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Results From a Controlled Study of the iPad Fractions Game Motion Math

Michelle M. Riconscente[1,2]

Abstract

Although fractions knowledge is essential for future success in mathematics, data show that most U.S. students fail to become proficient in fractions. With the advent of mobile technologies such as iPad tablets, new kinds of interactions with subject matter have become possible that have potential for improving learning. The present study used an experimental repeated measures crossover design to investigate whether the iPad fractions game Motion Math would improve fourth graders' fractions knowledge and attitudes. In results from 122 participants, students' fractions test scores improved an average of over 15% after playing Motion Math for 20 min daily over a 5-day period, representing a significant increase compared to a control group. In addition, children's self-efficacy for fractions, as well as their liking of fractions, each improved an average of 10%, representing a statistically significant increase compared to a control group. Implications for the design and study of interactive games are discussed.

Keywords

educational technology, fractions, app, experimental design

[1] Rossier School of Education, University of Southern California
[2] New York Hall of Science, Queens, NY, USA

CURRENT CONTACT INFO
Michelle M. Riconscente, PhD
CEO, Designs for Learning, Inc.
michelle@designs-for-learning.com

Corresponding Author:
Michelle M. Riconscente, Sara Lee Schupf Center for Play, Science and Technology Learning, New York Hall of Science, 47-01 111th Street Queens, NY 11368, USA.
Email: mriconscente@nysci.org

The topic of fractions has long been a challenging one for children (Hiebert, 1985; Newton, 2008; Wu, 2010). Research by the National Center for Educational Statistics shows that only 13% of U.S. fifth graders are proficient in fractions (Princiotta, Flanagan, & Germino Hausken, 2006). Even though the topic of fractions appears early in children's mathematics trajectory, success or failure at this juncture carries long-term consequences. The extent to which students master fractions is a strong predictor of later mathematical success, particularly in algebra, which serves a "gatekeeper" role for access to higher education (U.S. Department of Education, 1997).

It is in this context that instructional designers are increasingly turning to new technologies and creating learning experiences that leverage the unique interactive features of devices such as the iPad. Although hundreds of iPad apps on the market claim to improve learning, no published studies were found of controlled experiments that tested the effectiveness of an educational iPad app for increasing learning outcomes. This report describes a controlled study of the iPad app Motion Math, a game aimed at improving students' ability to relate various fraction representations to the number line. The developers of the game believe that by gaining skill at this central aspect of number sense, students will develop a better understanding of fractions that will transfer to improved performance on the kinds of questions posed on state and national standardized tests. Although little research has examined the relation between learning and iPad interactions, from a learning science perspective, there are several reasons to expect that the experience of playing Motion Math could foster learning. In the game, which relies on the "tilt" feature of the iPad, the player is continuously presented with fractions problems to solve, instant feedback, rewards, and increasing levels of challenge. The question is whether this combination of experiences does more than help players get better at the game itself. The present study sought to determine whether the game increased children's fractions knowledge, improved their attitudes toward fractions, and was perceived as fun and helpful by participants.

Review of the Literature

In the United States, most students are initially introduced to fractions in the second or third grade. By fifth grade, they are learning how to perform arithmetic operations such as addition and subtraction with fractions. However, the instructional approaches engaged in most classrooms are not proving effective for the vast majority of students. Research shows that classroom instruction tends to focus on fractions concretely as "parts of a whole" (Misquitta, 2011). This instruction tends to generate an inaccurate conceptualization which subsequently becomes an obstacle to students' abstract reasoning about fractions. For instance, it is difficult for students to reconcile the part-whole model with the fact that fractions are continuous and infinitely divisible (Behr, Lesh, Post, & Silver, 1983; Hiebert & Tonnessen, 1978). Moreover, students' prior knowledge of whole numbers has been theorized to lead to misinterpretations of the representational form of numerator over denominator, which students often understand simply as two whole numbers (Gallistel & Gelman, 1992).

> The model for introduction currently in most of our textbooks is that of regions (square, circle, and line). This enables us to talk of shares, but the result is a tangible amount (slice of pizza, cube of chocolate) which does not neatly fit within the operations of addition, subtraction, multiplication, and division. How can you multiply two pieces of pizza? (Hart, 2000, pp. 53–54)

Understanding fractions abstractly means recognizing that fractions are numbers, and as such they can be placed on a number line. Yet, most children fail to develop this knowledge (National Mathematics Advisory Panel, 2008; Siegler, Thompson, & Schneider, 2011a, 2011b), as can be seen in their reliance on procedural knowledge rather than on conceptual understanding when solving problems with fractions (Wong & Evans, 2007).

Case and colleagues have suggested that the mental number line is essential to early numerical understanding (Case & Griffin, 1990; Case & Okamoto, 1996). Siegler and Ramani (2009) theorized that the mental number line facilitates arithmetic problem solving by providing a "retrieval structure that improves encoding, storage, and retrieval of numerical information by organizing the information around the numbers' magnitudes" (p. 555). Likewise, renowned mathematician Wu has published widely on the value of teaching children the relation between fractions and the number line:

> [T]he use of the number line has the immediate advantage of conferring coherence on the study of numbers in school mathematics: decimals are rightfully restored as fractions of a special kind, and positive and negative fractions all become points on the number line. In particular, whole numbers are now points on the number line too ***and the arithmetic of whole numbers, in this new setting, is now seen to be entirely analogous to the arithmetic of fractions. (. . .)*** It must be said that this coherence has been largely absent from school mathematics for a long time. (Wu, 2008, p. 4; emphasis original)

In a recent report to the U.S. Department of Education's Institute of Education Sciences, Siegler and colleagues (2010) reviewed the available research on fractions teaching and learning. Among the five recommendations they forwarded for effective fractions instruction is the suggestion to "[h]elp students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward" (p. 19). The same report emphasizes that there is a dearth of rigorous experimental studies that examine the effectiveness of specific approaches to fractions instruction. With postsecondary success so closely bound to early mathematics achievement, identifying research-based ways to support students' fractions learning is of highest priority (U.S. Department of Education, 1997; Wu, 2008).

Importantly, children's struggles with understanding fractions often go hand in hand with low motivation and negative attitudes toward mathematics (Ashcraft, 2002; Stipek et al., 1998). Numerous studies of students' motivation have shown that low self-efficacy—that is, lack of confidence in one's ability to successfully complete academic tasks—is associated with lower achievement (Bandura, 1997; Pajares, 1996). Research also documents the importance of expectations for success and valuing of academic tasks for increased persistence (Wigfield & Eccles, 2000). Moreover, children are more likely to develop long-term interest in material that they understand and see as relevant (Hidi & Renninger, 2006; Riconscente, 2010). It is thus reasonable to conclude that comprehensively addressing the fractions crisis will require approaches that improve students' fractions knowledge while simultaneously transforming negative attitudes into positive ones.

Successfully integrating learning with motivation is a long-standing hallmark of computer game design. In computer games, players are constantly challenged by increasingly difficult obstacles they must overcome to attain a goal. The kinds of obstacles they face are new, even counterintuitive, and players must discover the rules of play in order to win. Good games will "ramp" up the challenge, by adding new rules and constraints that must be figured out and surpassed. Failure is accepted as a natural part of game play. In contrast to most academic settings, where failure is rarely met with increased enthusiasm, games are designed such that failure and challenge strengthen motivation.

The last decade has seen an exponential surge in the creation of educational games to address learning challenges in a variety of domains (Habgood & Ainsworth, 2011). The theory and study of educational games is increasingly taking center stage at academic conferences, and several journals dedicated to games and learning have sprung up. Features of new technologies, especially mobile devices such as the iPad and iPhone, offer instructional designers more options for creating effective learning experiences that move beyond static presentation, limited interaction, and the walls and schedules of formal schooling. For example, the multipoint touch-sensitive iPad display can be "pinched" to zoom in or out on an image. The iPad also detects when it is being moved or tilted, and through Global Positioning System (GPS) data knows its position on the Earth.

Some researchers have leveraged the capability of these new technologies to create embodied learning experiences. This approach is grounded in the insight that "cognitive processes are deeply rooted in the body's interactions with the world" (Wilson, 2002). In other words, rather than residing entirely in one central location—the brain—many cognitive scientists are now exploring the ways that knowledge is rooted in our physical interactions with the world (Clark, 1999; Wilson, 2002). Recent studies have examined embodied cognition in relation to mathematics. For example, Alibali and Nathan (2011) found that teachers' and students' gestures were valid evidence of their mathematical understanding.

Scholars have begun to apply the theory of embodied cognition to create embodied learning experiences. For example, Ramani and Siegler (2008) showed that preschoolers gained mathematical knowledge by playing board games. They theorized that opportunities to physically interact with a number line by moving tokens along linear board games helped children to develop a mental number line by offering tangible clues about the order and magnitude of numbers. Large-scale virtual embodied installations have also been shown to significantly increase students' understanding of such topics as chemical titration (Tolentino et al., 2009) and geoscience (Johnson-Glenberg, Birchfield, Savvides, & Megowan-Romanowicz, 2010).

As promising as these new technologies are, educational games remain challenged by a tension between entertainment and learning. Habgood and Ainsworth (2011) suggest that successful educational games are those that establish an intrinsic link between a core game mechanic and the target learning content. In other words, the game play itself engages the player with the content (Malone & Lepper, 1987). This design perspective stands in contrast to many edutainment titles, in which the educational content is artificially injected into game play—as, for instance, when the player must solve a math problem to pass through doors in a castle. Unfortunately, this extrinsic approach to game design is still prevalent. A recent search yielded 115 apps and games developed for the iPad that purport to teach fractions. Upon closer examination, however, many of these "games" are better characterized as math quizzes; others contain explicit instruction, however nearly always the instruction and the game (quiz) are isolated from one another. Importantly, although one unpublished study was found on an iPod literacy game for young children, to date there has been no experimental research to ascertain whether these apps promote children's learning of mathematics.

To extend the literature reviewed here, the present study used a controlled experimental design to test whether the mobile learning game Motion Math improves children's fractions knowledge and attitudes. The most common use of the game is by individuals on their own without the assistance of parents or teachers. Therefore, rather than compare Motion Math to other apps or to classroom instruction, this study focused on whether or not Motion Math achieves its goals as a learning app. In other words, when young people play Motion Math, do they get better at fractions and come away with more positive attitudes toward fractions?

About Motion Math

Motion Math was designed to help children strengthen their understanding of the relationship between fractions, proportions, and percentages to the number line. The game plays on iPad, iPhone, and iPod devices, leveraging the "accelerometer" feature. In Motion Math, the player physically tilts the device to direct a falling star to the correct place on the number line at the bottom of the screen (see Figure 1). The stars fall one at a time, and each displays either a fraction, percentage, decimal, or pie shape. The correct response generates a rewarding audio and visual response; wrong answers trigger increasingly strong instructional hints, starting with an arrow pointing either left or right, moving to hatch marks that break the number line into the appropriate number of parts, and finally labels on the hatch marks (see Figure 2).

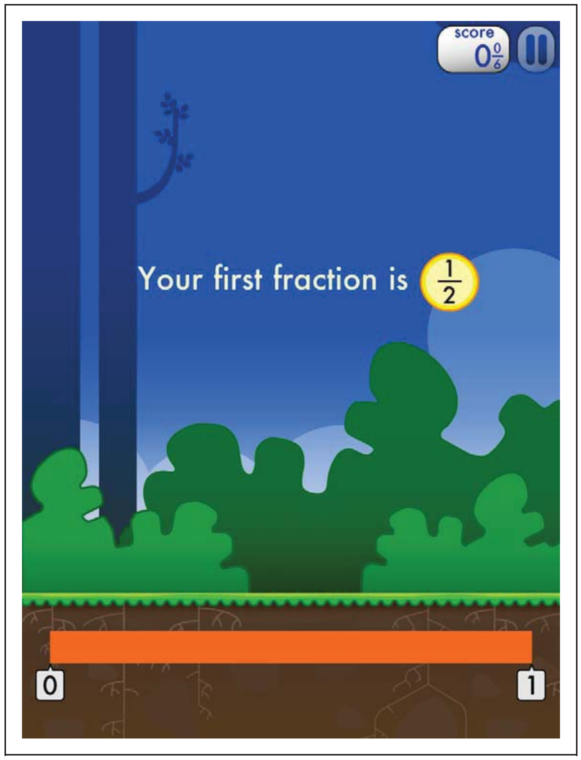

Figure 1. Star falling from top of screen. Player must tilt the device so the star falls at the correct location on the number line.

The game has three levels of difficulty, from beginner to hard, and each game difficulty mode consists of 24 increasingly challenging levels. Levels are generated according to an algorithm that increases the range of the denominator and changes the range of the number line. The visual and symbolic representations of the fractions also vary, and during more challenging game play representations are mixed together within each level. After completing every five levels or so, the player is presented with a "Less—Equals—More" challenge (see Figure 3). A comparison value appears on an unmarked number line above a bin marked "Equals." To the left and right of the Equals bin are a "Less" bin and "More" bin, respectively. For each fraction that falls from the sky, the player must tilt the device to drop it in its proper location. Audio, haptic, and visual feedback let the player know whether or not she has hit the mark.

From an instructional design perspective, Motion Math has several promising features. First, unlike most math "games" in which the content is interjected artificially, or learning is isolated from assessment and feedback, Motion Math makes fractions the focus of the game, and integrates feedback directly within game play for a

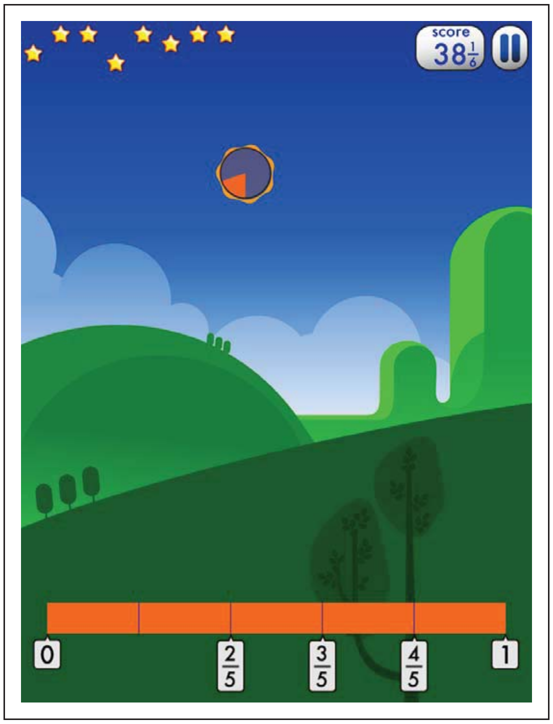

Figure 2. Wrong answers trigger increasingly explicit hints. This screen shows the last hint the player receives before losing the level.

continuous interactive experience. The visual interface, tone, and game mechanics were also designed to promote players' positive attitudes toward fractions. In addition, the game is an embodied learning experience in that physical action is tied directly to the target learning objectives.

Motion Math assumes a baseline of prior fractions knowledge. Specifically, the player must already have a basic understanding of the fraction representational form. In other words, Motion Math was not created with the goal of introducing the player to fractions. Rather, it focuses on developing and strengthening a player's understanding of how fractions are related to the number line. The expectation is that by fostering children's understanding of the relation between fractions and the number line, the game will lead to a better understanding of fractions, which in turn will transfer to fractions knowledge more generally.

Study Design and Hypotheses

On the basis of the literature review, the purpose of this study was to ascertain whether Motion Math increases learners' fractions knowledge and attitudes. There

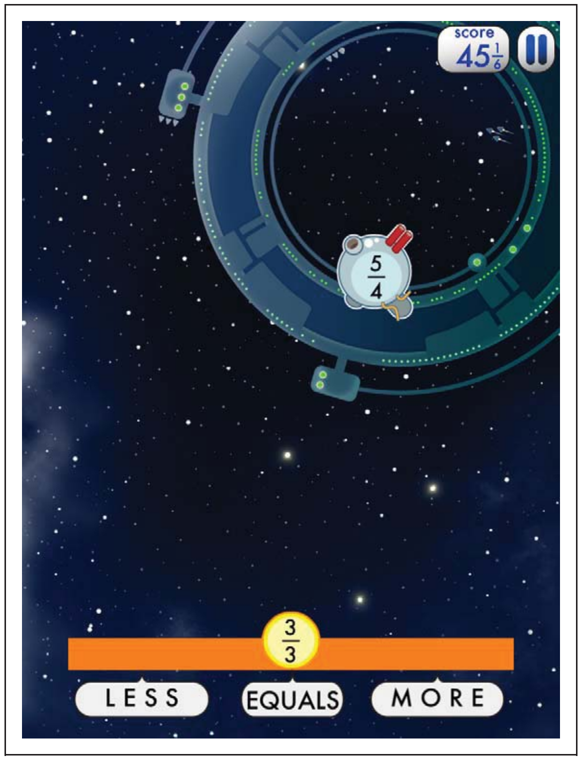

Figure 3. In Less—Equals—More levels, players must drop the falling fractions object into one of the three zones in relation to a comparison value.

were several reasons to expect that playing Motion Math would lead to these increases. Specifically, Motion Math utilizes the number line, which research and theory indicate as key to conceptual understanding of fractions. Second, the game mechanics are well designed to support instruction. For instance, players encounter a very high number of problems in a relatively short time span, thus offering many opportunities for practice. Third, the time factor could be expected to create a sense of urgency around the problems, providing the player with a compelling need to develop relevant schema. Finally, research shows that embodied learning experiences promote learning and engagement.

The purpose of the study was intentionally chosen over other possible purposes for several reasons. First, Motion Math was not designed to replace classroom instruction by a teacher, and therefore it would not have been appropriate to compare the game to classroom instruction. Second, comparing Motion Math to another fractions app would only enable conclusions relative to that one specific app, not to fractions apps more generally. At the time of this study, no apps were available that were

Table 1. Study Design: Repeated Measures Crossover Design.

Group	Day 1	Days 1–5	Day 6	Days 6–10	Day 10
1	Pretest	Motion Math	Midtest	Control	Posttest
2		Control		Motion Math	

sufficiently aligned with the same goals as Motion Math to represent an appropriate and valuable comparison point. Third, given the dearth of extant research on iPad games, and fractions games in general, a fundamental question was whether the app indeed attains its goal of helping players understand fractions better.

The present study implemented a repeated measures crossover experimental design. Students in the treatment condition played Motion Math daily for 20 min over five consecutive school days; the control group had regular mathematics instruction that did not target fractions. As shown in Table 1, this design uses two groups of children. For the first half of the study, one group serves as the treatment group and the other as the control group. Halfway through the study, the two groups switch. Four hypotheses were tested to determine whether playing Motion Math leads to increases in children's fractions knowledge and attitudes. First, it was hypothesized that children's scores would increase significantly after playing Motion Math, but remain constant over the control period. Second, children's posttest scores within each group were hypothesized to be significantly higher than their pretest scores. Third, both groups were hypothesized to have equivalent pretest and posttest scores; however, midtest scores for Group 1 were expected to be significantly higher than those of Group 2. Since change in scores between groups would provide a more robust test of this research question that would a simple comparison of midtest scores, it was also hypothesized that Group 1 and Group 2 midtest scores would differ after controlling for pretest scores. The fourth hypothesis related to the first research question was that the score trajectories from pretest to midtest to posttest would differ significantly between groups.

Finally, a descriptive analysis was conducted to explore the expectation that participants would respond positively to the Motion Math game.

Method

It bears mentioning that the reason the study was conducted in a school setting was not to compare Motion Math to classroom instruction. Rather, conducting the study in a classroom made it possible to control the frequency and amount of time students played the game, to ensure comparable conditions of game play, and to ensure that adults did not intervene while children played the game. Because the purpose of the study was to test whether Motion Math on its own leads to gains in learning and fractions attitudes, it was important to minimize the effect that classroom instruction might have on students' fractions knowledge. Therefore, participating teachers

Table 2. Fractions Attitudes Scale Reliabilities.

	Cronbach's α		
Scale	Pretest	Midtest	Posttest
Fractions self-efficacy	.77	.84	.81
Fractions liking	.64	.80	.71
Fractions knowledge self-ratings	.58	.85	.81

agreed to refrain from teaching fractions to participating students for the duration of the study.

Participants

Participants were 122 fifth graders enrolled in two schools in southern California. School A serves two equally sized classes of fifth graders and School B has 5 fifth-grade classes. At each school, students were randomly assigned to classes. School A is an urban public school serving primarily Latino students from low-income homes; School B is a public elementary school serving mostly Caucasian and Latino students. All students at School A have the same math teacher whereas the two classes of students who participated from School B are taught by different math teachers. Students had all received fractions instruction in fourth grade. During the study, which took place at the start of the 2011–2012 school year, the teachers refrained from covering fractions in their mathematics classes. A summary of the two school sites is presented in Table 2. The majority of participants (84%) completed all three measures. Twenty children were absent on at least 1 day of testing and were dropped from the analysis due to incomplete data. Seven of these participants were from School A and 13 were from School B.

Study Measures

Fractions Knowledge. To assess fractions knowledge, a fractions test was created based on released items targeting fractions and number sense from the California Standards Test, the National Assessment of Educational Progress (NAEP), and Trends in International Mathematics and Science Study. Items were created to test children's abilities to solve problems that ranged from very similar to very different from the kinds of problems posed in Motion Math.

At School A, participants completed a paper version of the test, which comprised 34 items. Figure 4 shows a sample item adapted from a NAEP-released item. This item was used in both the paper and the iPad versions of the test. The complete set of test items are available from the author. Scoring of the paper tests was done by hand using visual estimation. The criteria for correct responses on the paper tests were based on comparisons to key positions on the number line. For instance, to earn

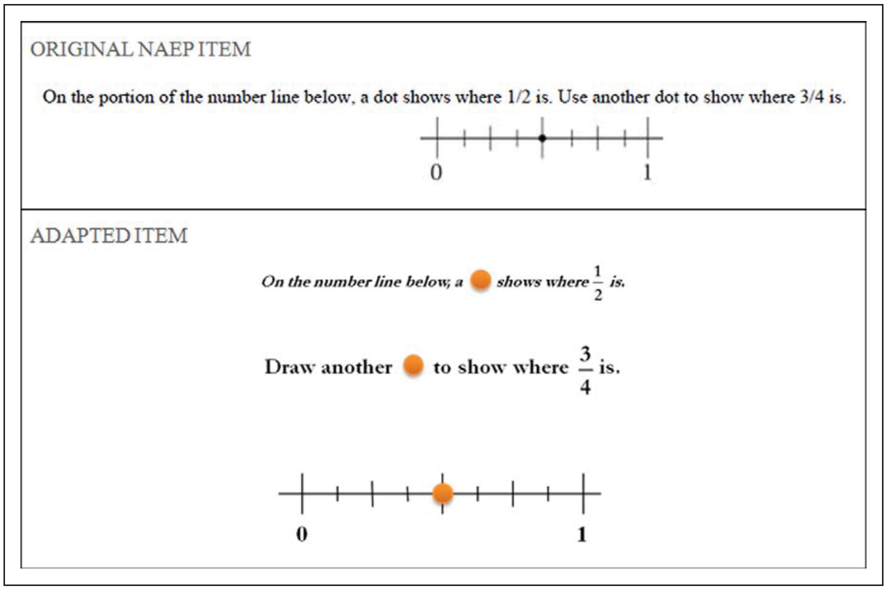

Figure 4. Example of original NAEP item and the adapted item administered in the present study. NAEP = National Assessment of Educational Progress.

credit for the question asking the participant to place 0.60 on the number line, it had to be visually slightly to the right of the halfway point and visually left of three quarters. Some paper questions asked children to place three different fractions on the same number line using symbols (e.g., on the number line below, draw a square to show where 1/2 is, draw a triangle to show where 1/3 is, and draw a circle to show where 1/4 goes). These items were scored based on correct ordering of the symbols as well as on location of placement on the number line. All the items were scored dichotomously.

At School B, participants completed the tests using the iPad. The iPad version of the test utilized a subset of 26 of the items due to the inability of the computerized interface to render certain question types. Scoring of iPad tests was conducted electronically; students were required to respond to each question before they could proceed to the next screen. Criteria for the online "place a dot" items was scored with a specific algorithm rather than visually. The number line was divided into 46 segments, each seven pixels wide. At seven pixels, children could make precise selections that were still visually distinct. Participants tapped on a dot to indicate their response and could revise their answer before moving forward. Children received full credit for placement within three dots of the exact response. On questions for which the correct answer was the endpoint of the scale (i.e., no visual estimation was necessary) participants only received credit for a precise response.

Fractions Attitudes. The fractions attitude measure comprised three subscales: fractions self-efficacy (e.g., ''I am good at fractions''), fractions liking (e.g., ''Fractions are fun''), and fractions-number line knowledge (e.g., ''I know where 1/2 goes on the number line''). The order of negative and positive response options was mixed to reduce response set threat to validity.

The attitudinal questions were presented in a format adapted from Susan Harter's (1981) work with young children to avoid social desirability threats to validity. For each question, participants were presented with two stick figures expressing opposite opinions about fractions and asked to circle the student who was most like them. Children then indicated whether they were ''a lot'' or ''a little'' like the student they had selected. Responses were coded into a 4-point Likert-type scale corresponding to *strongly disagree* (1) through *strongly agree* (4) and reverse coded if appropriate. Reliabilities are presented in Table 2.

Game Ratings. Four additional questions, using the same format as the fractions attitudes measures, were used to assess participants' perceptions of the Motion Math game itself. Children indicated whether they wanted to play the game again, whether the game was fun or boring, whether they thought their friends would like the game, and whether the game helped them learn fractions.

Procedures

This study used a repeated measures crossover design. At each study site, one class was randomly assigned to either Motion Math (Group 1) or control (Group 2) for the first week. In Week 2, the order switched. As shown in Table 1, all participants took the test 3 times, at the start, midpoint, and end of the study period. In this report, these three time points are referred to as pretest, midtest, and posttest, respectively. At both study sites, participating teachers agreed to refrain from teaching topics directly related to fractions during the study, since the purpose of the study was not to compare game play to classroom instruction but rather to ascertain whether the game is effective as a stand-alone instructional tool.

At School A, all participants completed paper-and-pencil tests on the first day of the study. They were instructed via a ''STOP HERE'' page in the booklet, as well as by their teacher, to wait after completing the attitudinal questions. When all participants were ready, the teacher began the timed section of the test. Children then had 10 min to complete the fractions items. School B followed a similar procedure, except that participants completed the measures using the iPads and were not timed. At both sites, for the next 5 days, students in Group 1 spent 20 min daily playing Motion Math. Children in Group 2 served as the control. After five school days, all participants completed all measures under the same conditions as on Day 1. The groups then switched treatment conditions. For the next five school days, children in Group 2 played Motion Math for 20 min daily while Group 1 participants served as the control. After Group 2 had played the game daily for five school days, all participants completed the measures for a third and final time. At the end of the study, all participants had experienced the same amount of exposure to Motion Math.

Results

To respond to the research questions, analyses were conducted to compare the groups to each other over time (i.e., between-subject comparisons) and to compare changes in scores within each group over time (i.e., within-subject comparisons). Data for the fractions attitude and game rating portions of the survey were aggregated across sites since all items were identical. However, to account for different administration conditions for the fractions knowledge tests, those data are presented separately for School A and School B.

Fractions Knowledge

School A Results. At School A, participants completed the test on paper and were given 10 min to complete the fractions test portion of the packet. Students completed each item in sequence as presented in the booklet. By including a time limit, the intent was to consider whether playing Motion Math would have an impact on *accuracy* of responses as well as *number* of items completed. However, the teacher reported that on the midtest and posttest, but not on the pretest, the children in both groups began to compete with one another to see how many questions they could answer in the allotted time. This competitive element introduced a confound, such that the number of items completed at the midtest and posttest, but not pretest, was due to a combination of students' fractions knowledge and the competitive element. To correct for this confound, pretest results were examined to determine how many items had been attempted by the majority of students, regardless of accuracy. Since the majority of students completed the first 20 items on the pretest, but not the remainder of the items, Items 1 through 20 were used as the basis for the analysis.

The two groups did not differ significantly at pretest, $t(43.37) = -.822$, $p = .415$. Levene's test for equality of variances indicated that the two groups had equivalent variances at midtest and posttest. Consistent with the hypothesis that Group 1, which played Motion Math first, would have higher fractions knowledge relative to Group 2 at the midtest, a one-tailed independent samples t-test was conducted. Results showed that at the midtest, Group 1 had a significantly higher average fractions score compared to Group 2, $t(52) = 2.670$, $p = .01$. At the posttest, both groups were again equivalent, based on results from a two-tailed independent samples t-test, $t(52) = 0.588$, $p = .559$. The effect sizes (ESs) presented in Table 3 were standardized by taking the average standard deviation for each group for the time points of interest. To ascertain whether scores at midtest were higher for Group 1 than for Group 2, an analysis of covariance (ANCOVA) was conducted using the pretest as a covariate. Results showed that the effect of treatment condition was significant, $F(1, 53) =$

Table 3. Descriptive Data and Effect Sizes for School A Fractions Knowledge.

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Post-Pre	
Group	*n*	*M*	*SD*	*M*	*SD*	ES	*t*	*M*	*SD*	ES	*t*	ES	*t*
1	28	.29	.20	.57	.24	1.27	8.879***	.55	.23	−.08	.752	1.21	7.355***
2	24	.35	.29	.40	.22	.20	2.129	.51	.27	.45	2.954**	0.57	3.962**

Note. ES = effect size; *M* = mean; *SD* = standard deviation.
**$p < .01$. ***$p < .001$.

25.194, $p < .001$, η^2 (ES) = .326. These results show that gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. In other words, controlling for students' initial fractions knowledge, the Motion Math group's midtest score was significantly higher than the control group's. A more stringent comparison of the Motion Math and control group's fractions knowledge was conducted using all three pretest variables as covariates. An ANCOVA test showed that even after controlling for initial levels of fractions knowledge, liking, and self-efficacy, the Motion Math group performed significantly better than did the control group, $F(1, 42) = 24.640$, $p < .001$, η^2 (ES) = .387.

In addition to comparing the two groups to each other, tests were conducted to determine whether gains within each group were significant and to determine whether time interacted with treatment condition. Using Bonferroni adjustments to control for Type 1 error, paired sample *t*-tests for each group showed that the change in scores from pretest to midtest was statistically significant for Group 1, but not for Group 2. From midtest to posttest, the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 3. Figure 5 shows the gains by group.

Finally, to examine interaction effects, a repeated measures test was conducted using a general linear model. Within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results are consistent with the crossover treatment design of the study and showed that, despite being equivalent at the start and end of the study, the two groups had statistically different test trajectories, $F(1, 50) = 29.783$, $p < .001$.

School B Results. At School B, the fractions test was administered using the iPad device, and students were not timed. Scores for each student were calculated by summing the correct responses and dividing by the total number of questions. The two groups were statistically equivalent at pretest, $t(48) = -.692$, $p = .492$, and the variance at each time point did not differ significantly between groups. One-tailed independent samples *t*-tests found that the two groups were statistically equivalent at both midtest and posttest. Standardized ESs are presented in Table 4.

A more robust comparison of the two groups at the midtest was conducted using an ANCOVA with pretest scores as covariate. Results showed that the effect of

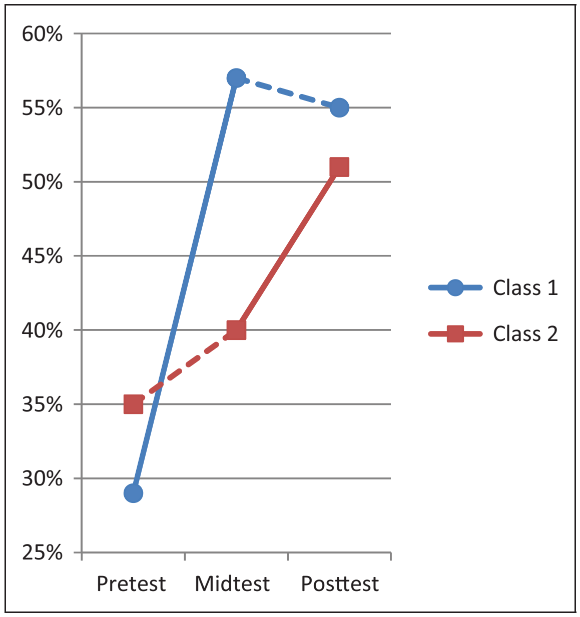

Figure 5. School A learning gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-group gains. Students at School A were given 10 min to respond to 20 questions.

Table 4. Descriptive Data and Effect Sizes for School B Fractions Knowledge.

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	*n*	*M*	*SD*	*M*	*SD*	ES	*t*	*M*	*SD*	ES	*t*	ES	*t*
1	26	.31	.144	.38	.209	0.40	2.446*	.40	.214	0.09	0.497	0.50	2.662**
2	24	.34	.168	.33	.181	0.06	0.214	.45	.205	0.62	3.421***	0.59	4.390***

Note. ES = effect size; *M* = mean; *SD* = standard deviation.
*$p < .05$. **$p < .01$. ***$p < .001$.

treatment condition was significant, $F(1, 53) = 4.427$, $p = .040$, η^2 (ES) = .078. In other words, gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. A second ANCOVA was conducted that added initial levels of fractions self-efficacy and fractions liking as covariates. The results showed that even after controlling for initial fractions knowledge, self-efficacy, and liking, Group 1 significantly outperformed Group 2 at the midtest, $F(1, 53) = 4.069$, $p = .049$, η^2 (ES) = .075.

Tests were also conducted to ascertain whether gains within each group were significant and to see whether time interacted with treatment condition. Paired

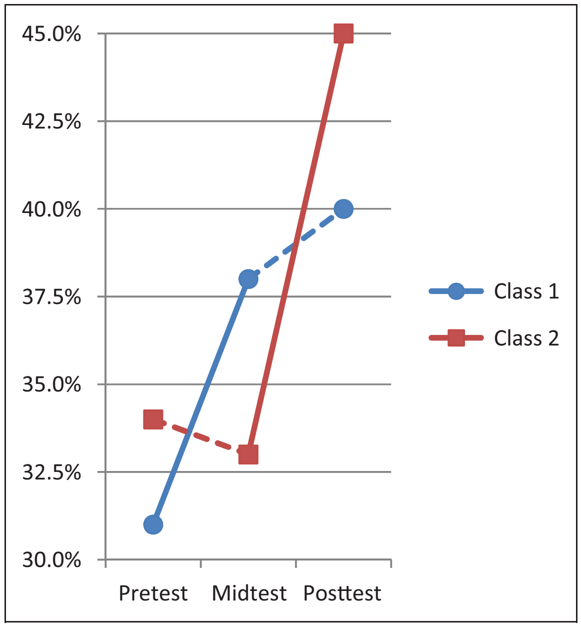

Figure 6. School B learning gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-group gains.

sample *t*-tests with Bonferroni adjustments showed the same pattern as was obtained for School A. Change in scores from pretest to midtest was statistically significant for Group 1 but not for Group 2. From midtest to posttest, the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 4. Figure 6 shows the gains by group.

Interaction effects were tested with a general linear model repeated measures analysis. Within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results support the hypothesis in a crossover design and demonstrated that the two groups had statistically different test trajectories, $F(1, 48) = 5.734$, $p = .021$, even though they were equivalent at the start and end of the study.

Fractions Attitudes: Self-Efficacy, Liking, and Knowledge Self-Ratings

The study also examined changes in students' reported knowledge self-ratings, their self-efficacy for fractions, and their liking of fractions. Similar to the hypotheses for fractions learning, the hypotheses were that students in Group 1 would show increases on average for these variables from pretest to midtest, and remain constant from midtest to posttest. In contrast, Group 2 scores were expected to remain

Table 5. Descriptive Data, Effect Sizes, and Paired-Sample Results for Self-Efficacy.

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	*n*	*M*	*SD*	*M*	*SD*	ES	*t*	*M*	*SD*	ES	*t*	ES	*t*
1	50	2.72	.838	3.04	.804	0.39	2.057*	3.11	.812	0.09	0.704	0.47	3.176**
2	44	2.75	.737	2.62	.872	0.16	−1.386	3.09	.870	0.54	3.885***	0.42	3.029**

Note. ES = effect size; *M* = mean; *SD* = standard deviation.
$^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

constant from pretest to midtest and to increase significantly from midtest to posttest. Since administrations of the fractions attitudes measures at both school sites were comparable, attitudes data were aggregated.

Fractions Self-Efficacy. The two groups had equivalent self-efficacy scores at the start of the study, $t(92) = -.112$, $p = .911$, and at all three time points did not differ significantly in variance. At the midtest, average self-efficacy for Group 1 was significantly higher than that of Group 2, $t(92) = 2.414$, $p = .009$. At the posttest, both groups were again equivalent, based on results from a two-tailed independent samples *t*-test, $t(92) = 0.140$, $p = .889$. Standardized ESs are shown in Table 5.

Controlling for pretest levels of self-efficacy, ANCOVA results showed that the effect of treatment condition was significant, $F(1, 94) = 7.357$, $p = .008$. In other words, self-efficacy gains for Group 1 from pretest to midtest were significantly higher than those for Group 2. Analyses were also carried out to determine whether gains within each group were significant. Using Bonferroni adjustments to control for Type 1 error, paired sample *t*-tests for each group showed that change in scores from pretest to midtest was statistically significant for Group 1 but not for Group 2. From midtest to posttest, the pattern was reversed, with Group 2 but not Group 1 posting significant gains, as shown in Table 5. Figure 7 shows the gains by group. Using a general linear model, within-subject contrasts yielded significant quadratic, but nonsignificant linear, contrasts. These results are consistent with the crossover treatment design of the study and showed that, despite being equivalent at the start and end of the study, the two groups had statistically different trajectories, $F(1, 92) = 7.558$, $p = .007$.

Fractions Liking. Analyses for fractions liking were identical to those for self-efficacy. Results are shown in Table 6 and Figure 8. The two groups were statistically equivalent at pretest and posttest and had equivalent variance at all three time points. At the midtest, Group 1 posted significantly higher fractions liking than did Group 2, $t(92) = 2.124$, $p = .018$. ANCOVA tests comparing midtest means with initial fractions liking as a covariate yielded a similar pattern of results, $F(1, 97) = 5.324$, $p = .023$. Pairwise comparisons using Bonferroni adjustments within each

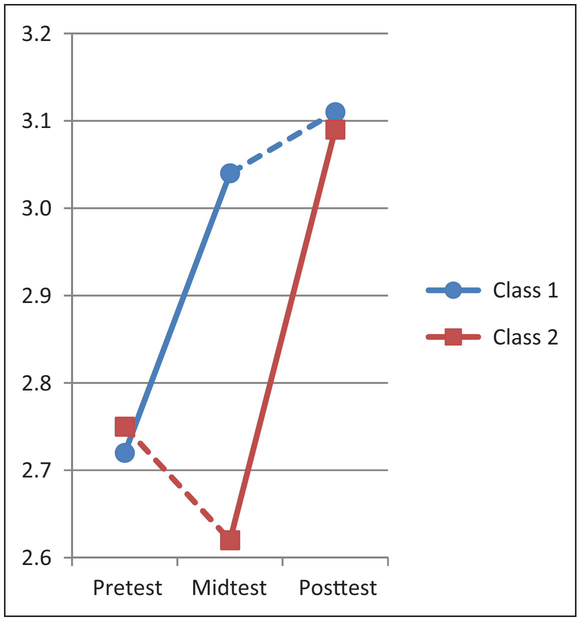

Figure 7. Self-efficacy gains over time for Group 1 (Motion Math first) and Group 2 (Regular first). Solid lines are statistically significant within-group gains.

Table 6. Descriptive Data, Effect Sizes, and Paired-Sample Results for Fractions Liking.

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	*n*	*M*	*SD*	*M*	*SD*	ES	*t*	*M*	*SD*	ES	*t*	ES	*t*
1	50	3.01	.945	3.28	.916	0.29	2.097*	3.31	.942	0.03	0.280	0.32	2.689**
2	44	2.97	.852	2.86	.985	0.12	−.852	3.44	.830	0.64	4.530***	0.56	3.643***

Note. ES = effect size; *M* = mean; *SD* = standard deviation.
**p* < .05. ***p* < .01. ****p* < .001.

group showed that Group 1 had statistically significant increases in fractions liking from pretest to midtest, $t(49) = 2.097$, $p = .02$, but not from midtest to posttest. The pattern was reversed for Group 2, with a significant increase from midtest to posttest, $t(43) = 4.530$, $p < .001$, but not from pretest to midtest. In other words, gains in fractions liking were associated with the period in which each group played Motion Math. Further confirming these patterns, quadratic within-subject contrasts were significant for treatment condition, $F(1, 92) = 9.571$, $p = .003$, indicating different trajectories for each group.

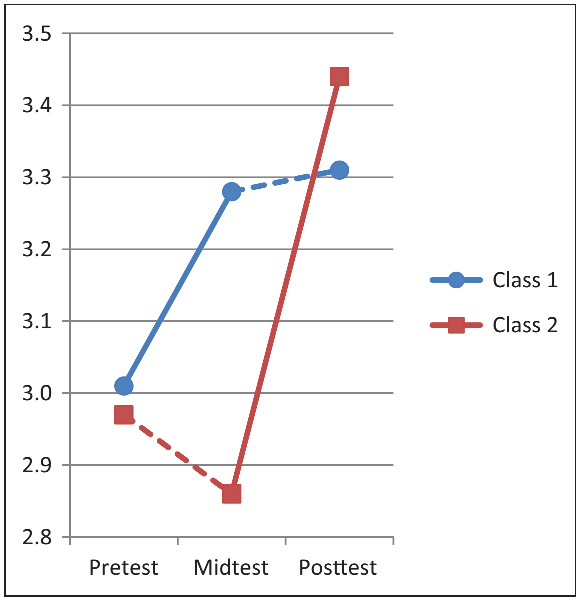

Figure 8. Liking gains over time for Group 1 and Group 2. Solid lines indicate statistically significant within-class gains.

Fractions Knowledge Self-Ratings. Results for fractions knowledge self-ratings are shown in Table 7 and Figure 9. The two groups were statistically equivalent at pretest and posttest and had equivalent variance at all three time points. At the midtest, Group 1 reported significantly higher fractions knowledge self-ratings compared to Group 2, $t(102) = 2.124$, $p = .018$. ANCOVA tests comparing midtest means with initial fractions knowledge self-rating as a covariate yielded a marginally significant result, $F(1, 102) = 3.261$, $p = .074$. For both groups, changes in fractions knowledge self-ratings did not differ significantly from pretest to midtest or from midtest to posttest. However, Group 1 did have overall gains from pretest to posttest. Trajectories were not significantly different for each group on this variable.

Students' Ratings of Motion Math

The third research question asked whether students would rate Motion Math positively after playing it for five consecutive school days. Responses to these questions strongly support students' positive attitudes of the game. One hundred percent of participants reported wanting to play the game again; nearly 80% strongly endorsed wanting to play the game again. One hundred percent of students also found the game fun and close to 80% strongly endorsed that statement. In response to the question asking whether their friends would like the game, 95% *agreed*, and 75%

Table 7. Descriptive Data, Effect Sizes, and Paired-Sample Results for Fractions Knowledge Self-Rating.

		Pretest		Midtest		Mid-Pre		Posttest		Post-Mid		Overall	
Group	*n*	*M*	*SD*	*M*	*SD*	ES	*t*	*M*	*SD*	ES	*t*	ES	*t*
1	55	3.01	.742	3.35	1.489	0.30	1.623	3.46	1.460	0.07	.446	0.41	2.284*
2	46	3.15	.784	2.92	1.500	0.20	−1.023	3.22	.671	0.28	1.292	0.10	0.495

Note. ES = effect size; *M* = mean; *SD* = standard deviation.
*$p < .05$.

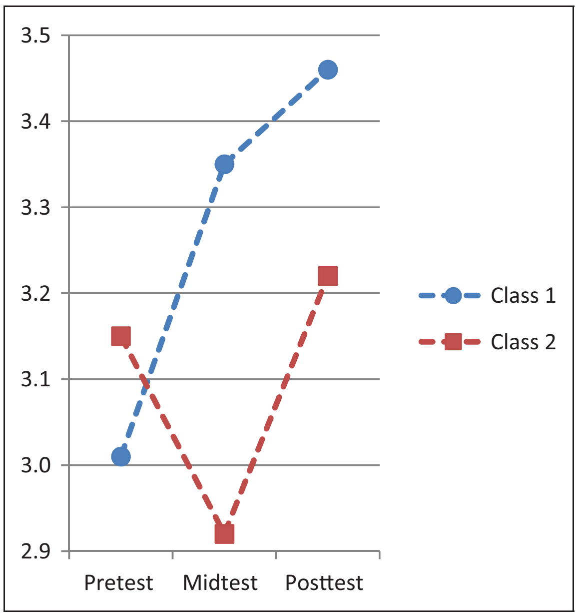

Figure 9. Knowledge self-rating changes over time for Group 1 and Group 2.

strongly agreed. Finally, 95% of students agreed that the game helped them learn fractions, with nearly two thirds strongly agreeing.

Discussion

The study posed three overarching research questions and hypotheses, which were tested using an experimental repeated measures crossover design. The data reported here suggest that a week of daily exposure to Motion Math resulted in improved fractions knowledge and increases in participants' fractions self-efficacy and liking. Moreover, children rated the game highly.

Fractions Knowledge

At both study sites, significant learning gains were tied to the period in which participants played Motion Math. Lending further strength to the findings is the fact that these patterns were produced by two different measures of fractions knowledge, at schools with two different demographic profiles, and in comparison with a control group. Importantly, the benefits accrued by the first groups to play the game did not shrink after several days of nonexposure to the game. Therefore, it is reasonable to conclude that the gains made were stable, at least in the short term.

Why did playing Motion Math improve participants' fractions knowledge? There are many possible answers to this question, and most likely several factors contribute to the game's effectiveness. One possible contributing factor is the instant feedback and scaffolding provided by the game. Another is that players tackle many fractions problems each time they play the game. Over the course of this study, each student encountered an average of over 770 math problems. This represents an incredible amount of fractions practice. The motivation generated by the engaging game mechanic may have been crucial to this benefit. In other words, the entertainment value of the game provided students the motivation necessary to persist in the extensive practice needed to attain mastery and automaticity. The timed nature of the game is another important feature, in that a time constraint adds a sense of urgency to problem solving, and may also foster rapid schema development. Importantly, the embodied nature of Motion Math game play is a key consideration in its effectiveness. Having to physically tilt the device repeatedly may have helped students develop their mental number line. This explanation is consistent with the Ramani and Siegler (2008), and Siegler and Ramani (2009) studies reviewed earlier.

The success of Motion Math in promoting learning opens up many questions for future exploration regarding the relation between the game design and learning. Important insights would be gained from in-depth analysis of game play data, such as how many problems students correctly solved on the first "bounce," the trajectories of game level attained, or whether specific problems serve as particularly strong predictors of fractions proficiency. The extent to which assessment of fractions proficiency could be embedded within game play has intriguing implications for student testing.

The gains observed from pretest to posttest indicate that the game served an instructional function. One question that emerges from these results is the extent to which the game was primarily teaching students to relate fractions to the number line, or activating and reinforcing students' prior knowledge. A related question is the extent to which, and the processes by which, experience with fractions estimation transfers to other kinds of fractions knowledge and problem solving. Future research could tease apart students' conceptual understanding from procedural understanding, and probe the extent to which game play brings students to a deeper conceptual understanding of fractions as numbers. Other topics for additional research include identifying game design features most responsible for learning gains. For instance, do specific problem sequences lead to conceptual insights more quickly or effectively than do others?

Another set of questions regards ways to optimize learners' interactions with Motion Math. For example, is the game most effective for students who are at a certain level of understanding of fractions? One of the participating teachers in the present study observed that students who had not yet grasped the basic idea of a fraction did not appear to improve their game play, whereas other students gave signs of making new connections and insights. An interesting consideration is how slight modifications to the game might lead to improved learning for a broader range of students. Motion Math was designed for students with a baseline understanding of the fractions representation form. For students with limited prior knowledge who may not spontaneously extract patterns from game play, introducing explicit sequences may prove effective. For instance, an introductory level might start with a sequence of unit fractions with increasing denominators (i.e., 1/2, 1/3, 1/4, 1/5, and 1/6) and explicitly draw the player's attention to the inverse relationship between the denominator and the distance from zero on the number line. Other instructional sequences include using a constant denominator with an incremental decrease or increase in the numerator; or maintaining the same fraction while changing the labels on the number line endpoints. Adding a whole number "refresher" level could serve as an effective scaffold to bridge students' intuitive understanding of whole numbers to an understanding of fractions as numbers (Wu, 2008). Changing the number line more often may foster more flexible schema development. Motion Math already includes levels in which the number line spans from -1 to zero. Including additional levels with number lines that span zero, such as one anchored at -1 and $+1$, could help reinforce the concept that fraction magnitude is independent of sign.

Wu (2008) asserted that fractions research should be integrated with the study of fractions instruction. Though Motion Math was created as a stand-alone game, it may be that even minimal instructional support exponentially increases the educational value of game play. Simple instructional prompts from a teacher, parent, or older peer could unlock insights for students. Future research should explore the value added by a variety of instructional wraparounds. Another consideration is the length of exposure to the game. This study took place over 10 school days. Over longer periods of time, do students develop deeper conceptual understanding?

Fractions Self-Efficacy, Liking, and Knowledge Self-Ratings

The study also investigated changes in students' attitudes toward fractions. The results for fractions self-efficacy and liking were highly similar to those for learning. Increases in these attitudes were associated with the period in which students played Motion Math and persisted after game play. These results are arguably just as important as the learning results, given the strong connection documented in the research literature between students' attitudes and subsequent learning. Developing positive affect for fractions should benefit students as they encounter new challenges in mathematics.

Gains in students' self-efficacy show that their confidence in their ability to solve fractions problems improved as a result of playing Motion Math. Importantly, the study asked students to rate their self-efficacy for fractions in general, not for their ability to succeed at the game. In other words, these outcomes suggest that the experience of playing Motion Math gave students a sense of confidence in fractions that extends beyond game play. One explanation for this outcome is the fact that mastery experiences are a well-documented source of self-efficacy (Usher & Pajares, 2008). The design of Motion Math enabled students to experience and be rewarded for success each time they correctly solved a problem. This stands in contrast to the kind of feedback students receive in school, where they are scored on overall assignments or tests. In this context, correctly solving a single problem is unlikely to contribute to a student's self-efficacy. A game environment also promotes persistence, a key motivation indicator. Several features of game play are likely to contribute to persistence. First, failure is an accepted part of game play, whereas in school tasks it is often reason for students to withdraw from the task. Everyone expects to have to fail many times before winning in a game. Second, a well-balanced game maintains the right proportion of challenge to skill level, so that persistence is rewarded by new successes, which in turn fuels more persistence. The game also gives students multiple chances to succeed at each problem. Related to this is that scaffolding communicates to the student that the game is "on their side" and thus encourages risk taking and persistence. Finally, the entertainment value of the game is probably responsible to a large degree for students' persistence.

Students also reported liking fractions more after playing Motion Math. Similar to the results obtained for learning and self-efficacy, these increases remained when students no longer played the game. In a game environment, the positive experience of game play is likely to become associated with the material, much like, in a negative fashion, bad experiences with math class lead to negative affect toward math. One reason for increased liking is the fact that learners tend to have more positive attitudes toward topics they feel they understand and can succeed in. Many schoolchildren believe that fractions are too hard to understand. By offering students opportunities to succeed at solving fractions problems, Motion Math also provided students with experiences that contributed to more positive fractions affect. The importance of this outcome cannot be underestimated for students' future decisions to persist in and succeed at mathematics.

In contrast, students' self-ratings of their fractions knowledge did not yield significant changes, although there were significant differences between the two groups at the midpoint of the study. There are several possible explanations for this outcome. Reliability was very low ($\alpha = .58$) for this 3-item measure, which was created for the present study to assess whether students might rate themselves more highly after game play on their ability to solve the specific kinds of questions played in the game. The low reliability may be the reason for the nonsignificant changes observed here. It may also be that the instrument, beyond being unreliable, was not a valid measure of self-rated knowledge due to language comprehension. Many students from School A provided inconsistent responses to the negatively worded item for this measure. Another possible explanation is that prior to playing Motion Math, students had never been asked to place a fraction or decimal on the number line, and therefore did not understand the questions. The fact that reliabilities increased to acceptable levels after the experience of game play suggest that once students began playing the game, the question made more sense and they were able to respond more accurately and consistently.

Ratings of Motion Math

Students' ratings of Motion Math were overwhelmingly positive and demonstrate that the game is successful in providing students with an entertaining experience. Outcomes regarding enjoyment of the game and interest in playing it again are all the more powerful in light of the evidence of learning documented here. After playing the game for 20 min a day, 5 days in a row, students' enthusiasm for the game remained high. The fact that students still wanted to play the game after doing so for five consecutive school days suggests that the game mechanic has hit the mark with a powerful "addictive" quality. Perhaps most importantly, in Motion Math, learning is seamlessly integrated with assessment. Game play and assessment are one and the same. Future research could probe the keys to the game's entertainment value, including interactive features, visual, haptic, and audio rewards, the tilt-based game mechanic, and the handheld nature of device interaction.

Limitations

Like every research study, the present investigation had several limitations that should be considered in interpreting and applying the results. First, the study did not compare Motion Math to traditional classroom instruction or to other educational apps. Therefore, the study did not enable conclusions to be drawn regarding the effectiveness of Motion Math relative to other approaches to learning fractions. Second, the study did not collect information on individual student demographics or academic achievement. Therefore, it was not possible to examine whether certain groups of students (e.g., by gender, ethnicity, and prior knowledge) responded more favorably than others to the Motion Math game. Third, game play data were not analyzed in this study. As a result, although the data enable valid claims about *whether* students learned, we do not know *why* they learned. Moreover, it is highly likely that certain students benefited more than others from exposure to the game. In addition, 16% of students did not complete all assessments and therefore were not included in the analyses. The inclusion of data from these students, who may have higher rates of absenteeism than their peers, may have changed the outcomes of the study. Finally, it could be argued that Motion Math was successful in promoting students' fractions learning because of the large number of problems students solved. In other words, if Motion Math is essentially fractions practice, then research should be conducted to determine whether it is more effective than other kinds of practice, for instance work sheets or flash cards.

Another consideration is the effect the iPad itself may have had on the outcomes obtained here. Anecdotal evidence suggested that most students in this study did not own their own iPad. It is therefore possible that students' reported enthusiasm for fractions and for Motion Math could be explained in part by their enthusiasm for the device itself. Even if the iPad was responsible for the attitudes outcomes, it is unlikely that the use of the iPad per se was responsible for fractions learning. The excitement at using the iPad may have given students the motivation they needed to persist at the game, which in turn led to fractions learning. The question is whether students would continue to engage effectively with the game once the novelty of the technology subsides. Finally, the fact that students participated in the context of the school day may have influenced the quality of their engagement with the game. It is possible that the quality of engagement in informal settings would result in smaller learning and attitudes gains.

Future Research

The present study offers a foundation for future research into these issues. Emergent mobile devices make it possible for players to interact physically with devices in revolutionary ways. Much is yet to be learned about how the new affordances of these devices can support learning, and how our conception of learning itself might change in light of these new ways of interacting with the world around us.

With attention to protection to individuals' privacy, research could use GPS data to determine where people are most likely to make use of mobile learning apps. Are they typically at home, work, waiting in line at the grocery store? Does location of use relate to aspects of mobile learning app usage? How is location of game play related to game features and to learning? For example, it may be that individuals are less likely to engage in games that require conspicuous physical interaction in public spaces. In addition, brief games are perhaps most appropriate for learning that happens best with distributed practice, whereas games that focus on shifting strong misconceptions may demand longer play periods, and consequently be less aligned with spontaneous usage.

Much is to be learned about how individual differences affect the way that learners interact physically with mobile devices. In the present study, differences were observed in the extent to which individuals were willing to dramatically tilt the device. While some students tended to use exaggerated movements, others were reticent to engage physically with the device. Future research, and the development of embodied learning apps, should consider the interplay between individual characteristics and game features.

Mobile learning apps offer tremendous potential for dynamic assessment that is embedded in game play (Riconscente & Vattel, 2013). In other words, in well-designed apps, learning could be tracked without the need for external tests and quizzes. Research and development is already underway in this direction; however, this approach to assessment is yet in its infancy.

Motion Math is grounded in research and theory on the field of mathematics and on how people learn. To be effective, developers of learning apps will need to invest substantial resources to deeply probe the target learning material, concepts, and skills, and explicitly shape game mechanics to reflect the target learning.

Conclusion

The well-documented difficulties children face when learning fractions call for new approaches to teaching this topic. To date, the present study is the first to hold the promise of iPad apps up to experimental evidence. This project investigated Motion Math, a mobile learning game that leverages the physical interaction features of the iPad to engage students in a fractions learning experience. In this controlled experiment, Motion Math significantly improved participants' fractions knowledge and attitudes. Moreover, children's ratings of the game were quite positive. This study thus offers evidence that Motion Math successfully integrates learning and entertainment, and in the process boosts players' attitudes toward fractions.

Authors' Note

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Author Biography

Michelle M. Riconscente, PhD, has over 20 years experience at the intersection of learning, technology, and assessment. Her current design and research interests include: extending evidence-centered design to create games, simulation, and interactive experiences that successfully promote learning and motivation; designing embedded assessments of cognitive and affective outcomes in interactive environments; and conducting program evaluations for a variety of educational technology initiatives.