \Delta v_B)]}{2t} + (p_A - c_A) \left\{ \frac{E[q'(p_A, \theta)s_A]}{2t} - \frac{\text{Var}[q(p_A, \theta)]}{2t} \right\} = 0.
\]

Thus, if \(p_A > p_B\),

\[
\frac{E[q(p_B, \theta)(\Delta v_A - \Delta v_B)]}{2t} < 0,
\]

since \(\theta\) is associated. So, we get a contradiction. Note that we also get a contradiction if \(p_A = p_B\). Thus we conclude that if \(p_A, p_B \in \hat{P}\) satisfy (9) for \(i = A\), then it has to be true that \(p_A < p_B\). Similarly, we can show that for \(p_B \geq c_B\) and for every \(p_A, p_B \in \hat{P}\) that satisfy (9) for \(i = B\), we have that \(p_B < p_A\). Thus we conclude that no equilibrium exists for values of \(p_B \geq c_B\).

Now let us prove existence. Note that as \(p_A \rightarrow c_A\) in (9) for \(i = A\), we have that \(p_B \rightarrow c_A\), and as \(p_A \rightarrow c_B\) (from the previous paragraph), we know that \(p_B \rightarrow \alpha_A > c_B\). Similarly, from (9) for \(i = B\), note that as \(p_A \rightarrow c_A\)

\[
\frac{E[q(p_B, \theta)(\Delta v_B - \Delta v_A)]}{2t} + (p_B - c_B) \left\{ \frac{E[q'(p_B, \theta)s_B]}{2t} - \frac{\text{Var}[q(p_B, \theta)]}{2t} \right\} = 0,
\]

which implies that \(p_B \rightarrow \alpha_B > c_B\) since the first term of the right-hand side must be negative. Finally, from (9) for \(i = B\), as \(p_A \rightarrow c_B\), \(p_B \rightarrow c_B\). Finally, note that the quasi-best response functions, (9) for \(i \in \{A, B\}\), are differentiable and therefore continuous for every \((p_A, p_B) \in [c_A, c_B]\) (see also Lemma 1).

(ii) It follows from substituting \(F^*_i\) for \(i \in \{A, B\}\) in the expected market share and the fact that in any pure-strategy Nash equilibrium, \(c_A < p^*_A < p^*_B < c_B\).

**Proof of Lemma 1.** Note that (A4) allows us to express (9) as a function of \(\sigma \equiv \text{Var}[h(\theta)]\) and \(\bar{\theta} \equiv E[h(\theta)]\)

\[
\xi^i(p) = \left( v(p_i) - v(p_j) - \phi_v^i(p_i) \right) \sigma - h^i(p_i)(p_i - c_i) \bar{\theta} \left\{ t + \bar{\theta} \frac{TS_i(p_i)}{3} - \bar{\theta} \frac{TS_i(p_j)}{3} \right\}, \tag{26}
\]

where \(p \equiv (p_i, p_j)\), \(h^i(p) \equiv -\frac{\phi_v^i(p)}{\pi_v^i(p)}\), \(\phi_v^i(p) \equiv \frac{q_v(p)^2(p - c_i)}{\pi_v^i(p)}\), \(q_v(p) \equiv -\frac{\partial h_v(p)}{\partial p}\), \(\pi_v^i(p) \equiv q_v(p)(p - c_i)\),
and $TS_i (p) \equiv v (p) + \pi_i (p)$ for $i \in \{A, B\}$ and $j \neq i$.

(i) $\frac{\partial R^i (p_A)}{\partial p_A} > 0$: In Proposition 4 we show that $-\frac{\partial \xi^A}{\partial p_A} > \frac{\partial \xi^B}{\partial p_B}$ and in part (ii) of this proposition we show that $\frac{\partial \xi^B}{\partial p_B} > 0$ thus, we just need to show that $\frac{\partial E}{\partial p_B} > 0$, where $\xi^i$ is equal to (26) for $i \in \{A, B\}$. Note that

$$\frac{\partial E}{\partial p_B} = q_v (p_B) \sigma + h^A (p_A) (p_A - c_A) \bar{\theta}^2 \cdot \frac{TS' (p_B)}{3} > 0,$$

since $TS' (p_B) > 0$ for $p_B < c_B$. Then from the Implicit Function Theorem we conclude that $\frac{\partial R^i (p_A)}{\partial p_A} > 0$ for $(p_A \times p_B) \in [c_A, c_B]^2$.

(ii) $\frac{\partial R^B (p_A)}{\partial p_A} > 0$: In Proposition 4 we show that $\frac{\partial \xi^A}{\partial p_B} < -\frac{\partial \xi^B}{\partial p_B}$. In part (i) we showed that $\frac{\partial \xi^B}{\partial p_B} > 0$. Thus, we just need to show that $\frac{\partial \xi^B}{\partial p_B} > 0$ for $(p_A \times p_B) \in [c_A, c_B]^2$. Note that for $p_B < c_B$, $|h^B (p_B) (p_B - c_B)| < 1$, then

$$\frac{\partial E}{\partial p_A} = q_v (p_A) \sigma + h^B (p_B) (p_B - c_B) \bar{\theta}^2 \cdot \frac{TS' (p_A)}{3} > 0,$$

since $3\sigma > \bar{\theta}^2$. Then from the Implicit Function Theorem we conclude that $\frac{\partial R^B (p_A)}{\partial p_A} > 0$ for $(p_A \times p_B) \in [c_A, c_B]^2$.

**Proof of Proposition 4.** We split the proof in two steps: (i) using the result in (ii), we show that the objective function of firm $i$ is single-peaked so that it has a global maximum. (ii) For $p^*_i$ and $p^*_j$ defined by (9) for $i \in \{A, B\}$, we show that there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p^*_i$ and $p_j \neq p^*_j$, such that the first order conditions are satisfied.

(i) We show that the objective function of firm $i$ is single-peaked so that it has a global maximum. We fix firm $j$’s strategy to $F_j = F_j^*$ and $p_j = p_j^*$, and solve the two-variable optimization problem of firm $i$ sequentially. First we show that for any $p_i \in \mathcal{P}$, firm $i$ chooses $F_i$ to maximize its profits. The first order condition with respect to $F_i$ yield

$$2 F_i^* = t + E \left[ v (p_i, \theta) - v (p_j^*, \theta) \right] + F_j^* - E \left[ \pi_i (p_i, \theta) \right]. \quad (27)$$

The profit is quadratic and strictly concave in $F_i$ and the unique solution is given by (27). Next, firm $i$ chooses $p_i$ to maximize its profits (we substitute $F_i^* (p_i)$ in the objective function of firm $i$)

$$E \left[ s \left( p_i, F_i^* (p_i), p_j^*, F_j^*; \theta \right) \left( \pi_i (p_i, \theta) + F_i^* (p_i) \right) \right], \quad (28)$$

37
where $4t \cdot E \left[ s \left( p_i, F_i^* \left( p_i \right), p_j^*, F_j^*; \theta \right) \right] = t + E \left[ v \left( p_i, \theta \right) - v \left( p_j^*, \theta \right) \right] + F_j^* + E \left[ \pi_i \left( p_i, \theta \right) \right]$. The derivative of (28) with respect to $p_i$, after using the envelope theorem and (27), is

$$T_1 \left( p_i; \theta \right) + T_2 \left( p_i; \theta \right) + 2t E \left[ q' \left( p_i, \theta \right) \left( p_i - c_i \right) s_i \left( p_i, F_i, p_j, F_j; \theta \right) \right],$$

where

$$T_1 \left( p_i; \theta \right) \equiv E \left[ \left( v \left( p_i, \theta \right) - v \left( p_j^*, \theta \right) \right) q \left( p_i, \theta \right) \right] - E \left[ \left( v \left( p_i, \theta \right) - v \left( p_j^*, \theta \right) \right) \right] E \left[ q \left( p_i, \theta \right) \right]$$

and

$$T_2 \left( p_i; \theta \right) \equiv E \left[ q \left( p_i, \theta \right) \right] E \left[ \pi_i \left( p_i, \theta \right) \right] - E \left[ q \left( p_i, \theta \right) \right] \left( \pi_i \left( p_i, \theta \right) \right)$$

for $p_i \in \mathcal{P}$. Note that for any $\theta \in \Theta$, $s \left( p_i, F_i^* \left( p_i \right), c_j, F_j^*; \theta \right)$ is strictly decreasing with respect to $p_i$ for any $p_i > c_i$. Note that if there exists a $\tilde{p}_i \in \mathcal{P}$ and a $\theta \in \Theta$, such that $s \left( \tilde{p}_i, F_i^* \left( \tilde{p}_i \right), c_j, F_j^*; \theta \right) = 0$, then from (A3), it follows that for any $p_i \geq \tilde{p}_i$ the profit is zero for all $\theta \in \Theta$. We show that (29) is single peaked for each firm $i \in \{A, B\}$.

1. Suppose $i = A$ and $j = B$. Note that $T_1 \left( p_i; \theta \right) > (\leq 0$ if $p_i < (>)p_j^*$, by increasing difference property and the fact that $\theta$ is strictly associated. Likewise, $T_2 \left( p_i; \theta \right) \leq 0$ for any $p_i \geq c_i$. Finally, note that the last term of (29) is negative for $p_i \in (c_i, \tilde{p}_i)$. So if $p_i \geq p_j^*$, (29) is negative. From the second part of the proof, we know that there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$ such that (29) is zero; by continuity, we conclude that (29) is negative for $p_i > p_j^*$. To show that (29) is positive for any $p_i < p_j^*$, note that (29) is positive for $p_i = c_i$ and we know that there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$ such that (29) is zero, by continuity, (29) is positive for any $p_i < p_j^*$.

2. Suppose $i = B$ and $j = A$. Note that (29) is negative for all $p_i > c_i$. Since there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$ such that (29) is zero, by continuity, it follows that (29) is negative for all $p_i > p_j^*$. Similarly, note that (29) is positive for $p_i \leq p_j^*$, and since there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$ such that (29) is zero, (29) is positive for $p_i \leq p_j^*$.

We conclude that (28) is single-peaked in $p_i$ for $p_i \in [\min \{c_i, c_j\}, \tilde{p}_i)$ and hence has a unique maximum at $p_i = p_i^*$.

(iii) We now show that there do not exist $(p_i, p_j) \in (c_A, c_B)^2$ with $p_i \neq p_i^*$ and $p_j \neq p_j^*$ such that (29) is zero. To prove this claim we show that in equilibrium $\frac{\partial R^A(p_A)}{\partial p_A} > \frac{\partial R^B(p_A)}{\partial p_A}$.

\[\text{Note that continuity follows from Lemma 1.}\]
i.e.,

$$\frac{-\partial \xi^A/\partial p_A}{\partial \xi^A/\partial p_B} > \frac{-\partial \xi^B/\partial p_A}{\partial \xi^B/\partial p_B}. \quad (30)$$

From Lemma 1 (upward sloping quasi-best-response functions for any \((p_i, p_j) \in (c_A, c_B)^2\)) and (30) it follows that there do not exist \((p_i, p_j) \in (c_A, c_B)^2\) with \(p_i \neq p_i^*\) and \(p_j \neq p_j^*\) such that (29) is zero. Before we show the proof for (30), first, we introduce the following assumption.

**Assumption A1.** \(c_B < \rho(c_A)\), where \(\rho(\cdot)\) is such that

$$c_B < c_A - \frac{q_v(c_B)}{2q_v(c_A) + q_v'(c_B)}.$$  

We first show that \(-\partial \xi^A/\partial p_A > \partial \xi^B/\partial p_A\). Next we show that \(-\partial \xi^B/\partial p_B > \partial \xi^A/\partial p_B\).

(ii.a) \(-\partial \xi^A/\partial p_A > \partial \xi^B/\partial p_A\). We need to show that

$$q_v(p_A) \sigma + h^B(p_B)(p_B - c_B) \frac{TS'(p_A)}{3} < (q_v(p_A) + \phi_v(p_A)) \sigma +$$

$$\left[ \frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) \right] \bar{\theta} \cdot T R_A + h^A(p_A)(p_A - c_A) \frac{TS'(p_A)}{3},$$

where \(T R_A \equiv \left\{ t + \bar{\theta} \frac{TS(p_A)}{3} - \bar{\theta} \frac{TS(p_B)}{3} \right\}\).

Note that \(-h^B(p_B)(p_B - c_B) < 1\) and that \(TS'(p_A) = q_v'(p_A)(p_A - c_A)\) thus we need to show that

$$0 < \frac{\bar{\theta}^2 q_v(p_A)}{3} \left( \frac{q_v'(p_A)(p_A - c_A)}{\pi'(p_A)} \right) + \phi_v(p_A) \sigma + \left[ \frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) \right] \bar{\theta} \cdot T R_A.$$  

In Lemma A1 we show that \(\frac{\partial}{\partial p_A} h^A(p_A)(p_A - c_A) > 0\). Given that \(q'(p_A)(p_A - c_A) > -q(p_A)\) and \(T R_A > 0\), it is enough to show that

$$-\bar{\theta}^2 \frac{q_v(p_A)^2}{3\pi'(p_A)} + \phi_v(p_A) \sigma > 0,$$

which follows from Lemma A2.

(ii.b) \(-\partial \xi^B/\partial p_B > \partial \xi^A/\partial p_B\). We need to show that

39
0 < \tilde{\theta}^2 q_v'(p_A)(p_A - c_A) q_v'(p_B)(p_B - c_B) \frac{\pi_v'(p_A)}{3} + \phi_v'(p_B)\sigma + \left[ \frac{\partial}{\partial p_B} h_B(p_B)(p_B - c_B) \right] \tilde{\theta} . TR_B

= \frac{q_v'(p_B)(p_B - c_B)\tilde{\theta}^2}{\pi_v'(p_B)} q_v'(p_B)(p_B - c_B),

where \( TR_B \equiv \{ t + \tilde{\theta}^{TS(p_B)} - \tilde{\theta}^{TS(p_A)} \} \).

In Lemma A3 we show that if \( c_B \) satisfies (A-A1), \( \frac{\partial}{\partial p_B} h_B(p_B)(p_B - c_B) > 0 \). Next, note that by (A-A1) it follows that \( -\frac{q'(p_A)(p_A - c_A)}{\pi_v'(p_A)} < \frac{1}{2} \) and \( -\frac{q'(p_B)(c_B - p_B)}{\pi_v'(p_B)} < \frac{1}{2} \) for \( p_A \in (c_A, c_B) \) if \( c_B \) satisfies (A-A1). Thus, it is enough if we show that

\[
0 < -\frac{\tilde{\theta}^2 q_v'(p_B)(p_B - c_B)}{3} + \phi_v'(p_B)\sigma,
\]

which follows from Lemma A2 and \( 3\sigma > \tilde{\theta}^2 \).

**Lemma A1.** \( \frac{\partial}{\partial p_A} h_A(p_A)(p_A - c_A) > 0 \).

**Proof of Lemma A1.** We need to show that

\[
\frac{\partial}{\partial p_A} h_A(p_A)(p_A - c_A) = -\frac{\partial}{\partial p_A} \frac{q_v'(p_A)(p_A - c_A)}{\pi_v'(p_A)} > 0,
\]

which is equal to

\[
= -\frac{q_v'(p_A)\pi_v'(p_A) + (p_A - c_A)\left\{ q_v''(p_A)q_v(p_A) - 2q_v'(p_A)^2 \right\}}{\pi_v'(p_A)^2}.
\]

From (A2) it follows that \( \{ q_v''(p_A)q_v(p_A) - 2q_v'(p_A)^2 \} > 0 \). Thus, \( \frac{\partial}{\partial p_A} h_A(p_A)(p_A - c_A) > 0 \).

**Lemma A2.** \( \left\{ \phi_v'(p_A) - \frac{q_v(p_A)^2}{\pi_v'(p_A)} \right\} > 0 \)

**Proof Lemma A2.** Note first that

\[
\phi_v'(p_A) - \frac{q_v(p_A)^2}{\pi_v'(p_A)} = \frac{\pi_v'(p_A)q(p_A)\pi_v'(p_A) + \pi_v'(p_A)q'(p_A)\pi_v'(p_A)}{\pi_v'(p_A)^2} = \frac{-\pi_A(p_A)q(p_A)\pi_v''(p_A)}{\pi_v'(p_A)^2} - \frac{q_A(p_A)^2}{\pi_v'(p_A)^2}.
\]

Substituting \( \pi_v'(p_A) = q(p_A) + q'(p_A)(p_A - c_A) \) and \( \pi_v''(p_A) = 2q'(p_A) + q''(p_A)(p_A - c_A) \) in (31),
\[
\phi_i^{\alpha_i} (p_A) - \frac{q_v (p_A)^2}{\pi'_A (p_A)} = \frac{q (p_A) q' (p_A)^2}{\pi'_A (p_A)^2} (p_A - c_A)^2 \left\{ 2 - \frac{q'' (p_A) q (p_A)}{q' (p_A)^2} \right\} > 0,
\]

which follows from (A2).

**Lemma A3.** \( \frac{\partial}{\partial p_B} h^B (p_B) (p_B - c_B) > 0. \)

**Proof of Lemma A3.** Note that

\[
\frac{\partial}{\partial p_B} h^B (p_B) (p_B - c_B) = -\frac{\partial}{\partial p_B} \frac{q_v (p_B) (p_B - c_B)}{\pi^B_v (p_B)} - \frac{q_v (p_B) (q_v (p_B) - q'_v (p_B) (p_B - c_B)) + (c_B - p_B) q''_v (p_B) q_v (p_B)}{\pi^B_v (p_B)^2} > 0.
\]

Now, \( (q_v (p_B) - q'_v (p_B) (p_B - c_B)) \) > 0 if \( c_B \) satisfies (A-A1) and \( p_B \in (c_A, c_B) \). Thus, we conclude that \( \frac{\partial}{\partial p_B} h^B (p_B) (p_B - c_B) > 0. \)

**Proof of Corollary 3.** From Proposition 4 we know that as \( p_A \to c_A \) in (9) for \( i = A \), we have that \( p_B \to c_A \) and as \( p_A \to c_B, p_B \to \alpha_A > c_B \). Similarly, from (9) for \( i = B \), as \( p_B \to c_B \) we have that \( p_A \to c_B \), while as \( p_A \to c_A, p_B \to \alpha_B > c_A \). As \( c_A, c_B \) tends to \( c \), \( \alpha_A \) and \( \alpha_B \) also tend to \( c \). Thus, the two best response functions intersect each other only at \( p_i = c_i \).

**Proof of Proposition 5.** We split the proof in five steps. (i) We first show that no solution exist for \( p_B > c \) (ii) using the result in (iii), we show that the objective function of firm \( i \) is single-peaked so that it has a global maximum. (iii) We show that the slope of the implicit functions defined by (11) is positive. (iv) We show existence. (v) We show that for \( p^*_i \) and \( p^*_j \) defined by (11) for \( i \in \{A, B\} \), we show that there do not exist \( (p_i, p_j) \in (c_A, c_B)^2 \) with \( p_i \neq p^*_i \) and \( p_j \neq p^*_j \), such that the first order conditions are satisfied.

The proof to show (ii) is very similar to the Proof of Proposition 4 so we omit it.

(i) We first show that no solution exist for \( p_B > c \): we show that for every \( p_A, p_B \in \tilde{\mathcal{P}} \) that satisfy (11) for \( i = A \), it must be true that \( \varphi (p_A, p_B; \theta) > 0, \forall \theta \in \Theta \), where \( \varphi (p_A, p_B; \theta) \equiv v (p_A, \theta) - \alpha v (p_B, \theta) \). Similarly, we show that for every \( p_A, p_B \in \tilde{\mathcal{P}} \) that satisfy (11) for \( i = B \), and for \( p_B > c_B, \varphi (p_A, p_B; \theta) < 0, \forall \theta \in \Theta \).

Lets first show that for every \( p_A, p_B \in \tilde{\mathcal{P}} \) that satisfy (11) for \( i = A \), \( \varphi (p_A, p_B; \theta) > 0 \) \( \forall \theta \in \Theta \). If \( p_A, p_B \in \tilde{\mathcal{P}} \) satisfy (11) for \( i = A \), then
\[
\xi^A (p_A, p_B) \equiv E \left[ q(p_A, \theta) \left( \hat{\Delta} v_A - \hat{\Delta} v_B \right) \right] + (p_A - c) \left\{ 2t \cdot E \left[ s_A (p_A, F_A^*, p_B, F_B^*, \theta) q'(p_A, \theta) \right] - \text{Var} [q(p_A, \theta)] \right\} < 0.
\]

Therefore

\[
E \left[ q(p_A, \theta) (\hat{\Delta} v_A - \hat{\Delta} v_B) \right] > 0,
\]

which by (A4) is equal to

\[
- \frac{\partial v(p_A)}{\partial p_A} (v(p_A) - \alpha v(p_B)) \text{Var}[h(\theta)] > 0,
\]

implying that \(v(p_A) - \alpha v(p_B) > 0\). Thus we conclude that for every \(p_A, p_B \in \hat{P}\) that satisfy (11) for \(i = A\), \((p_A, p_B) \in \{p_A, p_B | \varphi(p_A, p_B; \theta) > 0 \ \forall \theta \in \Theta\}\).

Now let us show that for every \(p_A, p_B \in \hat{P}\) that satisfy (11) for \(i = B\), it must be true that \(-\varphi(p_A, p_B; \theta) > 0 \ \forall \theta \in \Theta\) for \(p_B > c_B\), otherwise

\[
\xi^B (p_A, p_B) \equiv E \left[ q(p_B, \theta) \left( \hat{\Delta} v_B - \hat{\Delta} v_A \right) \right] + (p_B - c_B) \left\{ 2t \cdot E \left[ s_B (p_B, F_B^*, p_A, F_A^*, \theta) q'(p_B, \theta) \right] - \alpha \text{Var} [q(p_B, \theta)] \right\} = 0,
\]

would imply that

\[
E \left[ q(p_B, \theta) \left( \hat{\Delta} v_B - \hat{\Delta} v_A \right) \right] > 0,
\]

which would be a contradiction. Thus we conclude that for every \(p_A, p_B \in \hat{P}\) that satisfy (11) for \(i = B\) for \(p_B > c\) it is true that \((p_A, p_B) \in \{p_A, p_B | -\varphi(p_A, p_B; \theta) > 0, \ \forall \theta \in \Theta\}\).

Thus we conclude that no equilibrium exists for \(p_B > c\), which implies that if there exists any equilibrium of the game it must be in the set \(\hat{\Omega}(p_A, p_B)\), where \(\hat{\Omega}(p_A, p_B) \equiv \{p_A, p_B \in \hat{P} | (p_A, p_B) \in [c, \tilde{\alpha}_A] \times [\gamma_B, c]\}\), where \(\tilde{\alpha}_A\) is such that \((\tilde{\alpha}_A, c)\) satisfies (11) for \(i = B\).

(iii) Next, we show that for \((p_A, p_B) \in \hat{\Omega}(p_A, p_B)\) the slope of the implicit functions defined by (11) for \(i \in \{A, B\}\), \(\tilde{R}^i (p) : \hat{P} \to \hat{P}\), where \(\tilde{R}^i (\tilde{p}^i_A) = \tilde{p}^i_B\) is such that \(\tilde{p}^i_A\) and \(\tilde{p}^i_B\) satisfy (11) for \(i \in \{A, B\}\), is positive.

- \(\frac{\partial \tilde{R}^A (p_A)}{\partial p_A} > 0\): In (v) we show that \(-\frac{\partial \tilde{E}^A}{\partial p_A} > \frac{\partial \tilde{E}^B}{\partial p_A}\), and in part (iii - b) of this proposition we show that \(\frac{\partial \tilde{E}^A}{\partial p_A} > 0\). Thus, we need to show that \(\frac{\partial \tilde{E}^B}{\partial p_B} > 0\). Note that
\[
\frac{\partial \tilde{\xi}^A}{\partial p_B} = \alpha E[\pi' (p_A, \theta) q (p_B, \theta)] - \alpha E[\pi' (p_A, \theta)] E[q (p_B, \theta)]
\]

\[
- \alpha (p_A - c) (p_B - c) E[q' (p_A, \theta)] E[q' (p_B, \theta)]
\]

\[
= \alpha \text{Cov}(\pi' (p_A, \theta), q (p_B, \theta)) - \alpha (p_A - c) (p_B - c) \frac{E[q' (p_A, \theta)] E[q' (p_B, \theta)]}{3} > 0.
\]

The last inequality always holds for \( p_B \leq c \) and \( p_A \geq c \). Also, notice that by (A5) \( \text{Cov}(\pi' (p_A, \theta), q (p_B, \theta)) = \pi' (p_A) q(p_B) \text{Var}[h(\theta)] > 0 \). Then from the Implicit Function Theorem we conclude that \( \frac{\partial \tilde{R}^A(p_A)}{\partial p_A} \) is positive for \( (p_A, p_B) \in \bar{\Omega} (p_A, p_B) \).

- \( \frac{\partial \tilde{R}^B(p_A)}{\partial p_A} > 0 \): In (v) we show that \( \frac{\partial \tilde{\xi}^A}{\partial p_B} < - \frac{\partial \tilde{\xi}^B}{\partial p_B} \). In part \( (ii) - a \) of this proposition we showed that \( \frac{\partial \tilde{\xi}^A}{\partial p_B} > 0 \). Thus we just need to show that \( \frac{\partial \tilde{\xi}^B}{\partial p_A} > 0 \). Taking derivatives of (11) for \( i = B \) with respect to \( p_A \)

\[
\frac{\partial \tilde{\xi}^B}{\partial p_A} = \alpha \pi' (p_A) q(p_A) \text{Var}[h(\theta)] - \alpha (p_A - c) (p_B - c) \frac{E[q' (p_A, \theta)] E[q' (p_B, \theta)]}{3} > 0.
\]

The last inequality always holds for \( p_B \leq c \) and \( p_A \geq c \). Then, from the Implicit Function Theorem we conclude that \( \frac{\partial \tilde{R}^B(p_A)}{\partial p_A} \) is positive for \( (p_A, p_B) \in \bar{\Omega} (p_A, p_B) \).

(iv) Note that as \( p_A \to c \) in (11) for \( i = A \),

\[
\tilde{\xi}^A (c, p_B) = (v(c) - \alpha v(p_B)) \cdot q(c) \cdot \text{Var}[h(\theta)] < 0.
\]

It is not true that \( p_B \to c \), since \( \varphi (c, c; \theta) > 0 \) for all \( \theta \in \Theta \). Thus, from part (ii),\(^{58}\) we conclude that \( p_B \to \tilde{\gamma}_B < c \), as \( p_A \to c \), where \( \tilde{\gamma}_B \) is such that \( \tilde{\xi}^A (c, \tilde{\gamma}_B) = 0 \), that is

\[
E \left[ \varphi (c, \tilde{\gamma}_B; \theta) \right] = \frac{E \left[ q (c, \theta) \varphi (c, \tilde{\gamma}_B; \theta) \right]}{E \left[ q (c, \theta) \right]}.
\]

Similarly, as \( p_B \to c \) in (11) for \( i = B \),

\[
\tilde{\xi}^B (p_A, c) = (\alpha v(c) - v(p_A)) \cdot q(c) \cdot \text{Var}[h(\theta)] > 0.
\]

\(^{58}\)Note that \( \tilde{\xi}^A (c, p_B) > 0 \) if \( p_B = c \).
From part (ii) we conclude that \( p_A \rightarrow \tilde{\alpha}_A > c \), with \( \tilde{\xi}^B(\tilde{\alpha}_A, c) = 0 \).\(^{59}\) Likewise, as \( p_A \rightarrow c \) in (11) for \( i = B \), we have

\[
\tilde{\xi}^B(c, p_B) = (\alpha v(p_B) - v(c)) \cdot q(p_B) \cdot \text{Var}[h(\theta)] > 0
\]

\[
+ (p_B - c) \{ 2t \cdot E[s_B(p_B, F_B^*, c, F_A^*, \theta) q'(p_B, \theta)] - \alpha \text{Var}[q(p_B, \theta)] \}
\]

Note that if \( p_B = c, \tilde{\xi}^B(c, c) < 0 \), so from part (ii) we know that as \( p_A \rightarrow c \) in (11) for \( i = B \), \( p_B \rightarrow \tilde{\alpha}_B < c \), with \( \tilde{\xi}^B(c, \tilde{\alpha}_B) = 0 \). It is easy to show that \( \tilde{\alpha}_B > \tilde{\gamma}_B \). If we suppose that is not true, i.e., \( \tilde{\gamma}_B \geq \tilde{\alpha}_B \) then

\[
\tilde{\xi}^B(c, \tilde{\gamma}_B) = E[q(\tilde{\gamma}_B, \theta)] E[\varphi(c, \tilde{\gamma}_B; \theta)] - E[q(\tilde{\gamma}_B, \theta) \varphi(c, \tilde{\gamma}_B; \theta)]
\]

\[
+ (\tilde{\gamma}_B - c) \{ 2t \cdot E[s_B(\tilde{\gamma}_B, F_B^*, c, F_A^*, \theta) q'(\tilde{\gamma}_B, \theta)] - \alpha \text{Var}[q(\tilde{\gamma}_B, \theta)] \} > 0.
\]

From (34), \( \tilde{\gamma}_B \) is such that

\[
E[\varphi(c, \tilde{\gamma}_B; \theta)] = \frac{E[q(c, \theta) \varphi(c, \tilde{\gamma}_B; \theta)]}{E[q(c, \theta)]}.
\]

Thus,

\[
\tilde{\xi}^B(c, \tilde{\gamma}_B) \geq \frac{E[q(\tilde{\gamma}_B, \theta)]}{E[q(c, \theta)]} E[q(c, \theta) \varphi(c, \tilde{\gamma}_B; \theta)] - E[q(\tilde{\gamma}_B, \theta) \varphi(c, \tilde{\gamma}_B; \theta)] > 0,
\]

since \( \tilde{\gamma}_B < c \). So from part (ii) we conclude that \( \tilde{\alpha}_B > \tilde{\gamma}_B \). Finally, note that as \( p_B \rightarrow c \) in (11) for \( i = A \), \( p_A \rightarrow \tilde{\gamma}_A > c \), since

\[
\tilde{\xi}^A(p_A, c) = (v(p_A) - \alpha v(c)) \cdot q(p_A) \cdot \text{Var}[h(\theta)] < 0
\]

\[
+ (p_A - c) \{ 2t \cdot E[s_A(p_A, F_A^*, c, F_B^*, \theta) q'(p_A, \theta)] - \alpha \text{Var}[q(p_A, \theta)] \} < 0,
\]

and \( \tilde{\xi}^A(c, c) > 0 \), where \( \tilde{\gamma}_A \) is such that

\[
\tilde{\xi}^A(\tilde{\gamma}_A, c) = E[q(\tilde{\gamma}_A, \theta) \varphi(\tilde{\gamma}_A, c; \theta)] - E[q(\tilde{\gamma}_A, \theta)] E[\varphi(\tilde{\gamma}_A, c; \theta)] \]

\(^{59}\)Note that \( \tilde{\xi}^B(p_A, c) < 0 \) if \( p_A = c \).
where

\[ \tilde{\omega} \lesssim \sum_{i} \text{Var} \{ q \left( \tilde{\gamma}_A, \theta \right) \} \]

is greater than the slope defined for

\[ \tilde{\gamma}_A \rightarrow \tilde{\gamma}_A \] in (11) for \( i = B \), \( p_B \rightarrow \omega_B < c \). Thus, both curves cross each other at least once in the set \( \tilde{\Omega}(p_A, p_B) \).

(v) Finally, we show that for \( p_i^* \) and \( p_j^* \) defined by (11) for \( i \in \{A, B\} \), we show that there do not exist \((p_i, p_j) \in (c_A, c_B)^2\) with \( p_i \neq p_i^* \) and \( p_j \neq p_j^* \), such that the first order conditions are satisfied. We show show that in equilibrium the slope of the implicit function defined by (11) for \( i = A \) is greater than the slope defined for \( i = B \), that is, \( \frac{\partial \tilde{R}^A(p_A)}{\partial p_A} |_{p_A = p_A^*} > \frac{\partial \tilde{R}^B(p_A)}{\partial p_A} |_{p_A = p_A^*} \).

From (iii) we know that

\[
\frac{\partial \tilde{R}^A(p_A)}{\partial p_A} = \frac{N(p_A, p_B; \theta, t)}{\psi(p_A) - 1} \left\{ \frac{E[q(p_A, p_B; \theta)]}{3} - \frac{\alpha E[\pi(p_B, \theta)]}{3} \right\} - \alpha E \left[ q(p_B, \theta) \right] + \frac{\alpha E[\pi(p_A, \theta)]}{E[\pi(p_A, \theta)]},
\]

where \( \psi(p) = \frac{E[q(p, \theta)]}{E[\pi(p, \theta)]} \), \( \phi(p) = \frac{E[q(p, \theta)\psi(p-c)]}{E[\pi(p, \theta)]} \)

\[
N(p_A, p_B; \theta, t) \equiv \psi'(p_A) \left\{ t + \frac{E[\varphi(p_A, p_B; \theta)]}{3} - \frac{E[\alpha \pi(p_B, \theta)]}{3} - \frac{2E[\pi(p_A, \theta)]}{3} \right\}
\]

\[
+ \left( \psi(p_A) - 1 \right) \left\{ -\frac{E[q(p_A, \theta)]}{3} - \frac{2E[\pi'(p_A, \theta)]}{3} \right\} - E[q(p_A, \theta)] - E[\pi'(p_A, \theta)] + \frac{E[\pi'(p_A, \theta)] q(p_A, \theta)}{E[\pi'(p_A, \theta)]} + \phi'(p_A).
\]

Similarly, we have

\[
\frac{\partial \tilde{R}^B(p_A)}{\partial p_A} = \frac{\psi(p_B) - 1}{3} \left\{ \frac{E[q(p_A, \theta)]}{3} - \frac{E[q(p_A, \theta)]}{3} \right\} - E[q(p_A, \theta)] + \frac{E[q(p_A, \theta)]}{E[q(p_A, \theta)]},
\]

where \( \psi(p) = \frac{E[q(p, \theta)]}{E[\pi(p, \theta)]} \), \( \phi(p) = \frac{E[q(p, \theta)\psi(p-c)]}{E[\pi(p, \theta)]} \)
where

\[
D(p_A, p_B; \theta, t) \equiv \psi'(p_B) \left\{ t - \frac{E[\varphi(p_A, p_B; \theta)]}{3} - \frac{2E[\alpha \pi(p_B, \theta)]}{3} - \frac{E[\pi(p_A, \theta)]}{3} \right\} +
\]

\[
(\psi(p_B) - 1) \left\{ - \frac{E[\alpha q(p_B, \theta)]}{3} - \frac{2E[\alpha \pi'(p_B, \theta)]}{3} \right\} - \alpha E[q(p_B, \theta)] - \alpha E[\pi'(p_B, \theta)]
+ \frac{\alpha E[\pi'(p_B, \theta)] q(p_B, \theta)}{E[\pi'(p_B, \theta)]} + \alpha \phi'(p_B).
\]

Let us first analyze both denominators, which are both positive. In particular, we can show that the denominator of (40), \(D(p_A, p_B; \theta, t)\), is greater than the denominator of (39). That is,

\[
- (\psi(p_A) - 1) \left\{ \frac{E[\alpha q(p_B, \theta)]}{3} - \frac{E[\alpha \pi'(p_B, \theta)]}{3} \right\} - \alpha E[q(p_B, \theta)] + \frac{\alpha E[\pi'(p_A, \theta)] q(p_B, \theta)}{E[\pi'(p_B, \theta)]} < \]

\[
\psi'(p_B) \left\{ t - \frac{E[\varphi(p_A, p_B; \theta)]}{3} - \frac{2E[\alpha \pi(p_B, \theta)]}{3} - \frac{E[\pi(p_A, \theta)]}{3} \right\} +
\]

\[
(\psi(p_B) - 1) \left\{ - \frac{E[\alpha q(p_B, \theta)]}{3} - \frac{2E[\alpha \pi'(p_B, \theta)]}{3} \right\} - \alpha E[q(p_B, \theta)] - \alpha E[\pi'(p_B, \theta)]
+ \frac{\alpha E[\pi'(p_B, \theta)] q(p_B, \theta)}{E[\pi'(p_B, \theta)]} + \alpha \phi'(p_B).
\]

From (A5),

\[
0 < \psi'(p_B) \left\{ t - \frac{E[\varphi(p_A, p_B; \theta)]}{3} - \frac{2E[\alpha \pi(p_B, \theta)]}{3} - \frac{E[\pi(p_A, \theta)]}{3} \right\} + \]

\[
(1 - \psi(p_B)) E[\alpha q(p_B, \theta)] + \left\{ 1 - \frac{2\psi(p_B)}{3} - \frac{\psi(p_A)}{3} \right\} E[\alpha \pi'(p_B, \theta)] (p_B - c)
\]

\[
+ \frac{\alpha \phi'(p_B) - \alpha E[\pi'(p_B, \theta)]}{\geq 0}.
\]

Note that \(E[\pi'(\cdot)]\) is decreasing and \(\phi''(\cdot)\) is increasing with respect to \(p_i\). For \(p_B = c\), \(\phi''(c_B) > E[\pi''(c_B, \theta)]\) (see Lemma A4). Thus, \(\alpha \phi'(p_B) - \alpha E[\pi'(p_B, \theta)] \geq 0\) if \(\alpha \in \mathcal{O} \equiv \{\alpha \mid \phi'(\tilde{\gamma}_B) > E[\pi'(\tilde{\gamma}_B, \theta)]\} \).

Now let us analyze both numerators. In particular, let us show that the numerator of
Lemma A4. \( \psi \) is greater than the numerator of \( \psi' \). That is, let us show that

\[
- (\psi(p_B) - 1) \left\{ \frac{E[q(p_A, \theta)] - E[\pi'(p_A, \theta)]}{3} \right\} - E[q(p_A, \theta)] + \frac{E[\pi'(p_B, \theta) q(p_A, \theta)]}{E[\pi'(p_B, \theta)]} <
\]

\[
\psi'(p_A) \left\{ t + \frac{E[\varphi(p_A, p_B, \theta)]}{3} - \frac{E[\alpha \pi(p_B, \theta)]}{3} - \frac{2E[\pi(p_A, \theta)]}{3} \right\} +
\]

\[
(\psi(p_A) - 1) \left\{ - \frac{E[q(p_A, \theta)]}{3} - \frac{2E[\pi'(p_A, \theta)]}{3} \right\} - E[q(p_A, \theta)] - E[\pi'(p_A, \theta)]
\]

\[
+ \frac{E[\pi'(p_A, \theta) q(p_A, \theta)]}{E[\pi'(p_A, \theta)]} + \phi'(p_A).
\]

From (A5),

\[
0 < \psi'(p_A) \left\{ t + \frac{E[\varphi(p_A, p_B, \theta)]}{3} - \frac{E[\alpha \pi(p_B, \theta)]}{3} - \frac{2E[\pi(p_A, \theta)]}{3} \right\} +
\]

\[
- \psi(p_A) \frac{2E[q'(p_A, \theta) (p_A - c_A)]}{3} - \frac{E[q'(p_A, \theta) (p_A - c_A)]}{3} + \psi(p_B) \frac{E[q'(p_A, \theta) (p_A - c_A)]}{3}
\]

\[
+ \phi'(p_A) - \psi(p_A) E[q(p_A, \theta)],
\]

where \( \psi(p_B) \equiv (1 - \psi(p_B)) < 1 \), where \( \{ \phi'(p_A) - \psi(p_A) E[q(p_A, \theta)] \} \geq 0 \) follows from Lemma A4.

**Lemma A4.** \( \phi'(p_A) - \psi(p_A) E[q(p_A, \theta)] \) > 0 where \( \psi^A(p_A) \equiv \frac{E[q_A(p_A, \theta)]}{E[\pi^A(p_A, \theta)]} \).

**Proof of Lemma A4.**

Note first that

\[
\phi^A(p_A) - E[q_A(p_A, \theta)] \psi(p_A) = \frac{E[\pi^A(p_A, \theta) q(p_A, \theta)] E[\pi^A(p_A, \theta)] + E[\pi^A(p_A, \theta) q'(p_A, \theta)] E[\pi^A(p_A, \theta)]}{E[\pi^A(p_A, \theta)]^2}
\]

\[
- \frac{E[\pi^A(p_A, \theta) q(p_A, \theta)] E[\pi^A(p_A, \theta)]}{E[\pi^A(p_A, \theta)]^2} - \frac{E[q_A(p_A, \theta)]^2}{E[\pi^A(p_A, \theta)]}
\]

Substituting

\[
E[\pi^A(p_A, \theta)] = E[q(p_A, \theta) + q'(p_A, \theta) (p_A - c_A)]
\]

and \( E[\pi^A(p_A, \theta)] = E[2q'(p_A, \theta) + q''(p_A, \theta) (p_A - c_A)] \) in (41)

\[
\phi^A(p_A) - E[q_A(p_A, \theta)] \psi(p_A) =
\]
\[
\begin{align*}
&\left\{ \frac{2E[q(p_A, \theta)q'(p_A, \theta)]}{E[\pi'(p_A, \theta)]^2} E[q(p_A, \theta)](p_A - c_A) - \frac{E[2q'(p_A, \theta)] E[q(p_A, \theta)]^2}{E[\pi'(p_A, \theta)]^2} (p_A - c_A) \right\} \\
&\quad + \left\{ \frac{E[q(p_A, \theta)]^2}{E[\pi'(p_A, \theta)]} - \frac{E[q_A(p_A, \theta)]^2}{E[\pi'(p_A, \theta)]} \right\} \\
&\quad \geq 0
\end{align*}
\]
\[
\begin{align*}
&\left\{ \frac{2E[q(p_A, \theta)q'(p_A, \theta)]}{E[\pi'(p_A, \theta)]^2} E[q'(p_A, \theta)](p_A - c_A)^2 - \frac{E[q''(p_A, \theta)] E[q(p_A, \theta)]^2}{E[\pi'(p_A, \theta)]^2} (p_A - c_A)^2 \right\} \\
&\quad \equiv C1
\end{align*}
\]

From (A4)

\[
C1 = \frac{E[h(\theta)^2]}{E[\pi'(p_A, \theta)]^2} (p_A - c_A)^2 q(p_A)q'(p_A)^2 \left\{ 2 - \frac{q''(p_A)q(p_A)}{q'(p_A)} \right\} > 0,
\]

which follows from (A2).

**Proof of Proposition 6.** We first show the “if” and then the “only if” part of the proof. Suppose that condition (13) is satisfied for \( p_i = c_i \). The first order conditions for firm \( i \in \{A, B\} \) are

\[
\begin{align*}
[p_i] E \left\{ s(v_i(p_i, \theta) - F_i, v_j(p_j, \theta) - F_j) \pi''(p_i, \theta) \right. \\
&\quad - s_1(v_i(p_i, \theta) - F_i, v_j(p_j, \theta) - F_j) q(p_i, \theta) \left[ \pi^i(p_i, \theta) + F_i \right] \} = 0
\end{align*}
\]

and

\[
\begin{align*}
[F_i] E \left\{ s(v_i(p_i, \theta) - F_i, v_j(p_j, \theta) - F_j) - \\
&\quad s_1(v_i(p_i, \theta) - F_i, v_j(p_j, \theta) - F_j) \left[ \pi^i(p_i, \theta) + F_i \right] \} = 0.
\end{align*}
\]

From (43) and (42), marginal-cost pricing is an equilibrium if

\[
E \left[ s_1(v_i(c_i, \theta) - F^*_i, v_j(c_j, \theta) - F^*_j) q(c_i, \theta) \right] = E \left[ s_1(v_i(c_i, \theta) - F^*_i, v_j(c_j, \theta) - F^*_j) \right] E \left[ s(v_i(c_i, \theta) - F^*_i, v_j(c_j, \theta) - F^*_j) q(c_i, \theta) \right],
\]

which is true if (13) is satisfied for \( p_i = c_i \) and \( i \in \{A, B\} \). We can prove this last statement subtracting both sides by \( E \left[ s_1(v_i(c_i, \theta) - F^*_i, v_j(c_j, \theta) - F^*_j) \right] E[q(c_i, \theta)] \).

Let's show the “only if” part. Suppose that any pure-strategy Nash equilibrium involves
marginal-cost-based 2PT. From (42) and (43)

\[ E \left[ s \left( v_i(c_i, \theta) - F_i^* - v_j(c_j, \theta) - F_j^* \right) q(c_i, \theta) \right] \times \left\{ 1 - \frac{E \left[ s_1 \left( v_i(c_i, \theta) - F_i^* - v_j(c_j, \theta) - F_j^* \right) q(c_i, \theta) \right] E \left[ s \left( v_i(c_i, \theta) - F_i^* - v_j(c_j, \theta) - F_j^* \right) q(c_i, \theta) \right]}{E \left[ s \left( v_i(c_i, \theta) - F_i^* - v_j(c_j, \theta) - F_j^* \right) q(c_i, \theta) \right] E \left[ s_1 \left( v_i(c_i, \theta) - F_i^* - v_j(c_j, \theta) - F_j^* \right) q(c_i, \theta) \right]} \right\} = 0 \]

thus, the second term of the left side of (44) is equal to 1, which implies that condition (13) is satisfied for \( p_i = c_i \) for \( i \in \{A, B\} \).

**Proof of Corollary 5.** The problem of each firm is

\[
\max_{p_i, F_i} \ s \left( u_i(p_i, F_i), u_j(p_j, F_j) \right) \left[ \pi_i(p_i) + F_i \right].
\]

where the utility offered by firm \( i \in \{A, B\} \) is \( u_i(p_i, F_i) = v_i(p_i) - F_i \). We solve the two-variable optimization problem of firm \( i \) sequentially. First we show that for any \( p_i \in \mathcal{P} \), firm \( i \) chooses \( F_i \) to maximize its profits. The first order condition with respect to \( F_i \) yields

\[
\pi_i(p_i) + F_i^* = \frac{s \left( u_i(p_i, F_i^*), u_j(p_j, F_j) \right)}{s_1 \left( u_i(p_i, F_i^*), u_j(p_j, F_j) \right)}.
\]

The profit is strictly log-concave in \( F_i \), and the unique solution is implicitly determined by (45). Next, firm \( i \) chooses \( p_i \) to maximize its maximum profits (we substitute \( F_i^* \) \( p_i \) in the objective function of firm \( i \))

\[
s \left( p_i, F_i^*(p_i), p_j, F_j \right) \left[ \pi_i(p_i) + F_i^*(p_i) \right].
\]

The derivative of (46) with respect to \( p_i \), after using the Envelope Theorem, is

\[
q'(p_i)(p_i - c_i) s \left( p_i, F_i^*(p_i), p_j, F_j \right)
\]

Given \( p_j \in \mathcal{P} \) and \( F_j \geq 0 \), it follows from (45) and the Implicit Function Theorem that \( s \left( p_i, F_i^*(p_i), p_j, F_j \right) \) is strictly decreasing with respect to \( p_i \) for any \( p_i > c_i \). Thus, if there exists a \( \tilde{p}_i \) \( (p_j, F_j) \in \mathcal{P}, \) such that \( s \left( \tilde{p}_i, F_i^*(\tilde{p}_i), p_j, F_j \right) = 0 \), then, for any \( p_i \geq \tilde{p}_i \) the profit is zero. Then, for any \( p_j \in \mathcal{P} \) and \( F_j \geq 0 \), note that (47) is positive for \( p_i < c_i \) and it is negative for any \( p_i \in (c_i, \tilde{p}_i(p_j, F_j)) \). Thus, (46) is single-peaked in \( p_i \) and reaches a unique maximum at \( p_i = c_i \). Analogously, the profit function for firm \( j \neq i \) in terms of \( p_j \) is single-peaked in \( p_j \) and reaches a unique maximum at \( p_j = c_j \). Thus the equilibrium is unique, and it follows
that $F^*_i$ is implicitly determined by:

$$F^*_i = \frac{s(v^*_i, v^*_j)}{s(v^*_i, v^*_j)}.$$  \hspace{1cm} (48)

where $v^*_i \equiv v_i(c_i) - F^*_i$.

Next, we show that $F^*_i$, in equilibrium, is well defined. Using the dual approach, we show that there exist $F^*_i, F^*_j$ that satisfy (48) for $j \neq i$ and $i, j \in \{A, B\}$.

To simplify the notation, let $u_i \equiv v_i(c_i) - F_i$. Then we show that there exist $u_i = v^*_i$ and $u_j = v^*_j$ that satisfy

$$\frac{s(u_i, u_j)}{s(v^*_i, v^*_j)} - v_i(c_i) + u_i = 0,$$  \hspace{1cm} (49)

for $u_i, u_j \in U$ and $i \in \{A, B\}$. First, note that from the analogue of (49) for firm $j$ and the Implicit Function Theorem there exists a function $R^j(u_i) : U \to U$, where $R^j(u_i) = u_j$ is such that $v_i$ and $u_j$ satisfy (49) for $j \neq i$. Note that

$$\frac{\partial R^j(u_i)}{\partial u_i} = -\gamma_2(R^j(u_i), u_i) \frac{1}{1 + \gamma_1(R^j(u_i), u_i)} > 0,$$  \hspace{1cm} (50)

where $\gamma(u_i, u_j) \equiv \frac{s(u_i, u_j)}{s(v^*_i, v^*_j)}$, $\gamma_2(u_i, u_j) < 0$ and $\gamma_1(u_i, u_j) > 0$ for $u_i, u_j \in U$. We show in two steps that there exists a unique $u_i = v^*_i$ that satisfies (49).

1. First note that as $u_i \to 0$, $R^j(u_i) \to \alpha > 0$: For $u_i = 0$, $R^j(0)$ is implicitly defined by $u_j$ such that

$$v_j(c_j) - u_j = \gamma(u_j, 0).$$  \hspace{1cm} (51)

Note that the RHS of (51) is increasing with respect to $u_j$, and as $u_j \to 0$, the RHS of (51) $\to \gamma(0, 0) > 0$. Similarly, note that the LHS of (51) is decreasing with respect to $u_j$, and as $u_j \to 0$, the LHS of (51) $\to v_j(c_j) > \gamma(0, 0)$, and as $u_j \to v_j(c_j)$, the LHS of (51) $\to 0$. Thus, as $u_i \to 0$, from (50) and continuity we conclude that there is a unique value $R^j(u_i) > 0$ that satisfies

$$v_j(c_j) - R^j(u_i) = \gamma(R^j(u_i), u_i).$$

2. By assumption, we know that $v_i(c_i) > \gamma(0, 0) > \gamma(0, R^j(0))$. Then, as $u_i \to 0$, the LHS of (49) $\to \gamma(0, R^j(0)) - v_i(c_i) < 0$. Similarly, as $u_i \to v_i(c_i)$, the LHS of (49) $\to s(v_i(c_i), R^j(v_i(c_i))) > 0$. Thus existence follows.

$^60$\(U\) is the set of feasible utility offered to consumers defined by $U = [0, v(c)]$.  

50
Remember that the profit is strictly log-concave in $F_i$, thus, we conclude that there exist $F^*_i, F^*_j$ implicitly determined by (48).

**Proof of Proposition 7.** For completeness of the proof we first prove that marginal cost based-2PT is not an equilibrium, i.e., condition (13) is not satisfied. Let us suppose is not true, i.e., $p^*_i = p^*_j = c$ and $F_i = F_j = F^*$. Then, note that in equilibrium

$$s(v(\theta), v(\theta)) \equiv \frac{e^{v(\theta)}}{2e^{v(\theta)} + e^{u_0}}, \quad \phi(v(\theta)) = \frac{1}{E[s(v(\theta), v(\theta))] - \frac{(1 - s(v(\theta), v(\theta)))}{E[s_1(v(\theta), v(\theta))]}},$$

is monotonic increasing with respect to $\theta$. Thus, if $\theta$ is associated, (13) is not satisfied, since $q(c, \theta)$ is also monotonic increasing with respect to $\theta$.

We now show that in any symmetric equilibrium, $p^* > c$. Note that the first order conditions are

$$E[s(u_i(\theta), u_j(\theta)) \pi'(p_i, \theta) - s_1(u_i(\theta), u_j(\theta)) q(p_i, \theta) (\pi(p_i, \theta) + F_i)] = 0 \quad (52)$$

and

$$E[s(u_i(\theta), u_j(\theta)) - s_1(u_i(\theta), u_j(\theta)) (\pi(p_i, \theta) + F_i)] = 0, \quad (53)$$

where $u_i(\theta) \equiv v(p_i, \theta) - F_i$ for $i, j \in \{A, B\}$ and $j \neq i$. In a symmetric equilibrium,

$$F^* = \frac{E[s(u^*(\theta), u^*(\theta))] - E[s_1(u^*(\theta), u^*(\theta)) q(p^*, \theta)]}{E[s_1(u^*(\theta), u^*(\theta))]}(p^* - c), \quad (54)$$

where $u^*(\theta) = v(p^*, \theta) - F^*$. Let $h(\theta) = s(u^*(\theta), u^*(\theta))$, and $h_1(\theta) \equiv s_1(u^*(\theta), u^*(\theta))$. Thus from (52) and (54),

$$\begin{cases} \frac{E[h(\theta) q'(p^*, \theta)] - E[h_1(\theta) (q(p^*, \theta)^2)] + \frac{E[h_1(\theta) q(p^*, \theta)]^2}{E[h_1(\theta)]}}{G} (p^* - c) \quad (55) \\ + E[h(\theta) q(p^*, \theta)] - E[h_1(\theta) q(p^*, \theta)] \frac{E[h(\theta)]}{E[h_1(\theta)]} = 0. \end{cases}$$

Note that $h_1(\theta) = h(\theta) \cdot (1 - h(\theta))$, and that the last two terms of (55) are positive,

51
that is,

\[ E \left\{ q(p^*, \theta) \cdot \left( \frac{h(\theta)}{E[h(\theta)]} - \frac{h(\theta)(1 - h(\theta))}{E[h(\theta) \cdot (1 - h(\theta))]} \right) \right\} > 0 \]

if

\[ \text{Cov} \left\{ q(p^*, \theta), \left( \frac{h(\theta)}{E[h(\theta)]} - \frac{h(\theta)(1 - h(\theta))}{E[h(\theta) \cdot (1 - h(\theta))]} \right) \right\} > 0, \]

which holds if \( \theta \) is associated. Similarly, note that the first big bracket of (55) is negative, that is, \( G < 0 \) since

\[ \text{Cov} \left\{ s \in \{ L, H \}, \left( \frac{E[s \cdot (q(p^*, \theta) \theta)]}{E[s \cdot q(p^*, \theta)]} \right) \right\} > 0, \]

where \( k(\theta) \equiv \frac{s_1(u^*(\theta), u^*(\theta))}{E[s_1(u^*(\theta), u^*(\theta))]} \). Let \( m(\theta) = g(\theta) k(\theta) \) where \( g(\cdot) \) is the density function of \( \theta \). Note that \( m(\cdot) \) is a density function since \( \int m(\theta) d\theta = 1 \), then

\[ E[k(\theta) (q(p^*, \theta)^2)] - (E[k(\theta) q(p^*, \theta)])^2 \]

\[ = \int m(\theta) (q(p^*, \theta)^2) \ d(\theta) - \left( \int m(\theta) q(p^*, \theta) \ d(\theta) \right)^2 > 0 \]

by Lemma 2.2.1 in Tong (1980). This last inequality was originally due to Chebyshev, known as Chebyshev’s other inequality, or Kimball’s inequality. Thus, in equilibrium \( p^* > c \).

**Proof of Proposition 8.** For completeness of the proof we first show that marginal-cost-based 2PT is not an equilibrium. Next, we show that in any equilibrium, \( c_A < p^*_A < p^*_B < c_B \). Finally, we show existence. The first order conditions are

\[ [p_i] \sum_{k \in \{L, H\}} \lambda_k s(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j) \pi_i^j(p_i, \theta_k) \]

\[ - \sum_{k \in \{L, H\}} \lambda_k s_1(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j) q(p_i, \theta_k) (\pi_i(p_i, \theta_k) + F_i) = 0 \]

and

\[ [F_i] \sum_{k \in \{L, H\}} \lambda_k s(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j) \]

\[ - \sum_{k \in \{L, H\}} \lambda_k s_1(v(p_i, \theta_k) - F_i, v(p_j, \theta_k) - F_j) (\pi_i(p_i, \theta_k) + F_i) = 0. \]
Let
\[ s(u(p_i, F_i), u(p_j, F_j); \theta) = \begin{bmatrix} \lambda_L s(u(p_i, F_i), u(p_j, F_j), \theta_L) \\ \lambda_H s(u(p_i, F_i), u(p_j, F_j), \theta_H) \end{bmatrix}, \]
where \( u(p_i, F_i, \theta_k) \equiv v(c, \theta_k) - F_i \), and similarly for \( s_1(u(p_i, F_i), u(p_j, F_j); \theta) \). Let also
\[ q(p_i; \theta) = \begin{bmatrix} q(p_i, \theta_L) \\ q(p_i, \theta_H) \end{bmatrix}, \]
and similarly for \( q'(p_i; \theta) \) and \( q(p_i; \theta)^2 \). Finally, let
\[ \gamma(u(c_i, F_i), u(c_j, F_j); \theta_H) \equiv \gamma(u(c_i, F_i, \theta_H), u(c_j, F_j, \theta_H)). \]

(i). Here, we show that marginal-cost-based 2PT is not an equilibrium. Let us suppose is not true, i.e., suppose cost-based 2PT is an equilibrium. Then, from (56),
\[ F_i = \frac{s(u(c_i, F_i), u(c_j, F_j), \theta'; \theta)}{s_1(u(c_i, F_i), u(c_j, F_j), \theta'; \theta)} q(c; \theta) \] (58)
Note that from (57) and (58),
\[ \lambda (1 - \lambda) \frac{s_1(u(c_i, F_i, \theta_L), u(c_j, F_j, \theta_L)) s_1(u(c_i, F_i, \theta_H), u(c_j, F_j, \theta_H))}{A} (q(c_i, \theta_H) - q(c_i, \theta_L)) \]
\[ \times \{ \gamma(u(c_i, F_i), u(c_j, F_j), \theta_H) - \gamma(u(c_i, F_i), u(c_j, F_j), \theta_L) \} = 0, \]
where
\[ A \equiv s_1(u(c_i, F_i), u(c_j, F_j), \theta'; \theta) q(c; \theta), \]
which is a contradiction since
\[ \gamma(u(c_i, F_i), u(c_j, F_j), \theta_H) > \gamma(u(c_i, F_i), u(c_j, F_j), \theta_L) \]
for \( c_i < c_j \), given that \( \frac{s(c)}{s_1(c)} \) is increasing (following a similar strategy as in Quint, 2014, Theorem 1), and by the increasing differences property, \( v(c_i, \theta_H) - F_i - v(c_j, \theta_H) + F_j > v(c_i, \theta_L) - F_i - v(c_j, \theta_L) + F_j \).

(ii). Note that from (56) and (57), in equilibrium we have
\[ (p_i - c_i) \{ s(u_i, u_j; \theta') q'(p_i; \theta) - s_1(u_i, u_j; \theta') q(p_i; \theta)^2 \} \]
\[ + s_1(u_i, u_j; \theta') q(p_i; \theta) \times \left[ \frac{s_1(u_i, u_j; \theta') q(p_i; \theta)}{s_1(u_i, u_j; \theta') \cdot 1_{[2,1]}} \right] + \] (59)
\[ \frac{s(u_i, u_j; \theta)' q(p_i; \theta) - s_1(u_i, u_j; \theta)' q(p_i; \theta)}{s_1(u_i, u_j; \theta)' \cdot 1_{[2,1]}} \],

where \( u_i = u(p_i, F_i) \) and \( u_j = u(p_j, F_j) \). We first show that the expression inside the first big bracket is negative. Thus, for \( p_i > c_i \), the expression in the second big bracket must be positive. Now let’s analyze the first big bracket

\[
\begin{aligned}
&\left\{ \frac{s(u_i, u_j; \theta)' q(p_i; \theta) - s_1(u_i, u_j; \theta)' q(p_i; \theta)}{s_1(u_i, u_j; \theta)' \cdot 1_{[2,1]}} \right\},

\text{where} \quad -A + B \times C =

-D \cdot [q(p_i, \theta_H) - q(p_i, \theta_L)]^2 < 0

\text{and}

D \equiv \frac{\lambda (1 - \lambda) s_1(u(p_i, F_i, \theta_H), u(p_j, F_j, \theta_H)) s_1(u(p_i, F_i, \theta_L), u(p_j, F_j, \theta_L))}{s_1(u_i, u_j; \theta)' \cdot 1_{[2,1]}} > 0.

Now we show that the second big bracket is positive only if \( p_i < p_j \). That is,

\[
\begin{aligned}
&\left\{ \frac{s(u_i, u_j; \theta)' q(p_i; \theta) - s_1(u_i, u_j; \theta)' q(p_i; \theta)}{s_1(u_i, u_j; \theta)' \cdot 1_{[2,1]}} \right\},

&= D \cdot [q(p_i, \theta_H) - q(p_i, \theta_L)]

\times [\gamma(u(p_i, F_i), u(p_j, F_j); \theta_H) - \gamma(u(p_i, F_j), u(p_j, F_j); \theta_L)],

\text{since} \ \gamma(u_i, u_j) \ \text{is increasing with respect to} \ u_i, \ \text{and by the increasing differences property,} \ p_i < p_j. \ \text{Thus we conclude that there is no equilibrium in which} \ p_B > c_B. \ \text{Moreover, in any equilibrium,} \ c_A < p_A^* < p_B^* < c_B.

\textbf{Proof of Proposition 9.} The proof can be constructed following a similar strategy used in the proof of Proposition 5 in Armstrong and Vickers (2001). Suppose firm \( B \) offers a marginal-cost-based 2PT \((c_B, F_B^*)\). An upper bound on firm A’s profit is obtained by
assuming that $\theta$ is observed. The optimal method to generate profits for firm $A$ is by setting prices equal to the marginal cost and setting a fixed fee $F_A(\theta)$ such that

\[
\max_{F_A(\theta)} \left( \frac{1}{2} + \frac{v_A(\theta) - F_A(\theta) + v_B(\theta) + F_B^*}{2t} \right) F_A(\theta).
\]

Thus, from the first order conditions,

\[
\frac{1}{2} + \frac{v_A(\theta) - F_A^*(\theta) + v_B(\theta) + F_B^*}{2t} = \frac{F_A^*(\theta)}{2t}
\]

if $v_A(\theta) - v_B(\theta)$ is constant with respect to $\theta$, marginal-cost-based 2PTs ($c_A, F_A^*$) are optimal for firm $A$. Note that, provided that the market is fully covered, $F_A^*$ does not depend on $\theta$.

The proof can also be constructed by modifying the proof of Proposition 6 in Rochet and Stole (2002). Let us assume that $\theta$ is unidimensional and $q_i(\theta)$ satisfies the first order differential equation, $\dot{u}_i(\theta) = q_i(\theta)$. Note that the Hamiltonian for firm $i$ is characterized by (ignoring the monotonicity constraint)

\[
H(q_i, u_i, \theta, \lambda) = \left( \frac{1}{2} + \frac{u_i(\theta) - u_j(\theta)}{2t} \right) (S_i(q_i(\theta), \theta) - u_i(\theta)) f(\theta) + \lambda(\theta) q_i(\theta).
\]

The necessary conditions are: $\lambda(\theta) = \lambda(\bar{\theta}) = 0$,

\[
\left( \frac{1}{2} + \frac{u_i(\theta) - u_j(\theta)}{2t} \right) (S_i(q_i(\theta), \theta)) f(\theta) = -\lambda(\theta)
\]

and

\[
- \left( \frac{1}{2} + \frac{u_i(\theta) - u_j(\theta)}{2t} \right) f(\theta) + \frac{(S_i(q_i(\theta), \theta) - u_i(\theta)) f(\theta)}{2t} = -\lambda(\theta).
\]

Note that the marginal-cost-based 2PT $T_i(q_i) = q(c_i, \theta) c_i + F_i$ satisfies these constraints. In particular, note that this marginal-cost-based 2PT implies that $\lambda(\theta) = 0$ and that $v_i(\theta) - u_i(\theta) = F_i$, which requires $\dot{\lambda}(\theta) = 0$ for all $\theta$. Thus

\[
\frac{1}{2} + \frac{v_i(\theta) - F_i - v_j(\theta) - F_j}{2t} = \frac{F_i}{2t},
\]

which is feasible if $v_i(\theta) - v_j(\theta)$ is constant with respect to $\theta$.

To show the “only if” part, suppose marginal-cost-based 2PT is an equilibrium. From Proposition 2, $\text{Cov}(v_i(\theta) - v_j(\theta), q(c_i, \theta)) = 0$. We assumed that $q(\cdot)$ is strictly increasing with respect to $\theta$, which implies that $v_i(\theta) - v_j(\theta)$ is constant with respect to $\theta$. Note that if marginal-cost-based 2PT is a Nash equilibrium in a larger pricing space (e.g., general nonlinear pricing), it should also be a Nash equilibrium in a smaller space (e.g., 2PTs).
References


Colby, C. and Bell, K. (2016). The on-demand economy is growing, and not just for the young and wealthy. [Online; posted 14-April-2016].


One Click Retail (2018). Amazon market share.


