Project PAR: Promoting Algebra Readiness
A Fractions Curriculum

Center on Teaching and Learning
University of Oregon
Project PAR:
Promoting Algebra Readiness

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PAR: Promoting Algebra Readiness

Curriculum Sampler
Table of Contents

PAR Strand 1 Multiplication and Division of Whole Numbers - Lesson 6 ............. 1

PAR Strand 2 Fractions as Numbers – Lesson 6 ........................................... 17

PAR Strand 3 Addition and Subtraction of Fractions - Lesson 6....................... 32

PAR Strand 3 Addition and Subtraction of Fractions - Lesson 18.................... 47

PAR Strand 4 Multiplication and Division of Fractions - Lesson 10............... 59

PAR Alignment with Common Core State Standards Mathematics .................. 71
<table>
<thead>
<tr>
<th>Common Core State Standard (CCSS-M)</th>
<th>Foundational Knowledge and Skills</th>
<th>Bridging Knowledge and Skills</th>
<th>Target Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.NS.2</td>
<td>Use multiples as a strategy to determine factors, products, and quotients.</td>
<td>1.1.F.2 Model whole number multiplication and associated division within 100 using arrays, area models, and number lines.</td>
<td>1.1.1. Demonstrate conceptual understanding, procedural fluency, and application in multi-digit whole number multiplication and division. Verify answers using a variety of strategies.</td>
</tr>
<tr>
<td>6.NS.4</td>
<td>1.1.F.3 Understand the inverse relationship between multiplication and division using models, strategies, and by interpreting division as an unknown-factor problem.</td>
<td>1.1.F.4 Apply the commutative, associative, and/or distributive properties of multiplication as a strategy to multiply and divide.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1.F.7 Use the order of operations to evaluate expressions with parenthesis, brackets, and multiple representations of mathematical symbols to multiply and divide.</td>
<td>1.1.B.5 Assess the reasonableness of answers using mental computation and estimation strategies.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1.F.9 Model multidigit whole number multiplication and associated division using arrays, area models, and number lines.</td>
<td>1.1.B.6 Solve word problems involving single digit multiplication and associated division using a variable to stand for the unknown quantity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1.F.10 Understand multidigit whole number multiplication and associated division by explaining why these procedures work (including place value and properties of operations).</td>
<td>1.1.B.8 Solve two-step word problems involving multiplication and associated division using variables to stand for unknown quantities.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1.F.11 Solve word problems involving multidigit whole number multiplication and division.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 6: The Distributive Property as a Strategy to Multiply and Divide

S&S #1.1.F.4 Apply the commutative, associative, and/or distributive properties of multiplication as a strategy to multiply and divide.

**Lesson Objective(s)**

1. Apply the concept of the Distributive Property of Multiplication to compose and decompose products.
2. Apply the distributive relationship with appropriate numbers and symbols to area models.

**Teacher Materials:**
- Lesson 6 PowerPoint (PPT)
- Square color tiles (30)
- SmartPALs with blank paper inserted and markers (or sticky notes)
- Activity Practice 1 Key

**Student Materials:**
- Activity Practice 1
- Square color tiles (30 color tiles per student)
- SmartPALs with blank paper inserted and markers (or sticky notes)
- Two different color highlighters per student.
- Lesson 3 Anchor Chart: Number Properties of Multiplication

**Vocabulary**

- **Array** – items (objects, numbers, squares, etc.) arranged in rows and columns.
- **Commutative property of multiplication** – changing the order of the factors does not change the product.
- **Distributive property of multiplication** – the product of a number and a sum is equal to the sum of the individual products. (Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$.)
- **Equation** – a mathematical statement that shows two expressions are equivalent.
- **Factors** – the numbers multiplied to get a product.
- **Multiple** – the product of a counting number and another whole number.
- **Multiplication** – repeated addition or joining of equal groups. Multiplication is also the inverse of division.
- **Product** – the result of multiplying two or more numbers.
- **Variable** – a symbol for a number we do not know. It is usually a letter like $x, y, \text{ or } n$. 
Entry Task (<3min)

Display PPT #1. Have students complete the entry task on Activity Practice 1.

Answers
1. $7 \times 8 = 56$
2. $64 \div 8 = 8$
3. $10 \times 8 = 80$
4. $45 \div 5 = 9$

Activate Background Knowledge and Interest (<5min)

You have shown that you can represent multiplication and division with array models and equations. Let’s review some related math vocabulary.

What does an array model look like?

*Items or objects arranged in rows and columns.*

What is a multiple?

*The product of any counting number and another whole number.*

Which property states that changing the order of the factors does not change the product?

*The commutative property of multiplication.*

Previously, we talked about how properties are important characteristics of numbers that can help us solve equations. We used the Commutative Property to help us find missing factors in both division and multiplication equations. Today, we’ll learn about a new property called the Distributive Property that can help us find unknown products.

Ask 3 students to pass out 10 square color tiles to each student as you ask the following questions.
PAR Strand 1
Lesson 6: The Distributive Property as a Strategy to Multiply and Divide

**What is (student’s name) doing?**

*Passing out 10 color tiles to everyone.*

**What is (other student’s name) doing?**

*Passing out 10 square color tiles to everyone.*

**Can you think of another way you could say, “passing out”?**

*Lead students to saying the word, distributing (or tell students).*

**Using the word “distributing,” what is (other student’s name) doing?**

*Distributing 10 square color tiles to everyone.*

Yes. When you distribute something, you give pieces of it to many different people. These students distributed square color tiles to each of you. Another example is handing out or distributing papers or highlighters in class. In math, we have a number property that is about distributing.

Display PPT #2.

The Distributive Property states that factors in a multiplication problem can be decomposed (or split) into two or more parts, multiplied separately, and then added together to find the product. The result would be the same as multiplying the original factors to find the product. Using this property will help you use what you know about multiplication to find the product of more complex multiplication problems.

**I Can…**

Display PPT #3.

1. *Apply the concept of the Distributive Property of Multiplication to compose and decompose products.*

2. *Compose and decompose numbers while forming equations to show my “distributive” thinking.*
While we’ll use the Distributive Property to help us solve more complex problems in our next lesson, I’ll first show you how it works with some easier factors. For example, if I have a $3 \times 5$ sheet of stamps and I want to find the total number of stamps, or the product, I can build an area model to help me see the problem.

The sheet of stamps represents an area model. How many rows are there? (3) How many columns are there? (5)

If I want to know how many stamps are in the sheet, I can write an equation to solve the problem.

Click to display PPT #4: $3 \times 5 = y$

The model represents a $3 \times 5$ area and since I want to solve for the total number of stamps, I can use a variable to stand for the product. I can use any symbol such as $x, y, n$, or another letter to represent the variable. In this problem, I’m going to use “$y$” to represent the variable.

To find the total or product, I could count the squares, but in the complex problems we’ll do later there will be too many squares to count.

Instead, I’m going to use the distributive property to split or decompose the model into two parts. We can choose a column or row to split, so let’s use the column and split or decompose it into two parts. I have 5 columns. I can split or decompose it into 2 columns and 3 columns like this.
Display PPT #5:

**What do you noticed about the rows for each part?**

*Each part still has 3 rows or there is no change in the number of rows.*

I notice that the model has 3 rows of 2 squares, which I know equals 6.

Click to display PPT #5: I know that $3 \times 2 = 6$

**That leaves 3 rows of 3, which I know equals 9.**

Click to display PPT #5: I know that $3 \times 3 = 9$

**The two products, 6 and 9 are partial products of the original problem. I can add these two equations.**

Click to display PPT #5: $3 \times 5 = (3 \times 2) + (3 \times 3)$

$3 \times 5 = 6 + 9$

**And I can add the two partial products, 6 + 9 to get a total of 15.**

Click to display PPT #5: $3 \times 5 = 15$

So $y = 15$.

Click to display PPT #5: $y = 15$

**Did I get the same total that I would get if I multiplied 3 \times 5? (Yes.)**

That’s right! I “distributed” multiplication because I split or decomposed the factor 5 into 2 parts, 2 and 3. I then multiplied the parts by 3 because that was how many rows there were. After I multiplied to get the products, I then added the partial products to get the answer.

I decomposed this area model by splitting 5 columns into 2 columns of 3 rows and 3 columns of 3 rows. However, I could also split the model into 1 column of 3 rows and 4 columns of 3 rows.

Display PPT #6.

**How many squares are in 1 column of 3 rows? (3)**

Click to display PPT #6: I know that $3 \times 1 = 3$

**How many squares are in 4 columns of 3 rows? (12)**
PAR Strand 1  
Lesson 6: The Distributive Property as a Strategy to Multiply and Divide

Click to display PPT #6: I know that $3 \times 4 = 12$

**What is $3 + 12$? (15) So what does $y$ equal? (15)**

Click to display PPT #6: $3 \times 5 = 15$

**Is this the same total as $3 \times 5$? (Yes.)**

Click to display PPT #6: $y = 15$

No matter how I decompose the area model, adding the partial products will give me the total product.

Think-Pair-Share:

In my last two examples, I decomposed the area model by splitting the columns or the number in each row or group. What’s another way I could decompose this model?

*Possible answers:* You could decompose the area model by splitting the row horizontally instead of vertically to make two rows or groups of 5 columns. You could turn the area model on its side by changing the order of the factors and then decompose to make two groups of 5 rows. You could make groups of 5 instead of groups of 3.

One way to make two groups of 5 would be to use the Commutative Property to turn the area model on its side. Then we could make two groups of 5 rows. Another way is to make two groups of 5 columns. To make two groups of 5 columns, I can split 3 rows into two parts instead of splitting the factor 5.

Display PPT #7.

I now have 1 row of 5 squares, which I know equals 5.

My other group has 2 rows of 5 squares. How many squares are in this group? (10)

Now I can add the partial products to find the total.

Click to display PPT #7:

Once again, I get the same product, 15. Now I know that I can decompose using either factor 3 for the rows or 5 for the columns, to create new groups and find the product.
PAR Strand 1
Lesson 6: The Distributive Property as a Strategy to Multiply and Divide

Guided Practice

Give each student 30 Square color tiles, SmartPALs with blank paper inserted, and markers. Have students build area models with you on SmartPALs as you display your model (or you may build models on the desk and label with sticky notes).

Let’s build an area model together to find the product of 6 x 5 using the Distributive Property.

Give students a minute to start building their area model before you build your model. Monitor and check that students have a 6 x 5 tile area.

Write next to your model on a SmartPAL or sticky note: 6 x 5

How many tiles in each row? (5) How many tiles in each column? (6)

Now, we could count the squares, which could be very time consuming, or we can look for smaller area models of numbers we can multiply that we already know. Take a minute and look for a way to split or decompose your area model into two area models using factors where you already know the partial products. Remember that you can decompose using either factor, the rows or the columns.

![Diagram of area model split into two parts with partial products equations]

Monitor students as they split their area models into 2 area models. Have them write the partial products equations on the SmartPAL (or sticky notes), then add the partial products to find the answer.

Call on students who have different splits to share their strategy. Ideally, find one student who split vertically and one student who split horizontally. Have each student tell you how he/she split the area model as you split the teacher area model the same way. Have students bring up their example with the equations. Have the class confirm the equations. Repeat with another student.

One example student response follows.

*I split my area model into 2 rows of 5 squares, which I know is 10. Then there are 4 rows of 5 squares left over, which I know is 20. If I add up the two partial products, 10 + 20, I end up with the answer of 30.*
PAR Strand 1
Lesson 6: The Distributive Property as a Strategy to Multiply and Divide

If a student splits their area model into uneven groups without whole rows or columns, provide the following error correction:

We need to split our model into groups of whole rows or groups of whole columns to create partial products.

Display PPT #8: Distributive Thinking Template.

Have students write the equations for their model using the template on the front of Activity Practice 1. Monitor and assist as needed.

¬ Extend Down Option: Provide additional guided practice examples decomposing any or all of the following expressions: 8 × 8, 7 × 4, 9 × 5, 3 × 9, etc.

¬ Extend Up Option: For students needing a challenge, select more complex expressions such as: 12 × 12, 15 × 5, 20 × 6, etc.

Independent Practice

Have students find the Independent Practice on the back of Activity Practice 1. Instruct students to use 2 different color highlighters to split their area model into two parts. Have students write equations to show the Distributive Property of Multiplication. Monitor and assist as needed.

Teacher Note: Watch for multiplying the partial product instead of adding once the numbers have been distributed.

As you circulate, ask individual students the following questions.

Looking at one of your area models, what is the product?

Which two area models represent your product? How are these area models the same? How are these area models different?

How are the area models examples of the distributive property?

If you were to write a definition of the distributive property, what would it say?
Lesson Closure (<3min)

Have students find the Number Properties of Multiplication Anchor Chart that they began working on in Lesson 3.

Display PPT #9.

We are going to add an example of the Distributive Property to our number properties chart. In the Whole Numbers box, draw and write an example where you use the Distributive Property to split $8 \times 3$ into two parts and solve the equation.

Prompt students to draw models, label each part of the model, and solve the equation. Assist students with drawing area models as needed.

Exit Ticket (<3min)

Display PPT #10.

Have students complete the exit ticket on Activity Practice 1.
The Distributive Property as a Strategy to Multiply and Divide

PAR 1, Lesson 06

Entry Task

In your math journal write and complete the following equations. You may use your multiplication table to help you.

\[ 7 \times 8 = \square \]
\[ 64 \div 8 = \square \]
\[ 10 \times \square = 80 \]
\[ \square \div 5 = 9 \]

Distributive Property of Multiplication

This property states that factors in a multiplication problem can be split into two or more parts, multiplied separately, and then added together to find the product.

For example,

\[ 8 \times 7 = \square \]
I know that \( 8 \times 5 = 40 \), and
I know that \( 8 \times 2 = 16 \).
So, \( 8 \times 7 = 40 + 16 = 56 \)

I Can...

1. Apply the concept of the Distributive Property of Multiplication over Addition to compose and decompose products.
2. Compose and decompose numbers while forming equations to show my “distributive” thinking.

Stamps!

\[ 3 \times 5 = y \]

Distributive Thinking

\[ 3 \times 5 = y \]
\[ 3 \times 5 = (3 \times 2) + (3 \times 3) \]
\[ 3 \times 5 = 6 + 9 \]
\[ 3 \times 5 = 15 \]
\[ y = 15 \]
Distributive Thinking

**Distributive Thinking Template**

\[ 6 \times 5 = y \]

I know that \( \_	imes\_ = \_ \)

I know that \( \_	imes\_ = \_ \)

\[ 6 \times 5 = \_ + \_ = 30 \]

\[ y = \_ \]

Exit Ticket

- In three sentences or less, write a note to a friend saying what the Distributive Property is and what you can use it for.

**Number Properties of Multiplication**

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero Property</td>
<td>( a \times 0 = 0 )</td>
</tr>
<tr>
<td>Identity Property</td>
<td>( a \times 1 = a )</td>
</tr>
<tr>
<td>Commutative Property</td>
<td>( a \times b = b \times a )</td>
</tr>
<tr>
<td>Associative Property</td>
<td>When 3 or more factors are multiplied, the product is the same regardless of the grouping of the factors.</td>
</tr>
<tr>
<td>Distributive Property</td>
<td>The product of a number and a sum is equal to the sum of the individual products.</td>
</tr>
</tbody>
</table>

**Whole Numbers**

- \( 3 \times 5 = 15 \)
- \( 3 \times 10 = 30 \)
- \( 3 \times 15 = 45 \)

**Fractions**

- \( \frac{1}{2} \times \frac{2}{3} = \frac{1}{3} \)
- \( \frac{3}{4} \times \frac{2}{3} = \frac{1}{2} \)
- \( \frac{1}{6} \times \frac{3}{6} = \frac{1}{9} \)

**Whole Numbers and Fractions**

- \( 3 \times (\frac{1}{2} + \frac{2}{3}) = \frac{1}{2} \times 3 + \frac{2}{3} \times 3 \)
Lesson 6 Activity Practice 1

Name:____________________________

Entry Task

Complete the following equations. You may use your multiplication table to help you.

7 \times 8 = \square 

64 \div 8 = \square 

10 \times \square = 80 

\square = \div 5 = 9 

Distributive Thinking – Guided Practice

\[ 6 \times 5 = y \]

I know that \___ \times \___ = \___ 

I know that \___ \times \___ = \___ 

So \[ 6 \times 5 = \___ + \___ = \___ \]

\[ y = \]
### Distributive Property of Multiplication - Independent Practice

Directions: Use two different color highlighters to decompose the area into two parts. Write equations to show the Distributive Property of Multiplication.

**Example 1:**

$$7 \times 8 = y$$

I know that ____ × ____ = ____

I know that ____ × ____ = ____

So $$7 \times 8 = \_\_\_ + \_\_\_ = \_\_\_$$

$$y = \_\_\_$$

**Example 2:**

$$6 \times 9 = y$$

I know that ____ × ____ = ____

I know that ____ × ____ = ____

So $$6 \times 9 = \_\_\_ + \_\_\_ = \_\_\_$$

$$y = \_\_\_$$

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**Exit Ticket**

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PAR Strand 1
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Lesson 6 Activity Practice 1 Key

Name: _______________________________

Entry Task
Complete the following equations. You may use your multiplication table to help you.

\[7 \times 8 = 56\]
\[64 \div 8 = 8\]
\[10 \times 8 = 80\]
\[45 \div 5 = 9\]

Distributive Thinking – Guided Practice

\[2 \times 5 = 10\]
\[4 \times 5 = 20\]

\[6 \times 5 = y\]
I know that \(2 \times 5 = 10\)
I know that \(4 \times 5 = 20\)
So \(6 \times 5 = 10 + 20 = 30\)
\(y = 30\)
Distributive Property of Multiplication - Independent Practice

Directions: Use two different color highlighters to decompose the area into two parts. Write equations to show the Distributive Property of Multiplication.

Possible Answers:

- $6 \times 5 = y$
  - I know that $3 \times 8 = 24$
  - I know that $4 \times 8 = 32$
  - So $7 \times 8 = 24 + 32 = 56$
  - $y = 56$

- $6 \times 9 = y$
  - I know that $6 \times 5 = 30$
  - I know that $6 \times 4 = 24$
  - So $6 \times 9 = 30 + 24 = 54$
  - $y = 54$

Exit Ticket

The distributive property means that when I don’t know the answer to a multiplication problem, I can split (or decompose) one of the factors into a sum of 2 numbers. I can then multiply each of those numbers by the other factor, and add the products. That will be equivalent to multiplying the original factors. I can use this to make multiplication of larger numbers easier.
## PAR Scope and Sequence - Strand 2
revised 11/15

<table>
<thead>
<tr>
<th>Common Core State Standard (CCSS-M)</th>
<th>Foundational Knowledge and Skills</th>
<th>Bridging Knowledge and Skills</th>
<th>Target Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STRAND 2: FRACTIONS as NUMBERS</strong></td>
<td>Skills that support the standard and provide a starting point for PAR.</td>
<td>Pre-formal mathematical knowledge needed to bridge foundational with abstract mathematical reasoning.</td>
<td>Formal mathematical knowledge.</td>
</tr>
<tr>
<td>6.NS.6 Understand a rational number as a point on the number line.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1.F.1 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b).</td>
<td>2.1.B.2 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts.</td>
<td>2.1 Demonstrate understanding of fractions as a quantity with magnitude and a distance with length on a number line.</td>
<td></td>
</tr>
<tr>
<td>2.1.F.3 Partition fraction models into equal parts including number lines and strip diagrams.</td>
<td>2.1.B.4 Represent fractions as distances on a number line, (including special cases such as improper fractions, a/a, and a/1).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1.F.5 Model addition and subtraction of fractions as joining and separating parts referring to the same whole.</td>
<td>2.1.B.6 Explain that a fraction a/b is a multiple of 1/b. Understand a fraction a/b with a &gt; 1 as the sum of fractions 1/b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1.F.1 Identify two fractions as equivalent if they are the same size or the same point on the number line and explain using visual fraction models.</td>
<td>2.2.B.2 Compare two fractions with same numerator or denominator by reasoning about their size.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2.F.5 Express a fraction with a denominator of 10 as an equivalent fraction with a denominator 100.</td>
<td>2.2.B.3 Generate equivalent fractions using the number line.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2.F.6 Use decimal notation for fractions with denominators of powers of 10.</td>
<td>2.2.B.4 Generate equivalent fractions by multiplying by a fraction equal to one.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2.F.7 Compare two decimals to thousandths by reasoning about their size.</td>
<td>2.2.B.8 Model and compare fractions and decimals as equivalent representations on a number line.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
S&S #
2.1.B.4: Represent fractions as distances on a number line (including special cases such as improper fractions, $a/a$ and $a/1$).

**Lesson Objective(s)**

1. Model a fraction on a number line (as a point, a distance from zero and a distance between two points)
2. Describe a number line using a unit fraction or equal interval.
3. Identify that a number written as a fraction over itself is equal to 1.

**Teacher Materials:**

- Lesson 6 PowerPoint (PPT)
- Activity Practice 1
- Activity Practice 1 Key
- Activity Practice 2 Key

**Student Materials:**

- Activity Practice 1
- Activity Practice 2

**Vocabulary (new in bold)**

- Denominator – the number below the line in a fraction showing the number of parts the whole is divided into
- Fraction – a fraction $a/b$ is the quantity formed by a parts of size $1/b$.
- Interval – distance between two points on a number line
- Numerator – the number above the line in a fraction showing the number of parts of the whole
- Unit fraction – a fraction with a numerator of 1 (except where the denominator is 0)
- Unit interval – the distance between two consecutive whole numbers or any distance of 1, as measured by length, to describe the whole
- Whole numbers – counting numbers from zero to infinity
- Whole unit – all parts added together to equal one
Entry Task (<3min)

Display PPT #1

Have students complete the entry task on Activity Practice 1.

Answer: $\frac{1}{6}$ and $\frac{1}{5}$.

Display PPT #2 to share the answers.

In the first model, what is the unit interval? (0 to 1)

What are the intervals? ($\frac{1}{6}$) Yes, this means that each interval within the unit interval is $\frac{1}{6}$ of a unit.

What is the distance between the two points in the first model? ($\frac{1}{6}$ of a unit.)

The second model shows a number line with an interval where both points have been labeled. 1 $\frac{1}{5}$ is $\frac{1}{5}$ of a unit more than 1. What is the distance between the two points in the second model? ($\frac{4}{5}$ of a unit.)

Activate Background Knowledge and Interest (<5min)

Display on the doc cam: Activity Practice 1

Point to the first number line.

How many unit intervals are represented in this number line? (2 units.)

How many intervals are within each unit interval? (3)

Within each of these unit intervals, there are 3 equal intervals. What fraction represents each interval? ($\frac{1}{3}$)

What are three ways to represent a unit fraction?
PAR Strand 2
Lesson 6: Represent a Fraction \( \leq 1 \) as a Point, Distance and Distance Between

We can represent a unit fraction as a point, a distance from zero and a distance between two points.

I Can…

Display PPT #3

1. Represent a fraction as a point, as a distance from zero, and as a distance between two numbers on a number line that is greater than 1.

2. Describe a model using a unit fraction or intervals.

3. Identify that a number written as a fraction over itself is equal to 1.

Lesson (30 min)

Teacher Demonstration

Have students complete Activity Practice 1 with you as you demonstrate on the doc cam.

Today we are going to learn some more ways to represent fractions and describe the number line using unit fractions. Which fraction represents each interval within this unit interval? \( \frac{1}{3} \)

Label the first interval as \( \frac{1}{3} \) below the number line.

What should we label the second interval as? \( \frac{1}{3} \)

Label the second interval as \( \frac{1}{3} \) below the number line. Plot a point to represent \( \frac{2}{3} \).
PAR Strand 2
Lesson 6: Represent a Fraction ≤ 1 as a Point, Distance and Distance Between

If we start at 0, count 2 intervals away from 0, and plot a point on the number line, we have 2 one-thirds of a unit.

Each interval is $\frac{1}{3}$, and we have 2 joined intervals that represent the location as a point on the number line. We can describe the location as 2 one-thirds of a unit, 2 copies or intervals of $\frac{1}{3}$, or $\frac{2}{3}$.

Write the following above the point and have students do the same: $\frac{2}{3} = 2 \text{ copies of } \frac{1}{3}$

Display on the doc cam: 2nd number line and insert a segment from 0 to the 3rd tick mark.

Which fraction represents each interval within the unit interval? ($\frac{1}{3}$)

Is each interval within each unit interval the same length (or distance) on this number line? (Yes.)

Each interval is $\frac{1}{3}$ and we have a segment that represents 2 joined intervals on the number line. We can say that the segment represents 2 one-thirds of a unit, 2 copies or intervals of $\frac{1}{3}$ or what fraction? ($\frac{2}{3}$)

Write above the segment and have students do the same: $\frac{2}{3} = 2 \text{ copies of } \frac{1}{3}$.

We represented the fraction $\frac{2}{3}$ as a point (point to the first number line) and a distance from zero using a segment (point to the second number line).
PAR Strand 2
Lesson 6: Represent a Fraction \( \leq 1 \) as a Point, Distance and Distance Between

Display on the doc cam: 3rd number line and insert a segment as a distance between \( \frac{3}{3} \) and \( \frac{5}{3} \).

This segment represents a fraction as a distance between two points, 1 and \( \frac{5}{3} \). What fraction should we label each interval with? \( \left( \frac{1}{3} \right) \)

Label two intervals with \( \frac{1}{3} \) each below the number line.

Each interval is \( \frac{1}{3} \) and we have a segment that represents 2 joined intervals on the number line that are not from zero. We can say the segment represents 2 one-thirds of a unit, or 2 copies or intervals of what fraction? \( \left( \frac{1}{3} \right) \)

We can describe the distance as 2 one-thirds of a unit, 2 copies or intervals of \( \frac{1}{3} \), or \( \frac{2}{3} \).

Write above at the end of the segment and have students do the same: \( \frac{2}{3} = 2 \text{ copies of } \frac{1}{3} \)

Guided Practice

Have students turn Activity Practice 1 over and complete the number lines with you as you guide on the doc cam.

Display on the doc cam: Activity Practice 1 (side 2)

What fraction represents each interval? \( \left( \frac{1}{3} \right) \) So, how many intervals are within the unit interval? \( (3) \)
PAR Strand 2
Lesson 6: Represent a Fraction ≤ 1 as a Point, Distance and Distance Between

It’s your turn to count 3 intervals from zero and plot a point on the number line.

Student response:

If we start at 0, count 3 intervals to the right, and plot a point, what is the location of this point on the number line? \( \frac{3}{3} \text{ or } 1 \)

Label \( \frac{3}{3} \) above the point.

Each interval is \( \frac{1}{3} \) and we have joined 3 of the intervals to represent the location as a point on the number line.

Label first 3 intervals as \( \frac{1}{3} \) each below the number line and have students do the same.

What does the point on the number line represent?

3 one-thirds of a unit, 3 copies or intervals of \( \frac{1}{3} \text{ or } \frac{3}{3} \)

In previous lessons, we learned that a fraction is a way to represent division as the numerator (dividend) divided by the denominator (divisor). So \( \frac{3}{3} \) can be represented as 3 divided by 3 which equals 1. You can see that the point \( \frac{3}{3} \) on the number line is also 1.

Have students label \( \frac{3}{3} \) above the point on the number line.

Because all fractions can be represented as division, when the fraction is written as a fraction over itself, it is equal to 1. In this situation, 3 divided by 3 equals what? (1)

So what does \( \frac{3}{3} \) equal? (1)

Display the 2\text{nd} number line on the doc cam.

Let’s represent the same fraction as a distance from zero using a segment on the next number line.
PAR Strand 2
Lesson 6: Represent a Fraction ≤ 1 as a Point, Distance and Distance Between

It’s your turn to represent $\frac{3}{3}$ as a distance from 0 using a segment. How many intervals should we count from 0 to represent $\frac{3}{3}$? (3 intervals.)

Go ahead and count 3 intervals and represent the distance from zero using a segment.

What fraction represents the segment? ($\frac{3}{3}$)

Label $\frac{3}{3}$ above the segment.

What should we label each interval below the segment as? ($\frac{1}{3}$) It’s your turn to label each interval as $\frac{1}{3}$ below the number line.

Each interval is $\frac{1}{3}$ and how many intervals are joined to represent the segment? (3)

How can we describe this segment?

*We can describe it as 3 one-thirds of a unit, 3 copies or intervals of $\frac{1}{3}$ or $\frac{3}{3}$.

How else can we represent the fraction $\frac{3}{3}$ on the number line?

*We can represent it as a distance between two numbers.

Can we represent the fraction $\frac{3}{3}$ anywhere on a number line as a distance? (Yes.)

Have students work with a partner to model $\frac{3}{3}$ as a distance between any two points on the last number line. Have them label each interval with a fraction.

*Possible student response:

When students finish modeling the fraction as a distance, not from 0, have partners share their model and tell the class why the segment represents $\frac{3}{3}$ by describing the segment as 3 one-thirds, or 3 copies or intervals of $\frac{1}{3}$. 
That’s correct. Each interval is $\frac{1}{3}$ and when we join 3 intervals, it represents $\frac{3}{3}$ as a distance. Even though the segment does not start from 0, the segment represents $\frac{3}{3}$ or 1 as a distance. Who can tell why the segment $\frac{3}{3}$ is equal to 1?

Because $\frac{3}{3}$ can be represented as $3 \div 3$ which equals 1.

Is this the only way to represent $\frac{3}{3}$ as a distance, not from zero? (No.)

Call on other student pairs to share their responses and explain their model.

**Independent Practice**

Display PPT #4.

Have students complete Activity Practice 2 on their own. Have them model the fractions on the number line. Monitor and provide corrective feedback as needed.

**Lesson Closure (<3min)**

This lesson allowed us to see the three different ways that fractions can be represented on the number line.

What are the three different ways that fractions can be represented on the number line?

*Call on various students to offer different ways: As a point (or location), as distance from zero, or as a distance between two points.*

What are some ways we can describe the fraction $\frac{5}{5}$ using unit fractions?

*Call on different students to say it can be described as 5 one-fifths of a unit, or 5 copies of $\frac{1}{5}$, or 5 intervals of $\frac{1}{5}$.*

What whole number is equivalent to $\frac{5}{5}$? Why?

*Accept student responses such as $\frac{5}{5}$ equals 1 because a fraction is another way to represent division; the fraction $\frac{5}{5}$ means 5 divided by 5.*
Exit Ticket (<3min)

If time allows, display PPT #5. Otherwise, collect the completed number lines from the independent practice.

Display PPT #5.

Have students complete the exit ticket on the space provided on Activity Practice 2.

Exit ticket answer:

Correct response:

- $\frac{4}{4}$ as a point
- $\frac{4}{4}$ as a distance from 0
- $\frac{4}{4}$ as a distance between two points. Accept any model that represents 4 intervals of $\frac{1}{4}$ using a segment not from 0.
**ENTRY TASK**

What is the distance between the two points on the number lines?

- \[ \frac{1}{6} \]
- \[ \frac{1}{5} \]

**ENTRY TASK ANSWER**

What is the distance between the two points on each number line?

- \[ \frac{1}{6} \]
- \[ \frac{1}{5} \]

**I CAN...**

1. Represent a fraction as a point, as a distance from zero, and as a distance between two numbers on a number line that is greater than 1.

2. Describe the model using a unit fraction or intervals.

3. Identify that a number written as a fraction over itself is equal to 1.

**INDEPENDENT PRACTICE**

Represent:
- \( \frac{3}{4} \) as a point
- \( \frac{2}{6} \) as a distance from zero
- \( \frac{2}{2} \) as a distance between two points (NOT from zero)

**EXIT TICKET**

1. Draw a number line from 0 to 2.
2. Partition the number line to represent the fraction \( \frac{2}{4} \) three different ways.
   - As a point on the number line.
   - As a distance from zero.
   - As a distance, not from zero.
Lesson 6 Activity Practice 1 Key

Entry Task:
\[
\begin{align*}
\frac{1}{6} & \quad \frac{1}{5}
\end{align*}
\]

Name: __________________________

\[\frac{2}{3} = 2 \text{ copies of } \frac{1}{3}\]

\[\frac{2}{3} = 2 \text{ copies of } \frac{1}{3}\]

\[\frac{2}{3} = 2 \text{ copies of } \frac{1}{3}\]
Note: Students can make a segment to represent $\frac{1}{3}$ anywhere on the number.
Lesson 6 Activity Practice 2 Key

Name:__________________________

Represent $\frac{2}{3}$ as a point.

Represent $\frac{2}{6}$ as a distance from zero.

Represent $\frac{3}{3}$ as a distance between two points not from zero. (Accept any of the following segments.)
Exit Ticket

Correct response:

\[
\begin{array}{c}
\frac{4}{4} \text{ as a point} \\
0 \quad 1 \quad 2 \\
\end{array}
\]

\[
\begin{array}{c}
\frac{4}{4} \text{ as a distance from 0} \\
0 \quad 1 \quad 2 \\
\end{array}
\]

\[
\begin{array}{c}
\frac{4}{4} \text{ as a distance between two points. Accept any model that represents 4 intervals of } \frac{1}{4} \text{ using a segment not from 0.} \\
0 \quad 1 \quad 2 \\
\end{array}
\]
<table>
<thead>
<tr>
<th>Common Core State Standard (CCSS-M)</th>
<th>Foundational Knowledge and Skills</th>
<th>Bridging Knowledge and Skills</th>
<th>Target Knowledge and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.</td>
<td><strong>3.1.2.F.1 Generate multiple ways to decompose a fraction a/b into a sum of fractions greater than 1/b with the same denominators using visual and concrete models.</strong></td>
<td><strong>3.1.8.B.2 Represent addition and subtraction of fractions with like denominators as joining and separating parts referring to the same whole on a number line.</strong></td>
<td><strong>3.1 Demonstrate conceptual understanding, procedural fluency, and application in adding and subtracting fractions with like denominators ([a/b+c/b=(a+c)/b) and (a/b-c/b=(a-c)/b) ((b \neq 0) and (a &amp; c) are whole numbers)]. Verify answers using a variety of strategies.</strong></td>
</tr>
<tr>
<td>5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</td>
<td><strong>3.3.F.1 Model the sum of two fractions with unlike denominators as the length of two concatenated segments and estimate and assess the reasonableness of answers.</strong></td>
<td><strong>3.3.8.B.5 Add and subtract fractions with unlike denominators (including mixed numbers), by applying various equivalent fraction strategies.</strong></td>
<td><strong>3.3 Demonstrate conceptual understanding, procedural fluency and application in adding and subtracting fractions (including mixed numbers) with unlike denominators. Verify answers using a variety of strategies.</strong></td>
</tr>
<tr>
<td>3.2.B.2 Add and subtract a whole number and a fraction less than one ((q \ a/b)) by representing the whole number as an equivalent fraction with the same denominator as the fraction and explain why these procedures work.</td>
<td><strong>3.3.F.3 Generate equivalent fractions for two given fractions with unlike denominators by multiplying by a fraction equal to 1.</strong></td>
<td><strong>3.3.8.B.6 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases with unlike denominators using visual models and equations. Assess the reasonableness of answers using mental computation and estimation strategies.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

S&S # 3.1.B.3 Write equations to represent a fraction a/b as a sum of fractions with the same denominator (including improper fractions as whole numbers and fractions less than 1).

Lesson Objective(s)
1. Write addition and subtraction fraction equations that represent the inverse relationship between the two operations.
2. Explain the inverse relationship of addition and subtraction.

Teacher Materials: Student Materials:
• Lesson 6 PowerPoint (PPT) • Activity Practice 1 (inserted in SmartPAL)
• Activity Practice 1 Key • Activity Practice 2
• Activity Practice 2 Key

Vocabulary
• Addition - joining or combining two or more quantities to find a total or sum
• Denominator – the number below the line in a fraction showing the number of parts the whole is divided into
• Fraction - a fraction a/b is the quantity formed by a parts of size 1/b
• Improper fraction - a fraction with a numerator that is greater than its denominator
• Interval - distance between two points
• Inverse - the reverse of or opposite. Division is the opposite of multiplication. Addition is the opposite of subtraction
• Numerator – the number above the line in a fraction showing the number of parts of the whole
• Subtraction – separating two or more quantities to find a difference or amount left over
• Unit fraction - a fraction with a numerator of 1

Entry Task (<3min)
Display PPT #1.
Have students complete the Entry Task on Activity Practice 1 in the space provided.

Display PPT #2 to share the answer.
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

Activate Background Knowledge and Interest (<5min)

Display PPT #3.

Solve these problems in your head.

Call on different students to tell what number goes in each empty space.

You probably knew the answers to the first set of problems, but how did you figure out the answers to the second set of problems?

Accept answers such as you can use the same numbers to subtract by starting with the largest number (or sum or total), and subtracting one of the smaller numbers to determine the remaining number (or the difference).

You can take any addition equation and write two associated subtraction equations. This is because addition and subtraction are inverse operations. That means that they are opposite operations and they undo each other.

Display PPT #4: Inverse Operations

We can add two numbers such as 4 and 6 to get 10 and undo the addition with subtraction: 10 - 6 = 4 and 10 - 4 = 6.

Notice the addition equations have the same total. What is the total? \(10\)

Also notice the subtraction equations have the same beginning total amount. What is the beginning total amount? \(10\)

That's right. When writing equations using the inverse operation, the total from the addition equation becomes the beginning total amount in the subtraction equation.

I Can…

Display PPT #5.

1. Write equations that represent the inverse relationship for addition and subtraction.

2. Explain why addition and subtraction are inverse operations.
Teacher Demonstration

Have students place Activity Practice 1 into their SmartPAL and complete the steps as you display the PPTs.

We learned how whole numbers are closely related to rational numbers. Today, we’re going to extend the understanding of inverse operations with whole numbers to write fraction equations.

Display PPT #6.

This number line represents joined parts of unit fractions. What is the unit fraction? \( \frac{1}{5} \)

Tell me the equation that represents the number line.

\[
\frac{2}{5} + \frac{3}{5} = \frac{5}{5}
\]

Display PPT #7: \[ \frac{2}{5} + \frac{3}{5} = \frac{5}{5} \]

Monitor that students have written the equation in the space provided.

What fraction represents the total distance in this model? \( \frac{5}{5} \)

That’s right. The total distance of \( \frac{5}{5} \) is the result of combining \( \frac{2}{5} \) and \( \frac{3}{5} \). Now let’s use our knowledge of inverse operations to write associated subtraction equations for \( \frac{2}{5} + \frac{3}{5} = \frac{5}{5} \).

First, what is the inverse operation of addition? (Subtraction.)

The total in the addition equation is \( \frac{5}{5} \), so the beginning total will be \( \frac{5}{5} \) for the subtraction equation.

Display PPT #8.

Next, we subtract one of the addends or jumps. Let’s subtract the 2nd jump, or \( \frac{3}{5} \).

Display PPT #9.

What does \( \frac{5}{5} - \frac{3}{5} \) equal? \( \frac{2}{5} \)
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

Display PPT #10: $\frac{2}{5}$

Point to the jumps on the models and relate them to the equations as you say the following:

When we represent an addition equation on a number line, we use forward jumps since addition represents combining joined unit fractions to get a total.

When we undo this total using subtraction, we use backward jumps since subtraction separates the joined unit fractions.

Why are addition and subtraction inverse operations?

Because they are opposite operations and they undo each other.

We can write another subtraction equation that undoes the addition equation.

Display PPT #11.

We’ll still start with the total, which is $\frac{5}{5}$. Which jump should we subtract this time? (The first jump or $\frac{2}{5}$)

What does $\frac{5}{5} - \frac{2}{5}$ equal? $\frac{3}{5}$

Display PPT #12.

Guided Practice

Now it’s your turn to apply the inverse operation to an addition equation.

Display PPT #13.

Read the problem. $\frac{3}{8} + \frac{4}{8} =$

What does $\frac{3}{8} + \frac{4}{8}$ equal? $\frac{7}{8}$
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

If students do not provide the correct answer, have them model the equation on the number line using forward jumps, then answer the question.

Display PPT #14.

**For this addition equation, should we model with forward jumps or backward jumps?** *(Forward jumps.)*

**Your turn to model the addition equation and label the total, \( \frac{7}{8} \).**

Provide a short time for students to model the equation on the number line with jumps.

Display PPT #15.

**Your number line should look like this. Now, write an equation using the inverse operation. What operation should you use?** *(Subtraction.)*

**Which fraction do we start the subtraction equation with?** *(\( \frac{7}{8} \))

Display PPT #16: \( \frac{7}{8} - \)

**What fraction should we subtract from the total?** *(\( \frac{4}{8} \) or \( \frac{3}{8} \))

**We can subtract either fraction, but let’s start by subtracting \( \frac{4}{8} \). Go ahead and complete the subtraction equation.**

Display PPT #17: \( \frac{7}{8} - \frac{4}{8} = \frac{3}{8} \)

**Your turn to model the subtraction equation on the next number line and label \( \frac{3}{8} \).**

Provide a short time for students to model the number line that represents the equation \( \frac{7}{8} - \frac{4}{8} = \frac{3}{8} \). Provide feedback as needed.

Display PPT #18.

**Your number line should look like this.**

**There is another subtraction equation that represents the inverse operation. Write that equation next to the last number line and model the equation on the number line. Label the location with a fraction.**
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

Monitor as students write \( \frac{7}{8} - \frac{3}{8} = \frac{4}{8} \). If necessary, remind students that a subtraction equation starts with a total from the addition equation. Monitor and assist as needed.

Display PPT # 19.

**How are these addition and subtraction equations related?**

*The equations are related by the inverse operation, and they undo each other.*

**Independent Practice**

Have students use Activity Practice 2 to write two related subtraction equations for each given addition equation.

**Lesson Closure (<3min)**

*Today we worked with addition and subtraction fraction equations using the inverse operation. Tell me how the inverse operation works in writing equations.*

*Call on various students to offer the following ideas: The inverse operation is an opposite operation (such as, the inverse operation of addition is subtraction). When writing equations using the inverse operation of addition, we start with the total and subtract one of the addends.*

*If we’re given one addition fraction equation, how many subtraction equations can we write by using the inverse operation? (2)*

Display PPT #20.

*Just like an addition equation has two subtraction equations, each subtraction equation has two addition equations that we can write using the inverse operation.*

*Remember, the subtraction equation starts with the total, so the addition equation using the inverse operation should add two addends to equal the total.*

Display PPT #21.

Ask students to tell you the two addition equations using inverse operation.
Strand 3
Lesson 6: Represent Addition and Subtraction Equations as Inverse Relationships

Display PPT #22 to share the answer.

Exit Ticket (<3min)

If time allows, have students complete the problem in the exit ticket section of their notebook. Otherwise collect the independent practice as the exit ticket.

Display PPT #23.

Correct Answer:

\[
\frac{2}{6} + \frac{5}{6} = \frac{7}{6}
\]

\[
\frac{7}{6} - \frac{5}{6} = \frac{2}{6} \quad \frac{7}{6} - \frac{2}{6} = \frac{5}{6}
\]
Represent Addition and Subtraction Equations as Inverse Relationships

ENTRY TASK

1. Decompose $\frac{7}{6}$ into a whole number plus a fraction less than one.

2. Write the result as a mixed number.

ENTRY TASK ANSWER

1. Equation $\frac{7}{6} = 1 + \frac{1}{6}$

2. Mixed number $1\frac{1}{6}$

Inverse Operations

$4 + 6 = 10$  
$6 + 4 = 10$

$I$ CAN...

1. Write equations that represent the inverse relationship for addition and subtraction.

2. Explain why addition and subtraction are inverse operations.
Inverse Operation
\[ \frac{2}{5} + \frac{3}{5} = \frac{5}{5} \]
\[ \frac{5}{5} - \frac{2}{5} = \frac{3}{5} \]
\[ \frac{5}{5} - \frac{5}{5} = 0 \]

Inverse Operation
\[ \frac{3}{8} + \frac{4}{8} = \frac{7}{8} \]
\[ \frac{7}{8} - \frac{4}{8} = \frac{3}{8} \]
EXIT TICKET

1. Solve the equation.
2. Write two subtraction equations using the inverse operation.

\[
\begin{align*}
\frac{2}{6} + \frac{5}{6} &= \\
\frac{3}{10} + \frac{2}{10} &= \frac{5}{10} \\
\frac{5}{10} - \frac{2}{10} &= \frac{3}{10} \\
\end{align*}
\]
Lesson 6 Activity Practice 1 Key

Name: __________________________

Entry Task

\[
\frac{7}{6} = 1 + \frac{1}{6} = 1\frac{1}{6}
\]

\[
\frac{2}{5} + \frac{3}{5} = \frac{5}{5}
\]

\[
\frac{5}{5} - \frac{3}{5} = \frac{2}{5}
\]

\[
\frac{5}{5} - \frac{2}{5} = \frac{3}{5}
\]
\[
\frac{3}{8} + \frac{4}{8} = \frac{7}{8}
\]

\[
\frac{7}{8} - \frac{4}{8} = \frac{3}{8}
\]

\[
\frac{7}{8} - \frac{3}{8} = \frac{4}{8}
\]
For each addition equation, write 2 related subtraction equations using the inverse operation.

| Addition Equation: | 6 + 2 = 8  
9 9 9 9 9  |
|-------------------|------------------------|
|                   | 8 - 2 = 6  
9 9 9 9 9 9  |
|                   | 8 - 6 = 2  
9 9 9 9 9 9  |
| Subtraction Equations: | 8 9 9 9 9 9  |
|                   | 8 9 9 9 9 9  |
|                   | 8 9 9 9 9 9  |

| Addition Equation: | 3 + 5 = 8  
8 8 8 8 8 8  |
|-------------------|------------------------|
|                   | 8 - 5 = 3  
8 8 8 8 8 8  |
|                   | 8 - 3 = 5  
8 8 8 8 8 8  |
| Subtraction Equations: | 8 8 8 8 8 8  |
|                   | 8 8 8 8 8 8  |
|                   | 8 8 8 8 8 8  |

Exit Ticket

\[
\frac{2}{6} + \frac{5}{6} = \frac{7}{6} \quad \frac{7}{6} - \frac{5}{6} = \frac{2}{6} \quad \frac{7}{6} - \frac{2}{6} = \frac{5}{6}
\]
S&S # 3.3.F.2 For fractions \( \frac{a}{b} \) and \( \frac{c}{d} \) determine the common multiple by multiplying the denominators and partition a number line to model the renamed fractions.

**Lesson Objective(s)**

1. Understand that fractions with unlike denominators have different unit fractions and that finding a common denominator gives the same-sized unit fractions.
2. Partition number lines to generate equivalent fractions.

<table>
<thead>
<tr>
<th>Teacher Materials:</th>
<th>Student Materials:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Lesson 18 PowerPoint (PPT)</td>
<td>• Activity Practice 1</td>
</tr>
<tr>
<td>• Activity Practice 1</td>
<td>• Activity Practice 2</td>
</tr>
<tr>
<td>• Activity Practice 1 Key</td>
<td>• Optional: Multiplication Table</td>
</tr>
<tr>
<td>• Activity Practice 2 Key</td>
<td></td>
</tr>
</tbody>
</table>

**Key Vocabulary**

• Benchmark – locations \((0, \frac{1}{2}, 1)\) on the number line that are used to compare the size of fractions
• Denominator – the number below the line in a fraction showing the number of parts the whole is divided into
• Estimation - finding a value that is close to the right answer, usually with some thought or calculation involved
• Equivalent fraction - fractions with different numerators and denominators that have the same value; two fractions that are on the same location (point) of the number line from zero with different number of subdivided parts within the same whole
• Fraction - a fraction \( \frac{a}{b} \) is the quantity formed by a parts of size \( \frac{1}{b} \)
• Identity property of multiplication – a product of any number and one is the number
• Interval - distance between two points
• Numerator – the number above the line in a fraction showing the number of parts of the whole
• **Partitioning** - dividing objects/number line into parts
• Unit fraction - a fraction with a numerator of 1
Strand 3
Lesson 18: Represent Common Denominators on a Number Line

**Entry Task (<3min)**

Display PPT #1.

Have students complete the Entry Task on Activity Practice 1.

Display PPT #2 to share the answer.

**Activate Background Knowledge and Interest (<5min)**

Display PPT #3.

> Take a look at these two fractions. Do these fractions have like or unlike denominators? (*Unlike denominators.)*

> That’s right. When fractions have unlike denominators, do they have the same unit fraction or do they have different unit fractions? (*Different unit fractions.)*

> Tell me the unit fraction for each fraction. (*\(\frac{1}{4} \text{ and } \frac{1}{6}\))*

Display PPT #4.

> That’s correct. When fractions have unlike denominators, they have unlike unit fractions. If we want to add or subtract fractions, we need a common unit fraction.

\(\frac{1}{4} \text{ and } \frac{1}{6}\) have a common denominator of 12. This means that \(\frac{1}{12}\) is the common unit fraction for \(\frac{1}{4}\) and \(\frac{1}{6}\).

If necessary, remind students how to locate a common denominator on a multiplication table.
Strand 3
Lesson 18: Represent Common Denominators on a Number Line

I Can…

Display PPT #5:

1. Explain that finding a common denominator gives the same-sized unit fractions.

2. Partition number lines to generate equivalent fractions.

Lesson (30 min)

Teacher Demonstration

Teacher Note: Model how to generate an equivalent fraction with a common denominator by partitioning the number line. This lesson should be a review of how to generate equivalent fractions on number lines in Strand 2.

Have students complete the steps on Activity Practice 1 along with you as you demonstrate.

Display on the doc cam: Activity Practice 1:

\[
\frac{1}{4} + \frac{5}{6} = \]

Can we add these fractions without renaming them? (No.) Why?

Because they have unlike denominators.

Can we estimate the sum of these fractions without renaming them? (Yes.) How?

We can compare each fraction to determine the closest benchmark, and then add the benchmarks.

The fraction \( \frac{1}{4} \) is closest to which benchmark?

Call on students to offer various ideas. Some students may say 0, or \( \frac{1}{2} \), and other students may note that it is an equal distance between 0 and \( \frac{1}{2} \).

When a fraction is an equal distance from both benchmarks, we'll round up to the larger benchmark. Keep in mind that the estimated sum will be greater because the larger benchmark is being used. So what benchmark will we use for \( \frac{1}{4} \) and \( \frac{5}{6} \)?

The fraction \( \frac{5}{6} \) is closest to which benchmark? (1)
So, what is the estimated sum of these fractions? (1 1/2)

Now that we have an estimated the sum, we can model the fractions on the number lines. Take a look at the first number line. Which fraction should we represent on this number line? (1/4)

Let’s represent 1/4 as a point on the first number line and label the point above the number line.

It’s your turn to represent 5/6 as a point on the 2nd number line and label it.

Do 1/4 and 5/6 have the same size unit fraction? (No.)

What is the unit fraction of 1/4? (1/4)

What is the unit fraction of 5/6? (1/6)

That’s correct. When fractions have unlike denominators, their unit fractions are different sizes.

When can we add two fractions?

When fractions have common or like denominators.

That’s correct. When fractions have like denominators, the fractions have the same size unit fraction or a common unit fraction.

What are the denominators of these two fractions? (4 and 6)

What is the common denominator of 4 and 6? (12)
Strand 3
Lesson 18: Represent Common Denominators on a Number Line

Have students use the multiplication table if necessary. If students also identify other common multiples, let them know that these would work also, but it’s easier to use the lowest common multiple.

Because the common denominator of these fractions is 12, we need to generate an equivalent fraction of $\frac{1}{4}$ with a denominator of 12.

We learned to partition each interval into the same number of parts when generating an equivalent fraction.

How should we partition each interval to generate 12 equal intervals?

*Partition each interval into 3 equal intervals to create 12 intervals.*

Model partitioning each interval into 3 equal intervals and have students to do the same.

We also need to generate an equivalent fraction of $\frac{5}{6}$ with a denominator of 12. How should we partition each interval to generate 12 equal intervals?

*Partition each interval into 2 equal intervals to create 12 intervals.*

Partition each interval into 2 equal intervals to create 12 intervals and have students to do the same.

How many intervals are on the bottom half of each number line? (12)

What is the common unit fraction for these number lines now? (\(\frac{1}{12}\))

Do both number lines have the same size unit fraction or common unit fraction? (Yes.)

Indicate the unit fraction $\frac{1}{12}$ as a distance from 0 on both number lines. Count the intervals as you say:

The common unit fraction is $\frac{1}{12}$. So, we can rename the fraction $\frac{1}{4}$ as (count the intervals) $\frac{3}{12}$. 
Strand 3
Lesson 18: Represent Common Denominators on a Number Line

Label $\frac{3}{12}$ below $\frac{1}{4}$ on the number line.

These fractions are equivalent because they represent the same point on the number line.

We can rename the fraction $\frac{5}{6}$ as what fraction? ($\frac{10}{12}$)

Label $\frac{10}{12}$ below $\frac{5}{6}$ on the number line.

How do we know that $\frac{5}{6}$ and $\frac{10}{12}$ are equivalent?

They represent the same point on the number line.

Let’s rename $\frac{1}{4} + \frac{5}{6}$ at the top with the equivalent fractions: $\frac{3}{12} + \frac{10}{12}$.

Add $\frac{3}{12} + \frac{10}{12}$ to the equation.

Since both fractions have a common unit fraction or common denominator, can we now add or subtract these fractions? (Yes.)

What does $\frac{3}{12} + \frac{10}{12}$ equal? ($\frac{13}{12}$)

Add $\frac{13}{12}$ to the equation.

We estimated the sum of these fractions to be $1\frac{1}{2}$. Our answer is between which two benchmarks? (1 and $1\frac{1}{2}$.)

So our answer is reasonable. (Yes.)

Guided Practice

Display on the doc cam: the 2nd equation: $\frac{4}{6} + \frac{3}{8} = \square + \square = \square$

Read the equation. ($\frac{4}{6} + \frac{3}{8} =$)

Your turn to estimate the sum of these two fractions using benchmarks. The fraction $\frac{4}{6}$ is closest to which benchmark? ($\frac{1}{2}$)

The fraction $\frac{3}{8}$ is closest to which benchmark? ($\frac{1}{2}$)

What is the estimated sum? (1)
Strand 3
Lesson 18: Represent Common Denominators on a Number Line

It’s your turn to represent each fraction as a point on a number line.

Provide a short time for students to represent the fractions on number lines.

How do we generate equivalent fractions on number lines?

*Partition number lines using the common denominator.*

That’s correct. We need to determine a common denominator. What is the common denominator of these fractions? (24)

The common denominator of 24 tells us that we need to generate an equivalent fraction of \(\frac{4}{6}\) with what denominator? (24)

How should we partition the fraction \(\frac{4}{6}\) to generate 24 equal intervals?

*Partition each interval into 4 equal intervals to create 24 intervals.*

It’s your turn to partition the number line with \(\frac{4}{6}\) to generate 24 equal intervals and then label the point below the number line.

Students should partition each interval into 4 equal intervals to create 24 intervals.

Go ahead and partition the fraction \(\frac{3}{8}\) to generate an equivalent fraction with the denominator of 24. Don’t forget to label the point below the number line.

Students should partition each interval into 3 equal intervals to create 24 intervals.

When fractions have a common denominator, do the fractions have the same unit fraction? (Yes.)

How can we tell if two fractions have the same unit fraction on number lines?

*They will have the same number of intervals that make up the whole unit.*

What is the common unit fraction for these two fractions? \(\frac{1}{24}\)

Excellent. So, we can rename \(\frac{4}{6} + \frac{3}{8}\) as what? \(\frac{16}{24} + \frac{9}{24}\)

Add \(\frac{16}{24} + \frac{9}{24}\) to the equation.

What is the sum of \(\frac{16}{24} + \frac{9}{24}\)? \(\frac{25}{24}\)
Add $\frac{25}{24}$ to the equation.

We estimated the sum of these fractions to be 1. Is our answer close to 1? (Yes.)

So our answer is reasonable. (Yes.)

Independent Practice

Have students complete the problems on Activity Practice 2. Have them partition the number lines using the common denominator (or common unit fraction) and label the equivalent fractions. Have them rename the expression with equivalent fractions with like denominators and solve it. Have students verify their answers are reasonable by estimating the sum and comparing it to their answer.

 Irvine Extend Down Option: Have students work with a partner to solve the first problem and then solve the last problem independently.

Lesson Closure (<3min)

When we round fractions to the nearest benchmark, what do we do if a fraction is an equal distance between two benchmarks?

Round to the larger benchmark.

When partitioning the number line to generate equivalent fractions of fractions with unlike denominators, what tells us how to partition each interval?

The common denominator or common unit fraction.

How can we justify that two fractions are equivalent on a number line?

If the fractions represent the same point or distance on the number line, they are equivalent.

Exit Ticket (<3min)

Collect the independent worksheet as an exit ticket.
Represent Common Denominators on a Number Line

**ENTRY TASK**
Use the benchmarks (0, ½, 1) to estimate the sum.

\[
\frac{2}{6} + \frac{5}{8}
\]

**ACTIVATE BACKGROUND KNOWLEDGE**

**I CAN ...**
1. Explain that finding a common denominator gives the same-sized unit fractions.
2. Partition number lines to generate equivalent fractions.
Entry Task

\[ \frac{2}{6} + \frac{5}{8} = 1 \]

\[ \frac{1}{2} + \frac{1}{2} = 1 \]
\[
\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12}
\]

\[
\frac{4}{6} + \frac{3}{8} = \frac{16}{24} + \frac{9}{24} = \frac{25}{24}
\]
Lesson 18 Activity Practice 2 Key

\[ \frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} \]

\[ \frac{3}{4} + \frac{2}{8} = \frac{6}{8} + \frac{2}{8} = \frac{8}{8} \]

PAR Strand 3
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<table>
<thead>
<tr>
<th>Common Core State Standard (CCSS-M)</th>
<th>Knowledge Representations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STRAND 4: MULTIPLICATION and DIVISION of FRACTIONS</strong></td>
<td></td>
</tr>
<tr>
<td>6.NS.1 Interpre and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations, to represent the problem.</td>
<td><strong>Foundational Knowledge and Skills</strong>&lt;br&gt;Skills that support the standard and provide a starting point for PAR.</td>
</tr>
<tr>
<td>4.1.F.1 Model and explain multiplication of a whole number by a fraction and fraction by a whole number on a number line and/or visual representation (i.e., ( a/b \times q )) to prove the product of ((a/b) \times (c/d)) is equal to (a/c). Model and explain multiplication of a fraction by a mixed number on the number line and/or visual representation.</td>
<td>4.1.B.2 Multiply a whole number by a fraction and a fraction by a whole number. Verify the product with various strategies (i.e., (a) copies of (1/b)) or fractional comparison).</td>
</tr>
<tr>
<td>4.1.F.4 Model and explain multiplication of a fraction by a fraction (including unit fractions) on a number line and/or visual representation to prove the product of ((a/b) \times (c/d)) is equal to (a/c). Model and explain multiplication of a fraction by a mixed number on the number line and/or visual representation.</td>
<td>4.1.B.5 Explain the effects of multiplying a fraction by a fraction less than or greater than 1.</td>
</tr>
<tr>
<td>4.1.F.5 Explain the effects of multiplying a fraction by a fraction greater than 1, less than 1, and equal to 1.</td>
<td>4.1.B.7 Solve word problems involving multiplication of a fraction by a fraction or mixed number using visual representations and equations to represent the problem.</td>
</tr>
<tr>
<td>4.3.F.1 Model and explain division of a whole number by a whole number where the quotient has a remainder and is written as a fraction ((m \div n = q \ a/n ) where (m &gt; n).</td>
<td>4.3.B.3 Divide a whole number by a unit fraction using the inverse operation strategy. Explain why this strategy works.</td>
</tr>
<tr>
<td>4.3.F.2 Model and explain division of a whole number by a unit fraction on a number line and/or visual representation.</td>
<td>4.3.B.5 Divide a unit fraction by a non-zero whole number using the inverse operation strategy. Explain why this strategy works.</td>
</tr>
</tbody>
</table>
Strand 4
Lesson 10: Model Multiplication Of Two Fractions That Are Less Than One

S&S # 4.1.F.4 Model and explain multiplication of a fraction by a fraction (including unit fractions) on a number line and/or visual representation to prove the product of \((a/b) \times (c/d)\) is equal to \(ac/bd\). Model and explain multiplication of a fraction by a mixed number on the number line and/or visual representation.

Lesson Objective(s)
1. Model multiplication of two fractions that are each less than 1 as an area model.
2. Solve multiplication equations involving fractions less than 1 using an area model.

Teacher Materials:  
- Lesson 10 PowerPoint (PPT)
- Activity Practice 1
- Activity Practice 1 Key
- Activity Practice 2 Key

Student Materials:  
- Activity Practice 1 (inserted in SmartPAL)
- Activity Practice 2

Vocabulary
- Area model - a pictorial way to representing multiplication where the length and width of a rectangle represent factors, and the area of the rectangle represents their product
- Equation – a mathematical statement that shows two expressions are equivalent
- Fraction - a fraction \(a/b\) as the quantity formed by \(a\) parts of size \(1/b\)
- Horizontal – the direction from left to right (parallel to the horizon)
- Multiplication – repeated addition or joining of equal groups. Multiplication is also the inverse of division
- Product – the result of multiplying two or more numbers
- Square units – the area of a plane figure
- Unit – a measurement of 1
- Vertical – the direction from top to bottom (forms a right angle with the horizon)

Entry Task (<3min)

Display PPT #1.

ENTRY TASK
Write an equation that represents the problem.

Jordan’s haircut costs $15 and Amy’s haircut costs $45. What fraction of the cost of Amy’s hair cut is the cost of Jordan’s haircut?
Strand 4  
Lesson 10: Model Multiplication Of Two Fractions That Are Less Than One

Display PPT #2 to share the answer.

Activate Background Knowledge and Interest (<5min)

Display on the doc cam: Activity Practice 1

We've been using this template to model multiplication of fractions to help us visually see and understand what it means. When we model multiplication on a number line, we've been using this number line at the top.

Point to the number line at the top.

What is the length of this number line? (4 units.)

Point to the area model.

We have also modeled multiplication of fractions as an area using this area model.

What is the length of this area model? (4 units.)

What is the width of this area model? (4 units.)

What is the area of each square in the grid? (1 square unit.)

What is the total area of the grid? (16 square units.)

Great job! We'll be using these models in the next several lessons.

I Can…

Display PPT #3.

1. Model multiplication of two fractions that are less than 1.

2. Solve multiplication equations involving two fractions that are less than 1.
Lesson (30 min)

Teacher Demonstration

Have students place Activity Practice 1 into their SmartPALs and complete the problems with you as you demonstrate on the doc cam.

Today we are going to model multiplication of two fractions that are each less than one and determine the product using the area model.

Display on the doc cam: Activity Practice 1 and write on the equation line \( \frac{1}{3} \times \frac{1}{4} \).

Read this expression. \( \left( \frac{1}{3} \times \frac{1}{4} \right) \)

Let’s represent \( \frac{1}{3} \) on the horizontal number line and \( \frac{1}{4} \) on the vertical number line.

Have students model the length and width as segments on the number line. Monitor and assist as needed.

Now that we have modeled the length and width as segments, let’s partition the grid by extending each third mark from the horizontal number line across 1 unit like this.

Let’s partition the grid again by extending each fourth mark from the vertical number line across 1 unit.

Tell me how many rectangles are in this 1 sq. unit? (12)

Insert a check mark in one rectangle to represent the area of \( \frac{1}{3} \) of a unit in length by \( \frac{1}{4} \) of a unit in width.
Strand 4
Lesson 10: Model Multiplication Of Two Fractions That Are Less Than One

This rectangle represents the area of $\frac{1}{3}$ of a unit in length by $\frac{1}{4}$ of a unit in width.

What fraction represents the rectangle with the check mark? $\left(\frac{1}{12}\right)$

That tells us that each rectangle equals $\frac{1}{12}$ sq. unit and that $\frac{1}{12}$ is the product of $\frac{1}{3} \times \frac{1}{4}$. So, we can say that $\frac{1}{3} \times \frac{1}{4}$ equals $\frac{1}{12}$.

Add to the equation line: $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

Who can tell me why $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$?

*Call on various students to explain that $\frac{1}{12}$ is represented in this model by a rectangle that measures $\frac{1}{3}$ by $\frac{1}{4}$ which is the same as $\frac{1}{3} \times \frac{1}{4}$.*

Erase the previous problem and have students erase their problem.

**Guided Practice**

Display on the doc cam: Activity Practice 1 and write on the equation line $\frac{2}{3} \times \frac{2}{4}$.

Read the expression. $\left(\frac{2}{3} \times \frac{2}{4}\right)$

Tell me how to represent this on the number lines.

*Call on students to explain $\frac{2}{3}$ of a unit is represented on the horizontal number line and $\frac{2}{4}$ of a unit is represented on the vertical number line. Partition the horizontal number line into thirds and the vertical number line into fourths.*

It’s your turn to partition and represent the fractions on the number line as segments and label them.

Monitor and assist as needed.

Now that we represented the fractions as segments, what do you do next?

*Partition the grid by extending each third mark and fourth mark across 1 unit.*

Yes. Extend each third mark and fourth mark across 1 unit.

Monitor as student partition the grid using third mark and fourth mark across 1 unit.
How many rectangles are in this 1 sq. unit? (12)

Insert check marks to represent the area of \( \frac{2}{3} \) of a unit (in length) by \( \frac{2}{4} \) of a unit (in width).

Monitor as students shade in 4 rectangles or insert 4 check marks, then display the model on the doc cam.

What fraction represents 1 rectangle? \( \frac{1}{12} \)

If one rectangle represents \( \frac{1}{12} \), then what fraction represents 4 rectangles? \( \frac{4}{12} \)

So, what is the product of \( \frac{2}{3} \times \frac{2}{4} \)? \( \frac{4}{12} \)

Add to the equation line: \( \frac{2}{3} \times \frac{2}{4} = \frac{4}{12} \)

Tell your partner why \( \frac{2}{3} \times \frac{2}{4} = \frac{4}{12} \) using the area model.

Allow student time to explain to their partner that \( \frac{4}{12} \) is represented in this model by a rectangle that measures \( \frac{2}{3} \) by \( \frac{2}{4} \) which is the same as \( \frac{2}{3} \times \frac{2}{4} \). Then call on various students to explain to the class.

**Independent Practice**

Pass out Activity Practice 2. Have students model \( \frac{1}{4} \times \frac{1}{5} \) and then \( \frac{3}{4} \times \frac{4}{5} \) on the area model and complete the equation with the product.

Circulate around the class and ask the following questions.

Look at the model. How many rectangles are in 1 sq. unit? (20)

How many rectangles with check marks do you have? (1 or 12)

What fraction represents your check marked rectangles? \( \frac{1}{20} \) or \( \frac{12}{20} \)

What equation represents your model? \( \frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \) or \( \frac{3}{4} \times \frac{4}{5} = \frac{12}{20} \)

**Extend Down:** Guide students to model the first expression, then have them model the last expression independently.
Lesson Closure (<3min)

We learned how to model multiplication of two fractions that are less than one today. How does the area model represent the denominators of the fraction equation?

Call on various students to tell you that the total number of rectangles represents the product of multiplying the denominators.

How does the area model represent the numerators of the fraction equation?

Call on various students to tell you that the check marked (or shaded) rectangles represent the product of multiplying the numerators.

How does the area model represent the product of the fraction equation?

Call on various students to tell you that the total number of check marked (or shaded) rectangles over the total number of rectangles represents the product.

Exit Ticket (<3min)

Review the completed independent practice as an exit ticket.
Model Multiplication of Two Fractions that are Less than One on an Area Model

 ENTRY TASK

Write an equation that represents the problem.

Jordan’s haircut costs $15 and Amy’s haircut costs $45. What fraction of the cost of Amy’s hair cut is the cost of Jordan’s haircut?

 ENTRY TASK ANSWER

Jordan’s haircut costs $15 and Amy’s haircut costs $45. What fraction of the cost of Amy’s hair cut is the cost of Jordan’s haircut?

\[ \frac{45}{3} = \frac{15}{3} \]

I CAN...

1. Model multiplication of two fractions that are less than 1.

2. Solve multiplication equations of two fractions that are less than 1.
Lesson 10 Activity Practice 1 Key

Name______________________________

Entry Task

Write an equation that represents this problem.

Jordan’s haircut costs $15 and Amy’s haircut costs $45. What fraction of the cost of Amy’s hair cut is the cost of Jordan’s haircut?

\[ \$45 \times \frac{1}{3} = \$15 \text{ (Jordan)} \]
Teacher Demonstration

Equation: \( \frac{1}{3} \times \frac{1}{4} = \frac{1}{12} \)

Guided Practice

Equation: \( \frac{2}{3} \times \frac{2}{4} = \frac{4}{12} \)
Lesson 10 Activity Practice 2 Key

Name____________________________

Equation: \( \frac{1}{4} \times \frac{1}{5} = \frac{1}{20} \)
Equation: \( \frac{3}{4} \times \frac{4}{5} = \frac{12}{20} \)
### Grade 3 Operations and Algebraic Thinking

<table>
<thead>
<tr>
<th>CCSS.Math.3.OA.A</th>
<th>Represent and solve problems involving multiplication and division.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.OA.1</td>
<td>Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3.OA.2</td>
<td>Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3.OA.3</td>
<td>Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>3.OA.4</td>
<td>Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 × __ = 3, 6 × 6 = ?</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Grade 4 Operations and Algebraic Thinking

<table>
<thead>
<tr>
<th>CCSS.Math.4.OA.A</th>
<th>Use the four operations with whole numbers to solve problems.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.OA.1</td>
<td>Interpret a multiplication equation as a comparison, e.g., interpret 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4.OA.2</td>
<td>Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4.OA.3</td>
<td>Solve multistep word problems posed with whole numbers having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Grade 5 Operations and Algebraic Thinking

<table>
<thead>
<tr>
<th>CCSS.Math.5.OA.A</th>
<th>Write and interpret numerical expressions.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.OA.1</td>
<td>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>5.OA.2</td>
<td>Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Grade 3 Number and Operations in Base Ten

<table>
<thead>
<tr>
<th>CCSS.Math.3.NBT</th>
<th>Use place value understanding and properties of operations to perform multi-digit arithmetic.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.NBT.1</td>
<td>Use place value understanding to round whole numbers to the nearest 10 or 100.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Grade 4 Number and Operations in Base Ten

<table>
<thead>
<tr>
<th>CCSS.Math.4.NBT</th>
<th>Use place value understanding and properties of operations to perform multi-digit arithmetic.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NBT.4</td>
<td>Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>4.NBT.6</td>
<td>Find whole-number quotients and remainders with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

### Grade 5 Number and Operations in Base Ten

<table>
<thead>
<tr>
<th>CCSS.Math.5.NBT</th>
<th>Perform operations with multi-digit whole numbers and with decimals to hundredths.</th>
<th>Strand 1</th>
<th>Strand 2</th>
<th>Strand 3</th>
<th>Strand 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NBT.5</td>
<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>5.NBT.6</td>
<td>Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Graduate Number and Operations – Fractions

<table>
<thead>
<tr>
<th>CCSS.Math.3.NF.A</th>
<th>Develop understanding of fractions as numbers.</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.NF.A.1</td>
<td>Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by parts of size 1/b.</td>
<td>√</td>
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<td>√</td>
</tr>
<tr>
<td>3.NF.A.2</td>
<td>Understand a fraction a/b as the number on the number line; represent fractions on a number line diagram.</td>
<td></td>
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<td>√</td>
<td></td>
</tr>
<tr>
<td>3.NF.A.3.A</td>
<td>Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.</td>
<td>√</td>
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</tr>
<tr>
<td>3.NF.A.3.B</td>
<td>Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.NF.A.3.C</td>
<td>Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</td>
<td></td>
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<td>√</td>
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</tr>
<tr>
<td>3.NF.A.3.D</td>
<td>Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.</td>
<td></td>
<td>√</td>
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</tr>
<tr>
<td>3.NF.A.3.E</td>
<td>Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</td>
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<td>√</td>
</tr>
</tbody>
</table>

Grade 4 Number and Operations – Fractions

<table>
<thead>
<tr>
<th>CCSS.Math.4.NF.A</th>
<th>Extend understanding of fraction equivalence and ordering.</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.NF.A.1</td>
<td>Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</td>
<td>√</td>
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</tr>
<tr>
<td>4.NF.A.2</td>
<td>Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols &gt;, =, or &lt;, and justify the conclusions, e.g., by using a visual fraction model.</td>
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<td>√</td>
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</tr>
<tr>
<td>CCSS.Math.4.NF.B</td>
<td>Build fractions from unit fractions.</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
</tr>
<tr>
<td>4.NF.B.3</td>
<td>Understand a fraction a/b with a &gt; 1 as a sum of fractions 1/b.</td>
<td>√</td>
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</tr>
<tr>
<td>4.NF.B.3.A</td>
<td>Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</td>
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<tr>
<td>4.NF.B.3.B</td>
<td>Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.</td>
<td>√</td>
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</tr>
<tr>
<td>4.NF.B.3.C</td>
<td>Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</td>
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<tr>
<td>4.NF.B.3.D</td>
<td>Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</td>
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<tr>
<td>4.NF.B.4</td>
<td>Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</td>
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<tr>
<td>4.NF.B.5</td>
<td>Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.</td>
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</tbody>
</table>

Grade 5 Number and Operations – Fractions

<table>
<thead>
<tr>
<th>CCSS.Math.5.NF.A</th>
<th>Use equivalent fractions as a strategy to add and subtract fractions.</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NF.A.1</td>
<td>Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.</td>
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</tr>
<tr>
<td>5.NF.A.2</td>
<td>Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.</td>
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<tr>
<td>CCSS.Math.5.NF.B</td>
<td>Apply and extend previous understandings of multiplication and division to multiply and divide fractions.</td>
<td>S1</td>
<td>S2</td>
<td>S3</td>
<td>S4</td>
</tr>
<tr>
<td>5.NF.B.3</td>
<td>Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
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<tr>
<td>5.NF.B.4</td>
<td>Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show 2/3 × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = (ac)/(bd)).</td>
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<tr>
<td>5.NF.B.4.A</td>
<td>Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
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<tr>
<td>5.NF.B.4.B</td>
<td>Interpret multiplication as scaling (resizing), by: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.</td>
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<tr>
<td>5.NF.B.5.B</td>
<td>Interpret multiplication as scaling (resizing), by: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n × a)/(n × b) to the effect of multiplying a/b by 1.</td>
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<tr>
<td>5.NF.B.6</td>
<td>Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
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<tr>
<td>5.NF.B.5.7</td>
<td>Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</td>
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<tr>
<td>5.NF.B.5.7.A</td>
<td>Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) × 4 = 1/12 because (1/12) × 4 = 1/3.</td>
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<tr>
<td>5.NF.B.5.7.B</td>
<td>Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 × (1/5) = 20 because 20 × (1/5) = 4.</td>
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</tr>
<tr>
<td>5.NF.B.5.7.C</td>
<td>Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.</td>
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</tbody>
</table>

**Grade 6 The Number System**

<table>
<thead>
<tr>
<th>CCSS.Math.6.NS.A</th>
<th>Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>S2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.NS.A.1</th>
<th>Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.NS.B.2</td>
<td>Fluently divide multi-digit numbers using the standard algorithm.</td>
</tr>
<tr>
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</tr>
<tr>
<td>6.NS.B.4</td>
<td>Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).</td>
</tr>
<tr>
<td>-------------------</td>
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</tr>
<tr>
<td>CCSS.Math.6.NS.C</td>
<td>Apply and extend previous understandings of numbers to the system of rational numbers.</td>
</tr>
<tr>
<td>6.NS.C.6</td>
<td>Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</td>
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<tr>
<td>-------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
</tbody>
</table>

73