Quantum Random Walks in Unique Lattice Designs with Asymmetries

Theo Barklund
TheoBarklund6.28@gmail.com

under the direction of
Prof. Mohamed Bourennane
&
Alexander Moritz

Quantum Information and Quantum Optics
Department of Physics
Stockholm University

Research Academy for Young Scientists
July 11, 2018
Abstract

Optical circuits that use photons instead of electrons are one of the most promising architectures for future quantum computers. Building a quantum computer is, however, difficult. This study focuses on quantum random walks in optical circuits to better understand how light moves on the smallest of scales. During this study 14 quantum random walk lattices are designed, and some are studied. One such design is the hexagonal lattice which promotes an interesting new way to do quantum random walks.
Acknowledgements

I want to thank my mentor Mohamed Bourennane and Alexander Moritz for providing both the resources and the support of this project. I also thank Alban Seguinard for helping explain many difficult concepts that I now understand. Erik Eldh, Filip Frick, and Jakob Broman also provided excellent support throughout the writing of this report. I am thankful to Rays – for excellence and its partners, Europaskolan, and Kjell och Märta Beijers Stiftelse for making this project possible.
Contents

1 Introduction 1

1.1 Previous Research 2

1.2 Aim of Study 2

2 Theory 2

2.1 Waveguides 2

2.2 Writing Waveguides with Femtosecond Laser 3

2.3 Quantum Mechanics - The Photon 4

2.4 Evanescent Field 6

2.5 Classic and Quantum Random Walks 6

3 Method 7

3.1 Designing Photonic Circuits 7

3.2 Printing Photonic Circuits 9

3.3 Quality Control 9

3.4 Exciting Waveguides and Gathering Data 10

4 Results 10

5 Discussion 11

5.1 Interpreting Results 11

5.2 The FSL Issue 11

5.3 Further Research 11

References 13
1 Introduction

Supercomputers have become an essential part of modern science, yet they still have one common flaw: they use technology which is limited to storing data as binary digits, simple ones and zeros. There is a promising new type of computer which aims to overcome this limitation. Quantum computers will be able to do computations on problems and data that traditional supercomputers will never be able to complete. Reinventing the binary bit, quantum computers use qubits which can have many different states at once, as opposed to traditional ones and zeros. [1]

Building a functioning quantum computer is difficult as it requires that a variety of new technological challenges are overcome. There are many ways to design a quantum computer, and each method introduces its own challenges, benefits, and drawbacks. One of the most promising quantum architectures is based upon photonic circuits which are capable of precisely guiding photons through waveguides [1]. The photons are guided utilizing a phenomenon called total internal reflection, similarly to how fibre cables work. Furthermore, each photon can carry more information, through its many quantum states, than an electron in a classic circuit [2].

In order to design something with photonic circuits, it is important that the circuits work as expected, that the simulations and predictions match the practical experiments. Ensuring the reliability of photonic circuits has, however, proven to be difficult. A significant factor to said difficulty being the probabilistic nature of quantum particles. The circuits can be tested using the quantum analogue of a process called random walk (RW). A RW is a process where a mathematical object starts at the zero coordinates of a lattice and then repeatedly moves in a random direction for a determined number of steps. The quantum analogue, the quantum random walk (QRW) works by using a quantum particle, such as a photon, instead of a mathematical object. The position of the photon at the end of the QRW is measured and if the paths which the photon is more likely to take can be predicted, then it would be possible to build a quantum computer. However, many odd observations have been made using QRWs and much still needs to be explored. [3]
1.1 Previous Research

Previous studies on quantum random walks in photonic circuits almost solely use symmetrical circuits. A study by James A. Grieve [4], and another study by Alberto Peruzzo, both demonstrate that when photons pass through and exit symmetrical photonic circuits their probabilistic paths will also be symmetrical [5]. A similar study by H. Tang, X-F. Lin, et al. show that in a 2-dimensional environment a quantum random walker is expected to create a distribution pattern with initially one bright spot, then four, and then nine as the evolution length increases [6]. Although the distribution pattern is more interesting in a 2-dimensional environment, it is still symmetrical and not very surprising.

1.2 Aim of Study

This study aims to design various symmetrical and asymmetrical lattices. Furthermore, the effect symmetrical, and asymmetrical lattices have on quantum random walkers will be studied experimentally.

2 Theory

Quantum mechanics deals with explaining the movement and behaviour of the smallest known objects in existence.

2.1 Waveguides

Waveguides are used to guide photons in photonic circuits by utilizing total internal reflection, similar to how fibre cables work. Total internal reflection works by constantly reflecting photons inside of the cable, keeping it contained. Snell’s Law,

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2, \]  \hspace{1cm} (1)
describes the relationship between the angle of incidence, the angle at which the light hits the side of the waveguide, and the angle of refraction, the angle at which the light is reflected. The angles $\theta_1$ and $\theta_2$ represent the angle of incidence and refraction respectively, $n_1$ and $n_2$ represent the refractive index of appropriate media [7]. The area between the media, where the refractive index changes, is called the boundary.

Refractive index is defined as shown in Equation 2 where $c_v$ is the speed of light in vacuum, $c_m$ is the speed of light in the relevant medium, and $n$ the refractive index [8],

$$n = \frac{c_v}{c_m}.$$  

Total internal reflection occurs when the angle of incidence exceeds the critical angle, $\theta_c$, and the light is travelling from a medium of a higher refractive index to a medium of lower refractive index [7]. The critical angle is equal to the angle of incidence in Snell’s Law, Equation 1, when the angle of refraction is equal to 90°,

$$\theta_c = \arcsin \left( \frac{n_2}{n_1} \right).$$  

### 2.2 Writing Waveguides with Femtosecond Laser

A femtosecond laser (FSL) fires rapid pulses with a pulse duration in the order of femtoseconds. To write the waveguides, the glass plate needs to be mounted on a movable platform, the translation stage. The focal point of the FSL would then be positioned inside of the glass plate. Once set up the translation stage could be programmed to move the glass plate, effectively changing the position of the focal point inside of the glass with a degree of accuracy that is within a couple of nanometers [9]. The heat from the FSL pulses changes the molecular structure of the borosilicate glass which makes it denser, which in turn give the material a greater refractive index [10]. The short pulses of the FSL help prevent the heating of glass surrounding the focal point because it gives heat time to spread out. Therefore, the change in refractive index is confined, and very precise
waveguides can be created [9].

2.3 Quantum Mechanics - The Photon

Photons exhibit both wavelike and particle-like properties, but not at the same time, this is called wave-particle duality [11]. An example of the wave-like properties of light is interference. The wavelike photon consists of a wave in the electric field and a perpendicular wave in the magnetic field, see Figure 1a. The electromagnetic wave, both the electric and magnetic wave together, can be added with the electromagnetic wave of another photon to form light of different intensities and phases, this is called interferometry. There are two major types of interference: constructive, and destructive, see Figure 1b, c [12].

![Figure 1a](image1.png)

(a)

![Figure 1b](image2.png)

(b)

![Figure 1c](image3.png)

(c)

Figure 1: a) An electromagnetic wave propagating in the positive $x$ direction. The blue wave shows oscillations in the magnetic field and the red wave shows oscillations in the electric field. b) Electromagnetic interference. The red waves represent the electric field, but magnetic field is not shown. In this case the lower two waves are added together to create another wave of greater intensity - constructive interference. c) When two electromagnetic waves that are 180° out of phase are added together they cancel out - destructive interference.
Polarisation is also an important property of the photon; it describes how the electromagnetic wave is rotated. Using the electric wave as the reference, note that the magnetic wave is always present, and perpendicular. In this case the electric wave oscillates vertically, and therefore the photon is vertically polarised [13]. When a photon passes a polarising beamsplitter (PBS) it may be reflected outwards or transmitted depending on how the PBS is set up, see Figure 2. A PBS can only differentiate between orthogonal directions of polarisation. [14]

Figure 2: Polarising Beamsplitter Cube. A beam of mixed light containing both vertically and horizontally polarised photons is shone into the cube which then splits the light into two separate beams of vertically, and horizontally polarised light respectively.

If the polarisation of the photon is rotated diagonally relative to the PBS then the electromagnetic wave will be rotated, in either direction, to match the orthogonal directions of the PBS. Rotating the electromagnetic wave changes the polarisation of the photon which will, after being rotated, be reflected or transmitted accordingly. It is probabilistic whether the electromagnetic wave is rotated to be vertical or horizontal. An electromagnetic wave cannot be split in vectors and continue propagating at a lower intensity; it will always adjust its polarisation to an orthogonal direction of the PBS. If the polarisation of the photon is almost vertical, then it is more likely for the electromagnetic wave to
rotate towards the vertical direction. However, if the polarisation is perfectly diagonal to the orthogonal directions of the PBS then it is entirely random whether the photon is reflected or transmitted. [14]

2.4 Evanescent Field

When photons are able to travel through a waveguide, they will hit the boundary of the waveguide at an angle greater than the critical angle resulting in total internal reflection. The wave-particle duality of the photon makes it impossible for the electromagnetic wave to be discontinuous at the boundary and as such the wave extends slightly outside of the waveguide. The part of the wave which extends outside of the waveguide, past the boundary, is called the evanescent field and it facilitates the photons travel between waveguides. If the evanescent field overlaps a waveguide, the photon could also appear in said waveguide when measured. [15]

2.5 Classic and Quantum Random Walks

A classic random walk is a mathematical process where an object, starting at the zero coordinates in any coordinate system, repeatedly chooses a direction to move in at random. A one-dimensional walker can only move along one axis, in both the positive and negative directions, while a two-dimensional walker can move along two axes in both directions. Every time the walker moves, a step is counted. In a classic 2-dimensional random walk, the walker has an increased probability, for any number of steps, of being closer to the starting position. This is because any turn the walker makes will slow its progress outwards, if not also revert it. [16, 17]

Quantum random walks are similar to classic random walks except that the object, the walker, is instead a quantum particle such as a photon. A quantum random walk can be achieved by designing a lattice of waveguides where the waveguide separation distance, see Figure 3, is short enough for the evanescent field to overlap multiple waveguides as to
enable the spreading of a photon. Three significant factors are contributing to the photons travel distance from the starting waveguide. The first factor is the waveguide separation distance, the closer they are, the more likely the photon is to spread. The second factor is the parallel distance which is the distance during which the waveguides are close to each other, and the third being the polarisation of the photon.\[6\]

![Waveguide Diagram](image)

Figure 3: The waveguide separation distance is defined as the distance between two waveguides, and the parallel distance is the length during which the waveguides are close together. Sometimes the parallel distance is also called evolution length.

### 3 Method

In the experiment, a variety of different lattice designs were designed for the sake of comparing their effects on a quantum random walker.

#### 3.1 Designing Photonic Circuits

On the first borosilicate glass plate, a set of square lattice grids with different waveguide separation distances were designed as shown in Figure 4. For the second borosilicate glass plate, a set of hexagonal lattices with different waveguide separation distances was designed. In the hexagonal grid, every waveguide has six neighbours which are equally spaced relative to each other and the chosen reference waveguide. Furthermore, in the second borosilicate glass plate an additional four square lattice grids with, and a hexagonal lattice was included. Defect lattices have removed waveguides to study how the quantum
walker is affected. Figure 5.

Figure 4: A set of five square lattices in borosilicate glass, with the same parallel distance, but varying wavelength separation. Not visible in image: all waveguides exit on the other side of the glass.

Figure 5: A set of 4 hexagonal lattices in borosilicate glass with equal parallel distances, but different waveguide separation, and an additional hexagonal lattice with three defects, removed waveguides. Not visible in image: four square lattice grids with defects and the waveguides exiting on the other side of the glass.
3.2 Printing Photonic Circuits

A borosilicate glass plate of width 25mm, length 50mm, and depth 1.1mm was mounted on a movable platform below the focal point of a static femtosecond pulse laser. A back-and-forth printing method where the laser writes in both directions was used. The table moved the glass plate at a constant speed of 30mm/s with half-second pauses between each waveguide.

3.3 Quality Control

The borosilicate glass plates were polished before inspection. Once they were polished, methanol was used to clean the glass edges. The glass plates were attached vertically under a microscope so that the microscope was looking into the waveguide openings. Then the waveguides were confirmed to be of the correct dimensions and to have clear openings so that they could be excited.

Figure 6: A picture of waveguide openings on the edge of the glass plate. A microscope at 500x magnification was used. The waveguide separation distance is \(8.75 \mu m\).
3.4 Exciting Waveguides and Gathering Data

The first borosilicate glass plate, the sample, was attached via suction to a translation station separate from the one used to write the waveguides. A visible green laser was then used to find and align the lattices with the laser mount. Once aligned, the green laser was swapped with a more powerful 810nm laser. Before the green laser was used to excite a lattice it was passed through a PBS to separate a known polarisation of light which was to be guided through the sample. After the laser had passed through the sample, a lens was used to focus the light into a camera which recorded the distribution of exiting light, indicating how the light had spread inside of the lattice.

Complications with the focus of the FSL made it nonviable to print the second borosilicate glass plate which was to contain the defect and hexagonal lattices.

4 Results

The square lattice with a waveguide separation distance of $7\mu m$ could be excited and photographed as seen in Figure 7. The other four square lattices were too blurry to photograph and due to complications with the FSL the hexagonal and square lattice with defect designs were never printed.

![Figure 7](image)

Figure 7: The pictures a, b, and c are from the square lattice with $7\mu m$ separation distance.
5 Discussion

For the purpose of comparing symmetrical, and asymmetrical QRWs this study has created unique designs for lattices including hexagon based, and square grid-based lattices with various defects.

5.1 Interpreting Results

The four bright spots match well with the previous research on 2-dimensional QRWs [6]. Although, a clearer image was expected. It is possible that each of the four bright spots in every picture consists of a couple of waveguides which are all getting excited. However, it is still clear that there are four regions towards which light tends to move. The slight differences between the pictures are due to slight variations in the alignment of the laser source and sample.

5.2 The FSL Issue

It was discovered that the FSL was improperly focused. The cause of the issue is unknown, however, the many mirrors which guide the laser to the translation station are being investigated. Printing with an improperly focused laser was determined to be nonviable. It is reasonable that the 4 square lattices which were too blurry to photograph were a result of the improperly focused laser. The reason that the 7µm square lattice gave considerably more clear pictures could be because the waveguides were tighter which made it easier for light to spread.

5.3 Further Research

Exciting a hexagonal lattice would be interesting because it initially promotes light spreading equally in all directions as opposed to a square lattice which promotes horizontal, and vertical spreading above diagonal.
Finding out how light spreads in various kinds of a lattices helps provide understanding and insight into what factors need to be considered when creating optical circuits for quantum computers. Further studies using laser printed waveguides in borosilicate glass are worthwhile because it is a cost and time effective way to study quantum mechanics at the most fundamental level.
References


