

# Quantifying the Urban Experience: Establishing Criteria for Performance Based Zoning

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## ABSTRACT

Exponential growth is pressuring cities around the world to reevaluate their management of new development. Challenges stemming from such growth, including crises in affordable housing and transit accessibility, see cities responding by constructing ever greater densities, a process limited by current zoning regulations. Traditional zoning frameworks are rigid and slow to adapt, making it difficult for development to keep pace with greater density requirements. By establishing certain baseline urban metrics tied to a set of performance standards, performance-based zoning allows cities to proactively accommodate growth and demand while mitigating potential negative externalities.

One of the primary challenges for a truly performance-based zoning methodology is defining and calibrating desired performance standards. Cities must consider criteria associated with, for example, comfort, mobility, and activity in such a way that defines achievable, goal-oriented benchmarks— a difficult task given the complexity of urban systems. In proposing a process for creating a performance-based zoning framework, this paper: 1) explores urban analysis methods, focusing on urban comfort, mobility, and activity, to create new urban performance datasets for Manhattan for use in establishing new zoning protocols, and 2) develops new visualization techniques that can more effectively engage stakeholders, make data understandable and accessible for decision-makers, and broaden the discussion to involve a wider range of non-specialist participants.

## Author Keywords

Urban analysis; spatial analysis; urban planning; zoning; performance-based zoning, outcome-based zoning.

## ACM Classification Keywords

I.6.4 SIMULATION AND MODELING Model Validation and Analysis.; I.6.6 SIMULATION AND MODELING Simulation Output Analysis.

## 1 INTRODUCTION

The crafting and timely updating of zoning regulations represent a perennial challenge for municipal governments, even

more so when said regulations attempt to guarantee that goals of sustainable growth are met and an equitable urban experience is ensured. The density of development required to keep pace with contemporary growth rates is unprecedented [1]. Traditional standards and practices for a city's continued function and evolution, largely based on historical patterns and outdated work-flows, are no longer adequate.

In response to exponential population growth and economic demand, cities are rezoning for ever-greater densities. Many zoning regulations, for example those of New York City, however, are based on prescriptive zoning, established more than 100 years ago as a Progressive-era reaction to industrialization. This legacy form of regulation is inadequate for proactively addressing rapid densification and is often complex, opaque, and notoriously slow and difficult to change.

Contemporary zoning must move away from traditional, prescriptive methodologies. One alternative is a performance-based system: establishing specific performance measures directly related to desired outcomes. When organized around computational methods, the process is data-driven, based on easily visualized analyses, and legible to non-specialist stakeholders. Ensuing increases in legibility and engagement will result in an innovative and responsive urban fabric, one capable of dynamic adjustment in response to changing spatial and temporal conditions.

### 1.1 Performance Based Zoning

Performance-based zoning requires two components: a description of the goal, and a set of standards that can be used to a) measure whether said goal is achieved and b) to set impact thresholds to which individual properties must adhere [3]. This stands in stark contrast with typical prescriptive zoning strategies where a stated goal exists without a paired measurable input correlating with desired outcomes. Instead, a set of prescriptive rules is applied, which very often do not directly relate to the desired outcome.

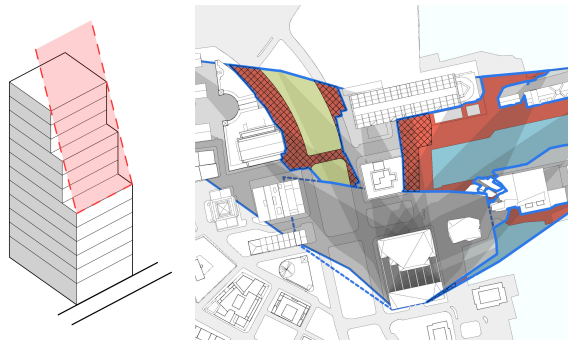
For example, one of New York City's stated goals in constructing its bulk regulations is "to provide for access of light and air to windows and for privacy... by controls over the spacing and height of buildings and other structures," (ZR 21-00 (f)) [7]. The rule established to regulate this outcome, the sky exposure plane (*Fig. 1*), does not measure light or air and

is based on the width of streets rather than light at windows. If the stated goal is to ensure access to light and air, why is the desirable amount of light not mandated, or even specified?

The core of performance-based zoning is metric-based criteria that directly relate to the stated goals, leveraged in order to evaluate potential and ensure desired outcomes. Explicit goals such as access to light, air, transit, public services, safety, and economic opportunity can be established and quantified. Performance-based zoning can have certain advantages over traditional zonings [8]: encouraging innovation in form and land-use; allowing for greater densities while mitigating impact on light and air; enabling dynamic adjustments; and increasing transparency through clear goals and metrics. Rather than relying on static, compartmentalized, and deterministic zoning rules, dynamic metrics offer an adaptable means of civic management, one better suited to the volatile transformations of the 21st century urban condition.

There have been attempts at a performance-based zoning framework in various forms since the 1950s, mostly concerning specific land-use and daylighting controls within otherwise traditional frameworks. Many applications have been abandoned due to the resources required to determine and enforce compliance, which present a greater administrative burden than traditional zoning. Little research has been conducted toward establishing a framework for performance-based zoning, or even in compiling an overview of municipalities that have implemented such criteria [3]. In contrast with previous attempts at performance-based zoning, we propose a framework and methodology that are based in rigorous urban analysis and thorough visualization of existing conditions to link performance to experience and comfort.

One previous study of note is "Simulation-based daylighting analysis procedure for developing urban zoning rules," in which the authors utilize annual climate based-daylight simulation methods to interrogate block typologies in New York City, demonstrating the benefits of a performance-based zoning framework [13]. Our work expands on this approach, including a daylighting measure paired with new analysis methods to drive a holistic approach to defining outcome criteria.



**Figure 1.** NYC Sky Exposure Plane (left), Boston shadow regulation analysis (right)

## 1.2 Framework & Contributions

While performance-based zoning is clear in concept, its crafting and implementation are technologically demanding. We propose the following as a framework toward realizing performance-based zoning:

1. Define clear goals based on desired outcomes.
2. Perform exhaustive, fine-grained urban analysis of several metrics to establish appropriate measurements.
3. Define performance criteria.
4. Test and calibrate criteria at the parcel and street segment level.
5. Create accessible and validated tools for assessment of compliance and enforcement.

This paper focuses on the first two steps of the proposed framework. While it is relatively easy to set individual performance criteria, like current daylighting requirements in Boston (*Fig. 1*) and China within a traditional zoning framework, it is challenging to define comprehensive performance-based criteria across an entire zoning code. To address this challenge, we ran an exploratory analysis of all tax lots (42,686) and street segments (13,237) in Manhattan, focusing on three categories of urban analysis that deal with how to control density, bulk and use in a performance-based system:

- Comfort: Urban daylight access - How should density and bulk be controlled?
- Mobility: Transit accessibility - Where might more density can be appropriate?
- Activity: Diversity of use and active uses over time - How will density be calibrated?

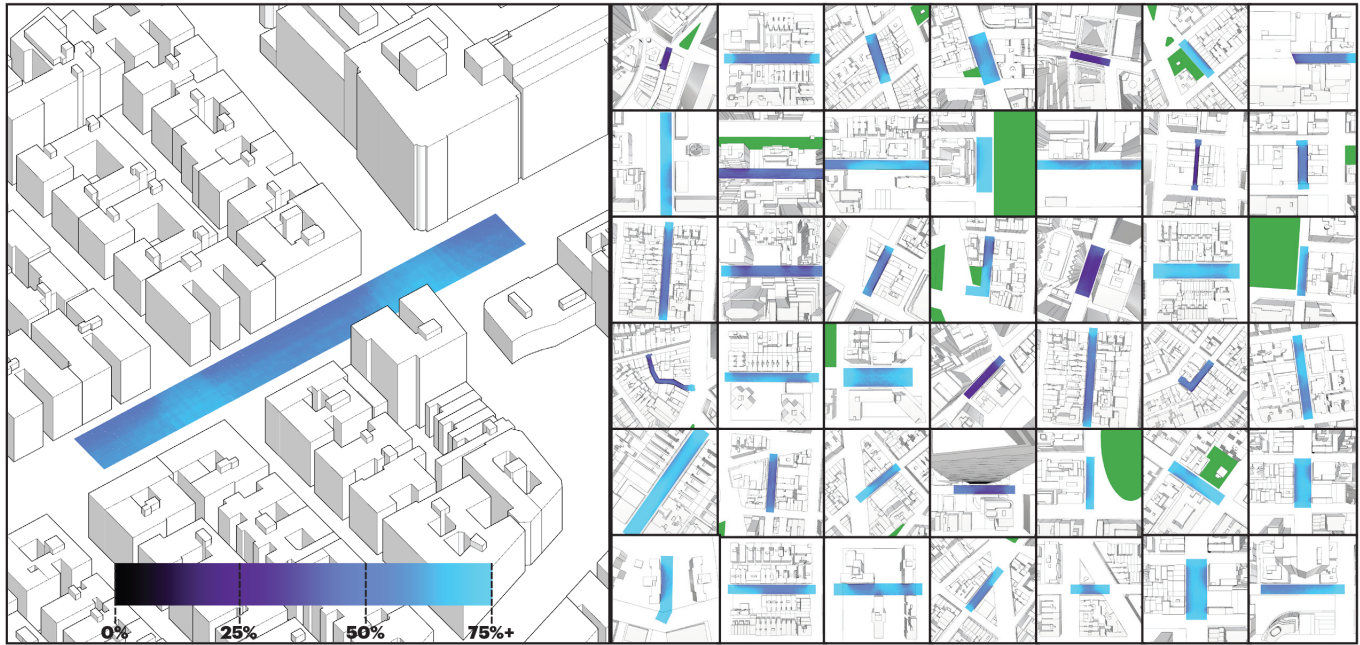
Through this process we create data-rich visualizations to better understand the qualities of urban space and create a new and unique dataset for Manhattan with visualization to facilitate stakeholder engagement in establishing performance-based criteria. We have made the datasets for mobility and diversity available for exploration through SimAUD and a custom built web app, link below, offering an example of how data can become interactive for stakeholder engagement.

<http://hay-stack.s3-website-us-east-1.amazonaws.com/#/?set=PerformanceBasedZoning>

## 2 URBAN DAYLIGHTING

The regulation of FAR<sup>1</sup>, height limits, and mandatory setbacks are the primary mechanisms in the traditional zoning toolkit, used to control the density and bulk of new development. However, we posit that relying on these legacy indicators is insufficient as they merely govern built form rather than outcome or the guarantee of daylight availability. To understand the expectations of street-level daylight embedded in current rules, we applied a daylight availability analysis to

<sup>1</sup>Floor Area Ratio or FAR, is the ratio of total building floor area to the area of its zoning lot. For example, on a 10,000 sqft lot with a maximum FAR of 3, the allowed built area is 30,000 sq ft.



**Figure 2.** Daylight Availability analysis of street segments in Manhattan. Broadway between 164th and 165th Streets (left) and 42 other segments (right). Lower Daylight outcomes exist in areas of increased Density (FAR), but correspond more directly to variations in form—offsets, setbacks, street wall heights, etc.

every street segment in Manhattan, teasing out insights into the relationship between daylight and density.

**Methodology**

We evaluate street-level daylight availability using a technique called sky-dome analysis [14], which measures the percentage of visible sky from a set of specified points. Methodology:

1. Choose an analysis surface and subdivide it (in this case every street segment in Manhattan (13,237)) [12].
2. Project a half-sphere skyward from each subdivision.
3. Subdivide the half-sphere by 1 degree increments and cast rays through subdivision centerpoints.
4. Test for occlusion with the built context [11] to return percentage of sky visible and amount of available daylight.

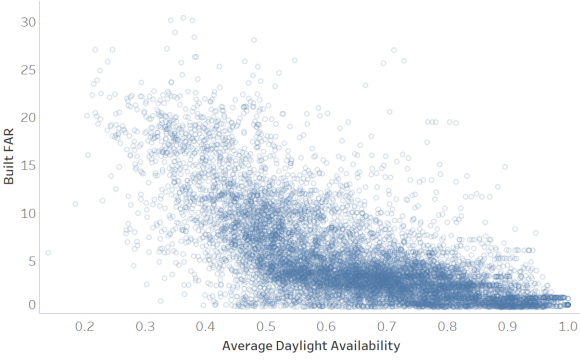
The resulting score is paired with density and land use data from the NYC Department of City Planning PLUTO [10] land-use database for adjacent buildings to explore relationships between daylight, density and use.

**Analysis**

Had the zoning regulations accomplished their intended goals, there should exist a causal relationship between Built FAR and the Average Daylight Score for a given street segment. Our findings demonstrate that one exists, but the correlation is much looser than anticipated. Our study returned a

coefficient of determination (R-squared value) of .45<sup>2</sup>, telling us that access to daylight decreases somewhat following increases in density, but a performance-based zoning methodology could guarantee more direct results (Fig. 3). Some very dense areas in Manhattan have high daylighting scores, while there are more sparse areas that score poorly. This is a testament to the need and opportunity for a more nuanced evaluation system for new development. Since limitations on FAR and height alone do not strongly correlate with daylight availability, we must turn to a more flexible, design-reactive criteria. The promise of performance-based zoning as applied to daylighting is this: we seek to demonstrate that under such

<sup>2</sup>The coefficient of determination, measured on a scale between 0 and 1, tells us how reliably we can predict the dependent variable (in this case Average Daylight Availability) based on knowledge of the independent variable (Contextual Built FAR).



**Figure 3.** Scatterplot demonstrating the moderate correlation between Density and Street-level Daylight Availability in Manhattan. A performance-based zoning code could create a more significantly causal relationship between input (Built Form) and outcome (Daylight Availability).

regulations buildings can, in a form-dependent manner, grow taller and larger without affecting the quality and quantity of daylight that permeates to the street. In order to set performance criteria, a comprehensive analysis of urban daylight availability is necessary. Urban daylighting, as a measure for managing density and bulk, can be paired with our Mobility metric to understand where density is appropriate, and with our Activity metrics to develop criteria for how that density is programmed and used.

### 3 MOBILITY

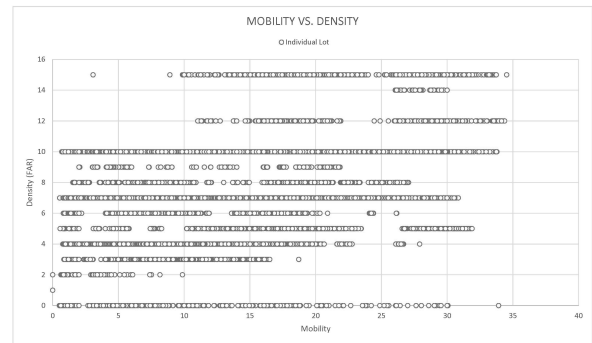
Locating urban density around quality public transportation, a strategy known as Transit Oriented Development (TOD), has many benefits: reduced commute times, minimized environmental impact, accessibility for the elderly, economic development, and increased overall quality of life [2]. The creation of a performance-based zoning metric around TOD will help to distribute benefits equally to citizens. Given the complexity of New York City, we must first achieve a thorough understanding of the city's current transit network.

This paper proposes a hybrid of the typical urban planning point/radius representation of travel with the multimodal evaluative tools of new platforms like Google Maps. By visualizing all sites accessible from a given point and incorporating all means of available transportation, we can produce a transit-oriented metric that is robust and intuitive. The metric provides a basis for the informed allocation of density and a more thorough understanding of mobility across the city.

#### Methodology

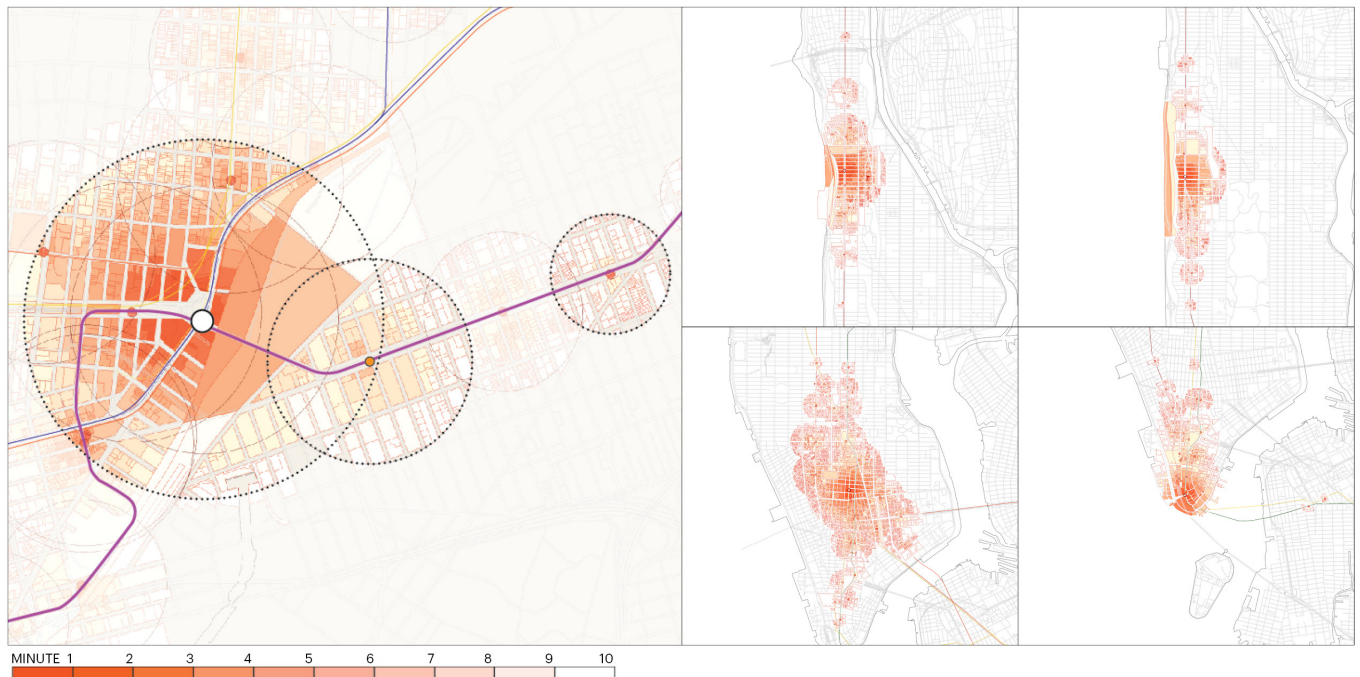
For every lot in Manhattan, a unique score is generated based on the distance to each individual subway station within a half

mile radius, a distance widely utilized in TOD standards, with the total score summing all available train lines<sup>3</sup> Generating mobility scores across Manhattan creates the opportunity to better align density and transit (currently, these two factors are not strongly correlated (Fig. 5)), and this methodology can be adapted to provide a more nuanced understanding of any one urban point's connection to the greater metropolitan whole (Fig. 4), outlining how far an individual can travel in a given amount of time, various transit types and stations available, the total count of buildings linked, and their respective uses and floor areas.



**Figure 5.** Scatter Plot demonstrating the weak correlation between Mobility and Density in Manhattan.

<sup>3</sup>Mobility Score Calculation: for all individual subway lines within 1/2 mile radius (2,640') of lot center point (P), Dn = distance of nth subway's closest stop to P, and Score (S) = sum(2640/Dn). The minimum value for Dn is limited to 1,320' in order to avoid extremely close subway stops from exerting an unreasonable influence on the score.



**Figure 4.** Diagrammatic View of Mobility Analysis (left). Mobility Analysis at various points along Broadway in Manhattan, contrasting the remarkable transit connectivity of downtown with the far less robust condition at the Northern portion of the island (right).

## Analysis

From an analysis standpoint, this fine-grained level of inquiry can provide City Planning departments with dynamic new tools to assess infrastructure, economic development, affordable housing, and preparations for the future. In particular, this study illustrates the dramatic difference in transit access between northern and southern Manhattan (Fig. 4). Within ten minutes, an individual at Times Square has access to thirteen subway lines and can access 5,550 buildings with over 45 million square feet of built area. At the intersection of 125th Street and Manhattan Avenue, a major commercial thoroughfare in Harlem, a person is limited to four subway lines and 3,500 buildings, totalling 25 million square feet. If Manhattan is to absorb significant additional density, the majority will likely need to be north of 59th Street and new transit options and cross-town subway linkages will be critical.

## 4 ACTIVITY

With Daylighting determining urban density and access to Mobility positioning it, the next step is defining corresponding Activity: what diversity of uses should be encouraged, during which hours of the day, and how would this represent better and more efficient use of space?

### 4.1 Diversity

The creation of vibrant urban places requires an overturning of traditional separations between residential, commercial, and manufacturing activities. By mixing functions like housing, shopping, offices, services, schools, cultural institutions, restaurants and entertainment, residents and visitors are drawn in for multiple reasons at various times of day [5].

Such mixed-use development supports dynamic neighborhoods and better aligns with the expectations of 21st century urban dwellers [2].

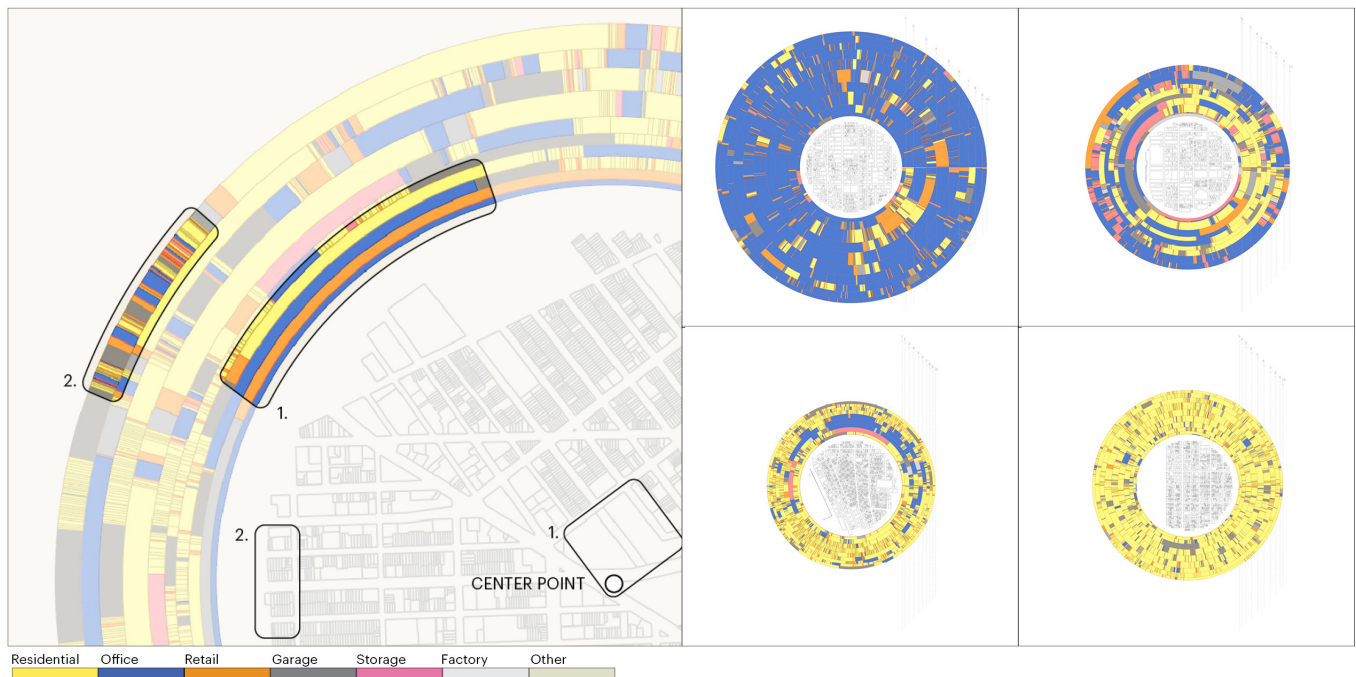
## Methodology

In order to foster the desired diversity of uses, it is first necessary to develop a methodology that quantifies existing conditions. Continuing Jane Jacobs' organicist conception of the city, this paper utilizes the Shannon-Wiener Diversity Index [6] (originally developed under the framework of information theory, but often applied to measure species diversity in native habitats) to calculate the *evenness* of how various Land Use Categories [9] are distributed within a ten minute walk of a given lot<sup>4</sup>. The resulting analysis provides both a high-level overview of the varying degrees of use diversity throughout Manhattan and a targeted assessment of any given lot in terms of urban place: density, number and size of buildings, programmatic diversity, and shifts in neighborhood character (Fig. 6).

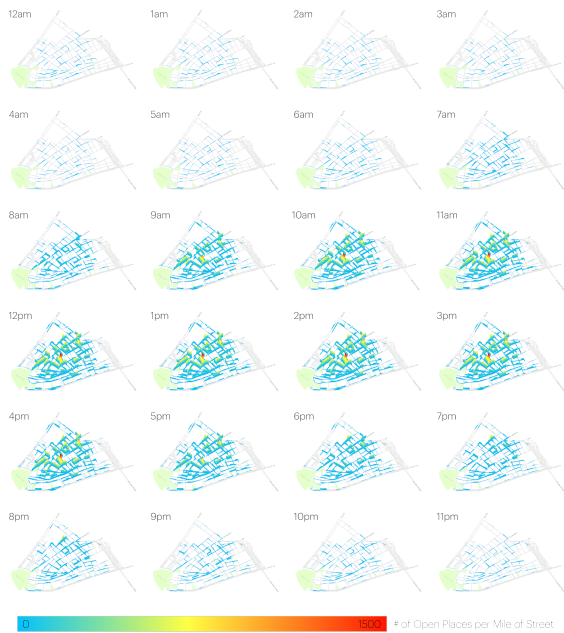
## Analysis

New York City features several prominent Central Business Districts of startling density, such as Midtown Manhattan (Fig. 6). While this currently fosters a disparity in correlation between diversity of use and density, the programmatic analysis undertaken in this paper provides planners and urbanists with a framework for injecting the desired degree of

<sup>4</sup>Given the following variables: H (Shannon's diversity index), S (sum of all Land Use Areas), Pi (the proportion of the Ith Land Use's Area to S), Eh (Evenness).  $H = -\sum(P_i \log(P_i))$   $E_h = H/H_{max} = H/\log(S)$



**Figure 6.** Diagrammatic View of Diversity Analysis (left). Diversity Analysis at various points in Manhattan, demonstrating a lack of diversity in the highest density neighborhoods. Clockwise from upper left: Midtown (high density, primarily office), Hell's Kitchen (medium density, mixed-use), Upper East Side (high density, primarily residential), West Village (low density, mixed use)



**Figure 7.** Number of open businesses per street segment for each hour of the day in the Financial District of New York City.

variety into urban places. When applied as part of a larger performance-based zoning metric, the quantification of use diversity can incentivize new development opportunities, encourage active, self-sustaining, 24-hour neighborhoods, or even track the growth or decline of individual neighborhoods in real time and dynamically adjust performance measures in response.

#### 4.2 Time

Performance-based zoning ensures desirable outcomes by limiting intensity of use. Intensity can be measured in decibel levels, where some degree of quiet is desired hours of daylight, where access to light is desired; or quantity of pollutants emitted, where clean air is desired. However, in most performance-based zoning, time is not included as part of the measurement. Because performance-based zoning seeks to regulate use in a manner more nuanced than the blunt tools of traditional zoning, time could be used as a means to set limits and measure outcomes.

Time has previously been excluded because there is very little data available. The land use maps maintained by municipalities or counties only contain data about general use, not the times when those uses are taking place. However, with new datasets being generated in both the public and private sector, planners can better understand how different districts within a town or city are used throughout the day.

#### Methodology

We used data from the Google Places API to analyze when different uses are most active in Manhattan [4], including businesses, attractions, parks, and public and private institutions. Many of these places identify their hours of operation, but those that are open to the public are more likely to display opening hours. For instance: a particular Starbucks loca-

tion will likely have opening hours, while a Starbucks' corporate headquarters will likely not. Unfortunately, the Google Places dataset is not comprehensive and cannot be used to quantify the absolute number of open businesses for any hour of the day. Even taking that flaw into account, the Places data can be used to understand the relative number of open businesses between neighborhoods within a city or across cities, and in doing so understand relative quantities of activity at different times of day. Additionally, Google assigns a primary type to each of the places which, despite not corresponding to traditional use types, can actually afford one a more granular image of the uses on a site.

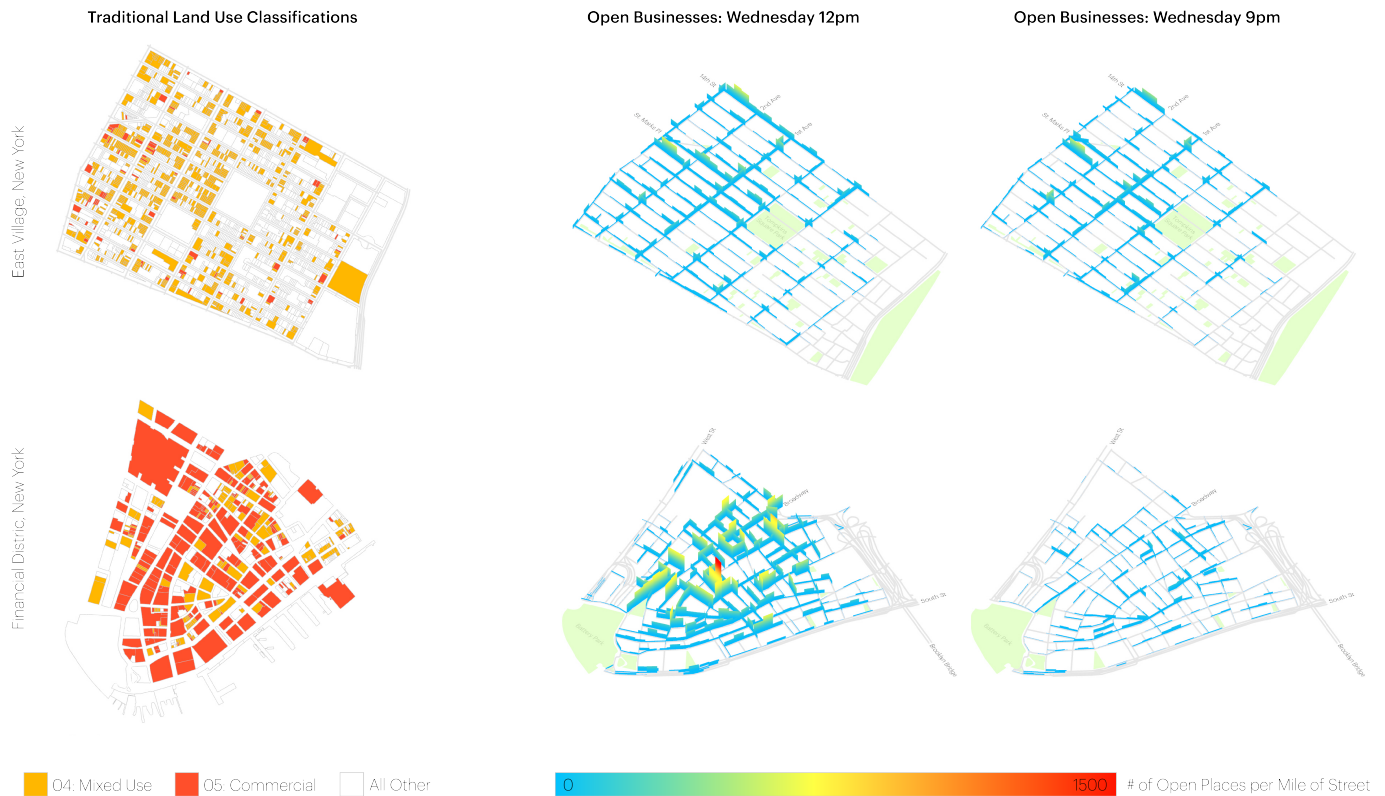
We associated the Google Places data with Manhattan street lines to concretize the impact that uses have on the public realm throughout the day. Each street is split into segments (approximately 50 meters), and the number of businesses open on that street segment are summed for every hour of the day (and normalized over the length). By aggregating the data to street segments instead of parcels, it allows us to understand how the intensity of use impacts public space. To visualize this data, we developed a representation technique that shows the quantity of places on a 3 dimensional map. Each street segment is extruded to reflect the density of open places on that specific segment for that specific hour of the day (Fig. 7). This allows the intensity of use to be visually compared between different neighborhoods for the same hour of the day. As an example, we have compared the Financial District with the East Village at both 12pm and 9pm (Fig. 8).

#### Analysis

A single land use category can include a broad range of activities that have different impacts on adjacent uses at different times of day. Commercial uses, for instance, could range from office districts to nightlife districts, with each seeing the most activity at different times of day. In Figure 9, we compare the traditional land use categories that represent commercial uses (commercial and mixed-use), with the time based analysis of commercial businesses using the Google Places data.

When looking at the traditional zoning map, the Financial District appears to have more commercial activity than the East Village based on the large area of commercial only parcels. The Google Places data confirms this in showing that the Financial District does have a higher quantity of open businesses at 12pm. However, when looking at the number of open businesses at 9pm, the East Village has more open businesses, demonstrating an increase in commercial activity later in the day and highlighting the breadth of the area's nightlife offerings.

These findings show that there is great potential in using time to inform performance-based zoning and as a means of regulating uses. Uses can be made adjacent based on the times at which they are active. For instance, if Midtown is largely vacant on weekends, there could be recreational or cultural uses that are more active on the weekend. They can take advantage of the reduced traffic to utilize the street space and be more accessible. Another potential example is regulating



**Figure 8.** Traditional land use data on the left is shown in contrast to time-based Google Places data on the right. The traditional land use only shows strict categories, while the time-based analysis shows the intensity of use and how that intensity changes throughout the course of the day.

potential nuisances based on time of day: rather than simply limiting noise to a specific maximum decibel level, regulators could set different maximum decibel levels for different times of day.

## 5 DISCUSSION & NEXT STEPS

### Relationships and Trends between our Metrics

A more nuanced approach to analyzing performance criteria means that performance-based zoning can take advantage of relationships between different metrics. For instance, both urban comfort and activity are time dependent. Therefore, it is suboptimal to prioritize sunlight in a park at a time that the park is sparsely used. Instead, times of direct sunlight can be coordinated with times of peak activity. Being more specific about when a park gets access to sunlight may make it possible to add significant amounts of density while also ensuring that sufficient sunlight reaches the park at the times of highest use.

Increased coordination can also improve the relationship between mobility and activity. Because planners are concerned about overburdening transit infrastructure, they often limit density through establishing bulk regulations and a maximum FAR. However, creating additional density for an area may be appropriate when there are different uses happening at different times of day. If these different users utilize transit at different times, transit no longer becomes the limiting factor in establishing a maximum density.

From our analysis of what we call "activity," we provide a more nuanced understanding of use than what is shown in traditional land use maps. While traditional land use classifications are intended to separate uses into single use districts, our criteria measure how uses come together to create a diverse ecosystem. By using these criteria in performance-based zoning, uses can be brought together in a manner that is mutually beneficial. Co-dependent businesses can be collocated, or uses can be diversified in a way that activates the street at all times of the day.

### Beyond Manhattan

While Manhattan's density and mix of uses make a good test case, there is much more to learn from expanding this research into the lower density boroughs of New York. For instance, transit accessibility becomes more diffuse and, as a result, more critical in zoning considerations. Some land uses, like manufacturing, have little presence in Manhattan, but are more commonplace in the outer boroughs. The diversity of uses varies widely in New York's other boroughs, from vibrant, mixed-use neighborhoods like Downtown Brooklyn to the almost exclusively residential neighborhoods in Staten Island that more closely resemble suburban towns. More precise tools for analyzing the built environment would allow this diversity to be more clearly understood.

### Including Other Transit Modes in Mobility Metric (biking, AVs, bus, etc.)

Future development of the mobility metrics presented in this paper will expand to include a wider array of transit options: bus, commuter rail, bike shares, and other systems currently unavailable in New York City (street cars, for one), and even future modes of transportation like autonomous vehicles. That said, our analysis reveals that nearly every point in Manhattan qualifies as TOD by traditional standards (access to at least a single subway stop within a half mile). This result makes clear the necessity of more nuanced TOD benchmarks, and the computational methods outlined in this paper can be adapted to incorporate new inputs such as differences in wait times, reliability, number of destinations, and overcrowding.

### Need for a True Urban Comfort Analysis

The metric for measuring daylight availability used in this paper is one part of a larger system for quantifying true urban comfort. An analysis harnessed to inform future zoning regulations should incorporate other metrics like wind, humidity, solar radiation and temperature levels in order to more comprehensively assess the experience of the individual at the street level and understand how that might be improved.

### From Analysis to Implementation

Performance-based zoning provides a proactive framework for cities to confront the challenges of the 21st century. Based on a flexible, analytic methodology, regulators can dynamically adjust metrics at the building, parcel and street segment level, responding to changes in the urban environment in real time rather than reactively addressing crises. The process of implementing performance-based zoning will require multiple steps (as listed in Section 1.2), and in this paper we focused on establishing goals and performing analysis. In developing a set of criteria and tools in order to accomplish our analysis, we illustrate a methodology for administering performance-based zoning enabled by digital workflows. While the administrative burden has historically been a significant hurdle to performance-based zoning, the prevalence of analytic software allows for automating the enforcement process for new developments. Instead of requiring officials to pour over print-outs of zoning drawings on a building by building basis, 3D models could be uploaded directly to performance analysis software that evaluates how the proposed buildings perform in a simulated urban environment. Such automation would overcome some difficulties that kept performance-based zoning from past implementation—namely, a lack of deep technological literacy or capacity on the part of the regulator.

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